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20 Years
JEE MAIN
Chapter-wise Solved Papers
(2002 - 21)

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M-1 – M-228

TREND ANALYSIS FOR JEE MAIN
YEAR 2011-2021

JEE Main Entrance Exam for admission into various engineering courses in different engineering colleges and institutes in the country has hit the imagination of the school-going students more than any other entrance test conducted at this level. Without argument, you need to be well-versed with the pattern as well as the level of the questions asked in the exam. A Chapterwise analysis of previous years' questions is called for here, with this objective in mind, we are giving below the chapter-wise analysis (break-up) of the questions asked in last 11 years' of JEE Main

PHYSICS

Ch. No.	Chapter Name	Number of Question(s) in											
		2011	2012	2013	2014	2015	2016 Ph-1	2017	2018	2019		2020	2021
										9-Jan (M)	9-April (M)	7-Jan (M)	24-Feb (M)
1	Physical World, Units and Measurements	1	2	1	2	1	2	1	1	–	1	–	1
2	Motion in a Straight Line	1	–	–	1	–	–	1	1	–	–	–	1
3	Motion in a Plane	1	2	1	–	1	–	–	–	1	1	0	0
4	Laws of Motion	–	1	2	2	1	–	–	1	1	1	0	2
5	Work, Energy and Power	–	1	1	1	1	2	2	4	2	2	2	1
6	System of Particles and Rotational Motion	2	–	3	1	2	2	2	3	2	2	3	1
7	Gravitation	1	1	1	1	1	1	1	–	1	1	1	3
8	Mechanical Properties of Solids	–	–	–	1	–	–	–	1	–	–	0	1
9	Mechanical Properties of Fluids	2	1	–	3	–	–	1	–	–	1	0	1
10	Thermal Properties of Matter	1	2	1	1	1	1	2	–	2	–	1	1
11	Thermodynamics	1	2	1	1	1	2	–	–	1	1	2	2
12	Kinetic Theory	1	–	–	1	1	–	2	1	1	2	1	0
13	Oscillations	2	2	2	1	2	1	1	1	–	1	0	1
14	Waves	1	1	1	1	1	2	–	1	1	2	1	0
15	Electric Charges and Fields	2	1	1	–	1	1	1	–	2	–	1	1
16	Electrostatic Potential and Capacitance	–	1	–	2	2	1	1	2	1	3	1	1
17	Current Electricity	1	1	2	1	2	1	2	3	4	1	1	2
18	Moving Charges and Magnetism	1	1	1	1	2	2	1	2	1	3	0	0
19	Magnetism and Matter	–	–	1	1	1	1	1	–	2	–	0	0
20	Electromagnetic Induction	1	2	1	–	–	–	1	–	1	1	3	0
21	Alternating Current	2	–	1	1	2	1	1	2	–	–	1	2
22	Electromagnetic Waves	–	1	1	2	–	1	–	1	1	1	1	1
23	Ray Optics and Optical Instruments	2	2	2	1	2	2	1	–	1	1	1	1
24	Wave Optics	1	1	2	1	1	1	2	2	2	1	2	2
25	Dual Nature of Radiation and Matter	1	1	1	1	1	1	2	–	1	1	1	1
26	Atoms	1	1	1	1	1	–	1	2	–	1	1	1
27	Nuclei	2	1	–	–	–	1	1	–	1	–	0	0
28	Semiconductor Electronics : Materials, Devices and Simple Circuits	1	1	1	1	1	3	1	1	1	1	1	2
29	Communication Systems	1	1	1	–	1	1	1	1	–	1	0	1
Total Questions		**30**	**30**	**30**	**30**	**30**	**30**	**30**	**30**	**30**	**30**	**25**	**30**

CHEMISTRY

Ch. No.	Chapter Name	Number of Question(s) in											
		2011	2012	2013	2014	2015	2016 Ph-1	2017	2018	2019		2020	2021
										9-Jan (M)	9-April (M)	7-Jan (M)	24-Feb (M)
1	Some Basic Concepts of Chemistry	1	–	2	–	1	1	2	1	–	1	1	1
2	Structure of Atom	1	2	1	1	1	1	2	0	1	1	1	1
3	Classification of Elements and Periodicity in Properties	1	–	1	–	1	1	–	0	1	1	1	1
4	Chemical Bonding and Molecular Structure	1	2	3	1	–	1	1	3	1	1	2	1
5	States of Matter	2	1	1	2	1	1		0	1	1	1	0
6	Thermodynamics	2	1	1	1	1	1	2	2	1	1	1	0
7	Equilibrium	2	2	1	1	1	1	1	3	2	–	1	2
8	Redox Reactions	–	–	1	–	1	–	1	1	–	–	1	1
9	Hydrogen	–	1		1	1	2		1	1	1	1	1
10	The s-Block Elements	2	1	2		1	1	1	0	1	1	–	1
11	The p-Block Elements (Group 13 & 14)	1	–	–	–		–	–	2	2	1	–	2
12	Organic Chemistry : Some Basic Principles and Techniques	2	–	2	1	2	2	1	0	–	2	–	0
13	Hydrocarbons	1	2	–	–	1	2	1	1	–	1	–	2
14	Environmental Chemistry	1	1	1	–	–	–	1	1	1	1	–	1
15	The Solid State	1	1	1	2	1	–	1	1	1	–	–	1
16	Solutions	1	2	1	1	1	1	1	1	2	2	1	1
17	Electrochemistry	1	1	2	3	1		1	1	1	1	1	0
18	Chemical Kinetics	1	1	1	1	1	1	1	1	1	1	1	1
19	Surface Chemistry	–	1	1	–	–	1	1	0	1	1	–	1
20	General Principles and Processes of Isolation of Elements	–	1		1	1	1	–	0	1	1	1	1
21	The p-Block Elements (Group 15, 16, 17 and 18)	1	1	–	–	3	2	1	1	–	1	1	0
22	The d and f-Block Elements	–	1	1	2	2	1	1	0	1	1	1	2
23	Coordination Compounds	1	1	1	1	2	2	1	1	1	2	1	1
24	Haloalkanes and Haloarenes		1	1	3	1	2	2	1	–	1	2	1
25	Alcohols, Phenols and Ethers	3	2	2	2		–	1	3	1	–	0	2
26	Aldehydes, Ketones and Carboxylic Acids	–	2	1	1	1	1	3	0	3	2	1	1
27	Amines	1		1	2	1	1	1	1	2	1	1	2
28	Biomolecules	1	1	1	1	1	1	1	2	1	1	1	1
29	Polymers	1	1		1	1	1	1	0	1	1	0	1
30	Chemistry in Everyday Life	1	–	–		1	1		0	–	–	1	0
31	Analytical Chemistry	–	–	–	1	–	–	–	2	1	1	2	0
Total Questions		**30**	**30**	**30**	**30**	**30**	**30**	**35**	**30**	**30**	**30**	**25**	**30**

MATHEMATICS

Ch. No.	Chapter Name	Number of Question(s) in												
		2011	2012	2013	2014	2015	2016 Ph-1	2016 Ph-2	2017	2018	2019		2020	2021
											9-Jan (M)	9-April (M)	7-Jan (M)	24-Feb (M)
1	Sets	0	1	0	0	0	1	0	1	1	–	–	–	–
2	Relations and Functions-1	1	0	0	0	0	0	0	1	0	–	–	–	–
3	Trigonometric Functions	1	1	2	1	1	2	1	1	1	1	2	1	–
4	Principle of Mathematical Induction	0	0	0	0	0	0	0	4	0	–	–	–	–
5	Complex Numbers and Quadratic Equations	2	1	2	2	2	2	1	0	1	2	1	2	2
6	Linear Inequalities	0	0	0	0	0	0	0	2	0	–	–	–	–
7	Permutations and Combinations	1	0	1	1	2	1	1	3	1	1	1	1	1
8	Binomial Theorem	1	1	2	0	2	2	1	0	1	1	1	0	1
9	Sequence and Series	2	2	2	2	2	2	2	0	2	2	2	2	2
10	Straight Lines & Pair of Straight Lines	1	2	2	2	0	1	0	3	2	1	1	2	1
11	Conic Sections	3	3	3	3	4	4	2	1	4	4	4	1	2
12	Limits and Derivatives	1	0	1	1	0	0	1	3	1	1	–	0	1
13	Mathematical Reasoning	1	1	1	1	1	1	1	3	1	1	1	1	1
14	Statistics	1	1	1	1	1	1	0	1	1	1	1	1	–
15	Probability-I	0	1	0	0	0	0	1	1	0	–	–	–	–
16	Relations and Functions-II	1	0	0	1	1	0	2	1	0	1	–	1	1
17	Inverse Trigonometric Functions	0	0	0	0	1	0	1	1	0	1	–	–	–
18	Matrices	1	0	0	0	1	2	1	2	0	–	–	0	1
19	Determinants	1	2	2	2	1	1	3	1	2	1	2	2	2
20	Continuity and Differentiability	2	2	1	1	1	1	1	1	1	1	2	3	1
21	Application of Derivatives	0	1	1	1	2	3	3	2	2	2	3	2	3
22	Integrals	1	2	3	2	2	1	2	2	2	2	3	1	3
23	Applications of Integrals	1	1	1	1	2	1	2	4	1	1	1	1	1
24	Differential Equations	2	1	1	1	1	1	0	0	1	1	1	1	1
25	Vector Algebra	2	2	1	1	1	1	1	3	1	1	1	1	1
26	Three Dimensional Geometry	2	2	2	2	2	2	2	2	2	3	2	1	2
27	Probability-II	2	1	1	3	0	0	0	2	1	1	1	1	2
28	Properties of Triangles	0	2	0	1	0	0	1	1	1	–	–	–	1
	Total Questions	**30**	**30**	**30**	**30**	**30**	**30**	**30**	**30**	**30**	**30**	**30**	**25**	**30**

JEE MAIN ONLINE 2021 Paper

Held on February 24, 2021 (Morning)

PHYSICS

CHAPTER-1 : PHYSICAL WORLD, UNITS AND MEASUREMENTS

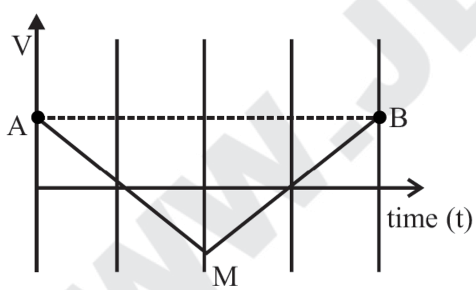
1. The work done by a gas molecule in an isolated system is given by, $W = \alpha\beta^2 e^{-\frac{x^2}{\alpha kT}}$, where x is the displacement k is the Boltzmann constant and T is the temperature, α and β are constants. Then the dimension of β will be :

(a) $[ML^2T^{-2}]$ (b) $[MLT^{-2}]$

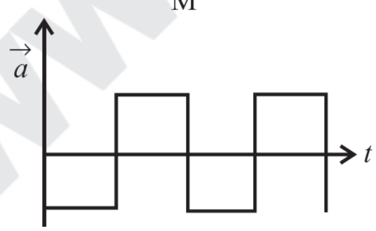
(c) $[M^2LT^2]$ (d) $[M^0LT^0]$

CHAPTER-2 : MOTION IN A STRAIGHT LINE

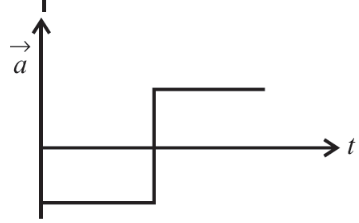
2. If the velocity-time graph has the shape AMB, what would be the shape of the corresponding acceleration-time graph?



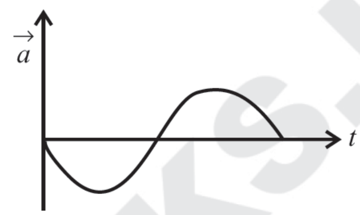
(a)



(b)



(c)



(d)

CHAPTER-4 : LAWS OF MOTION

3. The coefficient of static friction between a wooden block of mass 0.5 kg and a vertical rough wall is 0.2. The magnitude of horizontal force that should be applied on the block to keep it adhere to the wall will be _______ N.

[g = 10 ms^{-2}]

4. An inclined plane is bent in such a way that the vertical cross-section is given by $y = \frac{x^2}{4}$ where y is in vertical and x in horizontal direction. If the upper surface of this curved plane is rough with coefficient of friction $\mu = 0.5$, the maximum height in cm at which a stationary block will not slip downward is ______ cm.

CHAPTER-5 : WORK, ENERGY AND POWER

5. A ball with a speed of 9 m/s collides with another identical ball at rest. After the collision, the direction of each ball makes an angle of 30° with the original direction. The ratio of velocities of the balls after collision is $x : y$, where x is _________.

CHAPTER-6 : ROTATIONAL MOTION

6. Moment of inertia (M.I.) of four bodies, having same mass and radius, are reported as;
I_1 = M.I. of thin circular ring about its diameter,
I_2 = M.I. of circular disc about an axis perpendicular to the disc and going through the centre,
I_3 = M.I. of solid cylinder about its axis and
I_4 = M.I. of solid sphere about its diameter.
Then :

(a) $I_1 + I_3 < I_2 + I_4$ (b) $I_1 + I_2 = I_3 + \frac{5}{2}I_4$

(c) $I_1 = I_2 = I_3 > I_4$ (d) $I_1 = I_2 = I_3 < I_4$

CHAPTER-7 : GRAVITATION

7. Two stars of masses m and $2m$ at a distance d rotate about their common centre of mass in free space. The period of revolution is :

(a) $\frac{1}{2\pi}\sqrt{\frac{d^3}{3Gm}}$ (b) $2\pi\sqrt{\frac{d^3}{3Gm}}$

(c) $\frac{1}{2\pi}\sqrt{\frac{3Gm}{d^3}}$ (d) $2\pi\sqrt{\frac{3Gm}{d^3}}$

8. Four identical particles of equal masses 1 kg made to move along the circumference of a circle of radius 1 m under the action of their own mutual gravitational attraction. The speed of each particle will be :

(a) $\sqrt{\frac{G}{2}(1+2\sqrt{2})}$ (b) $\sqrt{G(1+2\sqrt{2})}$

(c) $\sqrt{\frac{G}{2}(2\sqrt{2}-1)}$ (d) $\frac{\sqrt{(1+2\sqrt{2})G}}{2}$

9. Consider two satellites S_1 and S_2 with periods of revolution 1 hr. and 8hr. respectively revolving around a planet in circular orbits. The ratio of angular velocity of satellite S_1 to the angular velocity of satellites S_2 is :
(a) 8 : 1 (b) 1 : 4 (c) 2 : 1 (d) 1 : 8

CHAPTER-8 : MECHANICAL PROPERTIES OF SOLIDS

10. If Y, K and η are the values of Young's modulus, bulk modulus and modulus of rigidity of any material respectively. Choose the correct relation for these parameters.

(a) $Y = \frac{9K\eta}{3K-\eta}$ N/m^2

(b) $\eta = \frac{3YK}{9K+Y}$ N/m^2

(c) $Y = \frac{9K\eta}{2\eta+3K}$ N/m^2

(d) $K = \frac{Y\eta}{9\eta-3Y}$ N/m^2

CHAPTER-9 : MECHANICAL PROPERTIES OF FLUIDS

11. A hydraulic press can lift 100 kg when a mass 'm' is placed on the smaller piston. It can lift _____ kg when the diameter of the larger piston is increased by 4 times and that of the smaller piston is decreased by 4 times keeping the same mass 'm' on the smaller piston.

CHAPTER-10 : THERMAL PROPERTIES OF MATTER

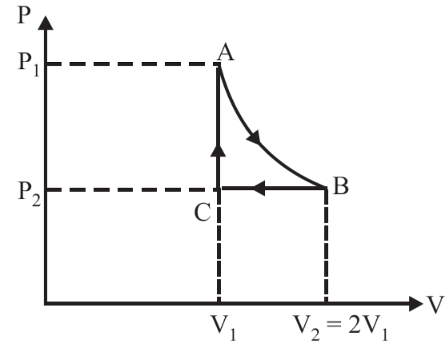
12. Each side of a box made of metal sheet in cubic shape is 'a' at room temperature 'T', the coefficient of linear expansion of the metal sheet is 'α'. The metal sheet is heated uniformly, by a small temperature ΔT, so that its new temperature is $T + \Delta T$. Calculate the increase in the volume of the metal box.

(a) $3a^3\alpha\Delta T$ (b) $4a^3\alpha\Delta T$

(c) $3\pi a^3\alpha\Delta T$ (d) $\frac{4}{3}\pi a^3\alpha\Delta T$

CHAPTER-11 : THERMODYNAMICS

13. n mole a perfect gas undergoes a cyclic process $ABCA$ (see (figure) consisting of the following processes.
A → B : Isothermal expansion at temperature T so that the volume is doubled from V_1 to $V_2 = 2V_1$ and pressure changes from P_1 to P_2.
B → C : Isobaric compression at pressure P_2 to initial volume V_1.
C → A : Isochoric change leading to change of pressure from P_2 to P_1.
Total work done in the complete cycle $ABCA$ is :



(a) 0 (b) $nRT\left(\ln 2+\frac{1}{2}\right)$

(c) $nRT \ln 2$ (d) $nRT\left(\ln 2-\frac{1}{2}\right)$

14. Match List-I with List-II:

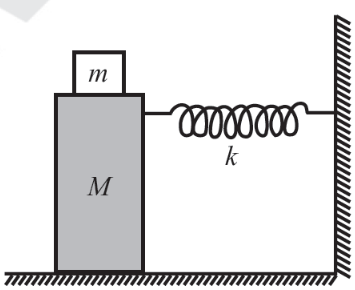
List-I	**List-II**
(A) Isothermal	(i) Pressure constant
(B) Isochoric	(ii) Temperature constant
(C) Adiabatic	(iii) Volume constant
(D) Isobaric	(iv) Heat content is constant

Choose the correct answer from the options given below:

(a) (A) → (i), (B) → (iii), (C) → (ii), (D) → (iv)
(b) (A) → (ii), (B) → (iii), (C) → (iv), (D) → (i)
(c) (A) → (ii), (B) → (iv), (C) → (iii), (D) → (i)
(d) (A) → (iii), (B) → (ii), (D) → (i), (D) → (iv)

CHAPTER-13 : OSCILLATIONS

15. In the given figure, a mass M is attached to a horizontal spring which is fixed on one side to a rigid support. The spring constant of the spring is k. The mass oscillates on a frictionless surface with time period T and amplitude A. When the mass is in equilibrium position, as shown in the figure, another mass m is gently fixed upon it. The new amplitude of oscillation will be :

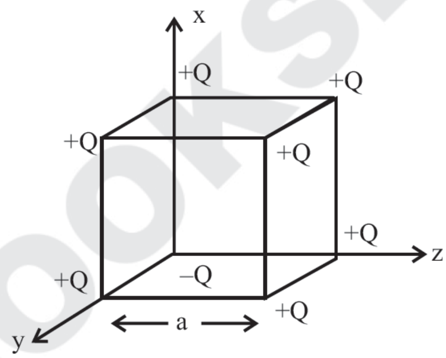


(a) $A\sqrt{\frac{M-m}{M}}$ (b) $A\sqrt{\frac{M}{M+m}}$

(c) $A\sqrt{\frac{M+m}{M}}$ (d) $A\sqrt{\frac{M}{M-m}}$

CHAPTER-15 : ELECTRIC CHARGES AND FIELDS

16. A cube of side 'a' has point charges $+Q$ located at each of its vertices except at the origin where the charge is $-Q$. The electric field at the centre of cube is :



(a) $\frac{3Q}{3\sqrt{3}\pi\varepsilon_0 a^2}(\hat{x}+\hat{y}+\hat{z})$

(b) $\frac{-2Q}{3\sqrt{3}\pi\varepsilon_0 a^2}(\hat{x}+\hat{y}+\hat{z})$

(c) $\frac{2Q}{3\sqrt{3}\pi\varepsilon_0 a^2}(\hat{x}+\hat{y}+\hat{z})$

(d) $\frac{Q}{3\sqrt{3}\pi\varepsilon_0 a^2}(\hat{x}+\hat{y}+\hat{z})$

CHAPTER-16 : ELECTROSTATIC POTENTIAL AND CAPACITANCE

17. Two equal capacitors are first connected in series and then in parallel. The ratio of the equivalent capacities in the two cases will be :

(a) 4 : 1 (b) 2 : 1 (c) 1 : 4 (d) 1 : 2

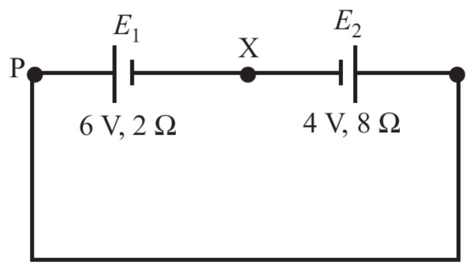
CHAPTER-17 : CURRENT ELECTRICITY

18. A current through a wire depends on time as $i = \alpha_0 t + \beta t^2$ where $\alpha_0 = 20$ A/s and $\beta = 8$ As^{-2}. Find the charge crossed through a section of the wire in 15 s.

(a) 2250 C (b) 11250 C

(c) 2100 C (d) 260 C

19. A cell E_1 of emf 6 V and internal resistance 2 Ω is connected with another cell E_2 of emf 4 V and internal resistance 8 Ω (as shown in the figure). The potential difference across points X and Y is :



(a) 10.0 V (b) 3.6 V
(c) 5.6 V (d) 2.0 V

CHAPTER-21 : ALTERNATING CURRENT

20. A resonance circuit having inductance and resistance 2×10^{-4} H and 6.28Ω respectively oscillates at 10 MHz frequency. The value of quality factor of this resonator is _____. [$\pi = 3.14$]

21. A common transistor radio set requires 12 V (D.C.) for its operation. The D.C. source is constructed by using a transformer and a rectifier circuit, which are operated at 220 V (A.C.) on standard domestic A.C. supply. The number of turns of secondary coil are 24, then the number of turns of primary are _______.

CHAPTER-22 : ELECTROMAGNETIC WAVES

22. An electromagnetic wave of frequency 5 GHz, is travelling in a medium whose relative electric permittivity and relative magnetic permeability both are 2. Its velocity in this medium is _____ $\times 10^7$ m/s.

CHAPTER-23 : RAY OPTICS AND OPTICAL INSTRUMENTS

23. The focal length f is related to the radius of curvature r of the spherical convex mirror by :

(a) $f = +\frac{1}{2}r$ (b) $f = -r$

(c) $f = -\frac{1}{2}r$ (d) $f = r$

CHAPTER-24 : WAVE OPTICS

24. In a Young's double slit experiment, the width of the one of the slit is three times the other slit. The amplitude of the light coming from a slit is proportional to the slit-width. Find the ratio of the maximum to the minimum intensity in the interference pattern.

(a) 1 : 4 (b) 3 : 1 (c) 4 : 1 (d) 2 : 1

25. An unpolarized light beam is incident on the polarizer of a polarization experiment and the intensity of light beam emerging from the analyzer is measured as 100 Lumens. Now, if the analyzer is rotated around the horizontal axis (direction of light) by 30° in clockwise direction, the intensity of emerging light will be ______ Lumens.

CHAPTER-25 : DUAL NATURE OF RADIATION AND MATTER

26. Given below are two statements :

Statement-I : Two photons having equal linear momenta have equal wavelengths.

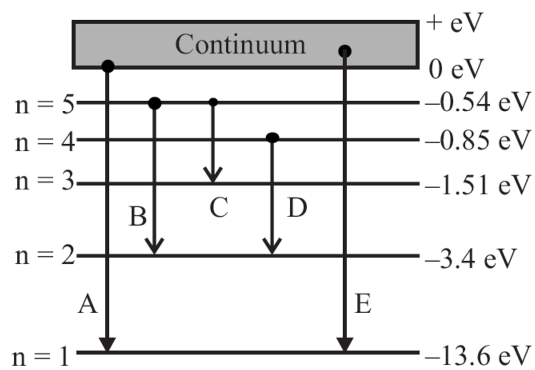
Statement-II: If the wavelength of photon is decreased, then the momentum and energy of a photon will also decrease.

In the light of the above statements, choose the correct answer from the options given below.

(a) Both Statement I and Statement II are true
(b) Statement I is false but Statement II is true
(c) Both Statement I and Statement II are false
(d) Statement I is true but Statement II is false

CHAPTER-26 : ATOMS

27. In the given figure, the energy levels of hydrogen atom have been shown along with some transitions marked A, B, C, D and E. The transitions A, B and C respectively represent :

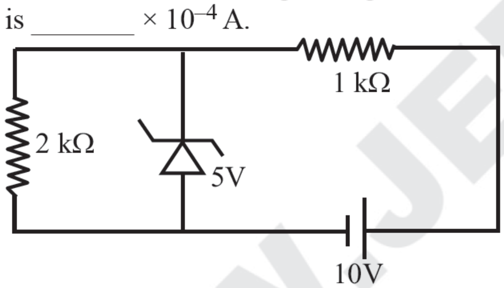


(a) The ionization potential of hydrogen, second member of Balmer series and third member of Paschen series.
(b) The first member of the Lyman series, third member of Balmer series and second member of Paschen series.
(c) The series limit of Lyman series, third member of Balmer series and second member of Paschen series.
(d) The series limit of Lyman series, second member of Balmer series and second member of Paschen series.

CHAPTER-28 : SEMICONDUCTOR ELECTRONICS :MATERIALS, DEVICES AND SIMPLE CIRCUITS

28. If an emitter current is changed by 4 mA, the collector current changes by 3.5 mA. The value of β will be :
(a) 7 (b) 0.5 (c) 0.875 (d) 3.5

29. In connection with the circuit drawn below, the value of current flowing through 2 kΩ resistor is _______ $\times 10^{-4}$ A.



CHAPTER-29 : COMMUNICATION SYSTEMS

30. An audio signal $v_m = 20 \sin 2\pi\ (1500\ t)$ amplitude modulates a carrier $v_c = 80 \sin 2\pi$ (100,000 t).
The value of percent modulation is ________ .

CHEMISTRY

CHAPTER-1 : SOME BASIC CONCEPTS OF CHEMISTRY

31. 4.5 g of compound A (MW = 90) was used to make 250 mL of its aqueous solution. The molarity of the solution is M is $x \times 10^{-1}$. The value of x is ________ (Rounded off to the nearest integer)

CHAPTER-2 : STRUCTURE OF ATOM

32. A proton and a Li^{3+} nucleus are accelerated by the same potential. If λ_{Li} and λ_p denote the de Broglie wavelengths of Li^{3+} and proton respectively, then the value of $\frac{\lambda_{Li}}{\lambda_p}$ is $x \times 10^{-1}$. The value of x is ________
(Rounded off to the nearest integer)
(Mass of Li^{3+} = 8.3 mass of proton)

CHAPTER-3 : CLASSIFICATION OF ELEMENTS AND PERIODICITY IN PROPERTIES

33. Consider the elements Mg, Al, S, P and Si, the correct increasing order of their first ionization enthalpy is :
(a) Mg < Al < Si < S < P
(b) Al < Mg < Si < S < P
(c) Mg < Al < Si < P < S
(d) Al < Mg < S < Si < P

CHAPTER-4 : CHEMICAL BONDING AND MOLECULAR STRUCTURE

34. Which of the following are isostructural pairs?
A. SO_4^{2-} and CrO_4^{2-}
B. $SiCl_4$ and $TiCl_4$
C. NH_3 and NO_3^-
D. BCl_3 and $BrCl_3$
(a) C and D only (b) A and B only
(c) A and C only (d) B and C only

CHAPTER-7 : EQUILIBRIUM

35. At 1990 K and 1 atm pressure, there are equal number of Cl_2 molecules and Cl atoms in the reaction mixture. The value of K_p for the reaction $Cl_2(g) \rightleftharpoons 2Cl(g)$ under the above conditions is $x \times 10^{-1}$. The value of x is ________ (Rounded off to the nearest integer)

36. For the reaction $A(g) \rightarrow (B)(g)$, the value of the equilibrium constant at 300 K and 1 atm is equal to 100.0. The value of $\Delta_r G$ for the reaction at 300 K and 1 atm in J mol^{-1} is $-xR$, where x is ________ (Rounded off to the nearest integer)
(R = 8.31 J mol^{-1} K^{-1} and ln 10 = 2.3)

CHAPTER-8 : REDOX REACTIONS

37. The reaction of sulphur in alkaline medium is given below:

$$S_8(s) + aOH^-(aq) \rightarrow bS^{2-}(aq) + cS_2O_3^{2-}(aq) + dH_2O(l)$$

The values of 'a' is ________. (Integer answer)

CHAPTER-9 : HYDROGEN

38. (A) $HOCl + H_2O_2 \rightarrow H_3O^+ + Cl^- + O_2$

(B) $I_2 + H_2O_2 + 2OH^- \rightarrow 2I^- + 2H_2O + O_2$

Choose the correct option.

(a) H_2O_2 acts as reducing and oxidising agent respectively in equation (A) and (B)
(b) H_2O_2 acts as oxidising agent in equation (A) and (B)
(c) H_2O_2 acts as reducing agent in equation (A) and (B)
(d) H_2O_2 act as oxidizing and reducing agent respectively in equation (A) and (B)

CHAPTER-10 : THE s-BLOCK ELEMENTS

39. Number of amphoteric compound among the following is ________

(A) BeO (B) BaO
(C) $Be(OH)_2$ (D) $Sr(OH)_2$

CHAPTER-11 : THE p-BLOCK ELEMENTS (GROUP-13 AND 14)

40. Al_2O_3 was leached with alkali to get X. The solution of X on passing of gas Y, forms Z. X, Y and Z respectively are:

(a) $X = Na[Al(OH)_4]$, $Y = SO_2$, $Z = Al_2O_3$
(b) $X = Na[Al(OH)_4]$, $Y = CO_2$, $Z = Al_2O_3 \cdot$
(c) $X = Al(OH)_3$, $Y = CO_2$, $Z = Al_2O_3$
(d) $X = Al(OH)_3$, $Y = SO_2$, $Z = Al_2O_3 \cdot H_2O$

41. Given below are two statements :

Statement I : Colourless cupric metaborate is reduced to cuprous metaborate in a luminous flame.

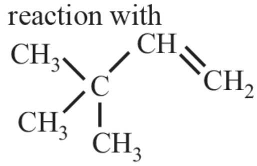
Statement II : Cuprous metaborate is obtained by heating boric anhydride and copper sulphate in a non-luminous flame.

In the light of the above statements, choose the most appropriate answer from the options given below :

(a) Statement I is true but Statement II is false
(b) Both Statement I and Statement II are false
(c) Statement I is false but Statement II is true
(d) Both Statement I and Statement II are true

CHAPTER-13 : HYDROCARBONS

42. What is the major product formed by HI on reaction with



(a) $\begin{array}{l} \quad\quad\ CH_3 \\ \quad\quad\ \ | \\ CH_3 - C - CH - CH_3I \\ \quad\quad\ \ |\quad\ \ | \\ \quad\quad\ CH_3 H \end{array}$

(b) $\begin{array}{l} \quad\quad\ CH_3 \\ \quad\quad\ \ | \\ CH_3 - C - CH - CH_3 \\ \quad\quad\ \ |\quad\ \ | \\ \quad\quad\ CH_3\ I \end{array}$

(c) $\begin{array}{l} \quad\quad\ CH_3 \\ \quad\quad\ \ | \\ CH_3 - C - CH - CH_3 \\ \quad\quad\ \ |\quad\ \ | \\ \quad\quad\ \ I\quad CH_3 \end{array}$

(d) $\begin{array}{l} CH_3 - CH - CH - CH_2 - CH_3 \\ \quad\quad\ \ |\quad\quad | \\ \quad\quad\ CH_3\ \ I \end{array}$

43. Which of the following reagent is used for the following reaction?

$$CH_3CH_2CH_3 \xrightarrow{?} CH_3CH_2CHO$$

(a) Manganese acetate
(b) Copper at high temperature and pressure
(c) Molybdenum oxide
(d) Potassium permanganate

CHAPTER-14 : ENVIRONMENTAL CHEMISTRY

44. The gas released during anaerobic degradation of vegetation may lead to:

(a) Ozone hole
(b) Acid rain
(c) Corrosion of metals
(d) Global warming and cancer

CHAPTER-15 : THE SOLID STATE

45. The coordination number of an atom in a body-centered cubic structure is ________.

[Assume that the lattice is made up of atoms.]

CHAPTER-16 : SOLUTIONS

46. When 9.45 g of $ClCH_2COOH$ is added to 500 mL of water, its freezing point drops by 0.5°C. The dissociation constant of $ClCH_2COOH$ is $x \times 10^{-3}$. The value of x is ________. (Rounded off to the nearest integer)

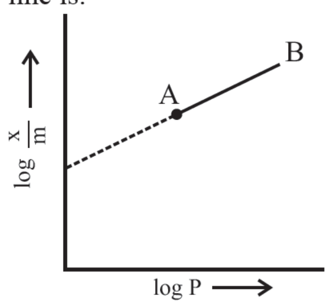
$[K_{f(H_2O)} = 1.86 \text{ K kg mol}^{-1}]$

CHAPTER-18 : CHEMICAL KINETICS

47. Gaseous cyclobutene isomerizes to butadiene in a first order process which has a 'k' value of 3.3×10^{-4} s^{-1} at 153 °C. The time in minutes it takes for the isomerization to proceed 40% to completion at this temperature is ________. (Rounded off to the nearest integer)

CHAPTER-19 : SURFACE CHEMISTRY

48. In Freundlich adsorption isotherm, slope of AB line is:



(a) log n with ($n > 1$)

(b) n with (n, 0.1 to 0.5)

(c) $\log\frac{1}{n}$ with ($n < 1$)

(d) $\frac{1}{n}$ with $\left(\frac{1}{n} = 0 \text{ to } 1\right)$

CHAPTER-20 : GENERAL PRINCIPLES AND PROCESSES OF ISOLATION OF ELEMENTS

49. Which of the following ore is concentrated using group 1 cyanide salt?

(a) Sphalerite (b) Calamine

(c) Siderite (d) Malachite

CHAPTER-22 : THE d-AND f-BLOCK ELEMENTS

50. The major components in "Gun Metal" are :

(a) Cu, Zn and Ni

(b) Cu, Sn and Zn

(c) Al, Cu, Mg and Mn

(d) Cu, Ni and Fe

51. The electrode potential of M^{2+}/M of 3d-series elements shows positive value of

(a) Zn (b) Fe (c) Co (d) Cu

CHAPTER-23 : CO-ORDINATION COMPOUNDS

52. The stepwise formation of $[Cu(NH_3)_4]^{2+}$ is given below

$$Cu^{2+} + NH_3 \xrightleftharpoons{K_1} [Cu(NH_3)_2]^{2+}$$

$$[Cu(NH_3)]^{2+} + NH_3 \xrightleftharpoons{K_2} [Cu(NH_3)_2]^{2+}$$

$$[Cu(NH_3)_2]^{2+} + NH_3 \xrightleftharpoons{K_3} [Cu(NH_3)_3]^{2+}$$

$$[Cu(NH_3)_3]^{2+} + NH_3 \xrightleftharpoons{K_4} [Cu(NH_3)_4]^{2+}$$

The value of stability constants K_1, K_2, K_3 and K_4 are 10^4, 1.58×10^3, 5×10^2 and 10^2 respectively. The overall equilibrium constants for dissociation of $[Cu(NH_3)_4]^{2+}$ is $x \times 10^{-12}$. The value of x is _______ (Rounded off to the nearest integer)

CHAPTER-24 : HALOALKANES AND HALOARENES

53. The product formed in the first step of the reaction of

$$CH_3-CH_2-\underset{}{\overset{Br}{\overset{|}{C}H}}-CH_2-\underset{\underset{Br}{|}}{CH}-CH_3$$

with excess $Mg/Et_2O(Et = C_2H_5)$ is :

(a) $\begin{array}{l} CH_3-CH_2-CH-CH_2-CH-CH_3 \\ \qquad\qquad\quad | \qquad\qquad\quad | \\ \qquad CH_3-CH-CH_2-CH-CH_2-CH_3 \end{array}$

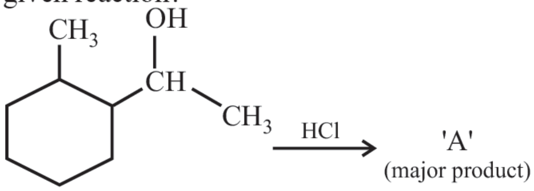
(b) $\begin{array}{l} CH_2-CH_2-CH-CH_2-CH-CH_2 \\ \qquad\qquad\quad | \qquad\qquad\quad | \\ CH_3-CH_2-CH-CH_2-CH-CH_3 \end{array}$

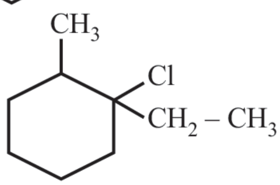
(c) $CH_3-CH\begin{array}{l}\diagup CH_2 \\ \quad | \\ \diagdown CH-CH_3\end{array}$

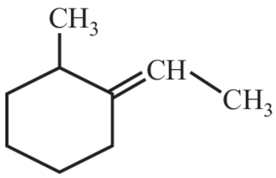
(d) $CH_3CH_2-\overset{MgBr}{\overset{|}{C}H}-CH_2-\underset{\underset{MgBr}{|}}{CH}-CH_3$

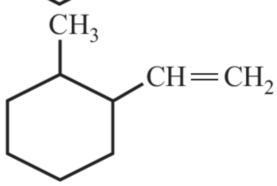
CHAPTER-25 : ALCOHOLS, PHENOLS AND ETHERS

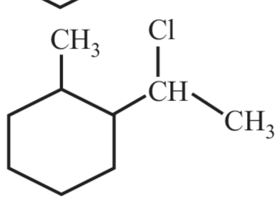
54. What is the final product (major) 'A' in the given reaction?



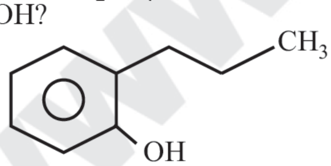
(a) 

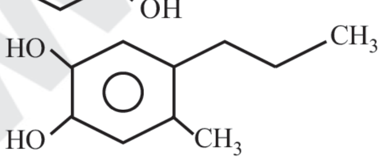
(b) 

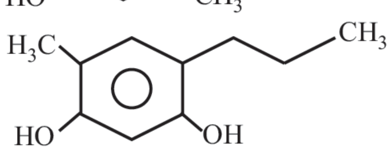
(c) 

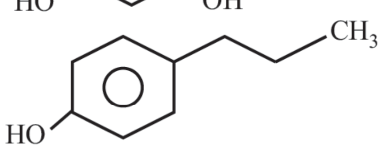
(d) 

55. Which of the following compound gives pink colour on reaction with phthalic anhydride in conc. H_2SO_4 followed by treatment with NaOH?

(a) 

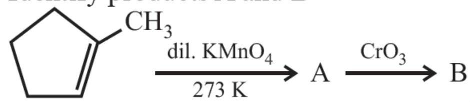
(b) 

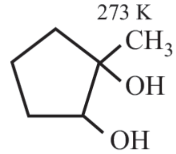
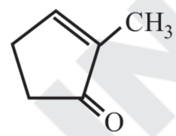
(c) 

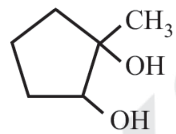
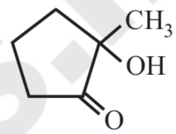
(d) 

CHAPTER-26 : ALDEHYDES, KETONES AND CARBOXYLIC ACIDS

56. Identify products A and B

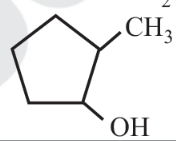
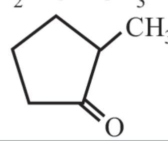


(a) A :  B : 

(b) A :  B : 

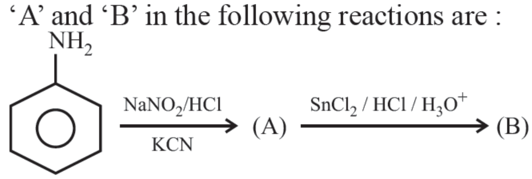
(c) A : $OHC-CH_2CH_2CH_2-\overset{\overset{O}{\|}}{C}-CH_3$

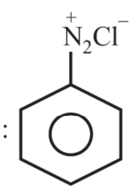
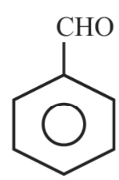
B : $HOOC-CH_2CH_2CH_2-\overset{\overset{O}{\|}}{C}-CH_3$

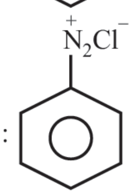
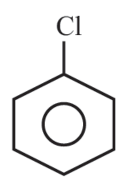
(d) A :  B : 

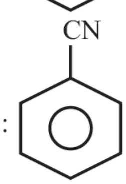
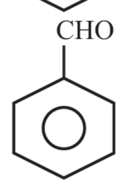
CHAPTER-27 : AMINES

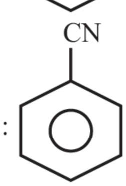
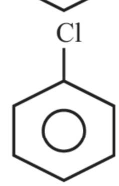
57. 'A' and 'B' in the following reactions are :



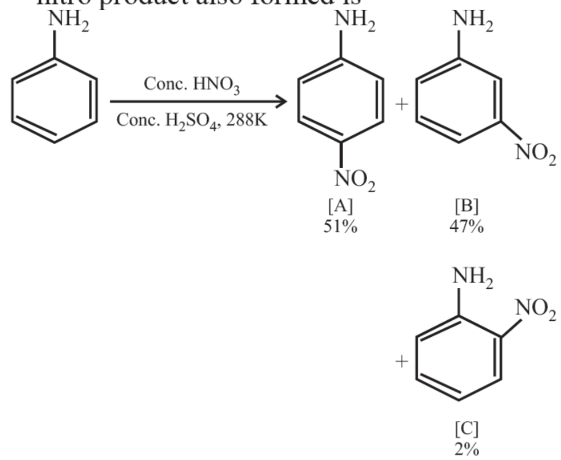
(a) (A) :  (B) : 

(b) (A) :  (B) : 

(c) (A) :  (B) : 

(d) (A) :  (B) : 

58. In the following reaction the reason why meta-nitro product also formed is



(a) low temperature
(b) $-NH_2$ group is highly meta-directive
(c) Formation of anilinium ion
(d) $-NO_2$ substitution always takes place at meta-position

CHAPTER-28 : BIOMOLECULES

59. Out of the following, which type of interaction is responsible for the stabilisation of α-helix structure of proteins?
(a) Ionic bonding
(b) Hydrogen bonding
(c) Covalent bonding
(d) vander Waals forces

CHAPTER-29 : POLYMERS

60. Match List-I with List-II.

List-I (Monomer Unit)	**List-II (Polymer)**
(A) Caprolactum	(i) Natural rubber
(B) 2-Chloro-1, 3-butadiene	(ii) Buna-N
(C) Isoperene	(iii) Nylon 6
(D) Acrylonitrile	(iv) Neoprene

Choose the correct answer from the options given below:
(a) (A)-(iv), (B)-(iii), (C)-(ii), (D)-(i)
(b) (A)-(ii), (B)-(i), (C)-(iv), (D)-(iii)
(c) (A)-(iii), (B)-(iv), (C)-(i), (D)-(ii)
(d) (A)-(i), (B)-(ii), (C)-(iii), (D)-(iv)

MATHEMATICS

CHAPTER-5 : COMPLEX NUMBERS AND QUADRATIC EQUATIONS

61. Let p and q be two positive numbers such that $p + q = 2$ and $p^4 + q^4 = 272$. Then p and q are roots of the equation:
(a) $x^2 - 2x + 2 = 0$ (b) $x^2 - 2x + 8 = 0$
(c) $x^2 - 2x + 136 = 0$ (d) $x^2 - 2x + 16 = 0$

62. If the least and the largest real values of α, for which the equation $z + \alpha|z - 1| + 2i = 0$ ($z \in C$ and $i = \sqrt{-1}$) has a solution, are p and q respectively, then $4(p^2 + q^2)$ is equal to ________.

CHAPTER-7 : PERMUTATIONS AND COMBINATIONS

63. A scientific committee is to be formed from 6 Indians and 8 foreigners, which includes at least 2 Indians and double the number of foreigners as Indians. Then the number of ways, the committee can be formed, is :
(a) 1625 (b) 575 (c) 560 (d) 1050

CHAPTER-8 : BINOMIAL THEOREM

64. The value of $-{}^{15}C_1 + 2.{}^{15}C_2 - 3.{}^{15}C_3 + ... -15.{}^{15}C_{15} + {}^{14}C_1 + {}^{14}C_3 + {}^{14}C_5 + ... + {}^{14}C_{11}$ is :
(a) $2^{16} - 1$ (b) $2^{13} - 14$
(c) 2^{14} (d) $2^{13} - 13$

CHAPTER-9 : SEQUENCE AND SERIES

65. If $e^{(\cos^2 x + \cos^4 x + \cos^6 x + ...\infty)\log_e 2}$ satisfies the equation $t^2 - 9t + 8 = 0$, then the value of $\dfrac{2\sin x}{\sin x + \sqrt{3}\cos x}\left(0 < x < \dfrac{\pi}{2}\right)$ is
(a) $2\sqrt{3}$ (b) $\dfrac{3}{2}$ (c) $\sqrt{3}$ (d) $\dfrac{1}{2}$

66. Let $A = \{x : x$ is 3-digit number$]$
$B = \{x : x = 9k + 2, k \in I\}$
and $C = \{x : x = 9k + \ell, k \in I, \ell \in I, 0 < \ell < 9\}$
for some ℓ $(0 < \ell < 9)$
If the sum of all the elements of the set $A \cap (B \cup C)$ is 274×400, then ℓ is equal to ______.

CHAPTER-10 : STRAIGHT LINES & PAIR OF STRAIGHT LINES

67. A man is walking on a straight line. The arithmetic mean of the reciprocals of the intercepts of this line on the coordinate axes is $\dfrac{1}{4}$. Three stones A, B and C are placed at the points (1, 1), (2, 2) and (4, 4) respectively. Then which of these stones is/are on the path of the man?
(a) A only (b) C only
(c) All the three (d) B only

CHAPTER-11 : CONIC SECTIONS

68. The locus of the mid-point of the line segment joining the focus of the parabola $y^2 = 4ax$ to a moving point of the parabola, is another parabola whose directrix is :

(a) $x = -\frac{a}{2}$ (b) $x = \frac{a}{2}$

(c) $x = 0$ (d) $x = a$

69. If one of the diameters of the circle $x^2 + y^2 - 2x - 6y + 6 = 0$ is a chord of another circle 'C', whose center is at (2, 1), then its radius is _______.

CHAPTER-12 : LIMITS AND DERIVATIVES

70. $\lim\limits_{n\to\infty} \tan\left\{\sum_{r=1}^{n} \tan^{-1}\left(\frac{1}{1+r+r^2}\right)\right\}$ is equal to ______.

CHAPTER-13 : MATHEMATICAL REASONING

71. The statement among the following that is a tautology is:

(a) $A \vee (A \wedge B)$

(b) $A \wedge (A \vee B)$

(c) $B \to [A \wedge (A \to B)]$

(d) $[A \wedge (A \to B)] \to B$

CHAPTER-16 : RELATIONS AND FUNCTIONS

72. Let $f : R \to R$ be defined as $f(x) = 2x - 1$ and $g : R - \{1\} \to R$ be defined as $g(x) = \dfrac{x - \frac{1}{2}}{x - 1}$.

Then the composition function $f(g(x))$ is :

(a) onto but not one-one

(b) both one-one and onto

(c) one-one but not onto

(d) neither one-one nor onto

CHAPTER-18 : MATRICES

73. Let M be any 3×3 matrix with entries from the set $\{0, 1, 2\}$. The maximum number of such matrices, for which the sum of diagonal elements of $M^T M$ is seven, is ______.

CHAPTER-19 : DETERMINANTS

74. The system of linear equations :

$3x - 2y - kz = 10$; $2x - 4y - 2z = 6$; $x + 2y - z = 5m$

is inconsistent if :

(a) $k = 3, m = \frac{4}{5}$ (b) $k \neq 3, m \in R$

(c) $k \neq 3, m \neq \frac{4}{5}$ (d) $k = 3, m \neq \frac{4}{5}$

75. Let $P = \begin{bmatrix} 3 & -1 & -2 \\ 2 & 0 & \alpha \\ 3 & -5 & 0 \end{bmatrix}$, where $\alpha \in R$. Suppose $Q = [q_{ij}]$ is a matrix satisfying $PQ = kI_3$ for some non-zero $k \in R$. If $q_{23} = -\frac{k}{8}$ and $|Q| = \frac{k^2}{2}$, then $\alpha^2 + k^2$ is equal to _____.

CHAPTER-20 : CONTINUITY AND DIFFERENTIABILITY

76. If $f : R \to R$ is a function defined by $f(x) = [x - 1] \cos\left(\frac{2x-1}{2}\right)\pi$, where [.] denotes the greatest integer function, then f is:

(a) discontinuous at all integral values of x except at $x = 1$

(b) continuous only at $x = 1$

(c) continuous for every real x

(d) discontinuous only at $x = 1$

CHAPTER-21 : APPLICATION OF DERIVATIVES

77. If the tangent to the curve $y = x^3$ at the point $P(t, t^3)$ meets the curve again at Q, then the ordinate of the point which divides PQ internally in the ratio 1 : 2 is :

(a) $-2t^3$ (b) 0 (c) $-t^3$ (d) $2t^3$

78. The function

$$f(x) = \frac{4x^3 - 3x^2}{6} - 2\sin x + (2x - 1)\cos x :$$

(a) increases in $\left[\frac{1}{2}, \infty\right)$

(b) increases in $\left(-\infty, \frac{1}{2}\right]$

(c) decreases in $\left[\frac{1}{2}, \infty\right)$

(d) decreases in $\left(-\infty, \frac{1}{2}\right]$

79. The minimum value of α for which the equation $\frac{4}{\sin x}+\frac{1}{1-\sin x}=\alpha$ has at least one solution in $\left(0,\frac{\pi}{2}\right)$ is ________.

CHAPTER-22 : INTEGRALS

80. If $\int\frac{\cos x-\sin x}{\sqrt{8-\sin 2x}}dx = a\sin^{-1}\left(\frac{\sin x+\cos x}{b}\right)+c,$ where c is a constant of integration, then the ordered pair (a, b) is equal to

(a) (– 1, 3) (b) (3, 1)
(c) (1, 3) (d) (1, – 3)

81. $\lim\limits_{x\to 0}\frac{\int_0^{x^2}(\sin\sqrt{t})\,dt}{x^3}$ is equal to :

(a) $\frac{2}{3}$ (b) $\frac{3}{2}$ (c) 0 (d) $\frac{1}{15}$

82. If $\int_{-a}^{a}(|x|+|x-2|)\,dx = 22$, $(a > 2)$ and $[x]$ denotes the greatest integer $\le x$, then $\int_{a}^{-a}(x+[x])\,dx$ is equal to ______.

CHAPTER-23 : APPLICATIONS OF INTEGRALS

83. The area (in sq. units) of the part of the circle $x^2 + y^2 = 36$, which is outside the parabola $y^2 = 9x$, is :

(a) $24\pi+3\sqrt{3}$ (b) $12\pi-3\sqrt{3}$
(c) $24\pi-3\sqrt{3}$ (d) $12\pi+3\sqrt{3}$

CHAPTER-24 : DIFFERENTIAL EQUATIONS

84. The population $P = P(t)$ at time 't' of a certain species follows the differential equation $\frac{dP}{dt} = 0.5\,P - 450$. If $P(0) = 850$, then the time at which population becomes zero is :

(a) $\log_e 18$ (b) $\log_e 9$
(c) $\frac{1}{2}\log_e 18$ (d) $2\log_e 18$

CHAPTER-25 : VECTOR ALGEBRA

85. Let three vectors $\vec{a},\vec{b}$ and $\vec{c}$ be such that $\vec{c}$ is coplanar with $\vec{a}$ and $\vec{b}, \vec{a}.\vec{c}=7$ and $\vec{b}$ is perpendicular to $\vec{c}$, where $\vec{a}=-\hat{i}+\hat{j}+\hat{k}$ and $\vec{b}=2\hat{i}+\hat{k}$, then the value of $2|\vec{a}+\vec{b}+\vec{c}|^2$ is _____.

CHAPTER-26 : THREE DIMENSIONAL GEOMETRY

86. The equation of the plane passing through the point (1, 2, –3) and perpendicular to the planes $3x + y - 2z = 5$ and $2x - 5y - z = 7$, is

(a) $3x - 10y - 2z + 11 - 0$
(b) $6x - 5y - 2z - 2 = 0$
(c) $11x + y + 17z + 38 = 0$
(d) $6x - 5y + 2z + 10 = 0$

87. The distance of the point (1, 1, 9) from the point of intersection of the line $\frac{x-3}{1}=\frac{y-4}{2}=\frac{z-5}{2}$ and the plane x + y + z = 17 is :

(a) $2\sqrt{19}$ (b) $19\sqrt{2}$ (c) 38 (d) $\sqrt{38}$

CHAPTER-27 : PROBABILITY

88. An ordinary dice is rolled for a certain number of times. If the probability of getting an odd number 2 times is equal to the probability of getting an even number 3 times, then the probability of getting an odd number for odd number of times is :

(a) $\frac{1}{32}$ (b) $\frac{5}{16}$ (c) $\frac{3}{16}$ (d) $\frac{1}{2}$

89. Let B_i $(i = 1, 2, 3)$ be three independent events in a sample space. The probability that only B_1 occur is α. Only B_2 occurs is β and only B_3 occurs is γ. Let p be the probability that none of the events B_i occurs and these 4 probabilities satisfy the equations $(\alpha - 2\beta)\,p = \alpha\beta$ and $(\beta - 3\gamma)\,p = 2\beta\gamma$ (All the probabilities are assumed to lie in the interval (0, 1)). Then $\frac{P(B_1)}{P(B_3)}$ is equal to ______.

CHAPTER-28 : PROPERTIES OF TRIANGLES

90. Two vertical poles are 150 m apart and the height of one is three times that of the other. If from the middle point of the line joining their feet, an observer finds the angles of elevation of their tops to be complementary, then the height of the shorter pole (in meters) is :

(a) $20\sqrt{3}$ (b) $25\sqrt{3}$ (c) 30 (d) 25

ANSWER KEY

1	(b)	**11**	(25600)	**21**	(440)	**31**	(2)	**41**	(b)	**51**	(d)	**61**	(d)	**71**	(d)	**81**	(a)
2	(b)	**12**	(a)	**22**	(15)	**32**	(2)	**42**	(c)	**52**	(1)	**62**	(10)	**72**	(c)	**82**	(3)
3	(25)	**13**	(d)	**23**	(a)	**33**	(b)	**43**	(c)	**53**	(d)	**63**	(a)	**73**	(540)	**83**	(c)
4	(25)	**14**	(b)	**24**	(c)	**34**	(b)	**44**	(d)	**54**	(a)	**64**	(b)	**74**	(d)	**84**	(d)
5	(1)	**15**	(b)	**25**	(75)	**35**	(5)	**45**	(8)	**55**	(a)	**65**	(d)	**75**	(17)	**85**	(75)
6	(c)	**16**	(b)	**26**	(d)	**36**	(1380)	**46**	(35)	**56**	(b)	**66**	(5)	**76**	(c)	**86**	(c)
7	(b)	**17**	(c)	**27**	(c)	**37**	(12)	**47**	(26)	**57**	(b)	**67**	(d)	**77**	(a)	**87**	(d)
8	(d)	**18**	(b)	**28**	(a)	**38**	(c)	**48**	(d)	**58**	(c)	**68**	(c)	**78**	(a)	**88**	(d)
9	(a)	**19**	(c)	**29**	(25)	**39**	(2)	**49**	(a)	**59**	(b)	**69**	(3)	**79**	(9)	**89**	(6)
10	(d)	**20**	(2000)	**30**	(25)	**40**	(b)	**50**	(b)	**60**	(c)	**70**	(1)	**80**	(c)	**90**	(b)

Hints and Solutions

PHYSICS

1. **(b)** $\dfrac{x^2}{\alpha kT} \rightarrow$ dimensionless

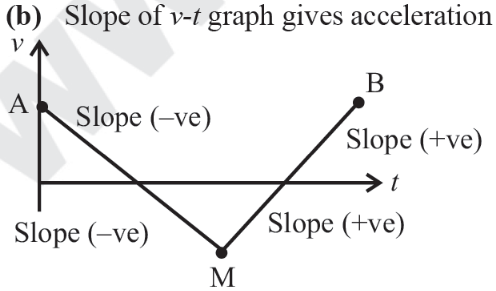
$$\Rightarrow [\alpha] = \frac{[x^2]}{[kT]} = \frac{L^2}{ML^2T^{-2}} = M^{-1}T^2$$

Now, $[W] = [\alpha]\,[\beta]^2$

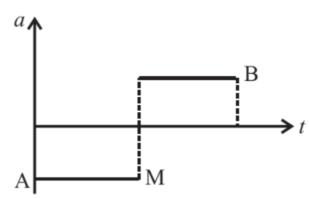
$$[\beta]^2 = \frac{[W]}{[\alpha]}$$

$$[\beta] = \left(\frac{ML^2T^{-2}}{M^{-1}T^2}\right)^{\frac{1}{2}} = MLT^{-2}$$

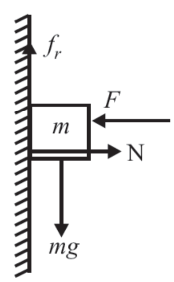
2. **(b)** Slope of v-t graph gives acceleration



As slope is negative from A to M. So, acceleration is negative and constant from A to M. From M to B, slope is positive and hence acceleration. So, correct graph will be as follow



3. **(25)** F.B.D. of the block is shown in the diagram



Since, block is at rest,

$\therefore f_r - mg = 0$ ... (i)

$F - N = 0$ ... (ii)

$f_r \;\; \mu N$

In limiting case,

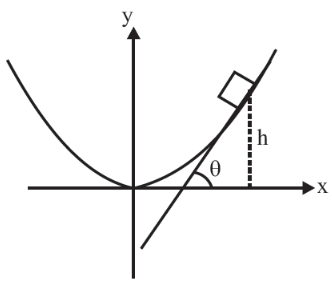
$f_r = \mu N = \mu F$ ... (iii)

Using equation (i) and (iii),

$$F = \frac{mg}{\mu}$$

$$\Rightarrow F = \frac{0.5 \times 10}{0.2} = 25\,N$$

4. (25)

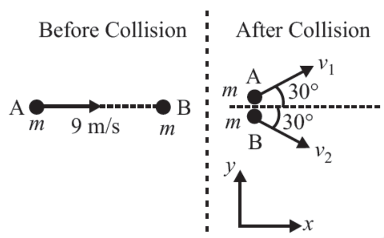


Block will experience maximum force due to friction when it is at maximum height, h at this height, if slope of the tangent is tan θ, then θ = Angle of repose.

$$\therefore \tan\theta = \frac{dy}{dx} = \frac{d}{dk}\left(\frac{x^2}{4}\right) = \frac{x}{2} = 0.5$$

$\Rightarrow x = 1$ and therefore $y = 0.25$ m = 25 cm (Assuming that x & y in the equation are given in meter)

5. (1)



From conservation of momentum along y-axis.

$$\vec{P}_{iy} = \vec{P}_{fy}$$

$$0 + 0 = mv_1 \sin 30^\circ\,\hat{j} + mv_2 \sin 30^\circ(-\hat{j})$$

$$mv_2 \sin 30^\circ = mv_1 \sin 30^\circ$$

$$v_2 = v_1 \text{ or } \frac{v_1}{v_2} = 1$$

6. (c) Moment of inertia of ring about its diameter,

$$I_1 = \frac{MR^2}{2}$$

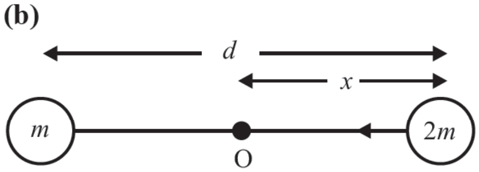
Moment of inertia of disc, $I_2 = \dfrac{MR^2}{2}$

Moment of inertia of solid cylinder, $I_3 = \dfrac{MR^2}{2}$

Moment of inertia of solid sphere, $I_4 = \dfrac{2}{5}MR^2$

$$\therefore I_1 = I_2 = I_3 > I_4$$

7. (b)



For point O to be the centre of mass of the system, moment about O should be zero.

$$\therefore 2mx = m(d - x)$$

$$\Rightarrow 3mx = md$$

$$\Rightarrow x = \frac{d}{3}$$

For equilibrium,

$$F_{\text{gravitational}} = F_{\text{centripetal}}$$

$$\therefore F = \frac{G(2m)m}{d^2} = (2m)\omega^2\left(\frac{d}{3}\right)$$

$$\Rightarrow \frac{Gm}{d^2} = \omega^2 \frac{d}{3}$$

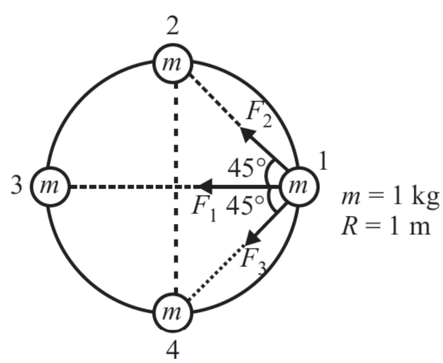
$$\Rightarrow \omega^2 = \frac{3Gm}{d^3}$$

$$\Rightarrow \omega = \sqrt{\frac{3Gm}{d^3}}$$

$\therefore$ Period of revolution,

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{d^3}{3Gm}}$$

8. (d)



Gravitational force acting between particle 1 and 3 is

$$F_1 = \frac{Gmm}{(2R)^2} = \frac{Gm^2}{4R^2}$$

Gravitational force acting between particle 1 and 2 is

$$F_2 = \frac{Gmm}{(\sqrt{2}R)^2} = \frac{Gm^2}{2R^2}$$

Gravitational force acting between 1 and 4 is

$$F_3 = \frac{Gmm}{(\sqrt{2}R)^2} = \frac{Gm^2}{2R^2}$$

Net force towards the centre,

$$F_{\text{net}} = F_1 + F_2 \cos 45^\circ + F_3 \cos 45^\circ$$

$$= \frac{Gm^2}{4R^2} + \frac{Gm^2}{2R^2}\frac{1}{\sqrt{2}} + \frac{Gm^2}{2R^2}\frac{1}{\sqrt{2}}$$

$$= \frac{Gm^2}{R^2}\left(\frac{1}{4} + \frac{1}{2\sqrt{2}} + \frac{1}{2\sqrt{2}}\right)$$

$$= \frac{Gm^2}{R^2}\left(\frac{1}{4} + \frac{1}{\sqrt{2}}\right) = \frac{Gm^2}{4R^2}\left(1 + 2\sqrt{2}\right)$$

At equilibrium $F_{\text{centre}} = F_{\text{centripetal}}$

$$F_{\text{net}} = \frac{Gm^2}{4R^2}(1+2\sqrt{2}) = \frac{mv^2}{R}$$

$$\Rightarrow v = \sqrt{\frac{(2\sqrt{2}+1)Gm}{4R}}$$

Put $m = 1$ kg, $R = 1$ m, we get

$$\Rightarrow v = \frac{\sqrt{G(1+2\sqrt{2})}}{2}$$

9. **(a)** Ratio $\frac{T_1}{T_2} = \frac{1}{8}$

$$\frac{\frac{2\pi}{\omega_1}}{\frac{2\pi}{\omega_2}} = \frac{1}{8} \qquad \left(\because T = \frac{2\pi}{\omega}\right)$$

$$\frac{\omega_1}{\omega_2} = \frac{8}{1}$$

10. **(d)** We know that

$Y = 3K(1 - 2\sigma)$

Here,

σ = Poisson's ratio

$$\sigma = \frac{1}{2}\left(1 - \frac{Y}{3K}\right) \qquad ...(i)$$

Also,

$Y = 2\eta(1 + \sigma)$

$$\sigma = \frac{Y}{2\eta} - 1 \qquad ...(ii)$$

From equation (i) and (ii), we have

$$\frac{1}{2}\left(1 - \frac{Y}{3K}\right) = \frac{Y}{2\eta} - 1$$

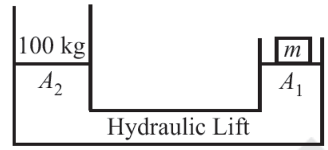
$$\Rightarrow 1 - \frac{Y}{3K} = \frac{Y}{\eta} - 2 \Rightarrow \frac{Y}{3K} = 3 - \frac{Y}{\eta}$$

$$\Rightarrow \frac{Y}{} \quad \frac{\eta - Y}{}$$

$$\Rightarrow \frac{\eta Y}{3K} = 3\eta - Y$$

$$\Rightarrow K = \frac{\eta Y}{9\eta - 3Y}$$

11. **(25600)** Using Pascal's law,



$$\frac{100 \times g}{A_2} = \frac{mg}{A_1}(A_1 < A_2) \qquad ...(i)$$

Let m mass can lift M_0 in second case then

$$\frac{M_0 g}{16A_2} = \frac{mg}{\frac{A_1}{16}} \quad \left(\because A = \frac{\pi d^2}{4}\right) \qquad ...(ii)$$

From equation (i) and (ii), we get

$$\frac{M_0}{16 \times 100} = 16$$

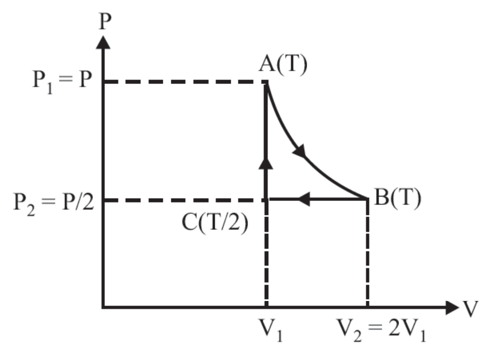
$M_0 = 25600$ kg

12. **(a)** $\Delta V = V\gamma\Delta T$

Since, $V = a^3$ and $\gamma = 3\alpha$

$\therefore \Delta V = 3a^3\alpha\Delta T$

13. **(d)**



Work done during isothermal process,

$$W_{AB} = nRT\ln\left(\frac{V_2}{V_1}\right)$$

$$W_{AB} = nRT\ln\left(\frac{2V}{V}\right) = nRT\ln 2$$

$$W_{BC} = nR(T_C - T_B) = nR\left(\frac{T}{2} - T\right) = -nR\frac{T}{2}$$

$W_{CA} = P\Delta V = 0 \qquad (\because \Delta V = 0)$

$\Rightarrow W_{\text{net}} = W_{AB} + W_{BC} + W_{CA}$

$$\Rightarrow W_{\text{net}} = nRT\ln 2 - \frac{nRT}{2}$$

$$\Rightarrow W_{\text{net}} = nRT\left(\ln 2 - \frac{1}{2}\right)$$

14. (b) (A) Isothermal ($\Delta T = 0$)
(A) $\rightarrow$ (ii)
(B) Isochoric ($\Delta V = 0$)
(B) $\rightarrow$ (iii)
(C) Adiabatic ($\Delta Q = 0$)
(C) $\rightarrow$ (iv)
(D) Isobaric ($\Delta P = 0$)
(D) $\rightarrow$ (i)

15. (b) Let the initial velocity of M is V.
On putting m on M, let velocity becomes V'.
Momentum of system remains conserved.

$\therefore p_i = p_f$

$\Rightarrow MV = (m + M)V'$

$\Rightarrow MA\omega = (m + M)A\omega \; (\because V = A\omega)$

$$\Rightarrow MA\sqrt{\frac{k}{M}} = (m+M)A'\sqrt{\frac{k}{m+M}} \quad \left(\begin{array}{l}\because k = M\omega^2 \\ \Rightarrow \omega = \sqrt{\dfrac{k}{M}}\end{array}\right)$$

$$\Rightarrow A' = A\sqrt{\frac{M}{M+m}}$$

16. (b) Electric field due to all charges will cancel out except two charges $+Q$ and $-Q$ placed along body diagonal.

$$\vec{E}_{-Q} = \frac{-Q}{4\pi\varepsilon_0 \frac{3a^2}{4}} \frac{(\hat{x}+\hat{y}+\hat{z})}{\sqrt{3}}$$

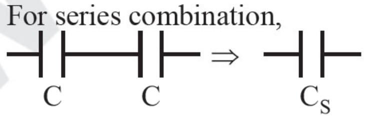
$$\vec{E}_{+Q} = \frac{-Q}{3\pi\varepsilon_0 a^2} \frac{(\hat{x}+\hat{y}+\hat{z})}{\sqrt{3}}$$

Net electric field at the centre of cube is

$$\vec{E}_{net} = \vec{E}_{-Q} + \vec{E}_{+Q}$$

$$= \frac{-2Q}{3\sqrt{3}\pi\varepsilon_0 a^2}(\hat{x}+\hat{y}+\hat{z})$$

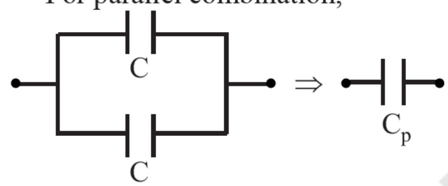
17. (c) For series combination,



$$\frac{1}{C_S} = \frac{1}{C} + \frac{1}{C}$$

$$\Rightarrow \boxed{C_S = \frac{C}{2}}$$

For parallel combination,



$C_p = C + C$

$\Rightarrow C_p = 2C$

$$\Rightarrow \frac{C_S}{C_p} = \frac{\left(\frac{C}{2}\right)}{2C} = \frac{1}{4} = 1:4$$

18. (b) Given, $i = \alpha_0 t + \beta t^2$
Put $\alpha_0 = 20$ and $\beta = 8$
We get $i = 20t + 8t^2$

Current, $i = \dfrac{dq}{dt}$

$$\Rightarrow \int dq = \int i dt$$

$$\Rightarrow q = \int_0^{15} (20t + 8t^2) dt$$

$$\Rightarrow q = \left(\frac{20t^2}{2} + \frac{8t^3}{3}\right)_0^{15}$$

$$\Rightarrow q = 20 \times \left(\frac{15^2 - 0^2}{2}\right) + \frac{8}{3}(15^3 - 0^3)$$

$$\Rightarrow q = 10 \times (15)^2 + \frac{8(15)^3}{3}$$

$\Rightarrow q = 2250 + 9000$

$\Rightarrow q = 11250$ C

19. (c) $I = \dfrac{6-4}{10} = \dfrac{1}{5}$ A

$$V_x + 4 + 8 \times \frac{1}{5} - V_y = 0$$

$V_x - V_y = -5.6$

$\Rightarrow |V_x - V_y| = 5.6$ V

20. (2000) Given : $L = 2 \times 10^{-4}$ H
$R = 6.28\ \Omega$
$\nu = 10$ MHz $= 10^7$ Hz
Since, quality factor,

$$Q = \omega_0 \frac{L}{R} = 2\pi\nu \frac{L}{R}$$

$$\therefore Q = 2\pi \times 10^7 \times \frac{2\times10^{-4}}{6.28}$$

$Q = 2 \times 10^3 = 2000$

21. **(440)** As we know,

$$\frac{N_P}{N_S} = \frac{V_P}{V_S}$$

Since, $N_S = 24$, $V_P = 220$ V and $V_S = 12$ V

$$\frac{N_P}{24} = \frac{220}{12}$$

$$N_P = \frac{220\times24}{12} = 440$$

22. **(15)** Given : Frequency of wave $\nu = 5$ GHz $= 5 \times 10^9$ Hz

Relative permittivity of the medium, $\varepsilon_r = 2$ and relative permeability of the medium, $\mu_r = 2$

Since speed of light in a medium is given by,

$$v = \frac{1}{\sqrt{\mu\varepsilon}} = \frac{1}{\sqrt{\mu_r\mu_0\varepsilon_r\varepsilon_0}} = \frac{c}{\sqrt{\mu_r\varepsilon_r}}$$

where c is speed of light is vacuum.

$$\therefore\ v = \frac{3\times10^8}{\sqrt{4}} = \frac{30\times10^7}{2}\text{m/s} = 15\times10^7 \text{ m/s}$$

23. **(a)** For convex mirror, focus is behind the mirror. So, its focal length (f) is positive.

$$\therefore\quad f = +\frac{r}{2}$$

24. **(c)** Let the amplitude of light wave coming from the narrower slit be A_1 and amplitude of light wave from the wider slit be A_2.

As amplitude $\propto$ Width of slit

$\therefore A_2 = 3A_1$

Maximum intensity occurs where constructive interference takes place and the minimum intensity where destructive interference takes place.

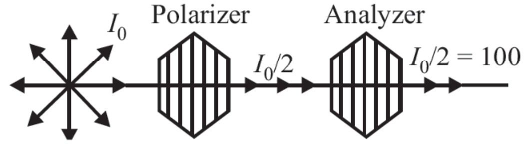
$\therefore A_{max} = 3A_1 + A_1 = 4A_1$

and $A_{min} = 3A_1 - A_1 = 2A_1$

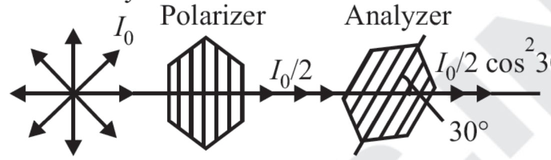
$\because$ Intensity $I \propto A^2$

$$\therefore\ \frac{I_{max}}{I_{min}} = \frac{A^2_{max}}{A^2_{min}} = \left(\frac{4A_1}{2A_1}\right)^2 = 4:1$$

25. **(75)** Assuming initially axis of polarizer and analyzer are parallel



When the angle between axis of polarizer and analyzer is 30°.



Now emerging intensity $= \frac{I_0}{2}\cos^2 30°$

$$= 100\left(\frac{\sqrt{3}}{2}\right)^2 = 100\times\frac{3}{4} = 75$$

26. **(d)** Momentum of photon,

$$p = \frac{h}{\lambda}$$

Here, h = Plank's constant,

λ = wavelength og light

Energy of photon $E = \frac{hc}{\lambda}$

If linear momentum are equal then wavelength also equal.

On decreasing wavelength, momentum and energy of photon increases.

27. **(c)** A corresponds to transition from $n = \infty$ to $n = 1$.

It is a series limit of Lymen series.

B corresponds to transition from $n = 5$ to $n = 2$.

It is a third member of Balmer series.

C corresponds to transition from $n = 5$ to $n = 3$, thus, it is second member of Paschen series.

28. **(a)** Given,

$\Delta I_e = 4$mA

$\Delta I_c = 3.5$ mA

For transistors,

$I_e = I_C + I_B$

$\therefore \Delta I_e = \Delta I_C + \Delta I_B$

$\Rightarrow 4mA = 3.5mA + \Delta I_B \Rightarrow \Delta I_B = 0.5mA$

Current gain for common emitter transistor,

$$\beta = \frac{\Delta I_C}{\Delta I_B} \Rightarrow \beta = \frac{3.5}{0.5}$$

$\Rightarrow \beta = 7$

29. (25) Current through 2 kΩ resistance is

$$I = \frac{5}{2\times10^{3}} = 2.5\times10^{-3}\,\text{A}$$

$$I = 25\times10^{-4}\,\text{A}$$

30. (25) % modulation $= \frac{A_m}{A_c}\times100$

$$= \frac{20}{80}\times100 = 25\%$$

CHEMISTRY

31. (2) $M = \frac{4.5/90}{250/1000} = 0.2 = 2\times10^{-1}.$

32. (2) $\lambda = \frac{h}{\sqrt{2mqV}}$

$$\frac{\lambda_{Li}}{\lambda_P} = \sqrt{\frac{m_P(e)V}{m_{Li}(3e)(V)}} \qquad m_{Li} = 8.3m_P$$

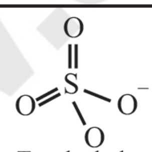
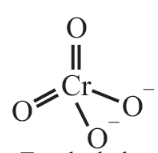
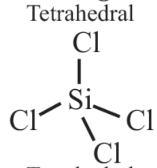
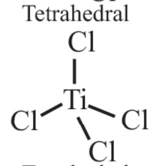
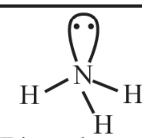
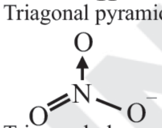
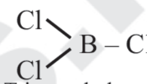
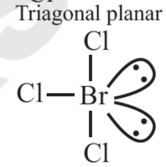
$$\frac{\lambda_{Li}}{\lambda_P} = \sqrt{\frac{1}{8.3\times3}} = \frac{1}{5} = 0.2 = 2\times10^{-1}$$

33. (b) Mg and P exhibit abnormal behaviour. Due to extra stability of half-filled and full-filled electronic configuration.

First ionisation enthalpy order is

Al < Mg < Si < S < P

34. (b) Isostructural means same structure.

Species	Structure
(A) SO_4^{2-}	 Tetrahedral
CrO_4^{2-}	 Tetrahedral
(B) $SiCl_4$	 Tetrahedral
$TiCl_4$	 Tetrahedral
(C) NH_3	 Triagonal pyramidal
NO_3^-	 Triagonal planar
(D) BCl_3	 Triagonal planar
$BrCl_3$	 T-shape

35. (5) $Cl_2(g) \rightleftharpoons 2Cl(g)$

Let mol of both of Cl_2 and Cl be x.

$$P_{Cl} = \frac{x}{2x}\times1 = \frac{1}{2}$$

$$P_{Cl_2} = \frac{x}{2x}\times1 = \frac{1}{2}$$

$$\therefore K_p = \frac{\left(\frac{1}{2}\right)^2}{\frac{1}{2}} = \frac{1}{2} = 0.5 = 5\times10^{-1}.$$

36. (1380) $\Delta G^\circ = RT\ln K_p$

$$= -R(300)(2)\ln(10)$$

$$= -R(300\times2\times2.3)$$

$$\Delta G^\circ = -1380R.$$

37. (12) $16e^- + S_8^- \longrightarrow 8S^{2-}$

$$12H_2O + S_8 \longrightarrow 4S_2O_3^{2-} + 24H^+ + 16e^-$$

$$2S_8 + 12H_2O \longrightarrow 8S^{2-} + 4S_2O_3^{2-} + 24H^+$$

For balancing in basic medium add OH^- equal to H^+.

$$2S_8 + 12H_2O + 24OH^- \longrightarrow 8S^{2-} + 4S_2O_8^{2-} + 24H_2O$$

$$2S_8 + 24OH^- \longrightarrow 8S^{2-} + 4S_2O_8^{2-} + 12H_2O$$

$$S_8 + 12OH^- \longrightarrow 4S^{2-} + 2S_2O_8^{2-} + 6H_2O$$

$\therefore a = 12.$

38. **(c)**

(A) $HOCl + H_3O_2 \rightarrow H_3O^+ + Cl^- + O_2$

In this equation, H_2O_2 is reducing chlorine from +1 to –1.

(B) $I_2 + H_2O_2 + 2OH^- \rightarrow 2I^- + 2H_2O + O_2$

In this equation, H_2O_2 is reducing iodine from 0 to –1.

Therefore, in both the reactions H_2O_2 acts as a reducing agent.

39. **(2)** BeO and $Be(OH)_2$ are amphoteric in nature while BaO and $Sr(OH)_2$ are basic in nature.

40. **(b)** $Al_2O_3(s) + 2NaOH(aq.) + 3H_2O(l)$

$$\longrightarrow \underset{(X)}{2Na[Al(OH)_4](aq)} \xrightarrow[(Y)]{CO_2}$$

$$Al_2O_3 \cdot xH_2O(s) + 2NaHCO_3(aq.)$$

41. **(b)**

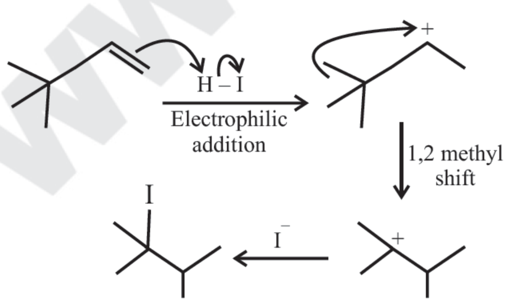
(I) Blue cupric metaborate $[Cu(BO_2)_2]$ is reduced to colourless cuprous metaborate $[Cu(BO_2)_2]$ in a luminous flame.

$$2Cu(BO_2)_2 + 2NaBO_2 + C \xrightarrow{\text{Luminous flame}}$$
$$2CuBO_2 + Na_2B_4O_7 + CO$$

(II) Cupric metaborate is obatined heating boric anhydride (B_2O_3) and copper sulphate in a non luminous flame.

$$B_2O_3 + CuSO_4 \xrightarrow{\text{Non-luminous flame}} \underset{\substack{\text{Cupric metaborate}\\ \text{(Blue-green)}}}{Cu(BO_2)_2} + SO_3$$

42. **(c)**



43. **(c)**

$$CH_3 - CH_2 - CH_3 \xrightarrow{MO_2O_3} CH_3 - CH_2 - CHO$$

44. **(d)** The gas, CH_4 evolved due to anaerobic degradation of vegetation which causes global warming and cancer.

45. **(8)**

46. **(35)** $ClCH_2COOH \rightleftharpoons ClCH_2COO^- + H^+$

Total dissociated = 1+ α

$\Rightarrow i = 1 + \alpha$

$\Delta T_f = ik_1 m$

$$\Rightarrow 0.5 = (1+\alpha)(1.86) \times \frac{9.45}{94.5} \times \frac{1000}{500}$$

$\Rightarrow 0.5 = (1+\alpha)(1.86)\ (0.2)$

$\Rightarrow 1 + \alpha = 1.34 \Rightarrow \alpha = 0.34$

$$ClCH_2COOH \rightleftharpoons ClCH_2COO^- + H^+$$

At $t = 0$	α	0	0
At time t	$C - C\alpha$	$C\alpha$	$C\alpha$

$$K_a = \frac{(C\alpha)^2}{C - C\alpha} = \frac{C\alpha^2}{1-\alpha}$$

$$\Rightarrow K_a = \frac{0.2 \times (0.34)^2}{1 - 0.34} = 0.035$$

$\therefore K_a = 35 \times 10^{-3}$.

47. **(26)** $k = \frac{1}{t} \cdot \ln \frac{[A]_0}{[A]_t}$

$$\Rightarrow t = \frac{1}{k} \ln\left(\frac{100}{60}\right) \Rightarrow t = \frac{1}{3.3 \times 10^{-4}}$$

$\Rightarrow t = 1548.49$ sec

$\Rightarrow t = 25.81$ min ≈ 26 min.

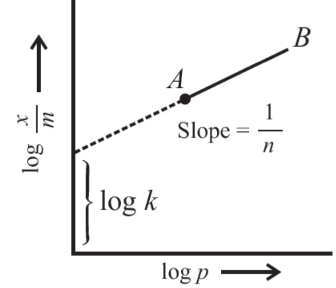
48. **(d)** Freundlich adsorption isotherm

$$\frac{x}{m} = kp^{1/n}$$

Taking logarithm

$$\log\left(\frac{x}{m}\right) = \log k + \frac{1}{n}\log p \quad \Rightarrow y = c + mx$$

$m = 1/n$ so slope will be equal to $1/n$.



$\frac{1}{n}$ can have value between 0 and 1.

49. **(a)** Sphalerite ore : ZnS
Calamine ore : $ZnCO_3$
Siderite ore : $FeCO_3$
Malachite ore : $Cu(OH)_2.CuCO_3$
In the froth flotation process (concentrated method), depressant helps to separates two sulphide ores by selective prevention of froth formation by one ore and allowing the other to come into froth.
For example, in case of an ore containing ZnS and PbS, NaCN is used as a depressant.
NaCN reacts with ZnS to form $Na_2[Zn(CN)_4]$, so it prevents ZnS from coming to the froth but allow PbS to come with the froth.

50. **(b)** The major components in "Gun Metal" are Cu : 87%; Zn : 3%; Sn : 10%.

51. **(d)** Only copper shows positive value for electrode potential of M^{2+}/M of 3*d*-series elements.

52. **(1)** $Cu^{2+} + NH_3 \xrightleftharpoons{K_1} [Cu(NH_3)]^{2+}$

$[Cu(NH_3)]^{2+} + NH_3 \xrightleftharpoons{K_2} [Cu(NH_3)_2]^{2+}$

$[Cu(NH_3)_2]^{2+} + NH_3 \xrightleftharpoons{K_3} [Cu(NH_3)_3]^{2+}$

$[Cu(NH_3)_3]^{2+} + NH_3 \xrightleftharpoons{K_4} [Cu(NH_3)_4]^{2+}$

$Cu^{2+} + 4NH_3 \xrightleftharpoons{K} [Cu(NH_3)_4]^{2+}$

So, $K = K_1 \times K_2 \times K_3 \times K_4$

$= 10^4 \times 1.58 \times 10^3 \times 5 \times 10^2 \times 10^2$

$\therefore K = 7.9 \times 10^{11}$.

Where K = Equilibrium constant for formation of $[Cu(NH_3)_4]^{2+}$

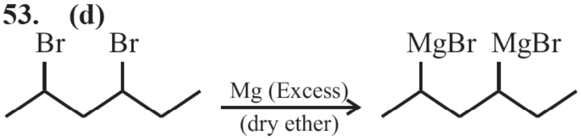
So, equilibrium constant (K') for dissociation of $[Cu(NH_3)_4]^{2+}$ is $\dfrac{1}{K}$

$$K' = \frac{1}{K}$$

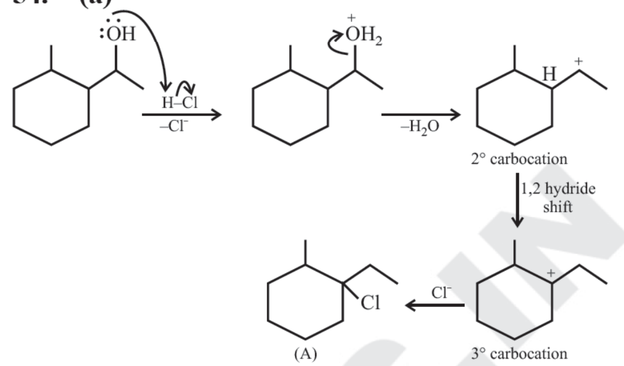
$$K' = \frac{1}{7.9 \times 10^{11}} = 1.26 \times 10^{-12} = (x \times 10^{-12})$$

So, $x = 1.26 \approx 1.0$.

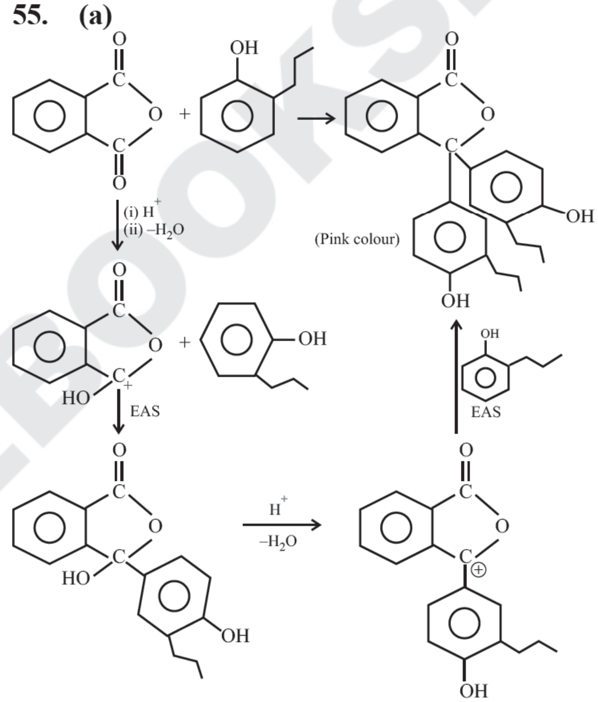
53. **(d)**



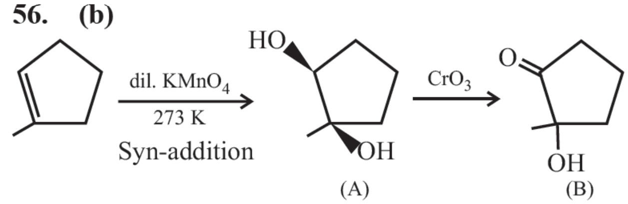
54. **(a)**



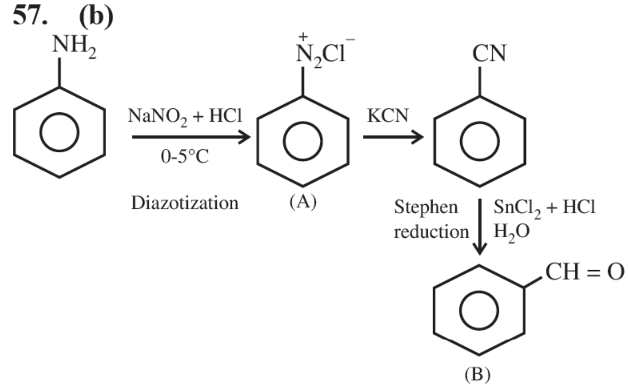
55. **(a)**



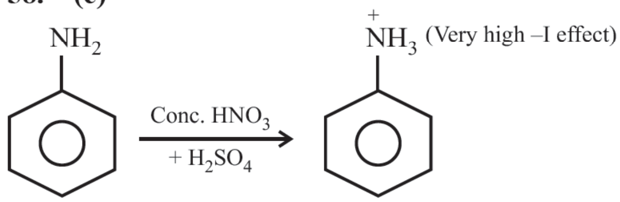
56. **(b)**



57. **(b)**



58. (c)



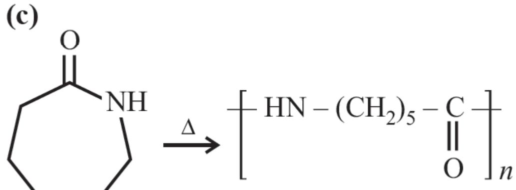
(Anilinium ion)

The positive anilinium nitrogen deactivates the benzene ring due to –I effect. It will behaves *m*-directing group as –I effect is more on -*o* and -*p* position and less on *m* position.
Therefore -*m* product is also formed along with -*o* and -*p* product.

59. (b) Hydrogen bonding is responsible for stabilisation of α-helix structure of proteins.

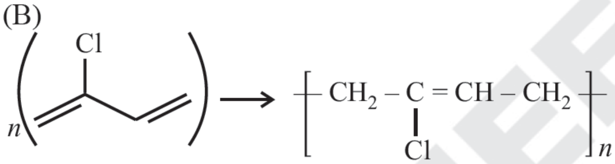
60. (c)

(A)



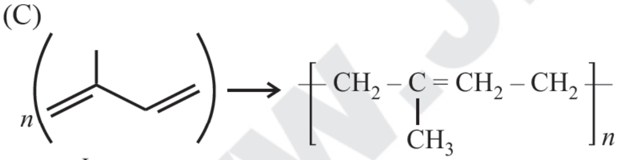
Nylon-6

(B)



2-chloro-1, 3-butadiene Neoprene (synthetic rubber)

(C)



Isoprene (2-Methylbuta-1, 3-diene) Polyisoprene (Natural rubber)

(D) $n\text{CH}_2 = \text{CH} - \text{CH} = \text{CH}_2 + \text{CH}_2 = \text{CH} - \text{CN}$
1,3-Butadiene Acrylonitrile

$$\xrightarrow[\text{Copolymerisation}]{\text{Peroxide}} \left[\text{CH}_2 - \text{CH} = \text{CH} - \text{CH}_2 - \text{CH}_2 - \underset{\text{CN}}{\underset{|}{\text{CH}}} \right]_n$$

Buna-N

MATHEMATICS

61. (d) We have $(p^2 + q^2)^2 - 2p^2q^2 = 272$

$((p + q)^2 - 2pq)^2 - 2p^2q^2 = 272$

$(4 - 2\,pq)^2 - 2p^2q^2 = 272$

$16 - 16pq + 2p^2q^2 = 272$

$(pq)^2 - 8pq - 128 = 0$

$pq = \dfrac{8 \pm 24}{2} = 16, -8$

$\therefore pq = 16 \quad (\because p, q > 0)$

$\therefore$ Required equation : $x^2 - (2)\,x + 16 = 0$

62. (10) Let $z = x + iy$

$x + iy + \alpha|x + iy - 1| + 2i = 0$

$\Rightarrow x + \alpha\sqrt{(x-1)^2 + y^2} + i(y+2) = 0 + 0i$

$\Rightarrow y + 2 = 0$ and $x + \alpha\sqrt{(x-1)^2 + y^2} = 0$

$\Rightarrow y = -2$ and $\alpha^2 = \dfrac{x^2}{x^2 - 2x + 5}$

Now, $\dfrac{x^2}{x^2 - 2x + 5} \in \left[0, \dfrac{5}{4}\right]$

$\therefore \alpha^2 \in \left[0, \dfrac{5}{4}\right] \Rightarrow \alpha \in \left[-\dfrac{\sqrt{5}}{2}, \dfrac{\sqrt{5}}{2}\right]$

$\therefore p = -\dfrac{\sqrt{5}}{2}; q = \dfrac{\sqrt{5}}{2}$

$\Rightarrow 4\,(p^2 + q^2) = 4\left(\dfrac{5}{4} + \dfrac{5}{4}\right) = 10$

63. (a)

Indians	Foreigners	Number of ways
2	4	${}^6C_2 \times {}^8C_4 = 1050$
3	6	${}^6C_3 \times {}^8C_6 = 560$
4	8	${}^6C_4 \times {}^8C_8 = 15$

Total number of ways = 1625

64. (b) $(-{}^{15}C_1 + 2.{}^{15}C_2 - 3.{}^{15}C_3 + ... - 15.{}^{15}C_{15}) + ({}^{14}C_1 + {}^{14}C_3 + ... + {}^{14}C_{11})$

$= \sum_{r=1}^{15} (-1)^r .r\,{}^{15}C_r + ({}^{14}C_1 + {}^{14}C_3 + ... + {}^{14}C_{11} + {}^{14}C_{13}) - {}^{14}C_3$

$= \sum_{r=1}^{15} (-1)^r 15.\,{}^{14}C_{r-1} + 2^{13} - 14$

$= 15\,(-{}^{14}C_0 + {}^{14}C_1 \,..... - {}^{14}C_{14}) + 2^{13} - 14$

$= 2^{13} - 14$

65. (d) $e^{(\cos^2 x + \cos^4 x + ...\infty)\ell n 2} = 2^{\cos^2 x + \cos^4 x + ...\infty} = 2^{\cot^2 x}$

Now $t^2 - 9t + 9 = 0 \Rightarrow t = 1, 8$

$\Rightarrow 2^{\cot^2 x} = 1, 8 \Rightarrow \cot^2 x = 0, 3$

$$\Rightarrow \frac{2\sin x}{\sin x+\sqrt{3}\cos x}=\frac{2}{1+\sqrt{3}\cot x}=\frac{2}{4}=\frac{1}{2}$$

66. **(5)** B and C will contain three digit numbers of the form $9k+2$ and $9k+\ell$ respectively. We need to find sum of all elements in the set $B \cup C$ effectively.

Now, $S(B\cup C)=S(B)+S(C)-S(B\cap C)$ where $S(k)$ denotes sum of elements of set k.

Also $B=\{101, 110, ...992\}$

$\therefore S(B)=\frac{100}{2}(101+992)=54650$

Case-I : If $\ell=2$

then $B\cap C=B$

$\therefore S(B\cup C)=S(B)$

which is not possible as given sum is $274\times 400=109600$.

Case-II : If $\ell\neq 2$

then $B\cap C=\phi$

$\therefore S(B\cup C)=S(B)+S(C)=400\times 274$

$$\Rightarrow 54650+\sum_{k=11}^{110} 9k+\ell=109600$$

$$\Rightarrow 9\sum_{k=11}^{110} k+\sum_{k=11}^{110}\ell=54950$$

$$\Rightarrow 9\left(\frac{100}{2}(11+110)\right)+\ell(100)=54950$$

$\Rightarrow 54450+100\ell=54950$

$\Rightarrow \ell=5$

67. **(d)** Let the line be $y=mx+c$

$\therefore$ x-intercept : $-\frac{c}{m}$

y-intercept : c

A.M. of reciprocals of the intercepts :

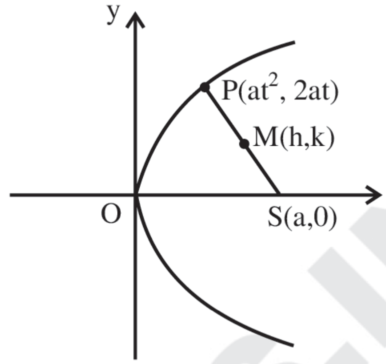
$$\frac{-\frac{m}{c}+\frac{1}{c}}{2}=\frac{1}{4}\Rightarrow 2(1-m)=c$$

Line : $y=mx+2(1-m)=c$

$\Rightarrow (y-2)-m(x-2)=0$

$\Rightarrow$ line always passes through (2, 2)

68. **(c)**



$$h=\frac{at^2+a}{2}, k=\frac{2at+0}{2}$$

$$\Rightarrow t^2=\frac{2h-a}{a} \text{ and } t=\frac{k}{a}$$

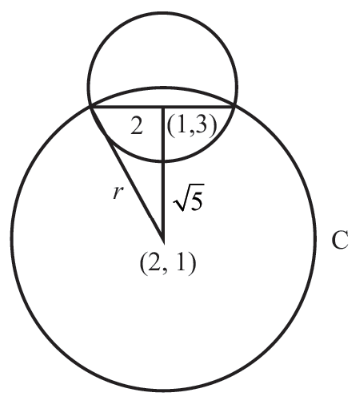
$$\Rightarrow \frac{k^2}{a^2}=\frac{2h-a}{a}$$

$\Rightarrow$ Locus of (h, k) is $y^2=a(2x-a)$

$$\Rightarrow y^2=2a\left(x-\frac{a}{2}\right)$$

Its directrix is $x-\frac{a}{2}=-\frac{a}{2}\Rightarrow x=0$

69. **(3)**



Given that $x^2+y^2-2x-6y+6=0$

center (1, 3) and radius = 2

Distance between (1, 3) and (2, 1) is $\sqrt{5}$

$\therefore (\sqrt{5})^2+(2)^2=r^2$

$\Rightarrow r=3$

70. **(1)** $\lim\limits_{n\to\infty}\tan\left(\sum_{r=1}^{n}\tan^{-1}\left(\frac{1}{1+r(r+1)}\right)\right)$

$$=\lim_{n\to\infty}\tan\left(\sum_{r=1}^{n}\tan^{-1}\left(\frac{r+1-r}{1+r(r+1)}\right)\right)$$

$= \tan\left(\lim_{n\to\infty}\sum_{r=1}^{n}\left[\tan^{-1}(r+1)-\tan^{-1}(r)\right]\right)$

$= \tan\left(\lim_{n\to\infty}\left(\tan^{-1}(n+1)-\frac{\pi}{4}\right)\right)$

$= \tan\left(\frac{\pi}{4}\right)=1$

71. **(d)** $(A \wedge (A \to B)) \to B$
$= (A \wedge (\sim A \vee B)) \to B$
$= ((A \wedge \sim A) \vee (A \wedge B)) \to B$
$= (A \wedge B) \to B$
$= \sim (A \wedge B) \vee B$
$= (\sim A \vee \sim B) \vee B$
$= T$

72. **(c)** $f(g(x)) = 2g(x) - 1 = 2\left(\frac{2x-1}{2(x-1)}\right)-1$

$= \frac{x}{x-1} = 1+\frac{1}{x-1}$

Range of $f(g(x)) = \mathbb{R} - \{1\}$
Range of $f(g(x))$ is not onto
& $f(g(x))$ is one-one
So, $f(g(x))$ is one-one but not onto.

73. **(540)** $\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}\begin{bmatrix} a & d & g \\ b & e & h \\ c & f & i \end{bmatrix}$

$a^2+b^2+c^2+d^2+e^2+f^2+g^2+h^2+i^2=7$

Case-I : Seven (1's) and two (0's)

$\frac{9!}{7!2!}=36$

Case-II: One (2) and three (1's) and five (0's)

$\frac{9!}{5!3!}=504$

$\therefore$ Total = 540

74. **(d)** $\Delta = \begin{vmatrix} 3 & -2 & -k \\ 2 & -4 & -2 \\ 1 & 2 & -1 \end{vmatrix} = 0$

$\Rightarrow 24 + 2(0) - k(8) = 0 \Rightarrow k = 3$

$\Delta_x = \begin{vmatrix} 10 & -2 & -3 \\ 6 & -4 & -2 \\ 5m & 2 & -1 \end{vmatrix}$

$= 10(8) - 2(-10m + 6) - 3(12 + 20m)$
$= 8(4 - 5m)$

$\Delta_y = \begin{vmatrix} 3 & 10 & -3 \\ 2 & 6 & -2 \\ 1 & 5m & -1 \end{vmatrix}$

$= 3(-6 + 10m) - 10(0) - 3(10m - 6)$
$= 0$

$\Delta_z = \begin{vmatrix} 3 & -2 & 10 \\ 2 & -4 & 6 \\ 1 & 2 & 5m \end{vmatrix}$

$= 3(-20m - 12) - 2(6 - 10m) + 10(8)$
$= -40m + 32 = 8(4 - 5m)$

For inconsistent,

$k = 3$ & $m \neq \frac{4}{5}$

75. **(17)** Given that
$PQ = kI$
$|P|.|Q| = k^3$
$\Rightarrow |P| = 2k \;\; 0 \Rightarrow P$ is an invertible matrix
$\because PQ = kI$
$\therefore Q = kP^{-1}I$ $\quad [\because P^{-1}P = I]$

$\therefore Q = \frac{\text{adj.P}}{2}$

$\because q_{23} = -\frac{k}{8}$

$\therefore \frac{(3\alpha+4)}{2} = -\frac{k}{8} \Rightarrow k = 4$

$\therefore |P| = 2k \Rightarrow k = 10 + 6\alpha$ ...(i)
Put value of k in (i)... we get $\alpha = -1$
$\therefore \alpha^2 + k^2 = 1 + 16 = 17$.

76. **(c)** For $x = n, n \in Z$

LHL $= \lim_{x\to n^-} f(x) = \lim_{x\to n^-} [x-1]\cos\left(\frac{2x-1}{2}\right)\pi = 0$

RHL $= \lim_{x\to n^+} f(x) = \lim_{x\to n^+} [x-1]\cos\left(\frac{2x-1}{2}\right)\pi = 0$

$f(n) = 0$
$\Rightarrow$ LHL = RHL = $f(n)$
$\Rightarrow f(x)$ is continuous for every real x.

77. **(a)** Slope of tangent at $P(t, t^3) = \left.\frac{dy}{dx}\right]_{(t,t^3)}$

$= (3x^2)^{x=t} = 3t^2$
So, equation tangent at $P(t, t^3)$:
$y - t^3 = 3t^2(x - t)$
For point of intersection with $y = x^3$

$x^3 - t^3 = 3t^2x - 3t^3$

$\Rightarrow (x - t)(x^2 + xt + t^2) = 3t^2(x - t)$

For $x \neq t$

$x^2 + xt + t^2 = 3t^2$

$\Rightarrow x^2 + xt - 2t^2 = 0 \Rightarrow (x - t)(x + 2t) = 0$

So, for $Q : x = -2t$, $Q(-2t, -8t^3)$

ordinate of required point :

$$\frac{2t^3 - 8t^3}{2+1} = -2t^3$$

78. (a) Given that

$$f(x) = \frac{4x^3 - 3x^2}{6} - 2\sin x + (2x-1)\cos x$$

$f'(x) = (2x^2 - x) - 2\cos x + 2\cos x$

$- \sin x\,(2x - 1)$

$= (2x - 1)(x - \sin x)$

For $x > 0$, $x - \sin x > 0$

$x < 0$, $x - \sin x < 0$

For $x \in (-\infty, 0] \cup \left[\frac{1}{2}, \infty\right), f'(x) \geq 0$

For $x \in \left[0, \frac{1}{2}\right], f'(x) \leq 0$

$\Rightarrow f(x)$ increases in $\left[\frac{1}{2}, \infty\right)$.

79. (9) Let $f(x) = \dfrac{4}{\sin x} + \dfrac{1}{1 - \sin x}$

$\Rightarrow f'(x) = 0 \Rightarrow \sin x = 2/3$

$\therefore f(x)_{\min} = \dfrac{4}{2/3} + \dfrac{1}{1 - 2/3} = 9$

$f(x)_{\max.} \to \infty$

$f(x)$ is continuous function

$\therefore \alpha_{\min} = 9$

80. (c) $\displaystyle\int \frac{\cos x - \sin x}{\sqrt{8 - \sin 2x}}\,dx$

$$\int \frac{\cos x - \sin x}{\sqrt{9 - (\sin x + \cos x)^2}}\,dx$$

Let $\sin x + \cos x = t$

$$\int \frac{dt}{\sqrt{9 - t^2}} = \sin^{-1}\frac{t}{3} + c$$

$$= \sin^{-1}\left(\frac{\sin x + \cos x}{3}\right) + c$$

So $a = 1$, $b = 3$.

81. (a) $$\lim_{x \to 0^+} \frac{\int_0^{x^2} \sin\sqrt{t}\,dt}{x^3} = \lim_{x \to 0^+} \frac{(\sin x)2x}{3x^2}$$

$$= \lim_{x \to 0^+}\left(\frac{\sin x}{x}\right) \times \frac{2}{3} = \frac{2}{3}$$

82. (3) We have,

$$\int_{-a}^{0} (-2x + 2)\,dx + \int_0^2 (x + 2 - x)\,dx + \int_2^a (2x - 2)\,dx = 22$$

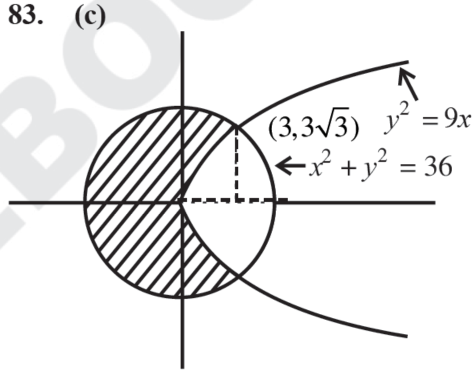
$\Rightarrow x^2 - 2x|_0^{-a} + 2x|_0^2 + x^2 - 2x|_2^a = 22$

$\Rightarrow a^2 + 2a + 4 + a^2 - 2a - (4 - 4) = 22$

$\Rightarrow 2a^2 = 18 \Rightarrow a = 3$

$$\therefore \int_3^{-3} (x + [x])dx = -(-3 - 2 - 1 + 1 + 2) = 3$$

83. (c)



Required area

$$= \pi \times (6)^2 - 2\int_0^3 \sqrt{9x}\,dx - 2\int_3^6 \sqrt{36 - x^2}\,dx$$

$$= 36\pi - 12\sqrt{3} - 2\left(\frac{x}{2}\sqrt{36 - x^2} + 18\sin^{-1}\frac{x}{6}\right)_3^6$$

$$= 36\pi - 12\sqrt{3} - 2\left(9\pi - 3\pi - \frac{9\sqrt{3}}{2}\right)$$

$$= 24\pi - 3\sqrt{3}$$

84. (d) Given that $\dfrac{dP}{dt} = 0.5P - 450$

$$\Rightarrow \int_0^t \frac{dp}{P - 900} = \int_0^t \frac{dt}{2}$$

$$\Rightarrow [\ell n\,|P(t) - 900|]_0^t = \left[\frac{t}{2}\right]_0^t$$

$$\Rightarrow \ell n\,|P(t) - 900| - \ell n\,|P(0) - 900| = \frac{t}{2}$$

$\Rightarrow \ell n\,|P(t)-900|-\ell n\,|50| = \dfrac{t}{2}$

For $P(t) = 0$

$\Rightarrow \ell n\left|\dfrac{900}{50}\right| = \dfrac{t}{2}$

$\Rightarrow t = 2\ell n18$

85. **(75)** Let $\vec{c} = \lambda(\vec{b}\times(\vec{a}\times\vec{b}))$

$= \lambda((\vec{b}.\vec{b})\vec{a}-(\vec{b}.\vec{a})\vec{b})$

$= \lambda(5(-\hat{i}+\hat{j}+\hat{k})+2\hat{i}+\hat{k})$

$= \lambda(-3\hat{i}+5\hat{j}+6\hat{k})$

$\vec{c}.\vec{a} = 7 \Rightarrow 3\lambda+5\lambda+6\lambda = 7$

$\lambda = \dfrac{1}{2}$

$\therefore 2\left|\left(\dfrac{-3}{2}-1+2\right)\hat{i}+\left(\dfrac{5}{2}+1\right)\hat{j}+(3+1+1)\hat{k}\right|^2$

$= 2\left(\dfrac{1}{4}+\dfrac{49}{4}+25\right) = 25+50 = 75$

86. **(c)** Normal vector :

$\begin{vmatrix} \hat{i} & j & \hat{k} \\ 3 & 1 & -2 \\ 2 & -5 & -1 \end{vmatrix} = -11\hat{i}-\hat{j}-17\hat{k}$

So, direction ratios of normal to the required plane are $< 11, 1, 17 >$

Plane passes through $(1, 2, -3)$

So, equation of plane :

$11(x-1) + 1\,(y-2) + 17\,(z+3) = 0$

$\Rightarrow 11x + y + 17z + 38 = 0$

87. **(d)** Let $\dfrac{x-3}{1} = \dfrac{y-4}{2} = \dfrac{z-5}{2} = t$

$\Rightarrow x = 3 + t,\ y = 2t + 4,\ z = 2t + 5$

For point of intersection with $x + y + z = 17$

$3 + t + 2t + 4 + 2t + 5 = 17$

$\Rightarrow 5t = 5 \Rightarrow t = 1$

$\Rightarrow$ Point of intersection is $(4, 6, 7)$

Distance between $(1, 1, 9)$ and $(4, 6, 7)$ is

$\sqrt{(4-1)^2+(6-1)^2+(7-9)^2}$

$= \sqrt{9+25+4} = \sqrt{38}.$

88. **(d)** ${}^nC_2\left(\dfrac{1}{2}\right)^n = {}^nC_3\left(\dfrac{1}{2}\right)^n \Rightarrow {}^nC_2 = {}^nC_3$

$\Rightarrow n = 5$

Probability of getting an odd number for odd number of times is

${}^5C_1\left(\dfrac{1}{2}\right)^5 + {}^5C_2\left(\dfrac{1}{2}\right)^5 + {}^5C_3\left(\dfrac{1}{2}\right)^5$

$= \dfrac{1}{2^5}(5+10+1) = \dfrac{1}{2}$

89. **(6)** Let $P(B_1) = p_1$, $P(B_2) = p_2$, $P(B_3) = p_3$

given that $p_1\,(1-p_2)\,(1-p_3) = \alpha$ ... (i)

$p_2\,(1-p_1)\,(1-p_3) = \beta$ ... (ii)

$p_3\,(1-p_1)\,(1-p_2) = \gamma$ ... (iii)

and $(1-p_1)\,(1-p_2)\,(1-p_3) = p$ ... (iv)

$\Rightarrow \dfrac{p_1}{1-p_1} = \dfrac{\alpha}{p}, \dfrac{p_2}{1-p_2} = \dfrac{\beta}{p}\ \&\ \dfrac{p_3}{1-p_3} = \dfrac{\gamma}{p}$

Also $\beta = \dfrac{\alpha p}{\alpha+2p} = \dfrac{3\gamma p}{p-2\gamma}$

$\Rightarrow \alpha p - 2\alpha\gamma = 3\alpha\gamma + 6p\gamma$

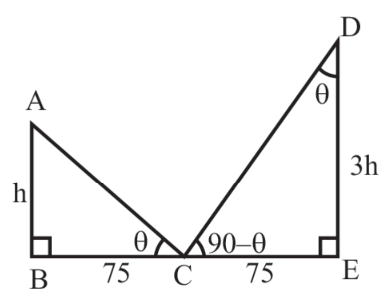
$\Rightarrow \alpha p - 6p\gamma - 5\alpha\gamma$

$\Rightarrow \dfrac{p_1}{1-p_1} - \dfrac{6p_3}{1-p_3} = \dfrac{5p_1p_3}{(1-p_1)(1-p_3)}$

$\Rightarrow p_1 - 6p_3 = 0$

$\Rightarrow \dfrac{p_1}{p_3} = 6$

90. **(b)**



In right angle ABC

and right ΔCDE

$\tan\theta = \dfrac{h}{75}$ and $\cot(90-\theta)$

$= \tan\theta = \dfrac{75}{3h}$

$\Rightarrow h^2 = \dfrac{(75)^2}{3}$

$h = 25\sqrt{3}\text{m}$

1 Physical World, Units and Measurements

1. Identify the pair whose dimensions are equal **[2002]**
(a) torque and work (b) stress and energy
(c) force and stress (d) force and work

2. Dimensions of $\frac{1}{\mu_0 \varepsilon_0}$, where symbols have their usual meaning, are **[2003]**
(a) $[L^{-1}T]$ (b) $[L^{-2}T^2]$
(c) $[L^2T^{-2}]$ (d) $[LT^{-1}]$

3. The physical quantities not having same dimensions are **[2003]**
(a) torque and work
(b) momentum and planck's constant
(c) stress and young's modulus
(d) speed and $(\mu_o \varepsilon_o)^{-1/2}$

4. Which one of the following represents the correct dimensions of the coefficient of viscosity? **[2004]**
(a) $[ML^{-1}T^{-1}]$ (b) $[MLT^{-1}]$
(c) $[ML^{-1}T^{-2}]$ (d) $[ML^{-2}T^{-2}]$

5. Out of the following pair, which one does NOT have identical dimensions? **[2005]**
(a) Impulse and momentum
(b) Angular momentum and planck's constant
(c) Work and torque
(d) Moment of inertia and moment of a force

6. The dimensions of magnetic field in M, L, T and C (coulomb) is given as **[2008]**
(a) $[MLT^{-1} C^{-1}]$ (b) $[MT^2 C^{-2}]$
(c) $[MT^{-1} C^{-1}]$ (d) $[MT^{-2} C^{-1}]$

7. A body of mass m = 3.513 kg is moving along the x-axis with a speed of 5.00 ms^{-1}. The magnitude of its momentum is recorded as **[2008]**
(a) 17.6 kg ms^{-1} (b) 17.565 kg ms^{-1}
(c) 17.56 kg ms^{-1} (d) 17.57 kg ms^{-1}

8. Two full turns of the circular scale of a screw gauge cover a distance of 1mm on its main scale. The total number of divisions on the circular scale is 50. Further, it is found that the screw gauge has a zero error of – 0.03 mm. While measuring the diameter of a thin wire, a student notes the main scale reading of 3 mm and the number of circular scale divisions in line with the main scale as 35. The diameter of the wire is **[2008]**
(a) 3.32 mm (b) 3.73 mm
(c) 3.67 mm (d) 3.38 mm

9. In an experiment the angles are required to be measured using an instrument, 29 divisions of the main scale exactly coincide with the 30 divisions of the vernier scale. If the smallest division of the main scale is half- a degree (= 0.5°), then the least count of the instrument is: **[2009]**
(a) half minute (b) one degree
(c) half degree (d) one minute

10. The respective number of significant figures for the numbers 23.023, 0.0003 and 2.1×10^{-3} are **[2010]**
(a) 5, 1, 2 (b) 5, 1, 5
(c) 5, 5, 2 (d) 4, 4, 2

11. A screw gauge gives the following reading when used to measure the diameter of a wire.
Main scale reading : 0 mm
Circular scale reading : 52 divisions
Given that 1mm on main scale corresponds to 100 divisions of the circular scale. The diameter of wire from the above data is **[2011]**
(a) 0.052 cm (b) 0.026 cm
(c) 0.005 cm (d) 0.52 cm

12. Resistance of a given wire is obtained by measuring the current flowing in it and the voltage difference applied across it. If the percentage errors in the measurement of the current and the voltage difference are 3% each, then error in the value of resistance of the wire is **[2012]**
(a) 6% (b) zero (c) 1% (d) 3%

13. A spectrometer gives the following reading when used to measure the angle of a prism.
Main scale reading : 58.5 degree
Vernier scale reading : 09 divisions
Given that 1 division on main scale corresponds to 0.5 degree. Total divisions on the Vernier scale is 30 and match with 29 divisions of the main scale. The angle of the prism from the above data is **[2012]**
(a) 58.59 degree (b) 58.77 degree
(c) 58.65 degree (d) 59 degree

14. Let $[\epsilon_0]$ denote the dimensional formula of the permittivity of vacuum. If M = mass, L = length, T = time and A = electric current, then: **[2013]**
(a) $\epsilon_0 = [M^{-1} L^{-3} T^2 A]$
(b) $\epsilon_0 = [M^{-1} L^{-3} T^4 A^2]$
(c) $\epsilon_0 = [M^1 L^2 T^1 A^2]$
(d) $\epsilon_0 = [M^1 L^2 T^1 A]$

15. The current voltage relation of a diode is given by $I = (e^{1000V/T} - 1)$mA, where the applied voltage V is in volts and the temperature T is in degree kelvin. If a student makes an error measuring ±0.01V while measuring the current of 5 mA at 300 K, what will be the error in the value of current in mA? **[2014]**
(a) 0.2 mA (b) 0.02 mA
(c) 0.5 mA (d) 0.05 mA

16. A student measured the length of a rod and wrote it as 3.50 cm. Which instrument did he use to measure it? **[2014]**
(a) A meter scale.
(b) A vernier calliper where the 10 divisions in vernier scale matches with 9 division in main scale and main scale has 10 divisions in 1 cm.
(c) A screw gauge having 100 divisions in the circular scale and pitch as 1 mm.
(d) A screw gauge having 50 divisions in the circular scale and pitch as 1 mm.

17. The period of oscillation of a simple pendulum is $T = 2\pi\sqrt{\frac{L}{g}}$. Measured value of L is 20.0 cm known to 1 mm accuracy and time for 100 oscillations of the pendulum is found to be 90 s using a wrist watch of 1s resolution. The accuracy in the determination of g is : **[2015]**
(a) 1% (b) 5% (c) 2% (d) 3%

18. A student measures the time period of 100 oscillations of a simple pendulum four times. The data set is 90 s, 91 s, 95 s, and 92 s. If the minimum division in the measuring clock is 1 s, then the reported mean time should be:
(a) 92 ± 1.8 s (b) 92 ± 3s **[2016]**
(c) 92 ± 1.5 s (d) 92 ± 5.0 s

19. A screw gauge with a pitch of 0.5 mm and a circular scale with 50 divisions is used to measure the thickness of a thin sheet of Aluminium. Before starting the measurement, it is found that wen the two jaws of the screw gauge are brought in contact, the 45th division coincides with the main scale line and the zero of the main scale is barely visible. What is the thickness of the sheet if the main scale reading is 0.5 mm and the 25th division coincides with the main scale line? **[2016]**
(a) 0.70 mm (b) 0.50 mm
(c) 0.75 mm (d) 0.80 mm

20. The following observations were taken for determining surface tensiton T of water by capillary method :
Diameter of capilary, $D = 1.25 \times 10^{-2}$ m
rise of water, $h = 1.45 \times 10^{-2}$ m
Using $g = 9.80$ m/s^2 and the simplified relation
$T = \frac{rhg}{2} \times 10^3$ N/m, the possible error in surface tension is closest to : **[2017]**
(a) 2.4% (b) 10% (c) 0.15% (d) 1.5%

21. The density of a material in the shape of a cube is determined by measuring three sides of the cube and its mass. If the relative errors in measuring the mass and length are respectively 1.5% and 1%, the maximum error in determining the density is: **[2018]**
(a) 2.5% (b) 3.5% (c) 4.5% (d) 6%

22. In the density measurement of a cube, the mass and edge length are measured as (10.00 ± 0.10) kg and (0.10 ± 0.01) m, respectively. The error in the measurement of density is: **[2019]**
(a) 0.01 kg/m^3 (b) 0.10 kg/m^3
(c) 0.31 kg/m^3 (d) 0.07 kg/m^3

23. The dimensions of $\frac{B^2}{2\mu_0}$, where B is magnetic field and μ_0 is the magnetic permeability of vacuum, is: **[2020]**
(a) MLT^{-2} (b) ML^2T^{-1}
(c) ML^2T^{-2} (d) $ML^{-1}T^{-2}$

Answer Key

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
(a)	(c)	(b)	(a)	(d)	(c)	(a)	(d)	(d)	(a)	(a)	(a)	(c)	(b)	(a)
16	17	18	19	20	21	22	23							
(b)	(d)	(c)	(d)	(d)	(c)	(c)	(d)							

Solutions

1. (a) Work $W = \vec{F}\cdot\vec{s} = Fs\cos\theta$

$\because\ \vec{A}\cdot\vec{B} = AB\cos\theta$

$= [MLT^{-2}][L] = [ML^2T^{-2}]$;

Torque, $\vec{\tau} = \vec{r}\times\vec{F} \Rightarrow \tau = rF\sin\theta$

$\because\ \vec{A}\times\vec{B} = AB\sin\theta$

$= [L]\,[MLT^{-2}] = [ML^2T^{-2}]$

2. (c) As we know, the velocity of light in free space is given by

$$C = \frac{1}{\sqrt{\mu_o\varepsilon_o}} = 3\times10^8[\text{m/s}]$$

$$\frac{1}{\mu_o\varepsilon_o} = C^2[\text{m/s}]^2$$

$= [LT^{-1}]^2$
$= [M^0L^2T^{-2}]$

3. (b) Momentum, $P = mv = [MLT^{-1}]$
Planck's constant,

$$h = \frac{E}{v}$$

$$= \frac{[ML^2T^{-2}]}{[T^{-1}]}$$

$= [ML^2T^{-1}]$

4. (a) According to, Stokes law,

$$F = 6\pi\eta rv \Rightarrow \eta = \frac{F}{6\pi r v}$$

$$\eta = \frac{[MLT^{-2}]}{[L][LT^{-1}]} \Rightarrow \eta = [ML^{-1}T^{-1}]$$

5. (d) Moment of Inertia, $I = MR^2$
$[I] = [ML^2]$

Moment of force, $\vec{\tau} = \vec{r}\times\vec{F}$

$\vec{\tau} = [L][MLT^{-2}] = [ML^2T^{-2}]$

6. (c) Magnitude of Lorentz formula $F = qvB\sin\theta$

$$B = \frac{F}{qv} = \frac{MLT^{-2}}{C\times LT^{-1}} = [MT^{-1}C^{-1}]$$

7. (a) Momentum, $p = m\times v$
Given, mass of a body = 3.513 kg speed of body
$= (3.513)\times(5.00) = 17.565$ kg m/s
$= 17.6$ (Rounding off)

8. (d) Least count of screw gauge = 0.01 mm

$\because\ \frac{0.5}{50}$ mm

Reading = [M.S.R. + C.S.R. × L.C.] – (zero error)
$= [3 + 35\times0.01] - (-0.03) = 3.38$ mm

9. (d) 30 Divisions of V.S. coincide with 29 divisions of M.S.

$\therefore$ 1 V.S.D $= \frac{29}{30}$ MSD

L.C. = 1 MSD – 1VSD

$= 1\text{ MSD} - \frac{29}{30}\text{ MSD}$

$= \frac{1}{30}\text{ MSD}$

$= \frac{1}{30}\times0.5° = 1$ minute.

10. (a) Number of significant figures in 23.023 = 5
Number of significant figures in 0.0003 = 1
Number of significant figures in $2.1\times10^{-3} = 2$
So, the radiation belongs to X-rays part of the spectrum.

11. (a) Least count, L.C. $= \frac{1}{100}$ mm

Diameter of wire $= MSR + CSR\times L.C.$
$\because$ 1 mm = 0.1 cm

$= 0 + \frac{1}{100}\times52 = 0.52$ mm $= 0.052$ cm

12. **(a)** According to ohm's law, V = IR

$R = \frac{V}{I}$

$\therefore$ Percentage error = $\frac{\text{Absolute error}}{\text{Measurement}} \times 10^2$

where, $\frac{\Delta V}{V} \times 100 = \frac{\Delta I}{I} \times 100 = 3\%$

then, $\frac{\Delta R}{R} \times 100 = \frac{\Delta V}{V} \times 10^2 + \frac{\Delta I}{I} \times 10^2$

$= 3\% + 3\% = 6\%$

13. **(c)** $\because$ Reading of Vernier = M.S.R. + V.S.R. × L.C.

M.S.R. = 58.5

V.S.R. = 09 division

L.C. of Vernier = 0.5°/30

Thus, $R = 58.5° + 9 \times \frac{0.5°}{30}$

$R = 58.65°$

14. **(b)** As we know, $F = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{R^2}$

$\Rightarrow \varepsilon_0 = \frac{q_1 q_2}{4\pi F R^2}$

Hence, $\varepsilon_0 = \frac{C^2}{N.m^2} = \frac{[AT]^2}{[MLT^{-2}][L^2]}$

$= [M^{-1} L^{-3} T^4 A^2]$

15. **(a)** The current voltage relation of diode is

$I = (e^{1000\, V/T} - 1)$ mA (given)

When, $I = 5mA, e^{1000\, V/T} = 6mA$

Also, $dI = (e^{1000\, V/T}) \times \frac{1000}{T}$

Error = ± 0.01 (By exponential function)

$= (6\, mA) \times \frac{1000}{300} \times (0.01) = 0.2$ mA

16. **(b)** Measured length of rod = 3.50 cm

For Vernier Scale with 1 Main Scale Division = 1 mm

9 Main Scale Division = 10 Vernier Scale Division,

Least count = 1 MSD – 1 VSD = 0.1 mm

17. **(d)** As, $g = 4\pi^2 \frac{L}{T^2}$

So, $\frac{\Delta g}{g} \times 100 = \frac{\Delta L}{L} \times 100 + 2\frac{\Delta T}{T} \times 100$

$= \frac{0.1}{20} \times 100 + 2 \times \frac{1}{90} \times 100 = 2.72 \simeq 3\%$

18. **(c)** $\Delta T = \frac{|\Delta T_1| + |\Delta T_2| + |\Delta T_3| + |\Delta T_4|}{4}$

$= \frac{2+1+3+0}{4} = 1.5$

As the resolution of measuring clock is 1.5 therefore the mean time should be 92 ± 1.5

19. **(d)** L.C. $= \frac{0.5}{50} = 0.01$ mm

Zero error = 5 × 0.01 = 0.05 mm (Negative)

Reading = (0.5 + 25 × 0.01) + 0.05 = 0.80 mm

20. **(d)** Surface tension, $T = \frac{rhg}{2} \times 10^3$

Relative error in surface tension,

$\frac{\Delta T}{T} = \frac{\Delta r}{r} + \frac{\Delta h}{h} + 0$

Percentage error

$100 \times \frac{\Delta T}{T} = \left(\frac{10^{-2} \times 0.01}{1.25 \times 10^{-2}} + \frac{10^{-2} \times 0.01}{1.45 \times 10^{-2}}\right)100$

= (0.8 + 0.689)

= (1.489) = 1.489% ≅ 1.5%

21. **(c)** Density (d) $= \frac{\text{Mass (M)}}{\text{Volume (V)}} = \frac{M}{L^3}$

$\therefore$ Error in density, $\frac{\Delta d}{d} = \frac{\Delta M}{M} + \frac{3\Delta L}{L}$

$= 1.5\% + 3(1\%) = 4.5\%$

22. **(c)** $d = \frac{M}{V} = \frac{M}{L^3} = ML^{-3}$

$\frac{\Delta d}{d} = \frac{\Delta M}{M} + 3\frac{\Delta L}{L}$

$= \frac{0.10}{10.00} + 3\left(\frac{0.01}{0.10}\right) = 0.31 \text{kgm}^{-3}$

23. **(d)** The quantity $\frac{B^2}{2\mu_0}$ is the energy density of magnetic field.

$\Rightarrow \left[\frac{B^2}{2\mu_0}\right] = \frac{Energy}{Volume} = \frac{\text{Force} \times \text{displacement}}{(\text{displacement})^3}$

$= \left[\frac{ML^2T^{-2}}{L^3}\right] = ML^{-1}T^{-2}$

2 Motion in a Straight Line

1. If a body looses half of its velocity on penetrating 3 cm in a wooden block, then how much will it penetrate more before coming to rest? **[2002]**
(a) 1 cm (b) 2 cm (c) 3 cm (d) 4 cm.

2. Speeds of two identical cars are u and $4u$ at the specific instant. The ratio of the respective distances in which the two cars are stopped from that instant is **[2002]**
(a) 1 : 1 (b) 1 : 4 (c) 1 : 8 (d) 1 : 16

3. From a building two balls A and B are thrown such that A is thrown upwards and B downwards (both vertically). If v_A and v_B are their respective velocities on reaching the ground, then **[2002]**
(a) $v_B > v_A$ (b) $v_A = v_B$
(c) $v_A > v_B$
(d) their velocities depend on their masses.

4. A car, moving with a speed of 50 km/hr, can be stopped by brakes after at least 6 m. If the same car is moving at a speed of 100 km/hr, the minimum stopping distance is **[2003]**
(a) 12m (b) 18m (c) 24m (d) 6m

5. A ball is released from the top of a tower of height h meters. It takes T seconds to reach the ground. What is the position of the ball at $\frac{T}{3}$ second **[2004]**
(a) $\frac{8h}{9}$ meters from the ground
(b) $\frac{7h}{9}$ meters from the ground
(c) $\frac{h}{9}$ meters from the ground
(d) $\frac{17h}{18}$ meters from the ground

6. An automobile travelling with a speed of 60 km/h, can brake to stop within a distance of 20m. If the car is going twice as fast i.e., 120 km/h, the stopping distance will be **[2004]**
(a) 60 m (b) 40 m (c) 20 m (d) 80 m

7. A car, starting from rest, accelerates at the rate f through a distance S, then continues at constant speed for time t and then decelerates at the rate $\frac{f}{2}$ to come to rest. If the total distance traversed is 15 S , then **[2005]**
(a) $S = \frac{1}{6} ft^2$ (b) $S = f\,t$
(c) $S = \frac{1}{4} ft^2$ (d) $S = \frac{1}{72} ft^2$

8. A particle is moving eastwards with a velocity of 5 ms^{-1}. In 10 seconds the velocity changes to 5 ms^{-1} northwards. The average acceleration in this time is **[2005]**
(a) $\frac{1}{2} ms^{-2}$ towards north
(b) $\frac{1}{\sqrt{2}} ms^{-2}$ towards north-east
(c) $\frac{1}{\sqrt{2}} ms^{-2}$ towards north-west
(d) zero

9. The relation between time t and distance x is $t = ax^2 + bx$ where a and b are constants. The acceleration is **[2005]**
(a) $-2bv^3$ (b) $-2abv^2$
(c) $2av^2$ (d) $-2av^3$

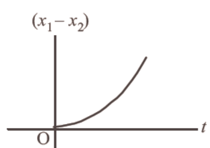
10. A particle located at $x = 0$ at time $t = 0$, starts moving along with the positive x-direction with a velocity 'v' that varies as $v = \alpha\sqrt{x}$. The displacement of the particle varies with time as **[2006]**

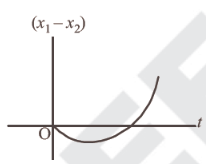
(a) t^2 (b) t (c) $t^{1/2}$ (d) t^3

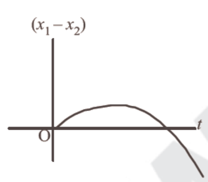
11. The velocity of a particle is $v = v_0 + gt + ft^2$. If its position is $x = 0$ at $t = 0$, then its displacement after unit time ($t = 1$) is **[2007]**

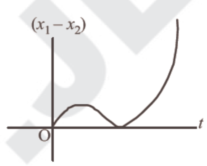
(a) $v_0 + g/2 + f$ (b) $v_0 + 2g + 3f$
(c) $v_0 + g/2 + f/3$ (d) $v_0 + g + f$

12. A body is at rest at $x = 0$. At $t = 0$, it starts moving in the positive x-direction with a constant acceleration. At the same instant another body passes through $x = 0$ moving in the positive x-direction with a constant speed. The position of the first body is given by $x_1(t)$ after time 't'; and that of the second body by $x_2(t)$ after the same time interval. Which of the following graphs correctly describes $(x_1 - x_2)$ as a function of time 't'? **[2008]**

(a) 

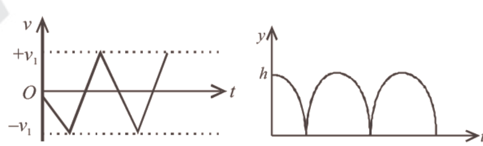
(b) 

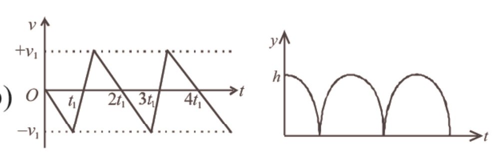
(c) 

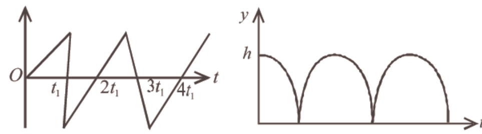
(d) 

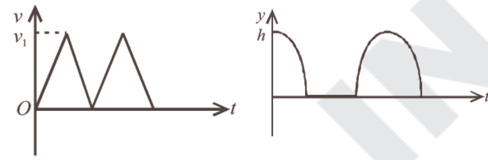
13. Consider a rubber ball freely falling from a height $h = 4.9$ m onto a horizontal elastic plate. Assume that the duration of collision is negligible and the collision with the plate is totally elastic.

Then the velocity as a function of time and the height as a function of time will be : **[2009]**

(a) 

(b) 

(c) 

(d) 

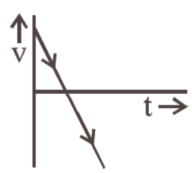
14. An object, moving with a speed of 6.25 m/s, is decelerated at a rate given by $\frac{dv}{dt} = -2.5\sqrt{v}$ where v is the instantaneous speed. The time taken by the object, to come to rest, would be: **[2011]**

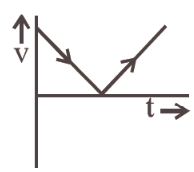
(a) 2 s (b) 4 s (c) 8 s (d) 1 s

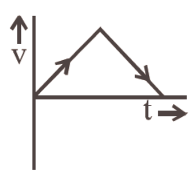
15. From a tower of height H, a particle is thrown vertically upwards with a speed u. The time taken by the particle, to hit the ground, is n times that taken by it to reach the highest point of its path. The relation between H, u and n is: **[2014]**

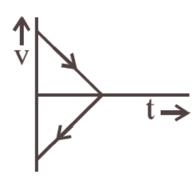
(a) $2gH = n^2u^2$ (b) $gH = (n-2)^2 u^2 d$
(c) $2gH = nu^2 (n-2)$ (d) $gH = (n-2)u^2$

16. A body is thrown vertically upwards. Which one of the following graphs correctly represent the velocity vs time? **[2017]**

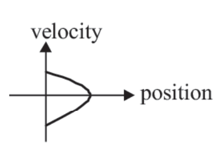
(a) 

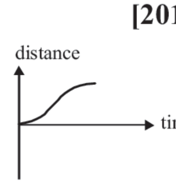
(b) 

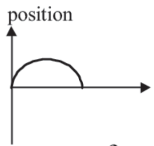
(c) 

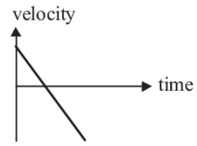
(d) 

17. All the graphs below are intended to represent the same motion. One of them does it incorrectly. Pick it up. **[2018]**

(a) 

(b) 

(c) 

(d) 

18. The stream of a river is flowing with a speed of 2 km/h. A swimmer can swim at a speed of 4 km/h. What should be the direction of the swimmer with respect to the flow of the river to cross the river straight? **[2019]**

(a) 90° (b) 150° (c) 120° (d) 60°

Answer Key

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
(a)	(d)	(b)	(c)	(a)	(d)	(d)	(c)	(d)	(a)	(c)	(b)	(b)	(a)	(c)
16	17	18												
(a)	(b)	(c)												

Solutions

1. **(a)** In first case

$u_1 = u\,;\, v_1 = \dfrac{u}{2}\,,\, s_1 = 3 \text{ cm},\, a_1 = ?$

Using, $v_1^2 - u_1^2 = 2a_1 s_1$ ...(i)

$\left(\dfrac{u}{2}\right)^2 - u^2 = 2 \times a \times 3$

$\Rightarrow\ a = \dfrac{-u^2}{8}$

In second case: Assuming the same retardation

$u_2 = u/2\,;\, v_2 = 0\,;\, s_2 = ?;\ a_2 = \dfrac{-u^2}{8}$

$v_2^2 - u_2^2 = 2a_2 \times s_2$ ...(ii)

$\therefore\ 0 - \dfrac{u^2}{4} = 2\left(\dfrac{-u^2}{8}\right) \times s_2$

$\Rightarrow\ s_2 = 1$ cm

2. **(d)** **For first car**

$u_1 = u,\, v_1 = 0,\, a_1 = -a,\, s_1 = s_1$

As $v_1^2 - u_1^2 = 2a_1 s_1$

$\Rightarrow\ -u^2 = -2as_1$

$\Rightarrow\ u^2 = 2as_1$

$\Rightarrow\ s_1 = \dfrac{u^2}{2a}$ ...(i)

For second car

$u_2 = 4u,\, v_1 = 0,\, a_2 = -a,\, s_2 = s_2$

$\therefore\ v_2^2 - u_2^2 = 2a_2 s_2$

$\Rightarrow\ -(4u)^2 = 2(-a)s_2$

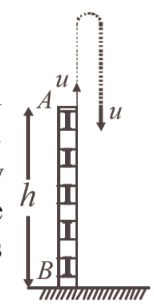
$\Rightarrow\ 16\,u^2 = 2as_2$

$\Rightarrow\ s_2 = \dfrac{8u^2}{a}$ ...(ii)

Dividing (i) and (ii),

$$\frac{s_1}{s_2} = \frac{u^2}{2a} \cdot \frac{a}{8u^2} = \frac{1}{16}$$

3. **(b)** Ball A is thrown upwards with velocity u from the building. During its downward journey when it comes back to the point of throw, its speed is equal to the speed of throw (u). So, for the journey of both the balls from point A to B.



We can apply $v^2 - u^2 = 2gh$.

As u, g, h are same for both the balls, $v_A = v_B$

4. **(c)** **Fir first case :** Initial velocity,

$u = 50 \times \dfrac{5}{18}\text{m/s},$

$v = 0, s = 6\text{m}, a = a$

Using, $v^2 - u^2 = 2as$

$\Rightarrow 0^2 - \left(50 \times \dfrac{5}{18}\right)^2 = 2 \times a \times 6$

$\Rightarrow -\left(50 \times \dfrac{5}{18}\right)^2 = 2 \times a \times 6$

$a = -\dfrac{250 \times 250}{324 \times 2 \times 6} \approx = -16 \text{ ms}^{-2}$.

Case-2 : Initial velocity, u = 100 km/hr

$= 100 \times \dfrac{5}{18}$ m/sec

$v = 0$, $s = s$, $a = a$

As $v^2 - u^2 = 2as$

$\Rightarrow 0^2 - \left(100 \times \dfrac{5}{18}\right)^2 = 2as$

$\Rightarrow -\left(100 \times \dfrac{5}{18}\right)^2 = 2 \times (-16) \times 5$

$s = \dfrac{500 \times 500}{324 \times 32} = 24\text{m}$

5. **(a)** We have $s = ut + \dfrac{1}{2}gt^2$,

$\Rightarrow \quad h = 0 \times T + \dfrac{1}{2}gT^2$

$\Rightarrow \quad h = \dfrac{1}{2}gT^2$

Vertical distance moved in time $\dfrac{T}{3}$ is

$h' = \dfrac{1}{2}g\left(\dfrac{T}{3}\right)^2 \Rightarrow h' = \dfrac{1}{2} \times \dfrac{gT^2}{9} = \dfrac{h}{9}$

$\therefore$ Position of ball from ground $= h - \dfrac{h}{9}$

$= \dfrac{8h}{9}$

6. **(d)** In first case speed,

$u = 60 \times \dfrac{5}{18}\text{m/s} = \dfrac{50}{3}\text{m/s}$

$d = 20$m,

Let retardation be a then

$(0)^2 - u^2 = -2ad$

or $\quad u^2 = 2ad \qquad \ldots$(i)

In second case speed, $u' = 120 \times \dfrac{5}{18}$

$= \dfrac{100}{3}$m/s

and $(0)^2 - u'^2 = -2ad'$

or $\quad u'^2 = 2ad' \qquad \ldots$(ii)

(ii) divided by (i) gives,

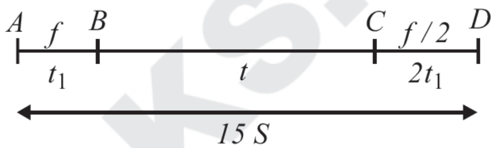
$4 = \dfrac{d'}{d} \Rightarrow d' = 4 \times 20 = 80\text{m}$

7. **(d)** Let car starts from A from rest and moves up to point B with acceleration f.

Distance, $AB = S = \dfrac{1}{2}ft_1^2$

Distance, $BC = (ft_1)t$

Distance, $CD = \dfrac{u^2}{2a} = \dfrac{(ft_1)^2}{2(f/2)} = ft_1^2 = 2S$



Total distance, $AD = AB + BC + CD = 15S$

$AD = S + BC + 2S$

$\Rightarrow \quad S + f\,t_1 t + 2S = 15\,S$

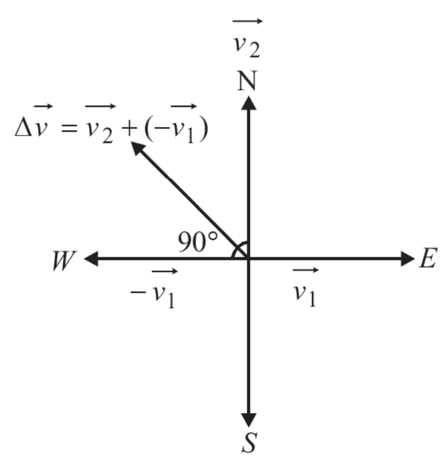
$\Rightarrow \quad f\,t_1 t = 12\,S \qquad$ .... (i)

$\dfrac{1}{2}f\,t_1^2 = S \qquad$ .... (ii)

Dividing (i) by (ii), we get $t_1 = \dfrac{t}{6}$

$\Rightarrow \quad S = \dfrac{1}{2}f\left(\dfrac{t}{6}\right)^2 = \dfrac{f\,t^2}{72}$

8. **(c)**



Initial velocity, $\vec{v}_1 = 5\hat{i}$,

Final velocity, $\vec{v}_2 = 5\hat{j}$,

Change in velocity $\Delta\vec{v} = (\vec{v}_2 - \vec{v}_1)$

$= \sqrt{v_1^2 + v_2^2 + 2v_1v_2\cos 90}$

$= \sqrt{5^2 + 5^2 + 0} = 5\sqrt{2}\text{m/s}$

[As $|v_1| = |v_2| = 5$ m/s]

Avg. acceleration $= \dfrac{\Delta \vec{v}}{t}$

$= \dfrac{5\sqrt{2}}{10} = \dfrac{1}{\sqrt{2}}\,\text{m/s}^2$

$\tan\theta = \dfrac{5}{-5} = -1$

which means θ is in the second quadrant. (towards north-west)

9. (d) Given, $t = ax^2 + bx$;

Diff. with respect to time (t)

$$\frac{d}{dt}(t) = a\frac{d}{dt}(x^2) + b\frac{dx}{dt} = a.2x\frac{dx}{dt} + \text{b.v.}$$

$\Rightarrow 1 = 2axv + bv = v(2ax + b)$ (v = velocity)

$2ax + b = \dfrac{1}{v}$.

Again differentiating, we get

$$2a\frac{dx}{dt} + 0 = -\frac{1}{v^2}\frac{dv}{dt}$$

$$\Rightarrow a = \frac{dv}{dt} = -2av^3 \left(\because \frac{dx}{dt} = v\right)$$

10. (a) $v = \alpha\sqrt{x}$,

$$\Rightarrow \frac{dx}{dt} = \alpha\sqrt{x}$$

$$\Rightarrow \frac{dx}{\sqrt{x}} = \alpha\, dt$$

Integrating both sides,

$$\int_0^x \frac{dx}{\sqrt{x}} = \alpha\int_0^t dt\,;\ \left[\frac{2\sqrt{x}}{1}\right]_0^x = \alpha[t]_0^t$$

$$\Rightarrow 2\sqrt{x} = \alpha t \ \Rightarrow x = \frac{\alpha^2}{4}t^2$$

11. (c) We know that, $v = \dfrac{dx}{dt}$

$\Rightarrow dx = v\, dt$

Integrating, $\displaystyle\int_0^x dx = \int_0^t v\, dt$

or $\displaystyle x = \int_0^t (v_0 + gt + ft^2)\, dt$

$$= \left[v_0 t + \frac{gt^2}{2} + \frac{ft^3}{3}\right]_0^t$$

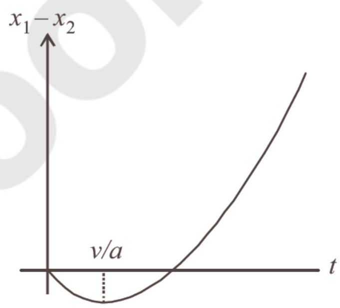
or, $x = v_0 t + \dfrac{gt^2}{2} + \dfrac{ft^3}{3}$

At $t = 1$, $x = v_0 + \dfrac{g}{2} + \dfrac{f}{3}$.

12. (b) **For the body starting from rest, distance travelled (x_1) is given by**

$x_1 = 0 + \dfrac{1}{2}at^2$

$\Rightarrow x_1 = \dfrac{1}{2}at^2$



For the body moving with constant speed

$x_2 = vt$

$\therefore\ x_1 - x_2 = \dfrac{1}{2}at^2 - vt$

at $t = 0, x_1 - x_2 = 0$

This equation is of parabola.

For $t < \dfrac{v}{a}$; the slope is negative

For $t = \dfrac{v}{a}$; the slope is zero

For $t > \dfrac{v}{a}$; the slope is positive

These characteristics are represented by graph (b).

13. (b) For downward motion $v = -gt$

The velocity of the rubber ball increases in downward direction and we get a straight line between v and t with a negative slope.

Also applying $y - y_0 = ut + \dfrac{1}{2}at^2$

We get $y - h = -\dfrac{1}{2}gt^2 \Rightarrow y = h - \dfrac{1}{2}gt^2$

The graph between y and t is a parabola with $y = h$ at $t = 0$. As time increases y decreases.

For upward motion.

The ball suffer elastic collision with the horizontal elastic plate therefore the direction of velocity is reversed and the magnitude remains the same.

Here $v = u - gt$ where u is the velocity just after collision.

As t increases, v decreases. We get a straight line between v and t with negative slope.

Also $y = ut - \frac{1}{2}gt^2$

All these characteristics are represented by graph (b).

14. (a) Given, $\frac{dv}{dt} = -2.5\sqrt{v}$

$\Rightarrow \frac{dv}{\sqrt{v}} = -2.5\ dt$

Integrating,

$\int_{6.25}^{0} v^{-1/2} dv = -2.5\int_0^t dt$

$\Rightarrow \left[\frac{v^{+1/2}}{(1/2)}\right]_{6.25}^{0} = -2.5[t]_0^t$

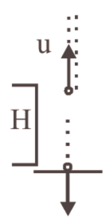
$\Rightarrow -2(6.25)^{1/2} = -2.5t$

$\Rightarrow -2 \times 2.5$

$= -2.5t$

$\Rightarrow t = 25$

15. (c) Speed on reaching ground



$v = \sqrt{u^2 + 2gh}$

Now, $v = u + at$

$\Rightarrow \sqrt{u^2 + 2gh} = -u + gt$

Time taken to reach highest point is

$t = \frac{u}{g}$,

$\Rightarrow t = \frac{u + \sqrt{u^2 + 2gH}}{g} = \frac{nu}{g}$

(from question)

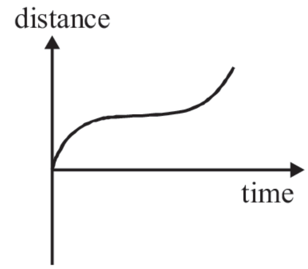
$\Rightarrow \quad 2gH = n(n-2)u^2$

16. (a) For a body thrown vertically upwards acceleration remains constant ($a = -g$) and velocity at anytime t is given by $V = u - gt$

During rise velocity decreases linearly and during fall velocity increases linearly and direction is opposite to each other.

Hence graph (a) correctly depicts velocity versus time.

17. (b) Graphs in option (c) position-time and option (a) velocity-position are corresponding to velocity-time graph option (d) and its distance-time graph is as given below. Hence distance-time graph option (b) is incorrect.

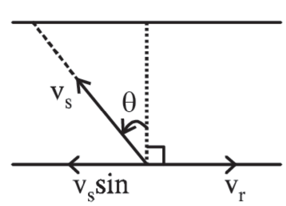


18. (c) To cross the river straight

$V_s \sin\theta = V_r \quad \therefore \sin\theta = \frac{v_r}{v_s} = \frac{2}{4} = \frac{1}{2}$

$\therefore \theta = 30°$

Direction of swimmer with respect to flow $= 90° + 30° = 120°$



3 Motion in a Plane

1. A boy playing on the roof of a 10 m high building throws a ball with a speed of 10m/s at an angle of 30º with the horizontal. How far from the throwing point will the ball be at the height of 10 m from the ground? **[2003]**

$[g = 10\text{m/s}^2, \sin 30^o = \frac{1}{2}, \cos 30^o = \frac{\sqrt{3}}{2}]$

(a) 5.20m (b) 4.33m
(c) 2.60m (d) 8.66m

2. The co-ordinates of a moving particle at any time 't' are given by $x = \alpha t^3$ and $y = \beta t^3$. The speed of the particle at time 't' is given by **[2003]**

(a) $3t\sqrt{\alpha^2+\beta^2}$ (b) $3t^2\sqrt{\alpha^2+\beta^2}$
(c) $t^2\sqrt{\alpha^2+\beta^2}$ (d) $\sqrt{\alpha^2+\beta^2}$

3. A projectile can have the same range 'R' for two angles of projection. If 'T_1' and 'T_2' to be time of flights in the two cases, then the product of the two time of flights is directly proportional to. **[2004]**

(a) R (b) $\frac{1}{R}$ (c) $\frac{1}{R^2}$ (d) R^2

4. Which of the following statements is **FALSE** for a particle moving in a circle with a constant angular speed ? **[2004]**

(a) The acceleration vector points to the centre of the circle
(b) The acceleration vector is tangent to the circle
(c) The velocity vector is tangent to the circle
(d) The velocity and acceleration vectors are perpendicular to each other.

5. A ball is thrown from a point with a speed 'v_0' at an elevation angle of θ. From the same point and at the same instant, a person starts running with a constant speed $\frac{'v_0'}{2}$ to catch the ball. Will the person be able to catch the ball? If yes, what should be the angle of projection θ? **[2004]**

(a) No (b) Yes, 30°
(c) Yes, 60° (d) Yes, 45°

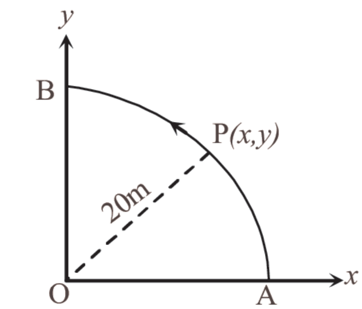
6. A particle has an initial velocity of $3\hat{i}+4\hat{j}$ and an acceleration of $0.4\hat{i}+0.3\hat{j}$. Its speed after 10 s is : **[2009]**

(a) $7\sqrt{2}$ units (b) 7 units
(c) 8.5 units (d) 10 units

7. A particle is moving with velocity $\vec{v} = k(y\hat{i}+x\hat{j})$, where k is a constant. The general equation for its path is **[2010]**

(a) $y = x^2$ + constant
(b) $y^2 = x$ + constant
(c) xy = constant
(d) $y^2 = x^2$ + constant

8. A point P moves in counter-clockwise direction on a circular path as shown in the figure. The movement of 'P' is such that it sweeps out a length $s = t^3 + 5$, where s is in metres and t is in seconds. The radius of the path is 20 m. The acceleration of 'P' when $t = 2$ s is nearly. **[2010]**



(a) $13 m/s^2$ (b) $12 m/s^2$
(c) $7.2 ms^2$ (d) $14 m/s^2$

9. For a particle in uniform circular motion, the acceleration $\vec{a}$ at a point P(R,θ) on the circle of radius R is (Here θ is measured from the x-axis) **[2010]**

(a) $-\frac{v^2}{R}\cos\theta\ \hat{i}+\frac{v^2}{R}\sin\theta\ \hat{j}$

(b) $-\frac{v^2}{R}\sin\theta\ \hat{i}+\frac{v^2}{R}\cos\theta\ \hat{j}$

(c) $-\frac{v^2}{R}\cos\theta\ \hat{i}-\frac{v^2}{R}\sin\theta\ \hat{j}$

(d) $\frac{v^2}{R}\hat{i}+\frac{v^2}{R}\hat{j}$

10. A water fountain on the ground sprinkles water all around it. If the speed of water coming out of the fountain is v, the total area around the fountain that gets wet is : **[2011]**

(a) $\pi\frac{v^4}{g^2}$ (b) $\frac{\pi}{2}\frac{v^4}{g^2}$

(c) $\pi\frac{v^2}{g^2}$ (d) $\pi\frac{v^2}{g}$

11. A boy can throw a stone up to a maximum height of 10 m. The maximum horizontal distance that the boy can throw the same stone up to will be **[2012]**

(a) $20\sqrt{2}$ m (b) 10 m

(c) $10\sqrt{2}$ m (d) 20 m

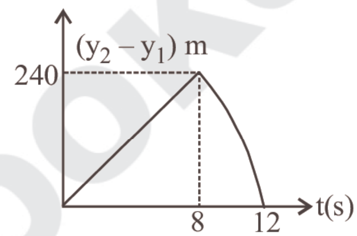
12. A projectile is given an initial velocity of $(\hat{i}+2\hat{j})$ m/s, where $\hat{i}$ is along the ground and $\hat{j}$ is along the vertical. If g = 10 m/s^2, the equation of its trajectory is : **[2013]**

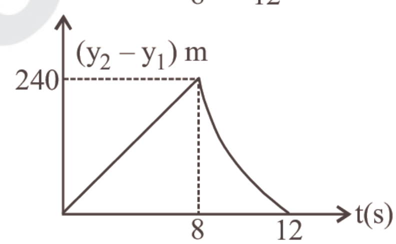
(a) $y = x - 5x^2$ (b) $y = 2x - 5x^2$
(c) $4y = 2x - 5x^2$ (d) $4y = 2x - 25x^2$

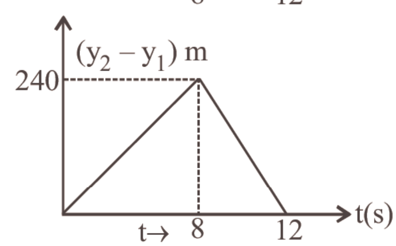
13. Two stones are thrown up simultaneously from the edge of a cliff 240 m high with initial speed of 10 m/s and 40 m/s respectively. Which of the following graph best represents the time variation of relative position of the second stone with respect to the first? **[2015]**

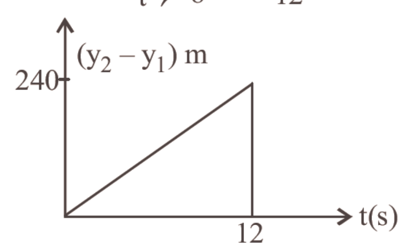
(Assume stones do not rebound after hitting the ground and neglect air resistance, take $g = 10 m/s^2$)

(*The figures are schematic and not drawn to scale*)

(a)


(b)


(c)


(d)


14. A particle is moving with a velocity $\vec{v} = K(y\hat{i} + x\hat{j})$, where K is a constant. The general equation for its path is:

(a) $y = x^2$ + constant (b) $y^2 = x$ + constant
(c) $y^2 = x^2$ + constant (d) xy = constant

[2019]

Answer Key

1	2	3	4	5	6	7	8	9	10	11	12	13
(d)	(b)	(a)	(b)	(c)	(a)	(a)	(d)	(c)	(a)	(d)	(b)	(b)
14												
(c)												

Solutions

1. (d) Horizontal range is required

$$R = \frac{u^2 \sin 2\theta}{g} = \frac{(10)^2 \sin(2 \times 30°)}{10} = 5\sqrt{3}$$

$= 8.66$ m

2. (b) Coordinates of moving particle at time 't' are

$x = \alpha t^3$ and $y = \beta t^3$

$$v_x = \frac{dx}{dt} = 3\alpha t^2 \text{ and } v_y = \frac{dy}{dt} = 3\beta t^2$$

$$\therefore\ v = \sqrt{v_x^2 + v_y^2} = \sqrt{9\alpha^2 t^4 + 9\beta^2 t^4}$$

$$= 3t^2\sqrt{\alpha^2 + \beta^2}$$

3. (a) A projectile have same range for two angle Let one angle be θ, then other is $90° - \theta$

$$T_1 = \frac{2u\sin\theta}{g}, T_2 = \frac{2u\cos\theta}{g}$$

$$\text{then, } T_1T_2 = \frac{4u^2\sin\theta\cos\theta}{g} = 2R$$

$$(\because\ R = \frac{u^2\sin^2\theta}{g})$$

Thus, it is proportional to R. (Range)

4. (b) Only option (b) is false since acceleration vector is always radial for uniform circular motion.

5. (c) Yes, Man will catch the ball, if the horizontal component of velocity becomes equal to the constant speed of man.

$$\frac{v_o}{2} = v_o\cos\theta$$

or $\theta = 60°$

6. (a) Given $\vec{u} = 3\hat{i} + 4\hat{j}$, $\vec{a} = 0.4\hat{i} + 0.3\hat{j}$, t=10s

From 1st equatoin of motion.

$$a = \frac{v-u}{t}$$

$\therefore\ v = at\ tu$

$$\Rightarrow v = (0.4\hat{i} + 0.3\hat{j}) \times 10 + (3\hat{i} + 4\hat{j})$$

$$\Rightarrow\ 4\hat{i} + 3\hat{j} + 3\hat{j} + 4\hat{j}$$

$$\Rightarrow\ v = 7\hat{i} + 7\hat{j}$$

$$\Rightarrow\ |\vec{v}| = \sqrt{7^2 + 7^2} = 7\sqrt{2} \text{ unit.}$$

7. (a) $v = k(yi + xj)$

$v = kyi + kxj$

$$\frac{dx}{dt} = ky, \frac{dy}{dt} = kx$$

$$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$$

$$\frac{dy}{dx} = \frac{kx}{ky}$$

$ydy = xdx$ ...(i)

Integrating equation (i)

$$\int ydy = \int x \cdot dx$$

$y^2 = x^2 + c$

8. (d) $s = t^3 + 5$, $r = 20$m, $t = 5$ sec

$$\text{velocity, } v = \frac{ds}{dt} = 3t^2$$

$$\text{Linear acceleration } a_t = \frac{dv}{dt} = 6t$$

$$\text{Centripetal acceleration } a_c = \frac{v^2}{R} = \frac{9t^4}{R}$$

At $t = 2s$, $at = 6 \times 2 = 12$ m/s^2

$$a_c = \frac{9 \times 16}{20} = 7.2 \text{ m/s}^2$$

Resultant acceleration

$$= \sqrt{a_t^2 + a_c^2}$$

$$= \sqrt{(12)^2 + (7.2)^2} = \sqrt{144 + 51.84}$$

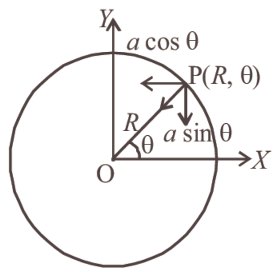
$$= \sqrt{195.84} = 14 \text{ m/s}^2$$

9. **(c)** Acceleration,

$$a = \frac{v^2}{R} \text{ towards the centre}$$

$$\vec{a} = -a_c \cos\theta(\hat{i}) \ -a_c \sin\theta(\hat{j})$$

$$= -\frac{v^2}{R}\cos\theta\,\hat{i} - \frac{v^2}{R}\sin\theta\,\hat{j}$$



10. **(a)** Let, total area around fountain

$$A = \pi R_{max}^2 \qquad ...(i)$$

Where $R_{max} = \dfrac{v^2 \sin 2\theta}{g} = \dfrac{v^2 \sin 90°}{g} = \dfrac{v^2}{g}$ ...(ii)

From equation (i) and (ii)

$$A = \pi\frac{v^4}{g^2}$$

11. **(d)** $R = \dfrac{u^2 \sin^2\theta}{g}$, $H = \dfrac{u^2 \sin^2\theta}{2g}$

H_{max} at $2\theta = 90°$

$$H_{max} = \frac{u^2}{2g}$$

$$\frac{u^2}{2g} = 10 \Rightarrow u^2 = 10g \times 2$$

$$R = \frac{u^2 \sin 2\theta}{g} \Rightarrow R_{max} = \frac{u^2}{g}$$

$$R_{max} = \frac{10 \times g \times 2}{g} = 20 \text{ metre}$$

12. **(b)** From equation, $\vec{v} = \hat{i} + 2\hat{j}$

$\Rightarrow x = t$ …(i)

$y = 2t - \dfrac{1}{2}(10t^2)$ …(ii)

From (i) and (ii), $y = 2x - 5x^2$

13. **(b)** $y_1 = 10t - 5t^2$; $y_2 = 40t - 5t^2$

for $y_1 = -240$m, $t = 8s$

$\therefore$ $y_2 - y_1 = 30t$ for $t \le 8s$.

for $t > 8s$,

$$y_2 - y_1 = 240 - 40t - \frac{1}{2}gt^2$$

14. **(c)** From given equation,

$$\vec{v} = k\left(y\hat{i} + x\hat{j}\right) = ky\hat{i} + kx\hat{j} = V_x\hat{i} + V_y\hat{j}$$

$$\frac{dx}{dt} = ky \text{ and } \frac{dy}{dt} = kx$$

Now, $\dfrac{\frac{dy}{dt}}{\frac{dx}{dt}} = \dfrac{x}{y} = \dfrac{dy}{dx} \Rightarrow ydy = xdx$

Integrating both sides we get $y^2 = x^2 + c$

4 Laws of Motion

1. A lift is moving down with acceleration a. A man in the lift drops a ball inside the lift. The acceleration of the ball as observed by the man in the lift and a man standing stationary on the ground are respectively **[2002]**
   (a) g, g (b) $g - a, g - a$
   (c) $g - a, g$ (d) a, g

2. When forces F_1, F_2, F_3 are acting on a particle of mass m such that F_2 and F_3 are mutually perpendicular, then the particle remains stationary. If the force F_1 is now removed then the acceleration of the particle is **[2002]**
   (a) F_1/m (b) F_2F_3/mF_1
   (c) $(F_2 - F_3)/m$ (d) F_2/m.

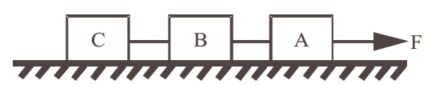
3. The minimum velocity (in ms^{-1}) with which a car driver must traverse a flat curve of radius 150 m and coefficient of friction 0.6 to avoid skidding is **[2002]**
   (a) 60 (b) 30 (c) 15 (d) 25

4. A solid sphere, a hollow sphere and a ring are released from top of an inclined plane (frictionless) so that they slide down the plane. Then maximum acceleration down the plane is for (no rolling) **[2002]**
   (a) solid sphere (b) hollow sphere
   (c) ring (d) all same.

5. Two forces are such that the sum of their magnitudes is 18 N and their resultant is 12 N which is perpendicular to the smaller force. Then the magnitudes of the forces are **[2002]**
   (a) 12 N, 6 N (b) 13 N, 5 N
   (c) 10 N, 8 N (d) 16N, 2N.

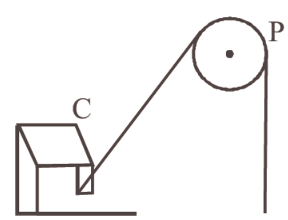
6. A light string passing over a smooth light pulley connects two blocks of masses m_1 and m_2 (vertically). If the acceleration of the system is $g/8$, then the ratio of the masses is **[2002]**
   (a) 8 : 1 (b) 9 : 7 (c) 4 : 3 (d) 5 : 3.

7. Three identical blocks of masses $m = 2$ kg are drawn by a force $F = 10.2$ N with an acceleration of 0. 6 ms^{-2} on a frictionless surface, then what is the tension (in N) in the string between the blocks B and C? **[2002]**



   (a) 9.2 (b) 3.4 (c) 4 (d) 9.8

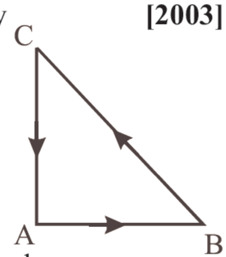
8. One end of a massless rope, which passes over a massless and frictionless pulley P is tied to a hook C while the other end is free. Maximum tension that the rope can bear is 360 N. With what value of maximum safe acceleration (in ms^{-2}) can a man of 60 kg climb on the rope? **[2002]**



   (a) 16 (b) 6 (c) 4 (d) 8

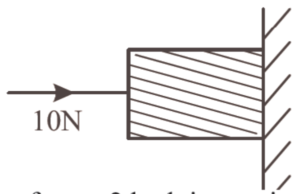
9. A spring balance is attached to the ceiling of a lift. A man hangs his bag on the spring and the spring reads 49 N, when the lift is stationary. If the lift moves downward with an acceleration of $5 m/s^2$, the reading of the spring balance will be **[2003]**
   (a) 24 N (b) 74 N (c) 15 N (d) 49 N

10. Three forces start acting simultaneously on a particle moving with velocity, $\vec{v}$. These forces are represented in magnitude and direction by the three sides of a triangle ABC. The particle will now move with velocity **[2003]**



(a) less than $\vec{v}$
(b) greater than $\vec{v}$
(c) $|\vec{v}|$ in the direction of the largest force BC
(d) $\vec{v}$, remaining unchanged

11. A horizontal force of 10 N is necessary to just hold a block stationary against a wall. The coefficient of friction between the block and the wall is 0.2. The weight of the block is **[2003]**



(a) 20 N
(b) 50 N
(c) 100 N
(d) 2 N

12. A marble block of mass 2 kg lying on ice when given a velocity of 6 m/s is stopped by friction in 10 s. Then the coefficient of friction is **[2003]**
(a) 0.02 (b) 0.03 (c) 0.04 (d) 0.06

13. A block of mass M is pulled along a horizontal frictionless surface by a rope of mass m. If a force P is applied at the free end of the rope, the force exerted by the rope on the block is **[2003]**

(a) $\frac{Pm}{M+m}$ (b) $\frac{Pm}{M-m}$

(c) P (d) $\frac{PM}{M+m}$

14. A light spring balance hangs from the hook of the other light spring balance and a block of mass M kg hangs from the former one. Then the true statement about the scale reading is **[2003]**
(a) both the scales read M kg each
(b) the scale of the lower one reads M kg and of the upper one zero
(c) the reading of the two scales can be anything but the sum of the reading will be M kg
(d) both the scales read $M/2$ kg each

15. A rocket with a lift-off mass 3.5×10^4 kg is blasted upwards with an initial acceleration of $10 m/s^2$. Then the initial thrust of the blast is
(a) 3.5×10^5 N (b) 7.0×10^5 N **[2003]**
(c) 14.0×10^5 N (d) 1.75×10^5 N

16. Two masses m_1 = 5g and m_2 = 4.8 kg tied to a string are hanging over a light frictionless pulley. What is the acceleration of the masses when left free to move? ($g = 9.8 m/s^2$) **[2004]**

(a) $5 m/s^2$
(b) $9.8 m/s^2$
(c) $0.2 m/s^2$
(d) $4.8 m/s^2$

17. A block rests on a rough inclined plane making an angle of 30° with the horizontal. The coefficient of static friction between the block and the plane is 0.8. If the frictional force on the block is 10 N, the mass of the block (in kg) is (take $g = 10 m/s^2$) **[2004]**
(a) 1.6 (b) 4.0 (c) 2.0 (d) 2.5

18. A smooth block is released at rest on a 45° incline and then slides a distance 'd'. The time taken to slide is 'n' times as much to slide on rough incline than on a smooth incline. The coefficient of friction is **[2005]**

(a) $\mu_k = \sqrt{1-\frac{1}{n^2}}$ (b) $\mu_k = 1-\frac{1}{n^2}$

(c) $\mu_s = \sqrt{1-\frac{1}{n^2}}$ (d) $\mu_s = 1-\frac{1}{n^2}$

19. A parachutist after bailing out falls 50 m without friction. When parachute opens, it decelerates at $2 m/s^2$. He reaches the ground with a speed of 3 m/s. At what height, did he bail out? **[2005]**
(a) 182 m (b) 91 m (c) 111 m (d) 293 m

20. An annular ring with inner and outer radii R_1 and R_2 is rolling without slipping with a uniform angular speed. The ratio of the forces experienced by the two particles situated on the inner and outer parts of the ring, $\frac{F_1}{F_2}$ is **[2005]**

(a) $\left(\frac{R_1}{R_2}\right)^2$ (b) $\frac{R_2}{R_1}$

(c) $\frac{R_1}{R_2}$ (d) 1

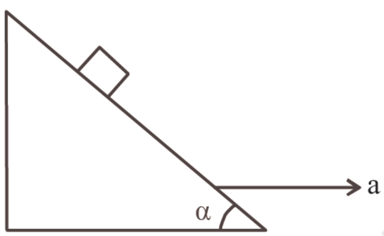
21. The upper half of an inclined plane with inclination ϕ is perfectly smooth while the lower half is rough. A body starting from rest at the top will again come to rest at the bottom if the coefficient of friction for the lower half is given by **[2005]**

(a) $2 \cos \phi$ (b) $2 \sin \phi$
(c) $\tan \phi$ (d) $2 \tan \phi$

22. A particle of mass 0.3 kg subject to a force $F = -kx$ with $k = 15$ N/m . What will be its initial acceleration if it is released from a point 20 cm away from the origin ? **[2005]**

(a) 15 m/s^2 (b) 3 m/s^2
(c) 10 m/s^2 (d) 5 m/s^2

23. A block is kept on a frictionless inclined surface with angle of inclination 'α'. The incline is given an acceleration 'a' to keep the block stationary. Then a is equal to **[2005]**



(a) g cosec α (b) $g / \tan \alpha$
(c) $g \tan \alpha$ (d) g

24. Consider a car moving on a straight road with a speed of 100 m/s . The distance at which car can be stopped is $[\mu_k = 0.5]$ **[2005]**

(a) 1000 m (b) 800 m
(c) 400 m (d) 100 m

25. A ball of mass 0.2 kg is thrown vertically upwards by applying a force by hand. If the hand moves 0.2 m while applying the force and the ball goes upto 2 m height further, find the magnitude of the force. (Consider $g = 10$ m/s^2). **[2006]**

(a) 4 N (b) 16 N
(c) 20 N (d) 22 N

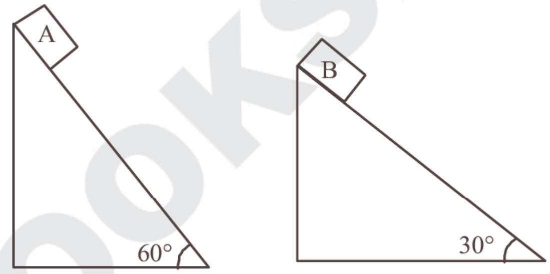
26. A player caught a cricket ball of mass 150 g moving at a rate of 20 m/s. If the catching process is completed in 0.1 s, the force of the blow exerted by the ball on the hand of the player is equal to **[2006]**

(a) 150 N (b) 3 N
(c) 30 N (d) 300 N

27. A block of mass m is connected to another block of mass M by a spring (massless) of spring constant k. The block are kept on a smooth horizontal plane. Initially the blocks are at rest and the spring is unstretched. Then a constant force F starts acting on the block of mass M to pull it. Find the force on the block of mass m. **[2007]**

(a) $\dfrac{MF}{(m+M)}$ (b) $\dfrac{mF}{M}$
(c) $\dfrac{(M+m)F}{m}$ (d) $\dfrac{mF}{(m+M)}$

28. Two fixed frictionless inclined planes making an angle 30° and 60° with the vertical are shown in the figure. Two blocks A and B are placed on the two planes. What is the relative vertical acceleration of A with respect to B? **[2010]**



(a) 4.9 ms^{-2} in horizontal direction
(b) 9.8 ms^{-2} in vertical direction
(c) Zero
(d) 4.9 ms^{-2} in vertical direction

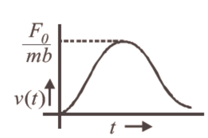
29. The minimum force required to start pushing a body up rough (frictional coefficient μ) inclined plane is F_1 while the minimum force needed to prevent it from sliding down is F_2. If the inclined plane makes an angle θ from the horizontal such that $\tan \theta = 2\mu$ then the ratio $\dfrac{F_1}{F_2}$ is **[2011 RS]**

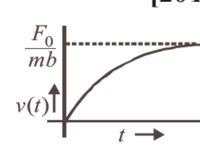
(a) 1 (b) 2 (c) 3 (d) 4

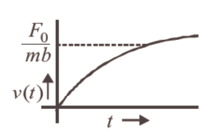
30. If a spring of stiffness 'k' is cut into parts 'A' and 'B' of length $\ell_A : \ell_B = 2 : 3$, then the stiffness of spring 'A' is given by **[2011 RS]**

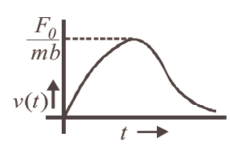
(a) $\dfrac{3k}{5}$ (b) $\dfrac{2k}{5}$ (c) k (d) $\dfrac{5k}{2}$

31. A particle of mass m is at rest at the origin at time $t = 0$. It is subjected to a force $F(t) = F_0 e^{-bt}$ in the x direction. Its speed $v(t)$ is depicted by which of the following curves? **[2012]**

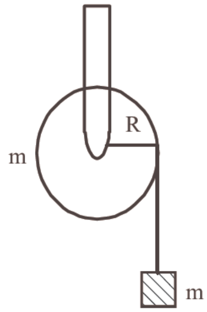
(a)


(b)


(c)


(d)


32. A mass 'm' is supported by a massless string wound around a uniform hollow cylinder of mass m and radius R. If the string does not slip on the cylinder, with what acceleration will the mass fall or release? **[2014]**



(a) $\frac{2g}{3}$

(b) $\frac{g}{2}$

(c) $\frac{5g}{6}$

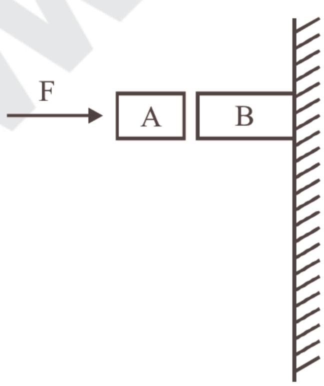
(d) g

33. A block of mass m is placed on a surface with a vertical cross section given by $y = \frac{x^3}{6}$. If the coefficient of friction is 0.5, the maximum height above the ground at which the block can be placed without slipping is: **[2014]**

(a) $\frac{1}{6}$m (b) $\frac{2}{3}$m

(c) $\frac{1}{3}$m (d) $\frac{1}{2}$m

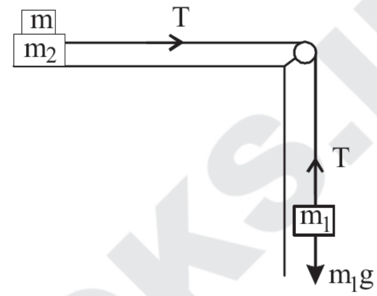
34. Given in the figure are two blocks A and B of weight 20 N and 100 N, respectively. These are being pressed against a wall by a force F as shown. If the coefficient of friction between the blocks is 0.1 and between block B and the wall is 0.15, the frictional force applied by the wall on block B is: **[2015]**



(a) 120 N (b) 150 N

(c) 100 N (d) 80 N

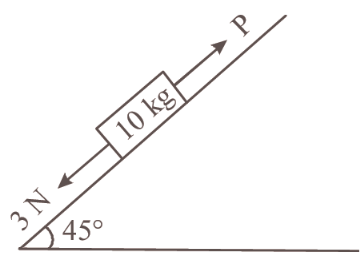
35. Two masses $m_1 = 5$ kg and $m_2 = 10$ kg, connected by an inextensible string over a frictionless pulley, are moving as shown in the figure. The coefficient of friction of horizontal surface is 0.15. The minimum weight m that should be put on top of m_2 to stop the motion is: **[2018]**



(a) 18.3 kg (b) 27.3 kg

(c) 43.3 kg (d) 10.3 kg

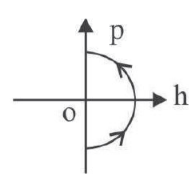
36. A block of mass 10 kg is kept on a rough inclined plane as shown in the figure. A force of 3 N is applied on the block. The coefficient of static friction between the plane and the block is 0.6. What should be the minimum value of force P, such that the block doesnot move downward? (take $g = 10$ ms^{-2}) **[2019]**

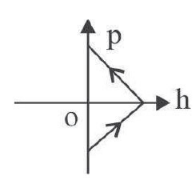


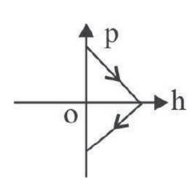
(a) 32 N (b) 18 N

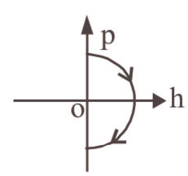
(c) 23 N (d) 25 N

37. A ball is thrown vertically up (taken as $+z$-axis) from the ground. The correct momentum-height (p-h) diagram is: **[2019]**

(a)


(b)


(c)


(d)


Answer Key

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
(c)	(a)	(b)	(d)	(b)	(b)	(b)	(c)	(a)	(d)	(d)	(d)	(d)	(a)	(b)
16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
(c)	(c)	(b)	(d)	(c)	(d)	(c)	(c)	(a)	(d)	(c)	(d)	(d)	(c)	(d)
31	32	33	34	35	36	37								
(c)	(b)	(a)	(a)	(b)	(a)	(d)								

Solutions

1. (c) **Case - I:** For the man standing in the lift, the acceleration of the ball

$\vec{a}_{bm} = \vec{a}_b - \vec{a}_m \Rightarrow a_{bm} = g - a$

Case - II: The man standing on the ground, the acceleration of the ball

$\vec{a}_{bm} = \vec{a}_b - \vec{a}_m \Rightarrow a_{bm} = g - 0 = g$

2. (a) When forces F_1, F_2 and F_3 are acting on the particle, it remains in equilibrium. Force F_2 and F_3 are perpendicular to each other,

$F_1 = F_2 + F_3$

$\therefore \quad F_1 = \sqrt{F_2^2 + F_3^2}$

The force F_1 is now removed, so, resultant of F_2 and F_3 will now make the particle move with force equal to F_1.

Then, acceleration, $a = \dfrac{F_1}{m}$

3. (b) The maximum velocity of the car is

$v_{max} = \sqrt{\mu rg}$

Here $\mu = 0.6$, $r = 150$ m, $g = 9.8$

$v_{max} = \sqrt{0.6 \times 150 \times 9.8} \simeq 30 \text{m/s}$

4. (d) This is a case of sliding (if plane is friction less) and therefore the acceleration of all the bodies is same.

5. (b) Let the two forces be F_1 and F_2 and let $F_2 < F_1$. R is the resultant force.

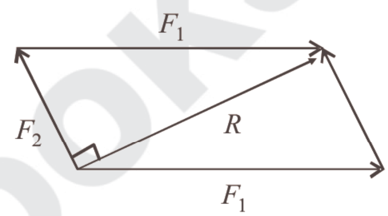
Given $F_1 + F_2 = 18$ ...(i)

From the figure $F_2^2 + R^2 = F_1^2$

$F_1^2 - F_2^2 = R^2$

$\therefore \quad F_1^2 - F_2^2 = 144$ ...(ii)

Only option (b) follows equation (i) and (ii).

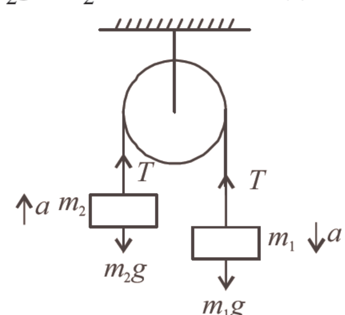


6. (b) **For mass m_1**

$m_1 g - T = m_1 a$ ...(i)

For mass m_2

$T - m_2 g = m_2 a$ ...(ii)



Adding the equations we get

$a = \dfrac{(m_1 - m_2)g}{m_1 + m_2}$

Here $a = \dfrac{g}{8}$

$\therefore \quad \dfrac{1}{8} = \dfrac{\dfrac{m_1}{m_2} - 1}{\dfrac{m_1}{m_2} + 1} \quad \Rightarrow \dfrac{m_1}{m_2} + 1 = 8\dfrac{m_1}{m_2} - 8$

$\Rightarrow \dfrac{m_1}{m_2} = \dfrac{9}{7}$

7. (b) Force = mass × acceleration

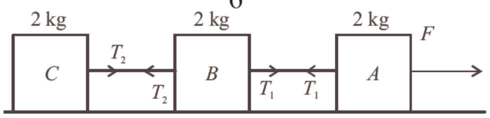
$\therefore \quad F = (m + m + m) \times a$

$F = 3\text{m} \times \text{a}$

$a = \frac{F}{3m}$

$\therefore\ a = \frac{10.2}{6} \text{m/s}^2$

$\therefore\ T_2 = ma = 2 \times \frac{10.2}{6} = 3.4\text{N}$



8. **(c)** Tension, $T = 360$ N

Mass of a man m = 60 kg

$mg - T = ma$

$\therefore\quad a = g - \frac{T}{m}$

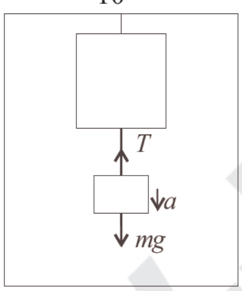
$= 10 - \frac{360}{60} = 4m/s^2$

9. **(a)** When lift is stationary, $W_1 = mg$ ...(i)

When the lift descends with acceleration, a

$W_2 = m(g - a)$

$W_2 = \frac{49}{10}(10 - 5) = 24.5\ N$

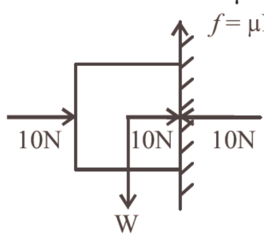


10. **(d)** Resultant force is zero, as three forces are represented by the sides of a triangle taken in the same order. From Newton's second law, $\vec{F}_{net} = m\vec{a}$.

Therefore, acceleration is also zero *i.e.*, velocity remains unchanged.

11. **(d)** Horizontal force, $N = 10$ N.

Coefficient of friction $\mu = 0.2$.



The block will be stationary so long as

Force of friction = weight of block

$\therefore\quad \mu N = W$

$\Rightarrow 0.2 \times 10 = W$

$\Rightarrow W = 2N$

12. **(d)** $u = 6$ m/s, $v = 0$, $t = 10s$, $a = ?$

Acceleration $a = \frac{v - u}{t}$

$\Rightarrow\quad a = \frac{0 - 6}{10}$

$\Rightarrow\quad a = \frac{-6}{10} = -0.6\text{m/s}^2$

The retardation is due to the frictional force

$\therefore\quad f = ma$

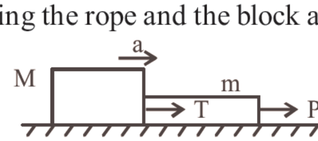
$\Rightarrow \mu N = ma$

$\Rightarrow \mu mg = ma$

$\Rightarrow\quad \mu = \frac{ma}{mg}$

$\Rightarrow \mu = \frac{a}{g} = \frac{0.6}{10} = 0.06$

13. **(d)** Taking the rope and the block as a system



we get $P = (m + M)a$

$\therefore$ Acceleration produced, $a = \frac{P}{m + M}$

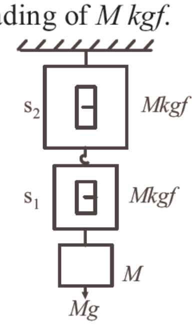
Taking the block as a system,

Force on the block, $F = Ma$

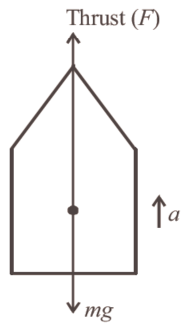
$\therefore\ F = \frac{MP}{m + M}$

14. **(a)** The Earth exerts a pulling force Mg. The block in turn exerts a reaction force Mg on the spring of spring balance S_1 which therefore shows a reading of M kgf.

As both the springs are massless. Therefore, it exerts a force of Mg on the spring of spring balance S_2 which shows the reading of M *kgf*.



15. (b) In the absence of air resistance, if the rocket moves up with an acceleration a, then thrust $F = mg + ma$



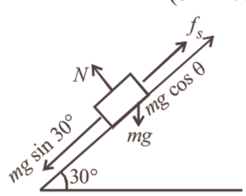
$\therefore\ F = m(g + a) = 3.5 \times 10^4\ (10+10) = 7 \times 10^5$ N

16. (c) Here, $m_1 = 5$kg and $m_2 = 4.8$ kg.
If a is the acceleration of the masses,
$m_1 a = m_1 g - T$ ...(i)
$m_2 a = T - m_2 g$ ...(ii)
Solving (i) and (ii) we get

$$a = \left(\frac{m_1 - m_2}{m_1 + m_2}\right) g$$

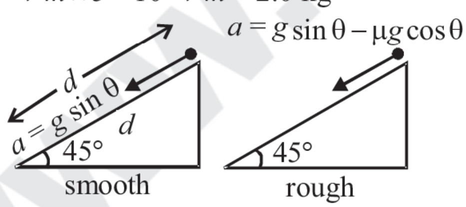
$$\Rightarrow \quad a = \frac{(5-4.8)\times 9.8}{(5+4.8)}\text{m/s}^2 = 0.2\ \text{m/s}^2$$

17. (c)



Since the body is at rest on the inclined plane,
$mg \sin 30° =$ Force of friction
$\Rightarrow m \times 10 \times \sin 30° = 10$
$\Rightarrow m \times 5 = 10 \Rightarrow m = 2.0$ kg

18. (b)



On smooth inclined plane, acceleration of the body $= g \sin \theta$. Let d be the distance travelled

$$\therefore \quad d = \frac{1}{2}(g\sin\theta)t_1^2,$$

$$t_1 = \sqrt{\frac{2d}{g\sin\theta}},$$

On rough inclined plane,

$$a = \frac{mg\sin\theta - \mu R}{m}$$

$$\Rightarrow \quad a = \frac{mg\sin\theta - \mu mg\cos\theta}{m}$$

$\Rightarrow \quad a = g \sin \theta - \mu_k g \cos \theta$

$$\therefore \quad d = \frac{1}{2}(g\sin\theta - \mu\hat{k}g\cos\theta)t_2^2$$

$$t_2 = \sqrt{\frac{2d}{g\sin\theta - \mu\hat{k}g\cos\theta}}$$

According to question, $t_2 = nt_1$

$$n\sqrt{\frac{2d}{g\sin\theta}} = \sqrt{\frac{2d}{g\sin\theta - \mu\hat{k}g\cos\theta}}$$

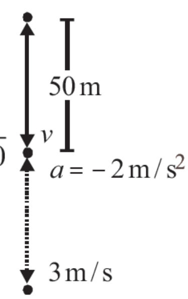
Here, μ is coefficient of kinetic friction as the block moves over the inclined plane.

$\therefore \quad \sin\theta = (\sin\theta - \mu\hat{k}\ \cos\theta)n^2$

$$\Rightarrow \quad n = \frac{1}{\sqrt{1-\mu_k}} \quad \Rightarrow \quad n^2 = \frac{1}{1-\mu_k}$$

$$\Rightarrow \quad \mu_k = 1 - \frac{1}{n^2}$$

19. (d) Initially, the parachutist falls under gravity.



$u = \sqrt{2gh}$

$u = \sqrt{2\times 9.8\times 50} = \sqrt{980}$

He reaches the ground with a speed of 3 m/s.

$\therefore \quad v = 3$m/s

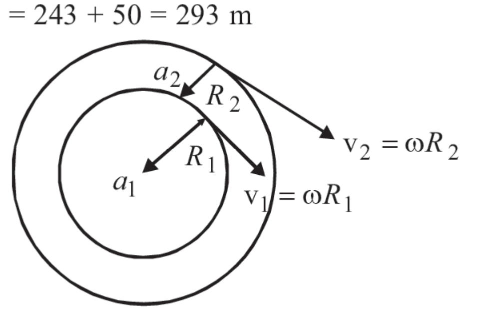
Using $v^2 - u^2 = 2aS$

$$\Rightarrow \quad S = \frac{v^2 - u^2}{2\times 2} = \frac{3^2 - 980}{4} \approx 243 \text{ m}$$

Initially he has fallen 50 m.

$\therefore$ Total height from where he bailed out $= 243 + 50 = 293$ m

20. (c)



Let m is the mass of each particle and ω is the angular speed of the annular ring.

$$a_1 = \frac{v_1^2}{R_1} = \frac{\omega^2 R_1^2}{R_1} = \omega^2 R_1$$

$$a_2 = \frac{v_2^2}{R_2} = \omega^2 R_2$$

Taking particle masses equal

$$\frac{F_1}{F_2} = \frac{ma_1}{ma_2} = \frac{mR_1\omega^2}{mR_2\omega^2} = \frac{R_1}{R_2}$$

ALTERNATE SOLUTION

The force experienced by any particle is only along radial direction.

Force experienced by the particle, $F = m\omega^2 R$

$$\therefore \frac{F_1}{F_2} = \frac{R_1}{R_2}$$

21. (d) For first half

acceleration = $g \sin \phi$;

For second half

acceleration = $-(g \sin \phi - \mu g \cos \phi)$

For the block to come to rest at the bottom, acceleration in I half = retardation in II half.

$$g \sin \phi = -(g \sin \phi - \mu g \cos \phi)$$

$$\Rightarrow \mu = 2 \tan \phi$$

ALTERNATE SOLUTION

According to work-energy theorem, $W = \Delta K = 0$

(Since initial and final speeds are zero)

$\therefore$ Workdone by friction + Work done by gravity = 0

$$i.e., \quad -(\mu\, mg \cos \phi)\frac{\ell}{2} + mg\ell \sin \phi = 0$$

$$\text{or} \quad \frac{\mu}{2}\cos \phi = \sin \phi \quad \text{or} \quad \mu = 2 \tan \phi$$

22. (c) Mass $(m) = 0.3$ kg

Force, $F = m.a = -kx$

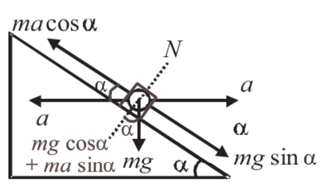
$\Rightarrow \quad ma = -15x$

$\Rightarrow \quad 0.3a = -15x$

$$\Rightarrow \quad a = -\frac{15}{0.3}x = \frac{-150}{3}x = -50x$$

$a = -50 \times 0.2 = 10 \text{m/s}^2$

23. (c) When the incline is given an acceleration a towards the right, the block receives a reaction ma towards left.



For block to remain stationary, Net force along the incline should be zero.

$$mg \sin \alpha = ma \cos \alpha \Rightarrow a = g \tan \alpha$$

24. (a) Given, initial velocity, $u = 100$m/s.

Final velocity, $v = 0$.

Acceleration, $a = \mu_k g = 0.5 \times 10$

$v^2 - u^2 = 2as$ or

$\Rightarrow \quad 0^2 - u^2 = 2(-\mu_k g)s$

$$\Rightarrow \quad -100^2 = 2 \times -\frac{1}{2} \times 10 \times s$$

$\Rightarrow \quad s = 1000$ m

25. (d) For the motion of ball, just after the throwing

$v = 0$, $s = 2$m, $a = -g = -10\text{ms}^{-2}$

$v^2 - u^2 = 2as$ for upward journey

$$\Rightarrow -u^2 = 2(-10) \times 2 \Rightarrow u^2 = 40$$

When the ball is in the hands of the thrower

$u = 0$, $v = \sqrt{40}$ ms^{-1}

$s = 0.2$ m

$v^2 - u^2 = 2as$

$$\Rightarrow 40 - 0 = 2\ (a)\ 0.2 \Rightarrow a = 100 \text{ m/s}^2$$

$$\therefore F = ma = 0.2 \times 100 = 20\ N$$

$$\Rightarrow N - mg = 20 \Rightarrow N = 20 + 2 = 22N$$

ALTERNATE SOLUTION

$$W_{hand} + W_{gravity} = \Delta K$$

$$\Rightarrow F(0.2) + (0.2)(10)(2.2) = 0 \Rightarrow F = 22\ N$$

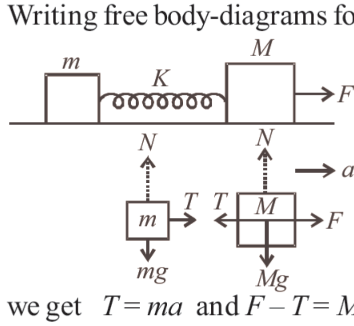
26. (c) Given, mass of cricket ball, $m = 150\,g = 0.15$ kg

Initial velocity, $u = 20$ m/s

Force,

$$F = \frac{m(v-u)}{t} = \frac{0.15(0-20)}{0.1} = 30\,N$$

27. (d) Writing free body-diagrams for m & M,



we get $T = ma$ and $F - T = Ma$

where T is force due to spring

$\Rightarrow \quad F - ma = Ma$ or, $F = Ma + ma$

$\therefore$ Acceleration of the system

$$a = \frac{F}{M+m}.$$

Now, force acting on the block of mass m is

$$ma = m\left(\frac{F}{M+m}\right) = \frac{mF}{m+M}.$$

If a is the acceleration along the inclined plane, then

28. **(d)** $mg \sin\theta = ma$

$\therefore\ a = g \sin\theta$

$\therefore$ Vertical component of acceleration $= g \sin^2\theta$

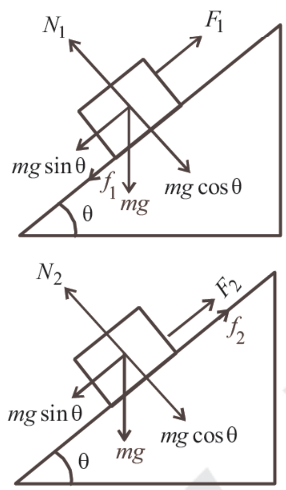
$\therefore$ Relative vertical acceleration of A with respect to B is

$$g(\sin^2 60 - \sin^2 30]$$

$$= g\left(\frac{3}{4} - \frac{1}{4}\right) = \frac{g}{2} = 4.9 \text{ m/s}^2$$

in vertical direction

29. **(c)**



When the body slides up the inclined plane, then

$$\text{mg}\sin\theta + f_1 = F_1$$

or, $F_1 = \text{mg}\sin\theta + \mu\text{mg}\cos\theta$

When the body slides down the inclined plane, then

$$\text{mg}\sin\theta - f_2 = F_2$$

or $F_2 = \text{mg}\sin\theta - \mu\text{mg}\cos\theta$

$$\therefore\ \frac{F_1}{F_2} = \frac{\sin\theta + \mu\cos\theta}{\sin\theta - \mu\cos\theta}$$

$$\Rightarrow\ \frac{F_1}{F_2} = \frac{\tan\theta + \mu}{\tan\theta - \mu} = \frac{2\mu + \mu}{2\mu - \mu} = \frac{3\mu}{\mu} = 3$$

30. **(d)** It is given $\ell_A : \ell_B = 2 : 3$

$$\ell_A = \frac{2\ell}{5},\ \ \ell_B = \left(\frac{3\ell}{5}\right)$$

$\therefore$ We know that $k \propto \frac{1}{\ell}$

If initial spring constant is k, then

$$k\ell = k_A \ell_A = k_B \ell_B$$

$$k\ell = k_A\left(\frac{2\ell}{5}\right)$$

$$k_A = \frac{5k}{2}$$

31. **(c)** Given that $F(t) = F_0 e^{-bt}$

$$\Rightarrow\ m\frac{dv}{dt} = F_0 e^{-bt}$$

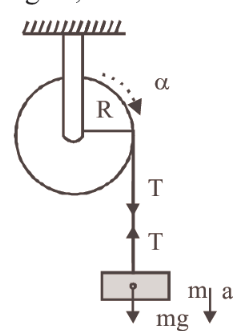
$$\frac{dv}{dt} = \frac{F_0}{m} e^{-bt}$$

$$\int_0^v dv = \frac{F_0}{m}\int_0^t e^{-bt}\,dt$$

$$v = \frac{F_0}{m}\left[\frac{e^{-bt}}{-b}\right]_0^t = \frac{F_0}{mb}\left[-\left(e^{-bt} - e^{-0}\right)\right]$$

$$\Rightarrow\ v = \frac{F_0}{mb}\left[1 - e^{-bt}\right]$$

32. **(b)** From figure,



Acceleration $a = R\alpha$ ...(i)

and $mg - T = ma$ ...(ii)

From equation (i) and (ii)

$$T \times R = mR^2\alpha = mR^2\left(\frac{a}{R}\right)$$

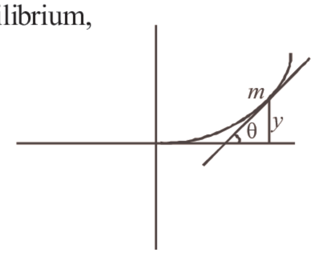
or $\quad T = ma$

$\Rightarrow\quad mg - ma = ma$

$\Rightarrow\quad a = \frac{g}{2}$

33. **(a)** At limiting equilibrium,

$$\mu = \tan\theta$$



$\tan\theta = \mu = \frac{dy}{dx} = \frac{x^2}{2}$ (from question)

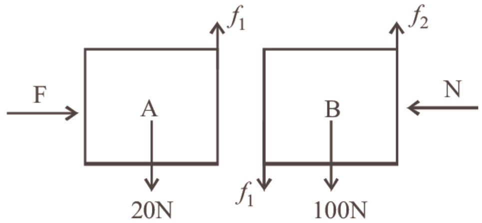
$\because$ Coefficient of friction $\mu = 0.5$

$\therefore \quad 0.5 = \frac{x^2}{2}$

$\Rightarrow \quad x = \pm 1$

Now, $y = \frac{x^3}{6} = \frac{1}{6} m$

34. (a)

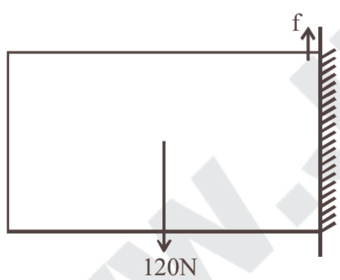


Assuming both the blocks are stationary

N = F

$f_1 = 20$ N

$f_2 = 100 + 20 = 120$N



Considering the two blocks as one system and due to equilibrium $f = 120$N

35. (b) Given : $m_1 = 5$kg; $m_2 = 10$kg; $\mu = 0.15$

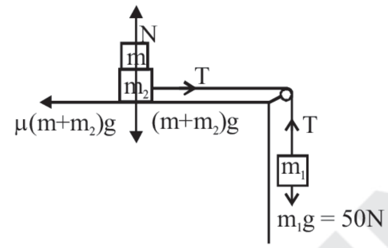
FBD for m_1, $m_1 g - T = m_1 a$

$\Rightarrow 50 - T = 5 \times a$

and, $T - 0.15\,(m + 10)g = (10 + m)a$

For rest $a = 0$

or, $50 = 0.15\,(m + 10)\,10$

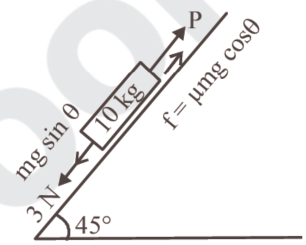


$\Rightarrow \quad 5 = \frac{3}{20}(m + 10)$

$\frac{100}{3} = m + 10 \;\; \therefore \;\; m = 23.3$kg;

close to option (b)

36. (a)



For equilibrium

$3 + mg\sin\theta = P + \mu mg\cos\theta$

$\Rightarrow 3 + 10 \times 10 \times \frac{1}{\sqrt{2}}$

$= P + 0.6 \times 10 \times 10 \times \cos 45°$

$\therefore \qquad P = 31.28 \approx 32$ N

37. (d) $v^2 = u^2 - 2gh$

or $v = \sqrt{u^2 - 2gh}$

Momentum, $p = mv$

$\therefore p = m\sqrt{u^2 - 2gh}$

Therefore graph between p and h cannot have straight line.

(b) and (c) are not possible.

During upward journey as h increases, p decreases and in downward journey as h decreases p increases. Therefore (d) is the correct option.

5 Work, Energy and Power

1. Consider the following two statements :**[2003]**
   A. Linear momentum of a system of particles is zero
   B. Kinetic energy of a system of particles is zero. Then
   (a) A does not imply *B* and *B* does not imply *A*
   (b) *A* implies *B* but *B* does not imply *A*
   (c) *A* does not imply *B* but *B* implies *A*
   (d) *A* implies *B* and *B* implies *A*
2. A wire suspended vertically from one of its ends is stretched by attaching a weight of 200N to the lower end. The weight stretches the wire by 1 mm. Then the elastic energy stored in the wire is **[2003]**
   (a) 0.2 J (b) 10 J
   (c) 20 J (d) 0.1 J
3. A spring of spring constant 5×10^3 N/m is stretched initially by 5cm from the unstretched position. Then the work required to stretch it further by another 5 cm is **[2003]**
   (a) 12.50 N-m (b) 18.75 N-m
   (c) 25.00 N-m (d) 6.25 N-m
4. A body is moved along a straight line by a machine delivering a constant power. The distance moved by the body in time 't' is proportional to **[2003]**
   (a) $t^{3/4}$ (b) $t^{3/2}$
   (c) $t^{1/4}$ (d) $t^{1/2}$
5. A particle moves in a straight line with retardation proportional to its displacement. Its loss of kinetic energy for any displacement x is proportional to **[2004]**
   (a) x (b) e^x
   (c) x^2 (d) $\log_e x$
6. A uniform chain of length 2 m is kept on a table such that a length of 60 cm hangs freely from the edge of the table. The total mass of the chain is 4 kg. What is the work done in pulling the entire chain on the table ? **[2004]**
   (a) 12 J (b) 3.6 J
   (c) 7.2 J (d) 1200 J
7. A force $\vec{F} = (5\vec{i} + 3\vec{j} + 2\vec{k})N$ is applied over a particle which displaces it from its origin to the point $\vec{r} = (2\vec{i} - \vec{j})m$. The work done on the particle in joules is **[2004]**
   (a) +10 (b) +7
   (c) –7 (d) +13
8. A body of mass 'm', accelerates uniformly from rest to 'v_1' in time 't_1'. The instantaneous power delivered to the body as a function of time 't' is **[2004]**
   (a) $\dfrac{mv_1t^2}{t_1}$ (b) $\dfrac{mv_1^2t}{t_1^2}$
   (c) $\dfrac{mv_1t}{t_1}$ (d) $\dfrac{mv_1^2t}{t_1}$
9. A particle is acted upon by a force of constant magnitude which is always perpendicular to the velocity of the particle, the motion of the particles takes place in a plane. It follows that **[2004]**
   (a) its kinetic energy is constant
   (b) its acceleration is constant
   (c) its velocity is constant
   (d) it moves in a straight line
10. The block of mass M moving on the frictionless horizontal surface collides with the spring of spring constant k and compresses it by length L. The maximum momentum of the block after collision is **[2005]**

(a) $\frac{kL^2}{2M}$ (b) $\sqrt{Mk}\ L$

(c) $\frac{ML^2}{k}$ (d) zero

11. A spherical ball of mass 20 kg is stationary at the top of a hill of height 100 m. It rolls down a smooth surface to the ground, then climbs up another hill of height 30 m and finally rolls down to a horizontal base at a height of 20 m above the ground. The velocity attained by the ball is **[2005]**

(a) 20 m/s (b) 40 m/s

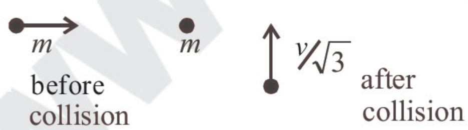
(c) $10\sqrt{30}$ m/s (d) 10 m/s

12. A body of mass m is accelerated uniformly from rest to a speed v in a time T. The instantaneous power delivered to the body as a function of time is given by **[2005]**

(a) $\frac{mv^2}{T^2}.t^2$ (b) $\frac{mv^2}{T^2}.t$

(c) $\frac{1}{2}\frac{mv^2}{T^2}.t^2$ (d) $\frac{1}{2}\frac{mv^2}{T^2}.t$

13. A mass 'm' moves with a velocity 'v' and collides inelastically with another identical mass . After collision the 1st mass moves with velocity $\frac{v}{\sqrt{3}}$ in a direction perpendicular to the initial direction of motion. Find the speed of the 2nd mass after collision. **[2005]**



(a) $\sqrt{3}v$ (b) v

(c) $\frac{v}{\sqrt{3}}$ (d) $\frac{2}{\sqrt{3}}v$

14. A bomb of mass 16kg at rest explodes into two pieces of masses 4 kg and 12 kg. The velolcity of the 12 kg mass is $4\ ms^{-1}$. The kinetic energy of the other mass is **[2006]**

(a) 144 J (b) 288 J

(c) 192 J (d) 96 J

15. A particle of mass 100g is thrown vertically upwards with a speed of 5 m/s. The work done by the force of gravity during the time the particle goes up is **[2006]**

(a) –0.5 J (b) –1.25 J

(c) 1.25 J (d) 0.5 J

16. The potential energy of a 1 kg particle free to move along the x-axis is given by

$$V(x) = \left(\frac{x^4}{4} - \frac{x^2}{2}\right) J.$$

The total mechanical energy of the particle is 2 J. Then, the maximum speed (in m/s) is **[2006]**

(a) $\frac{3}{\sqrt{2}}$ (b) $\sqrt{2}$

(c) $\frac{1}{\sqrt{2}}$ (d) 2

17. A 2 kg block slides on a horizontal floor with a speed of 4m/s. It strikes a uncompressed spring, and compresses it till the block is motionless. The kinetic friction force is 15N and spring constant is 10,000 N/m. The spring compresses by **[2007]**

(a) 8.5 cm (b) 5.5 cm

(c) 2.5 cm (d) 11.0 cm

18. An athlete in the olympic games covers a distance of 100 m in 10 s. His kinetic energy can be estimated to be in the range **[2008]**

(a) 200 J - 500 J

(b) 2×10^5 J - 3×10^5 J

(c) 20,000 J - 50,000 J

(d) 2,000 J - 5, 000 J

19. A block of mass 0.50 kg is moving with a speed of 2.00 ms^{-1} on a smooth surface. It strikes another mass of 1.00 kg and then they move together as a single body. The energy loss during the collision is **[2008]**

(a) 0.16 J (b) 1.00 J

(c) 0.67 J (d) 0.34 J

20. The potential energy function for the force between two atoms in a diatomic molecule is approximately given by $U(x) = \frac{a}{x^{12}} - \frac{b}{x^6}$, where a and b are constants and x is the distance between the atoms. If the dissociation energy of the molecule is $D = [U(x = \infty) - U_{\text{at equilibrium}}]$, D is **[2010]**

(a) $\frac{b^2}{2a}$ (b) $\frac{b^2}{12a}$

(c) $\frac{b^2}{4a}$ (d) $\frac{b^2}{6a}$

21. **Statement -1:** Two particles moving in the same direction do not lose all their energy in a completely inelastic collision. **[2010]**

Statement -2 : Principle of conservation of momentum holds true for all kinds of collisions.

(a) Statement -1 is true, Statement -2 is true ; Statement -2 is the correct explanation of Statement -1.

(b) Statement -1 is true, Statement -2 is true; Statement -2 is **not** the correct explanation of Statement -1

(c) Statement -1 is false, Statement -2 is true.

(d) Statement -1 is true, Statement -2 is false.

22. At time $t = 0$ a particle starts moving along the x-axis. If its kinetic energy increases uniformly with time 't', the net force acting on it must be proportional to **[2011 RS]**

(a) constant (b) t

(c) $\frac{1}{\sqrt{t}}$ (d) $\sqrt{t}$

23. This question has statement I and statement II. Of the four choices given after the statements, choose the one that best describes the two statements.

Statement - I: Apoint particle of mass m moving with speed υ collides with stationary point particle of mass M. If the maximum energy loss possible is given as $f\left(\frac{1}{2}mv^2\right)$ then $f = \left(\frac{m}{M+m}\right)$.

Statement - II: Maximum energy loss occurs when the particles get stuck together as a result of the collision. **[2013]**

(a) Statement - I is true, Statment - II is true, Statement - II is the correct explanation of Statement - I.

(b) Statement-I is true, Statment - II is true, Statement - II is not the correct explanation of Statement - II.

(c) Statement - I is true, Statment - II is false.

(d) Statement - I is false, Statment - II is true.

24. When a rubber-band is stretched by a distance x, it exerts restoring force of magnitude $F = ax + bx^2$ where a and b are constants. The work done in stretching the unstretched rubber-band by L is: **[2014]**

(a) $aL^2 + bL^3$ (b) $\frac{1}{2}\left(aL^2 + bL^3\right)$

(c) $\frac{aL^2}{2} + \frac{bL^3}{3}$ (d) $\frac{1}{2}\left(\frac{aL^2}{2} + \frac{bL^3}{3}\right)$

25. A particle of mass m moving in the x direction with speed $2v$ is hit by another particle of mass $2m$ moving in the y direction with speed **v**. If the collision is perfectly inelastic, the percentage loss in the energy during the collision is close to : **[2015]**

(a) 56% (b) 62%

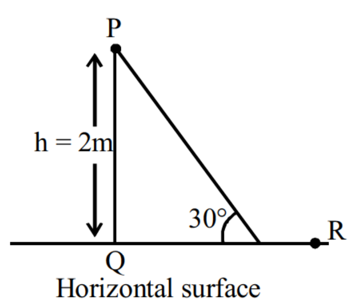
(c) 44% (d) 50%

26. A person trying to lose weight by burning fat lifts a mass of 10 kg upto a height of 1 m 1000 times. Assume that the potential energy lost each time he lowers the mass is dissipated. How much fat will he use up considering the work done only when the weight is lifted up? Fat supplies 3.8×10^7 J of energy per kg which is converted to mechanical energy with a 20% efficiency rate. Take $g = 9.8\ ms^{-2}$: **[2016]**

(a) 9.89×10^{-3} kg (b) 12.89×10^{-3} kg

(c) 2.45×10^{-3} kg (d) 46×10^4 J

27. A point particle of mass m, moves long the uniformly rough track PQR as shown in the figure. The coefficient of friction, between the particle and the rough track equals μ. The particle is released, from rest from the point P and it comes to rest at a point R. The energies, lost by the ball, over the parts, PQ and QR, of the track, are equal to each other, and no energy is lost when particle changes direction from PQ to QR. The value of the coefficient of friction μ and the distance x (= QR), are, respectively close to : **[2016]**



(a) 0.29 and 3.5 m (b) 0.29 and 6.5 m
(c) 0.2 and 6.5 m (d) 0.2 and 3.5 m

28. A body of mass m = 10^{-2} kg is moving in a medium and experiences a frictional force $F = -kv^2$. Its intial speed is $v_0 = 10\ ms^{-1}$. If, after 10 s, its energy is $\frac{1}{8}mv_0^2$, the value of k will be: **[2017]**

(a) 10^{-4} kg m^{-1} (b) 10^{-1} kg m^{-1} s^{-1}
(c) 10^{-3} kg m^{-1} (d) 10^{-3} kg s^{-1}

29. A time dependent force F = 6t acts on a particle of mass 1 kg. If the particle starts from rest, the work done by the force during the first 1 secand will be **[2017]**

(a) 9 J (b) 18 J
(c) 4.5 J (d) 22 J

30. A particle is moving in a circular path of radius a under the action of an attractive potential $U = -\frac{k}{2r^2}$. Its total energy is: **[2018]**

(a) $-\frac{k}{4a^2}$ (b) $\frac{k}{2a^2}$
(c) zero (d) $-\frac{3}{2}\frac{k}{a^2}$

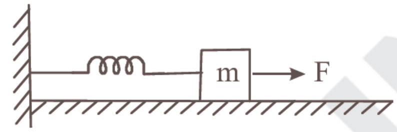
31. In a collinear collision, a particle with an initial speed v_0 strikes a stationary particle of the same mass. If the final total kinetic energy is 50% greater than the original kinetic energy, the magnitude of the relative velocity between the two particles, after collision, is: **[2018]**

(a) $\frac{v_0}{4}$ (b) $\sqrt{2}v_0$
(c) $\frac{v_0}{2}$ (d) $\frac{v_0}{\sqrt{2}}$

32. It is found that if a neutron suffers an elastic collinear collision with deuterium at rest, fractional loss of its energy is p_d; while for its similar collision with carbon nucleus at rest, fractional loss of energy is P_c. The values of P_d and P_c are respectively: **[2018]**

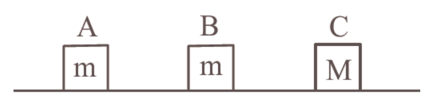
(a) (·89, ·28) (b) (·28, ·89)
(c) (0, 0) (d) (0, 1)

33. A block of mass m, lying on a smooth horizontal surface, is attached to a spring (of negligible mass) of spring constant k. The other end of the spring is fixed, as shown in the figure. The block is initally at rest in its equilibrium position. If now the block is pulled with a constant force F, the maximum speed of the block is: **[2019]**



(a) $\frac{2F}{\sqrt{mk}}$ (b) $\frac{F}{\pi\sqrt{mk}}$
(c) $\frac{\pi F}{\sqrt{mk}}$ (d) $\frac{F}{\sqrt{mk}}$

34. There block A, B and C are lying on a smooth horizontal surface, as shown in the figure. A and B have equal masses, m while C has mass M. Block A is given an inital speed v towards B due to which it collides with B perfectly inelastically. The combined mass collides with C, also perfectly inelastically $\frac{5}{6}$th of the initial kinetic energy is lost in whole process. What is value of M/m? **[2019]**



(a) 5 (b) 2
(c) 4 (d) 3

35. A uniform cable of mass 'M' and length 'L' is placed on a horizontal surface such that its $\left(\frac{1}{n}\right)^{th}$ part is hanging below the edge of the surface. To lift the hanging part of the cable upto the surface, the work done should be: **[2020]**

(a) $\frac{MgL}{2n^2}$ (b) $\frac{MgL}{n^2}$
(c) $\frac{2MgL}{n^2}$ (d) nMgL

36. A body of mass 2 kg makes an elastic collision with a second body at rest and continues to move in the original direction but with one fourth of its original speed. What is the mass of the second body? **[2020]**

(a) 1.0 kg (b) 1.5 kg
(c) 1.8 kg (d) 1.2 kg

Answer Key

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
(c)	(d)	(b)	(b)	(c)	(b)	(b)	(b)	(a)	(b)	(b)	(b)	(d)	(b)	(b)
16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
(a)	(b)	(d)	(c)	(d)	(a)	(c)	(d)	(c)	(a)	(b)	(a)	(a)	(c)	(c)
31	32	33	34	35	36									
(b)	(a)	(d)	(c)	(b)	(d)									

Solutions

1. (c) Kinetic energy of a system of particle is zero only when the speed of each particles is zero. This implies momentum of each particle is zero, thus linear momentum of the system of particle has to be zero.

Also if linear momentum of the system is zero it does not mean linear momentum of each particle is zero. This is because linear momentum is a vector quantity. In this case the kinetic energy of the system of particles will not be zero.

$\therefore$ A does not imply B but B implies A.

Given, force, $F = 200$ N extension of wire, $x = 1$mm.

2. (d) The elastic potential energy

$$= \frac{1}{2} \times \text{Force} \times \text{extension}$$

$$= \frac{1}{2} \times F \times x$$

$$= \frac{1}{2} \times 200 \times 0.001 = 0.1 \text{ J}$$

3. (b) Spring constant, $k = 5 \times 10^3$ N/m

Let x_1 and x_2 be the initial and final stretched position of the spring, then

$$\text{Work done, } W = \frac{1}{2}k\left(x_2^2 - x_1^2\right)$$

$$= \frac{1}{2} \times 5 \times 10^3 \left[(0.1)^2 - (0.05)^2\right]$$

$$= \frac{5000}{2} \times 0.15 \times 0.05 = 18.75 \text{ Nm}$$

4. (b) Power, $P = Fv = ma.v$

$$\Rightarrow \quad P = \frac{mdv}{dt} v = c = \text{contant}$$

$$\left(\because F = ma = \frac{mdv}{dt}\right)$$

$mv_0v = cdt$

Integrating both sides, we get

$$m\int_0^v vdv = c\int_0^t dt$$

$$\Rightarrow \quad \frac{1}{2}mv^2 = ct$$

$$\Rightarrow \quad \frac{v^2}{2} = \frac{c.t}{m}$$

$$\Rightarrow \quad v_2 = \frac{2c.t}{m}$$

$$\Rightarrow \quad v = \sqrt{\frac{2c}{m}} \times t^{1/2}$$

$$\Rightarrow \quad \frac{dx}{dt} = \sqrt{\frac{2c}{m}} \times t^{1/2} \quad \text{where } v = \frac{dx}{dt}$$

$$\Rightarrow \quad \int_e^x dx = \sqrt{\frac{2c}{m}} \times \int_0^t t^{1/2} dt$$

$$\Rightarrow \quad x = \sqrt{\frac{2c}{m}} \times \frac{2t^{3/2}}{3} \quad \Rightarrow \quad \text{x} \propto \text{t}^{3/2}$$

5. (c) Given : retardation $\propto$ displacement

i.e., $a = -kx$ [Here, k = constant]

But $a = v\dfrac{dv}{dx}$

$$\therefore \frac{vdv}{dx} = -kx \Rightarrow \int_{v_1}^{v_2} v\, dv = -\int_0^x kxdx$$

$$\Rightarrow \quad \left[\frac{v^2}{2}\right]_{v_1}^{v_2} = -k\left[\frac{x^2}{2}\right]_0^x$$

$$\Rightarrow \quad \left(v_2^2 - v_1^2\right) = -\frac{kx^2}{2}$$

$$\Rightarrow \quad \frac{1}{2}m\left(v_2^2 - v_1^2\right) = \frac{1}{2}m\left(\frac{-kx^2}{2}\right)$$

$\therefore$ Loss in kinetic energy, $\therefore \Delta K \propto x^2$

6. (b) Mass of over hanging part of the chain

$$m' = \frac{4}{2} \times (0.6)\text{kg} = 1.2 \text{ kg}$$

Weight of hanging part of the chain
$= 1.2 \times 10 = 12$ N
C.M. of hanging part = 0.3 m below the table
Workdone in putting the entire cha in on the table = $12 \times 0.30 = 3.6$ J.

7. (b) Given, Force, $\vec{F} = \left(5\hat{i} + 3\hat{j} + 2\hat{k}\right)$

Displacement, $x = \left(2\hat{i} - \hat{j}\right)$

Work done,

$$W = \vec{F} \cdot \vec{x} = (5\hat{i} + 3\hat{j} + 2\hat{k}) \cdot (2\hat{i} - \hat{j})$$

$$= 10 - 3 = 7 \text{ joules}$$

8. (b) Let a be the acceleration of body
Using, $v = u + at$

$$v_1 = 0 + at_1 \Rightarrow a = \frac{v_1}{t_1}$$

Velocity of the body at instant t,

$$v = at$$

$$\Rightarrow \quad v = \frac{v_1 t}{t_1}$$

Instantaneous powr, P = $\vec{F} \cdot \vec{v} = (m\vec{a}) \cdot \vec{v}$

$$= \left(\frac{mv_1}{t_1}\right)\left(\frac{v_1 t}{t_1}\right) = m\left(\frac{v_1}{t_1}\right)^2 t$$

9. (a) Work done by such force is always zero since force is acting in a direction perpendicular to velocity.

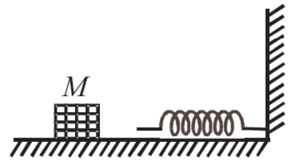
$\therefore$ From work-energy theorem = ΔK = 0
K remains constant.

10. (b) When the spring gets compressed by length L.

K.E. lost by mass M = P.E. stored in the compressed spring.

$$\frac{1}{2}Mv^2 = \frac{1}{2}kL^2$$

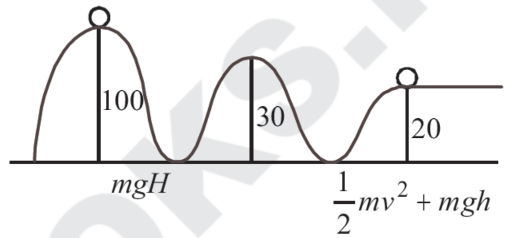
$$\Rightarrow \quad v = \sqrt{\frac{k}{M}} \cdot L$$



Momentum of the block, $= M \times v$

$$= M \times \sqrt{\frac{k}{M}} \cdot L = \sqrt{kM} \cdot L$$

11. (b)



Using conservation of energy,
Total energy at 100 m height
= Total energy at 20m height

$$\text{m}(10 \times 100) = m\left(\frac{1}{2}v^2 + 10 \times 20\right)$$

or $\frac{1}{2}v^2 = 800$ or $v = \sqrt{1600} = 40$ m/s

ALTERNATE SOLUTION

Loss in potential energy = gain in kinetic energy

$$m \times g \times 80 = \frac{1}{2}mv^2$$

$$10 \times 80 = \frac{1}{2}v^2$$

$v^2 = 1600$ or $v = 40$ m/s

12. (b) Using, $v = u + at$

$$\Rightarrow \quad v = 0 + aT$$

$$\Rightarrow \quad a = \frac{V}{T}$$

Force, $F = ma = \frac{mV}{T}$

Velocity acquired, $V = at = \frac{V}{T}t$

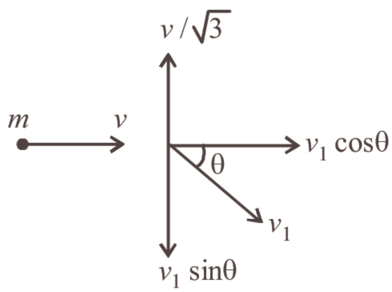
Instantaneous power, $P = Fv$

$$\Rightarrow \quad P = \frac{mV}{T} \times \frac{V}{T}t = \frac{mv^2}{T^2}t$$

13. **(d)** Considering conservation of momentum along x-direction,
$mv = mv_1 \cos\theta$ ...(1)
where v_1 is the velocity of second mass
In y-direction,

$$0 = \frac{mv}{\sqrt{3}} - mv_1 \sin\theta$$

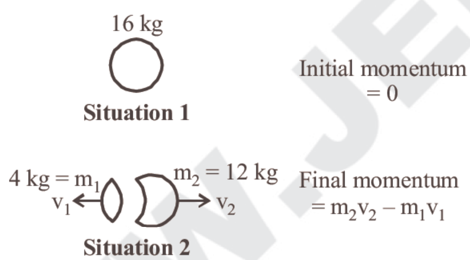
or $m_1 v_1 \sin\theta = \frac{mv}{\sqrt{3}}$ ...(2)



Squaring and adding eqns. (1) and (2) we get

$$v_1^2 = v^2 + \frac{v^2}{\sqrt{3}} \Rightarrow v_1 = \frac{2}{\sqrt{3}} v$$

14. **(b)** Let the velocity and mass of 4 kg piece be v_1 and m_1 and that of 12 kg piece be v_2 and m_2.



Applying conservation of linear momentum
$16 \times 0 = 4 \times v_1 + 12 \times 4$

$$\Rightarrow \quad v_1 = -\frac{12 \times 4}{4} = -12 \ ms^{-1}$$

Kinetic energy of 4 kg mass

$$\therefore K.E. = \frac{1}{2} m_1 v_1^2 = \frac{1}{2} \times 4 \times 144 = 288 \ J$$

15. **(b)** Given, Mass of the particle, $m = 100$g
Initial speed of the particle, $\mu = 5$ m/s
Final speed of the particle, $v = 0$
Work done by the force of gravity
= Loss in kinetic energy of the body.

$$= \frac{1}{2} m (v^2 - u^2) = \frac{1}{2} \times \frac{100}{1000} (0^2 - 5^2)$$

$= -1.25$ J

16. **(a)** Potential energy

$$V(x) = \frac{x^4}{4} - \frac{x^2}{2} \text{ joule}$$

For maxima of minima

$$\frac{dV}{dx} = 0 \Rightarrow x^3 - x = 0 \Rightarrow x = \pm 1$$

$$\Rightarrow \text{Min. P.E.} = \frac{1}{4} - \frac{1}{2} = -\frac{1}{4} \text{ J}$$

K.E.$_{(max.)}$ + P.E.$_{(min.)}$ = 2 (Given)

$$\therefore \text{K.E.}_{(max.)} = 2 + \frac{1}{4} = \frac{9}{4}$$

$$\text{K.E.}_{max.} = \frac{1}{2} m v_{max.}^2$$

$$\Rightarrow \frac{1}{2} \times 1 \times v_{max.}^2 = \frac{9}{4} \Rightarrow v_{max.} = \frac{3}{\sqrt{2}}$$

17. **(b)** Suppose the spring gets compressed by x before stopping.
kinetic energy of the block = P.E. stored in the spring + work done against friction.

$$\Rightarrow \frac{1}{2} \times 2 \times (4)^2 = \frac{1}{2} \times 10{,}000 \times x^2 + 15 \times x$$

$$\Rightarrow 10{,}000 x^2 + 30x - 32 = 0$$

$$\Rightarrow 5000 x^2 + 15x - 16 = 0$$

$$\therefore \quad x = \frac{-15 \pm \sqrt{(15)^2 - 4 \times (5000)(-16)}}{2 \times 5000}$$

$$= 0.055\text{m} = 5.5\text{cm}.$$

18. **(d)** The average speed of the athelete

$$v = \frac{5}{t} = \frac{100}{10} = 10 \text{ m/s}$$

$$\therefore \quad \text{K.E.} = \frac{1}{2} mv^2$$

Assuming the mass of athelet to 40 kg his average K.E would be

$$\text{K.E} = \frac{1}{2} \times 40 \times (10)^2 = 2000 \text{J}$$

Assuming mass to 100 kg average kinetic energy

$$\text{K.E.} = \frac{1}{2} \times 100 \times (10)^2 = 5000 \text{J}$$

19. **(c)** Initial kinetic energy of the system

$$\text{K.E}_i = \frac{1}{2} mu^2 + \frac{1}{2} M (0)^2$$

$$= \frac{1}{2} \times 0.5 \times 2 \times 2 + 0 = 1J$$

Momentum before collision

= Momentum after collision

$$m_1u_1 + m_2u_2 = (m+M) \times v$$

$$\therefore 0.5 \times 2 + 1 \times 0 = (0.5+1) \times v \Rightarrow v = \frac{2}{3} \text{m/s}$$

Final kinetic energy of the system is

$$\text{K.E}_f = \frac{1}{2}(m+M)v^2$$

$$= \frac{1}{2}(0.5+1) \times \frac{2}{3} \times \frac{2}{3} = \frac{1}{3}\text{J}$$

$\therefore$ Energy loss during collision

$$= \left(1 - \frac{1}{3}\right)\text{J} = 0.67\text{J}$$

20. **(d)** At equilibrium : $F = \frac{-dU(x)}{dx}$

$$\Rightarrow F = \frac{-d}{dx}\left[\frac{a}{x^{12}} - \frac{b}{x^6}\right]$$

$$\Rightarrow F = -\left[\frac{12a}{x^{13}} + \frac{6b}{x^7}\right]$$

$$\Rightarrow \frac{12a}{x^{13}} = \frac{6b}{x^7} \Rightarrow x = \left(\frac{2a}{b}\right)^{\frac{1}{6}}$$

$\therefore U_{\text{at equilibrium}} = \frac{a}{\left(\frac{2a}{b}\right)^2} - \frac{b}{\left(\frac{2a}{b}\right)} = -\frac{b^2}{4a}$ and

$$U_{(x=\infty)} = 0$$

$$\therefore D = 0 - \left(-\frac{b^2}{4a}\right) = \frac{b^2}{4a}$$

21. **(a)** In completely inelastic collision, all initial kinetic energy is not lost but loss in kinetic energy 15 as large as it can be. Linear momentum remain conserved in all types of collision. Statement -2 explains statement -1 correctly because applying the principle of conservation of momentum, we can get the common velocity and hence the kinetic energy of the combined body.

22. **(c)** K.E. $\propto t$

K.E. $= ct$ [Here, c = constant]

$$\Rightarrow \frac{1}{2}mv^2 = ct$$

$$\Rightarrow \frac{(mv)^2}{2m} = ct$$

$$\Rightarrow \frac{p^2}{2m} = ct \quad (\because p = mv)$$

$$\Rightarrow p = \sqrt{2ctm}$$

$$\Rightarrow F = \frac{dp}{dt} = \frac{d\left(\sqrt{2ctm}\right)}{dt}$$

$$\Rightarrow F = \sqrt{2\,cm} \times \frac{1}{2\sqrt{t}}$$

$$\Rightarrow F \propto \frac{1}{\sqrt{t}}$$

23. **(d)** Maximum energy loss $= \frac{P^2}{2m} - \frac{P^2}{2(m+M)}$

$$\left[\because \text{K.E.} = \frac{P^2}{2m} = \frac{1}{2}mv^2\right]$$

$$= \frac{P^2}{2m}\left[\frac{M}{(m+M)}\right] = \frac{1}{2}mv^2\left\{\frac{M}{m+M}\right\}$$

Statement II is a case of perfectly inelastic collision.

By comparing the equation given in statement I with above equation, we get

$$f = \left(\frac{M}{m+M}\right) \text{ instead of } \left(\frac{m}{M+m}\right)$$

Hence statement I is wrong and statement II is correct.

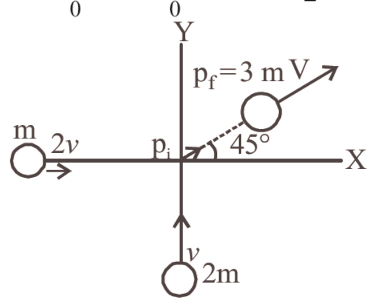
24. **(c)** Work done in stretching the rubber-band by a distance dx is

$$dW = F\,dx = (ax + bx^2)dx$$

Integrating both sides,

$$W = \int_0^L ax\,dx + \int_0^L bx^2\,dx = \frac{aL^2}{2} + \frac{bL^3}{3}$$

25. **(a)**



Initial momentum of the system

$$p_i = \sqrt{[m(2V)^2 \times 2m(2V)^2]}$$

$$= \sqrt{2}m \times 2V$$

Final momentum of the system = 3mV
By the law of conservation of momentum

$$2\sqrt{2}mv = 3mV \Rightarrow \frac{2\sqrt{2}v}{3} = V_{combined}$$

Loss in energy

$$\Delta E = \frac{1}{2}m_1V_1^2 + \frac{1}{2}m_2V_2^2 - \frac{1}{2}(m_1+m_2)V_{combined}^2$$

$$DE = 3mv^2 - \frac{4}{3}mv^2 = \frac{5}{3}mv^2 = 55.55\%$$

Percentage loss in energy during the collision $\simeq$ 56%

26. (b) $n = \frac{W}{input} = \frac{mgh \times 1000}{input} = \frac{10 \times 9.8 \times 1 \times 1000}{input}$

$$Input = \frac{98000}{0.2} = 49 \times 10^4 J$$

$$Fat\ used = \frac{49 \times 10^4}{3.8 \times 10^7} = 12.89 \times 10^{-3} kg.$$

27. (a) Work done by friction at QR = μmgx

In triangle, $\sin 30° = \frac{1}{2} = \frac{2}{PQ}$

$\Rightarrow PQ = 4m$

Work done by friction at PQ = $\mu mg \times$

$\cos 30° \times 4 = \mu mg \times \frac{\sqrt{3}}{2} \times 4 = 2\sqrt{3}\ \mu mg$

Since work done by friction on parts *PQ* and *QR* are equal,

$\mu mg x = 2\sqrt{3}$ umg

$\Rightarrow x = 2\sqrt{3} \cong 3.5m$

Using work energy theorem mg sin 30° × 4

$= 2\sqrt{3}\ \mu mg + \mu mgx$

$\Rightarrow 2 = 4\sqrt{3}\ \mu$

$\Rightarrow \mu = 0.29$

28. (a) Let V_f is the final speed of the body.
From questions,

$$\frac{1}{2}mV_f^2 = \frac{1}{8}mV_0^2 \quad \Rightarrow \quad V_f = \frac{V_0}{2} = 5\,m/s$$

$$F = m\left(\frac{dV}{dt}\right) = -kV^2 \quad \therefore \quad (10^{-2})\frac{dV}{dt} = -kV^2$$

$$\int_{10}^{5}\frac{dV}{V^2} = -100K\int_0^{10} dt$$

$$\frac{1}{5} - \frac{1}{10} = 100K(10) \qquad \text{or,} \quad K = 10^{-4}\ kgm^{-1}$$

29. (c) Using, $F = ma = m\frac{dV}{dt}$

$$6t = 1.\frac{dV}{dt} \qquad [\because \ m = 1 \text{ kg given}]$$

$$\int_0^v dV = \int 6t\,dt \quad V = 6\left[\frac{t^2}{2}\right]_0^1 = 3\,ms^{-1}$$

$[\because \ t = 1$ sec given$]$

From work-energy theorem,

$$W = \Delta KE = \frac{1}{2}m\left(V^2 - u^2\right) = \frac{1}{2} \times 1 \times 9 = 4.5\ J$$

30. (c) $F = -\frac{\partial u}{\partial r}\hat{r} = \frac{K}{r^3}\hat{r}$

Since particle is moving in circular path

$$F = \frac{mv^2}{r} = \frac{K}{r^3} \Rightarrow mv^2 = \frac{K}{r^2}$$

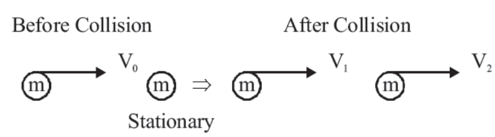
$$\therefore \ K.E. = \frac{1}{2}mv^2 = \frac{K}{2r^2}$$

Total energy = P.E. + K.E.

$$= -\frac{K}{2r^2} + \frac{K}{2r^2} = \text{Zero}$$

$$\left(\because P.E. = -\frac{K}{2r^2} \text{ given}\right)$$

31. (b)



$$\frac{1}{2}mv_1^2 + \frac{1}{2}mv_2^2 = \frac{3}{2}\left(\frac{1}{2}mv_0^2\right)$$

$$\Rightarrow v_1^2 + v_2^2 = \frac{3}{2}v_0^2 \qquad ....(i)$$

From momentum conservation

$mv_0 = m(v_1 + v_2)$ ....(ii)

Squarring both sides,

$(v_1 + v_2)^2 = v_0^2$

$\Rightarrow \ v_1^2 + v_2^2 + 2v_1v_2 = v_0^2$

$$2v_1v_2 = -\frac{v_0^2}{2}$$

$$(v_1 - v_2)^2 = v_1^2 + v_2^2 - 2v_1v_2 = \frac{3}{2}v_0^2 + \frac{v_0^2}{2}$$

Solving we get relative velocity between the two particles

$v_1 - v_2 = \sqrt{2}v_0$

32. (a) For collision of neutron with deuterium:

Applying conservation of momentum :

$mv + 0 = mv_1 + 2mv_2$ .....(i)

$v_2 - v_1 = v$ .....(ii)

$\because$ Collision is elastic, e = 1

From eqn (i) and eqn (ii) $v_1 = -\frac{v}{3}$

$$P_d = \frac{\frac{1}{2}mv^2 - \frac{1}{2}mv_1^2}{\frac{1}{2}mv^2} = \frac{8}{9} = 0.89$$

Now, For collision of neutron with carbon nucleus

Applying Conservation of momentum

$mv + 0 = mv_1 + 12mv_2$ ....(iii)

$v = v_2 - v_1$ ....(iv)

From eqn (iii) and eqn (iv)

$$v_1 = -\frac{11}{13}v$$

$$P_c = \frac{\frac{1}{2}mv^2 - \frac{1}{2}m\left(\frac{11}{13}v\right)^2}{\frac{1}{2}mv^2} = \frac{48}{169} \approx 0.28$$

33. (d) Maximum speed is at equilibrium position where

$$F = kx \Rightarrow x = \frac{F}{k}$$

From work-energy theorem,

$$W_F + W_{sp} = \Delta KE$$

$$F(x) - \frac{1}{2}kx^2 = \frac{1}{2}mv^2 - 0$$

$$F\left(\frac{F}{k}\right) - \frac{1}{2}k\left(\frac{F}{k}\right)^2 = \frac{1}{2}mv^2$$

$$\Rightarrow \frac{1}{2}\frac{F^2}{K} = \frac{1}{2}mv^2$$

or, $v_{max} = \frac{F}{\sqrt{mk}}$

34. (c) Kinetic energy of block A

$$k_1 = \frac{1}{2}mv_0^2$$

$\therefore$ From principle of linear momentum conservation

$$mv_0 = (2m+M)v_f \Rightarrow v_f = \frac{mv_0}{2m+M}$$

$$K.E_f = \frac{1}{6}K.E_i \quad \text{(given)}$$

$$\frac{1}{2}(2m+M)v_f^2 = \frac{1}{6}\times\frac{1}{2}mv_0^2$$

$$6(2m+M)\frac{m^2v_0^2}{(2m+M)^2} = mv_0^2$$

$$\Rightarrow 6m = 2m + M$$

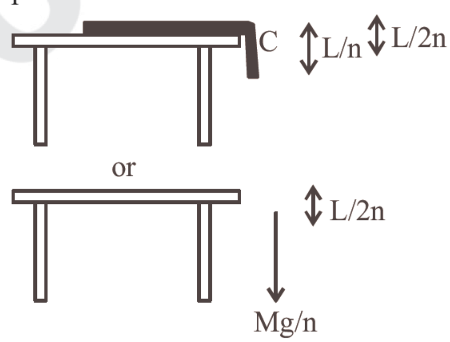
$$\Rightarrow 4m = M$$

35. (b) Length of hanging part = L/n

Mass of hanging part = M/n

Weight of hanging part = Mg/n

Let 'C' be the centre of mass of the hanging part.



The hanging part can be assumed to be a particle of weight Mg/n at a distance L/n below the table top. The work done in lifting it to the table top is equal to increase in its potential energy.

$$\therefore W = \left(\frac{Mg}{n}\right)\left(\frac{L}{n}\right)$$

$$\therefore W = \frac{MgL}{n^2}$$

36. (d) For head on elastic collision we have

$$V_1 = \frac{(m_1 - m_2)u_1}{m_1 + m_2} + \frac{2m_2u_2}{m_1 + m_2}$$

Here $m_1 = 2\text{kg}$, $u_1 = x$, $u_2 = 0$,

$v_1 = x/4$

$$\therefore \frac{x}{4} = \frac{(2-m_2)x}{2+m_2} \Rightarrow m_2 = 1.2\text{kg}$$

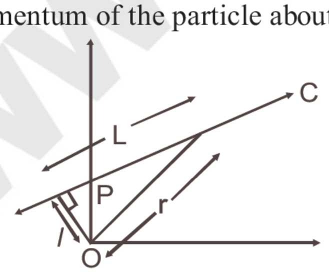
6 System of Particles and Rotational Motion

1. Initial angular velocity of a circular disc of mass M is ω_1. Then two small spheres of mass m are attached gently to diametrically opposite points on the edge of the disc. What is the final angular velocity of the disc? **[2002]**
(a) $\left(\frac{M+m}{M}\right)\omega_1$ (b) $\left(\frac{M+m}{m}\right)\omega_1$
(c) $\left(\frac{M}{M+4m}\right)\omega_1$ (d) $\left(\frac{M}{M+2m}\right)\omega_1$.

2. Two identical particles move towards each other with velocity $2v$ and v respectively. The velocity of centre of mass is **[2002]**
(a) v (b) $v/3$
(c) $v/2$ (d) zero

3. Moment of inertia of a circular wire of mass M and radius R about its diameter is **[2002]**
(a) $MR^2/2$ (b) MR^2
(c) $2MR^2$ (d) $MR^2/4$.

4. A particle of mass m moves along line PC with velocity v as shown. What is the angular momentum of the particle about P? **[2002]**



(a) mvL (b) mvl
(c) mvr (d) zero.

5. A circular disc X of radius R is made from an iron plate of thickness t, and another disc Y of radius $4R$ is made from an iron plate of thickness $\frac{t}{4}$. Then the relation between the moment of inertia I_X and I_Y is **[2003]**
(a) $I_Y = 32\, I_X$ (b) $I_Y = 16\, I_X$
(c) $I_Y = I_X$ (d) $I_Y = 64\, I_X$

6. A particle performing uniform circular motion has angular frequency is doubled & its kinetic energy halved, then the new angular momentum is **[2003]**
(a) $\frac{L}{4}$ (b) $2\,L$
(c) $4\,L$ (d) $\frac{L}{2}$

7. Let $\vec{F}$ be the force acting on a particle having position vector $\vec{r}$, and $\vec{T}$ be the torque of this force about the origin. Then **[2003]**
(a) $\vec{r}.\vec{T} = 0$ and $\vec{F}.\vec{T} \neq 0$
(b) $\vec{r}.\vec{T} \neq 0$ and $\vec{F}.\vec{T} = 0$
(c) $\vec{r}.\vec{T} \neq 0$ and $\vec{F}.\vec{T} \neq 0$
(d) $\vec{r}.\vec{T} = 0$ and $\vec{F}.\vec{T} = 0$

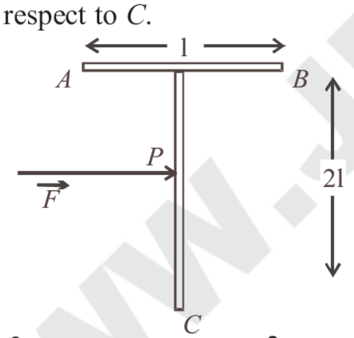
8. A solid sphere is rotating in free space. If the radius of the sphere is increased keeping mass same, which one of the following will not be affected ? **[2004]**
(a) Angular velocity
(b) Angular momentum
(c) Moment of inertia
(d) Rotational kinetic energy

9. One solid sphere A and another hollow sphere B are of same mass and same outer radii. Their moment of inertia about their diameters are respectively I_A and I_B such that **[2004]**
(a) $I_A < I_B$ (b) $I_A > I_B$
(c) $I_A = I_B$ (d) $\frac{I_A}{I_B} = \frac{d_A}{d_B}$
where d_A and d_B are their densities.

10. A body A of mass M while falling vertically downwards under gravity breaks into two parts; a body B of mass $\frac{1}{3}M$ and a body C of mass $\frac{2}{3}M$. The centre of mass of bodies B and C taken together shifts compared to that of body A towards **[2005]**
(a) does not shift
(b) depends on height of breaking
(c) body B
(d) body C

11. The moment of inertia of a uniform semicircular disc of mass M and radius r about a line perpendicular to the plane of the disc through the centre is **[2005]**
(a) $\frac{2}{5}Mr^2$ (b) $\frac{1}{4}Mr$
(c) $\frac{1}{2}Mr^2$ (d) Mr^2

12. A 'T' shaped object with dimensions shown in the figure, is lying on a smooth floor. A force '$\vec{F}$' is applied at the point P parallel to AB, such that the object has only the translational motion without rotation. Find the location of P with respect to C. **[2005]**

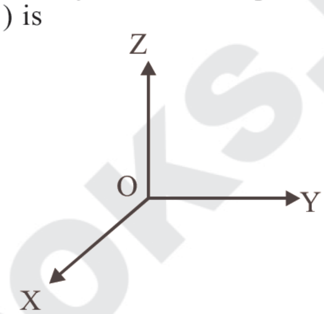


(a) $\frac{3}{2}\ell$ (b) $\frac{2}{3}\ell$
(c) ℓ (d) $\frac{4}{3}\ell$

13. Consider a two particle system with particles having masses m_1 and m_2. If the first particle is pushed towards the centre of mass through a distance d, by what distance should the second particle is moved, so as to keep the centre of mass at the same position? **[2006]**
(a) $\frac{m_2}{m_1}d$ (b) $\frac{m_1}{m_1+m_2}d$
(c) $\frac{m_1}{m_2}d$ (d) d

14. Four point masses, each of value m, are placed at the corners of a square $ABCD$ of side ℓ. The moment of inertia of this system about an axis passing through A and parallel to BD is **[2006]**
(a) $2m\ell^2$ (b) $\sqrt{3}m\ell^2$
(c) $3m\ell^2$ (d) $m\ell^2$

15. A force of $-F\hat{k}$ acts on O, the origin of the coordinate system. The torque about the point $(1, -1)$ is **[2006]**



(a) $F(\hat{i}-\hat{j})$ (b) $-F(\hat{i}+\hat{j})$
(c) $F(\hat{i}+\hat{j})$ (d) $-F(\hat{i}-\hat{j})$

16. A thin circular ring of mass m and radius R is rotating about its axis with a constant angular velocity ω. Two objects each of mass M are attached gently to the opposite ends of a diameter of the ring. The ring now rotates with an angular velocity ω' = **[2006]**
(a) $\frac{\omega(m+2M)}{m}$ (b) $\frac{\omega(m-2M)}{(m+2M)}$
(c) $\frac{\omega m}{(m+M)}$ (d) $\frac{\omega m}{(m+2M)}$

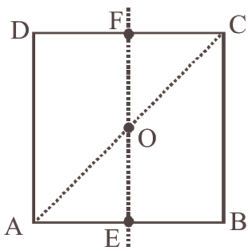
17. A circular disc of radius R is removed from a bigger circular disc of radius $2R$ such that the circumferences of the discs coincide. The centre of mass of the new disc is α / R form the centre of the bigger disc. The value of α is **[2007]**
(a) 1/4 (b) 1/3
(c) 1/2 (d) 1/6

18. A round uniform body of radius R, mass M and moment of inertia I rolls down (without slipping) an inclined plane making an angle θ with the horizontal. Then its acceleration is **[2007]**
(a) $\frac{g\sin\theta}{1-MR^2/I}$ (b) $\frac{g\sin\theta}{1+I/MR^2}$
(c) $\frac{g\sin\theta}{1+MR^2/I}$ (d) $\frac{g\sin\theta}{1-I/MR^2}$

19. Angular momentum of the particle rotating with a central force is constant due to **[2007]**

(a) constant torque
(b) constant force
(c) constant linear momentum
(d) zero torque

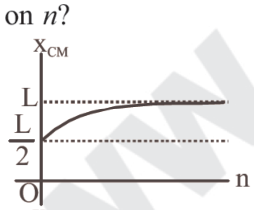
20. For the given uniform square lamina $ABCD$, whose centre is O, **[2007]**

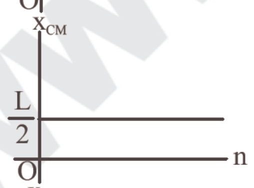


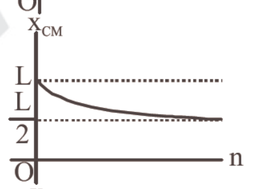
(a) $I_{AC} = \sqrt{2}\, I_{EF}$ (b) $\sqrt{2} I_{AC} = I_{EF}$

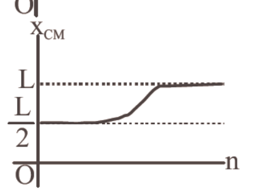
(c) $I_{AD} = 3 I_{EF}$ (d) $I_{AC} = I_{EF}$

21. A thin rod of length 'L' is lying along the x-axis with its ends at $x = 0$ and $x = L$. Its linear density (mass/length) varies with x as $k\left(\frac{x}{L}\right)^n$, where n can be zero or any positive number. If the position x_{CM} of the centre of mass of the rod is plotted against 'n', which of the following graphs best approximates the dependence of x_{CM} on n? **[2008]**

(a)


(b)


(c)


(d)


22. Consider a uniform square plate of side 'a' and mass 'M'. The moment of inertia of this plate about an axis perpendicular to its plane and passing through one of its corners is **[2008]**

(a) $\frac{5}{6}Ma^2$ (b) $\frac{1}{12}Ma^2$

(c) $\frac{7}{12}Ma^2$ (d) $\frac{2}{3}Ma^2$

23. A thin uniform rod of length l and mass m is swinging freely about a horizontal axis passing through its end. Its maximum angular speed is ω. Its centre of mass rises to a maximum height of **[2009]**

(a) $\frac{1}{6}\frac{l\omega}{g}$ (b) $\frac{1}{2}\frac{l^2\omega^2}{g}$

(c) $\frac{1}{6}\frac{l^2\omega^2}{g}$ (d) $\frac{1}{3}\frac{l^2\omega^2}{g}$

24. A mass m hangs with the help of a string wrapped around a pulley on a frictionless bearing. The pulley has mass m and radius R. Assuming pulley to be a perfect uniform circular disc, the acceleration of the mass m, if the string does not slip on the pulley, is: **[2011]**

(a) g (b) $\frac{2}{3}g$

(c) $\frac{g}{3}$ (d) $\frac{3}{2}g$

25. A thin horizontal circular disc is rotating about a vertical axis passing through its centre. An insect is at rest at a point near the rim of the disc. The insect now moves along a diameter of the disc to reach its other end. During the journey of the insect, the angular speed of the disc. **[2011]**

(a) continuously decreases
(b) continuously increases
(c) first increases and then decreases
(d) remains unchanged

26. A pulley of radius 2 m is rotated about its axis by a force $F = (20t - 5t^2)$ newton (where t is measured in seconds) applied tangentially. If the moment of inertia of the pulley about its axis of rotation is 10 kg-m^2 the number of

rotations made by the pulley before its direction of motion is reversed, is: **[2011]**

(a) more than 3 but less than 6
(b) more than 6 but less than 9
(c) more than 9
(d) less than 3

27. A loop of radius r and mass m rotating with an angular velocity ω_0 is placed on a rough horizontal surface. The initial velocity of the centre of the loop is zero. What will be the velocity of the centre of the loop when it ceases to slip? **[2013]**

(a) $\frac{r\omega_0}{4}$ (b) $\frac{r\omega_0}{3}$

(c) $\frac{r\omega_0}{2}$ (d) $r\omega_0$

28. A bob of mass m attached to an inextensible string of length l is suspended from a vertical support. The bob rotates in a horizontal circle with an angular speed ω rad/s about the vertical. About the point of suspension: **[2014]**

(a) angular momentum is conserved.
(b) angular momentum changes in magnitude but not in direction.
(c) angular momentum changes in direction but not in magnitude.
(d) angular momentum changes both in direction and magnitude.

29. Distance of the centre of mass of a solid uniform cone from its vertex is z_0. If the radius of its base is R and its height is h then z_0 is equal to : **[2015]**

(a) $\frac{5h}{8}$ (b) $\frac{3h^2}{8R}$

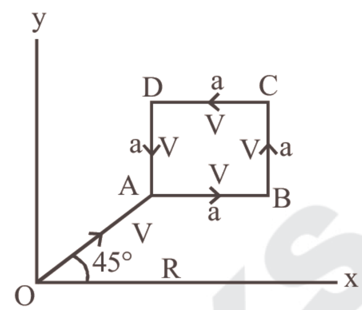
(c) $\frac{h^2}{4R}$ (d) $\frac{3h}{4}$

30. From a solid sphere of mass M and radius R a cube of maximum possible volume is cut. Moment of inertia of cube about an axis passing through its center and perpendicular to one of its faces is: **[2015]**

(a) $\frac{4MR^2}{9\sqrt{3}\pi}$ (b) $\frac{4MR^2}{3\sqrt{3}\pi}$

(c) $\frac{MR^2}{32\sqrt{2}\pi}$ (d) $\frac{MR^2}{16\sqrt{2}\pi}$

31. A particle of mass m is moving along the side of a square of side 'a', with a uniform speed v in the x-y plane as shown in the figure : **[2016]**



Which of the following statements is false for the angular momentum $\vec{L}$ about the origin?

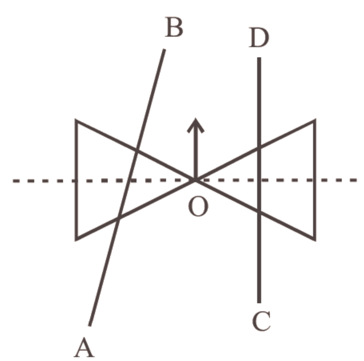
(a) $\vec{L} = mv\left[\frac{R}{\sqrt{2}} + a\right]\hat{k}$ when the particle is moving from B to C.

(b) $\vec{L} = \frac{mv}{\sqrt{2}}R\hat{k}$ when the particle is moving from D to A.

(c) $\vec{L} = -\frac{mv}{\sqrt{2}}R\hat{k}$ when the particle is moving from A to B.

(d) $\vec{L} = mv\left[\frac{R}{\sqrt{2}} - a\right]\hat{k}$ when the particle is moving from C to D.

32. A roller is made by joining together two cones at their vertices O. It is kept on two rails AB and CD, which are placed asymmetrically (see figure), with its axis perpendicular to CD and its centre O at the centre of line joining AB and CD (see figure). It is given a light push so that it starts rolling with its centre O moving parallel to CD in the direction shown. As it moves, the roller will tend to : **[2016]**



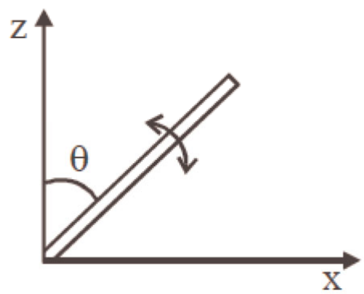
(a) go straight.
(b) turn left and right alternately.
(c) turn left.
(d) turn right.

33. The moment of inertia of a uniform cylinder of length ℓ and radius R about its perpendicular bisector is I. What is the ratio ℓ/R such that the moment of inertia is minimum? **[2017]**

(a) 1 (b) $\frac{3}{\sqrt{2}}$

(c) $\sqrt{\frac{3}{2}}$ (d) $\frac{\sqrt{3}}{2}$

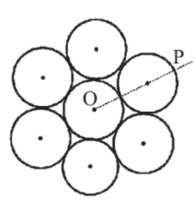
34. A slender uniform rod of mass M and length ℓ is pivoted at one end so that it can rotate in a vertical plane (see figure). There is negligible friction at the pivot. The free end is held vertically above the pivot and then released. The angular acceleration of the rod when it makes an angle θ with the vertical is **[2017]**



(a) $\frac{3g}{2\ell}\cos\theta$ (b) $\frac{2g}{3\ell}\cos\theta$

(c) $\frac{3g}{2\ell}\sin\theta$ (d) $\frac{2g}{2\ell}\sin\theta$

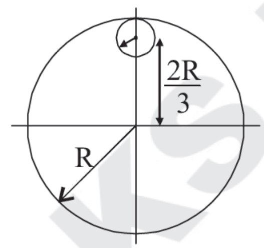
35. Seven identical circular planar disks, each of mass M and radius R are welded symmetrically as shown. The moment of inertia of the arrangement about the axis normal to the plane and passing through the point P is: **[2018]**



(a) $\frac{19}{2}MR^2$ (b) $\frac{55}{2}MR^2$

(c) $\frac{73}{2}MR^2$ (d) $\frac{181}{2}MR^2$

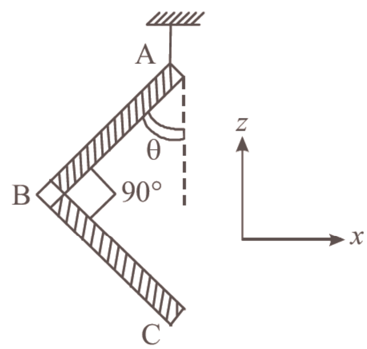
36. From a uniform circular disc of radius R and mass 9 M, a small disc of radius $\frac{R}{3}$ is removed as shown in the figure. The moment of inertia of the remaining disc about an axis perpendicular to the plane of the disc and passing, through centre of disc is : **[2018]**



(a) $4\ MR^2$ (b) $\frac{40}{9}\ MR^2$

(c) $10\ MR^2$ (d) $\frac{37}{9}MR^2$

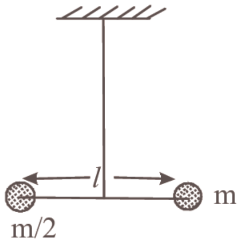
37. An L-shaped object, made of thin rods of uniform mass density, is suspended with a string as shown in figure. If AB = BC, and the angle made by AB with downward vertical is θ, then: **[2019]**



(a) $\tan\theta = \frac{1}{2\sqrt{3}}$ (b) $\tan\theta = \frac{1}{2}$

(c) $\tan\theta = \frac{2}{\sqrt{3}}$ (d) $\tan\theta = \frac{1}{3}$

38. Two masses m and $\frac{m}{2}$ are connected at the two ends of a massless rigid rod of length *l*. The rod is suspended by a thin wire of torsional constant *k* at the centre of mass of the rod-mass system (see figure). Because of torsional constant *k*, the restoring toruque is $\tau = k\theta$ for angular displacement θ. If the rod is rotated by θ_0 and released, the tension in it when it passes through its mean position will be: **[2019]**



(a) $\frac{3k\theta_0^2}{l}$ (b) $\frac{2k\theta_0^2}{l}$

(c) $\frac{k\theta_0^2}{l}$ (d) $\frac{k\theta_0^2}{2l}$

39. A stationary horizontal disc is free to rotate about its axis. When a torque is applied on it, its kinetic energy as a function of θ, where θ is the angle by which it has rotated, is given as $k\theta^2$. If its moment of inertia is I then the angular acceleration of the disc is **[2019]**

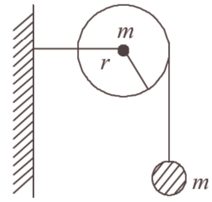
(a) $\frac{k}{4I}\theta$ (b) $\frac{k}{I}\theta$ (c) $\frac{k}{2I}\theta$ (d) $\frac{2k}{I}\theta$

40. The following bodies are made to roll up (without slipping) the same inclined plane from a horizontal plane : (i) a ring of radius R, (ii) a solid cylinder of radius $\frac{R}{2}$ and (iii) a solid sphere of radius $\frac{R}{4}$. If, in each case, the speed of the center of mass at the bottom of the incline is same, the ratio of the maximum heights they climb is : **[2019]**

(a) 4 : 3 : 2 (b) 10 : 15 : 7

(c) 14 : 15 : 20 (d) 2 : 3 : 4

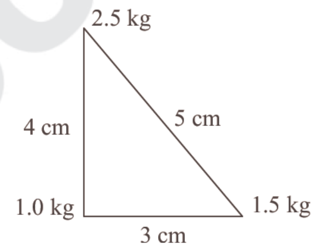
41. A s shown in the figure, a bob of mass *m* is tied by a massless string whose other end portion is wound on a fly wheel (disc) of radius r and mass m. When released from rest the bob starts falling vertically. When it has covered a distance of *h*, the angular speed of the wheel will be: **[2020]**



(a) $\frac{1}{r}\sqrt{\frac{4gh}{3}}$ (b) $r\sqrt{\frac{3}{2gh}}$

(c) $\frac{1}{r}\sqrt{\frac{2gh}{3}}$ (d) $r\sqrt{\frac{3}{4gh}}$

42. Three point particles of masses 1.0 kg, 1.5 kg and 2.5 kg are placed at three corners of a right angle triangle of sides 4.0 cm, 3.0 cm and 5.0 cm as shown in the figure. The center of mass of the system is at a point: **[2020]**



(a) 0.6 cm right and 2.0 cm above 1 kg mass
(b) 1.5 cm right and 1.2 cm above 1 kg mass
(c) 2.0 cm right and 0.9 cm above 1 kg mass
(d) 0.9 cm right an d 2.0 cm above 1 kg mass

43. The radius of gyration of a uniform rod of length l, about an axis passing through a point $\frac{l}{4}$ away from the centre of the rod, and perpendicular to it, is: **[2020]**

(a) $\frac{1}{4}l$ (b) $\frac{1}{8}l$

(c) $\sqrt{\frac{7}{48}}l$ (d) $\sqrt{\frac{3}{8}}l$

Answer Key

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
(c)	(c)	(a)	(d)	(d)	(a)	(d)	(b)	(a)	(a)	(c)	(d)	(c)	(c)	(c)
16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
(d)	(b)	(b)	(d)	(d)	(a)	(d)	(c)	(b)	(c)	(a)	(c)	(c)	(d)	(a)
31	32	33	34	35	36	37	38	39	40	41	42	43		
(a)	(c)	(c)	(c)	(d)	(a)	(d)	(c)	(d)	(c)	(a)	(d)	(c)		

Solutions

1. (c) Moment of inertia of circular disc

$$I_1 = \frac{1}{2}MR^2$$

When two small sphere are attached on the edge of the disc, the moment of inertia becomes

$$I_2 = \frac{1}{2}MR^2 + 2mR^2$$

When two small spheres of mass m are attached gently, the external torque, about the axis of rotation, is zero and therefore the angular momentum about the axis of rotation is constant.

$$\therefore\ I_1\omega_1 = I_2\omega_2 \Rightarrow \omega_2 = \frac{I_1}{I_2}\omega_1$$

$$\therefore \omega_2 = \frac{\frac{1}{2}MR^2}{\frac{1}{2}MR^2 + 2mR^2} \times \omega_1 = \frac{M}{M+4m}\omega_1$$

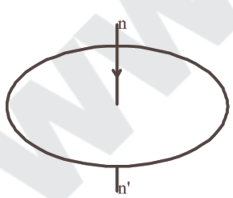
2. (c) The velocity of centre of mass of two particle system is given by

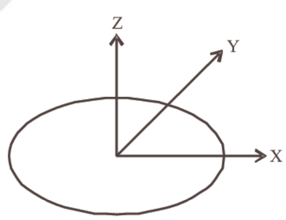
$$v_c = \frac{m_1v_1 + m_2v_2}{m_1 + m_2}$$

Here, $m_1 = m_2 = \text{m}$
and $v_1 = 2\text{v}$ and $\text{v}_2 = -\text{v}$

$$\therefore v_{cm} = \frac{m(2v)+m(-v)}{m+m} = \frac{v}{2}$$

3. (a) M. I of a circular wire about an axis nn' passing through the centre of the circle and perpendicular to the plane of the circle $= MR^2$





As shown in the figure, X-axis and Y-axis lie along diameter of the ring . Using perpendicular axis theorem

$I_X + I_Y = I_Z$

Here, I_X and I_Y are the moment of inertia about the diameter.

$\Rightarrow 2\,I_X = MR^2$ $\quad [\because I_X = I_Y$ (by symmetry) and $I_Z = MR^2]$

$$\therefore I_X = \frac{1}{2}MR^2$$

4. (d) Angular momentum (L)
= (linear momentum) × (perpendicular distance of the line of action of momentum from the axis of rotation)

As the particle moves with velocity V along line PC, the line of motion passes through P.

$\therefore$ L $= mv \times r$
$= mv \times 0$
$= 0$

5. (d) We know that density $(d) = \dfrac{mass(M)}{volume(V)}$

$\therefore\ M = d \times V = d \times (\pi R^2 \times t)$.

The moment of inertia of a disc is given by

$$I = \frac{1}{2}MR^2$$

$$\therefore\ I_x = \frac{1}{2}MxR_x^2 = \frac{1}{2}(d \times \pi R^2 \times t)R^2$$

$$= \frac{\pi d}{2}t \times R^4$$

$$I_y = \frac{1}{2}M_yR_y^2 = \frac{1}{2}\left[\pi\left(4R^2\right)\left(\frac{1}{4}\right)d\right] \times (4R)^2$$

$$\therefore\ \frac{I_X}{I_Y} = \frac{t_X R_X^4}{t_Y R_Y^4} = \frac{t \times R^4}{\frac{t}{4} \times (4R)^4} = \frac{1}{64}$$

6. (a) Rotational kinetic energy $= \dfrac{1}{2}I\omega^2$,

Angular momentum, $L = I\omega \Rightarrow I = \dfrac{L}{\omega}$

$$\therefore\ K.E. = \frac{1}{2}\frac{L}{\omega} \times \omega^2 = \frac{1}{2}L\omega$$

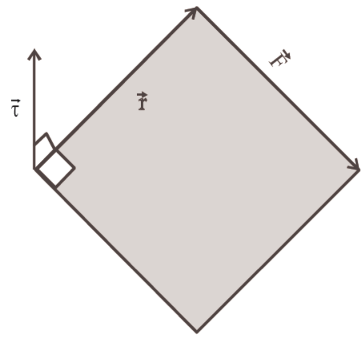
$$\text{L}' = \frac{2\text{K.E}}{\omega}$$

When ω is doubled and K.E is haled.
New angular momentum,

$$L' = \frac{\frac{2\text{K.E}}{2}}{2\omega}$$

$$\Rightarrow \therefore\ L' = \frac{L}{4}$$

7. **(d)** We know that $\vec{\tau} = \vec{r} \times \vec{F}$



Vector $\vec{\tau}$ is perpendicular to both $\vec{r}$ and $\vec{F}$. We also know that the dot product of two vectors which have an angle of 90° between them is zero.

$\therefore\ \vec{r} \cdot \vec{T} = 0$ and $\vec{F} \cdot \vec{T} = 0$

8. **(b)** Angular momentum will remain the same since no external torque act in free space.

9. **(a)** The moment of inertia of solid sphere A about its diameter $I_A = \frac{2}{5}MR^2$.

The moment of inertia of a hollow sphere B about its diameter $I_B = \frac{2}{3}MR^2$.

$\therefore I_A < I_B$

10. **(a)** The centre of mass of bodies B and C taken together does not shift as no external force acts. The centre of mass of the system continues its original path. It is only the internal forces which comes into play while breaking.

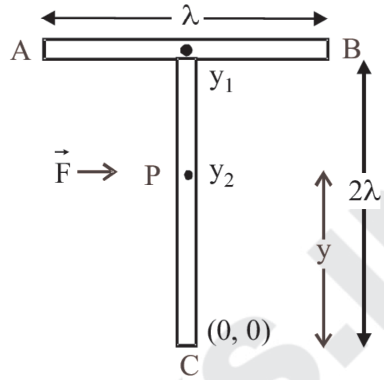
11. **(c)** The disc may be assumed as combination of two semi circular parts. Therefore, circular disc will have twice the mass of semicircular disc.

Moment of inertia of disc $= \frac{1}{2}(2m)r^2 = Mr^2$

Let I be the moment of inertia of the uniform semicircular disc

$$\Rightarrow 2I = 2Mr^2 \Rightarrow I = \frac{Mr^2}{2}$$

12. **(d)**

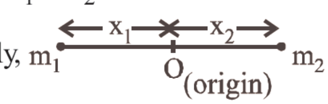


To have translational motion without rotation, the force $\vec{F}$ has to be applied at centre of mass. i.e. the point 'P' has to be at the centre of mass

Taking point C at the origin position, positions of y, and y_2 are $r_1 = 2l$, $r_2 = l$ and $ml = m$ and $m_2 = 2m$

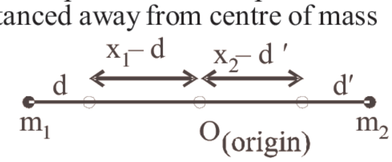
$$y = \frac{m_1 y_1 + m_2 y_2}{m_1 + m_2} = \frac{m \times 2\ell + 2m \times \ell}{3m} = \frac{4\ell}{3}$$

13. **(c)** Initially,



$$0 = \frac{m_1(-x_1) + m_2 x_2}{m_1 + m_2} \Rightarrow m_1 x_1 = m_2 x_2 \ ...(1)$$

Let the particles is displaced through distanced away from centre of mass

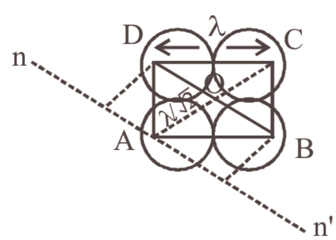


$$\therefore 0 = \frac{m_1(d - x_1) + m_2(x_2 - d')}{m_1 + m_2}$$

$$\Rightarrow 0 = m_1 d - m_1 x_1 + m_2 x_2 - m_2 d'$$

$$\Rightarrow d' = \frac{m_1}{m_2} d$$ [From (1).]

14. **(c)**



$I_{nn'}$ = $M.I$ due to the point mass at B + $M.I$ due to the point mass at D + $M.I$ due to the point mass at C.

$$I_{nn'} = m\left(\frac{\ell}{\sqrt{2}}\right)^2 + m\left(\frac{\ell}{\sqrt{2}}\right)^2 + m\left(\sqrt{2}\ell\right)^2$$

$$\Rightarrow I_{nn'} = 2 \times m\left(\frac{\ell}{\sqrt{2}}\right)^2 + m(\sqrt{2}\ell)^2$$

$$= m\ell^2 + 2m\ell^2 = 3m\ell^2$$

15. **(c)** Torque $\vec{\tau} = \vec{r} \times \vec{F} = (\hat{i} - \hat{j}) \times (-F\hat{k})$

$$= F[-\hat{i} \times \hat{k} + \hat{j} \times \hat{k}]$$

$$= F(\hat{j} + \hat{i}) = F(\hat{i} + \hat{j})$$

$$\left[\text{Since } \hat{k} \times \hat{i} = \hat{j} \text{ and } \hat{j} \times \hat{k} = \hat{i}\right]$$

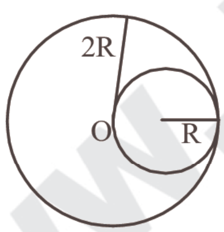
16. **(d)** Applying conservation of angular momentum $I'\omega' = I\omega$

$(mR^2 + 2MR^2)\omega' = mR^2\omega$

$\Rightarrow (m + 2m)R^2\omega' = mR^2\omega$

$$\Rightarrow \omega' = \omega\left[\frac{m}{m + 2M}\right]$$

17. **(b)** Let σ be the mass per unit area of the disc. Then the mass of the complete disc $= \sigma(\pi(2R)^2)$



The mass of the removed disc

$= \sigma(\pi R^2) = \pi\sigma R^2$

Let us consider the above situation to be a complete disc of radius $2R$ on which a disc of radius R of negative mass is superimposed. Let O be the origin. Then the above figure can be redrawn keeping in mind the concept of centre of mass as :

$$x_{c.m} = \frac{\left(6\pi(2R)^2\right) \times 0 + \left(-6\left(\pi R^2\right)\right)R}{4\pi\sigma R^2 - \pi\sigma R^2}$$

$$\therefore \; x_{c.m} = \frac{-\pi\sigma R^2 \times R}{3\pi\sigma R^2}$$

$$\therefore \; x_{c.m} = -\frac{R}{3} = \alpha R \Rightarrow \alpha = \frac{1}{3}$$

18. **(b)** Acceleration of the body rolling down an inclined plane is given by.

$$a = \frac{g\sin\theta}{1 + \frac{I}{MR^2}}$$

19. **(d)** We know Torque $\vec{\tau_c} = \frac{d\vec{L_c}}{dt}$

where $\vec{L_c}$ = Angular momentum about the center of mass of the body. Central forces act along the center of mass. Therefore torque about center of mass is zero.

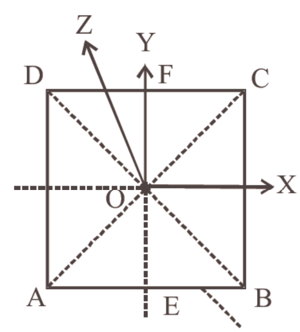
$$\therefore \tau = \frac{dL}{dt} = 0 \qquad \Rightarrow \vec{L_c} = \text{constt.}$$

20. **(d)** By the theorem of perpendicular axes,

$I = I_{EF} + I_{GH}$

Here, I is the moment of inertia of square lamina about an axis through O and perpendicular to its plane.

$\therefore \; I_{EF} = I_{GH}$ (By Symmetry of Figure)



$$\therefore \; I_{EF} = \frac{I}{2} \qquad \text{...(i)}$$

Again, by the same theorem $I = I_{AC} + I_{BD} = 2\, I_{AC}$ ($\therefore \; I_{AC} = I_{BD}$ by symmetry of the figure)

$$\therefore \; I_{AC} = \frac{I}{2} \qquad \text{...(ii)}$$

From (i) and (ii), we get, $I_{EF} = I_{AC}$.

21. **(a)** • The linear mass density $\lambda = k\left(\frac{x}{L}\right)^n$

Here $\frac{x}{L} \le 1$

With increase in the value of n, the centre of mass shift towards the end $x = L$ This is satisfied by only option (a).

$$x_{CM} = \frac{\int_0^L x\,dm}{\int_0^L dm} = \frac{\int_0^L x(\lambda dx)}{\int_0^L \lambda\,dx} = \frac{\int_0^L k\left(\frac{x}{L}\right)^n .xdx}{\int_0^L k\left(\frac{x}{L}\right)^n dx}$$

$$= \frac{k\left[\frac{x^{n+2}}{(n+2)L^n}\right]_0^L}{\left[\frac{k\,x^{n+1}}{(n+1)L^n}\right]_0^L} = \frac{L(n+1)}{n+2}$$

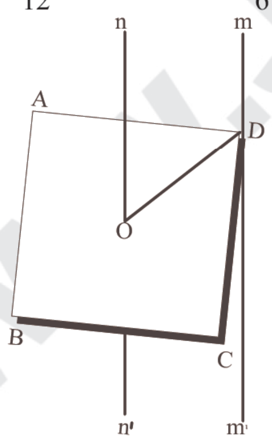
For $n = 0, x_{CM} = \frac{L}{2}$; $n = 1$,

$x_{CM} = \frac{2L}{3}$; $n = 2$, $x_{CM} = \frac{3L}{4}$;....

For $n \to \infty x_{cm} = L$

Moment of inertia of a square plate about an axis through its centre and perpendicular to its plane is.

22. **(d)** $I_{nn'} = \frac{1}{12}M(a^2 + a^2) = \frac{Ma^2}{6}$



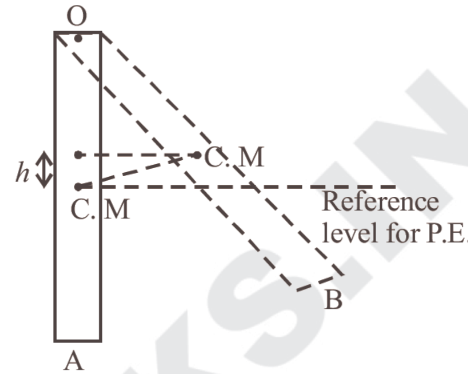
Also, $DO = \frac{DB}{2} = \frac{\sqrt{2}a}{2} = \frac{a}{\sqrt{2}}$

By parallel axes the orem, moment of inertia of plate about an axis through one of its corners.

$$I_{mm'} = I_{nn'} + M\left(\frac{a}{\sqrt{2}}\right)^2 = \frac{Ma^2}{6} + \frac{Ma^2}{2}$$

$$= \frac{Ma^2 + 3Ma^2}{6} = \frac{2}{3}Ma^2$$

23. **(c)**



The moment of inertia of the rod about O is $\frac{1}{3}m\ell^2$. The kinetic energy of the rod at position A = $\frac{1}{2}I\omega^2$ where I is the moment of inertia of the rod about O. When the rod is in position B, its angular velocity is zero. In this case, the energy of the rod is mgh where h is the maximum height to which the centre of mass (C.M) rises

Gain in potential energy = Loss in kinetic energy

$$\therefore\ mgh = \frac{1}{2}I\omega^2 = \frac{1}{2}\left(\frac{1}{3}ml^2\right)\omega^2$$

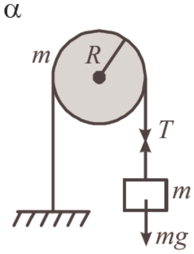
$$\Rightarrow\ h = \frac{\ell^2\omega^2}{6g}$$

24. **(b)** For translational motion,

$mg - T = ma$ .....(1)

For rotational motion,

T.R $= I\alpha$



$$\Rightarrow \text{T.R} = \frac{1}{2}mR^2\alpha$$

Also, acceleration, $a = R\alpha$

$$\therefore T = \frac{1}{2}mR\alpha = \frac{1}{2}ma$$

Substituting the value of T is equation (1)

we get mg - $\frac{1}{2}ma = ma \Rightarrow a = \frac{2}{3}g$

25. (c) Angular momentum, $L = I\omega \Rightarrow L = mr^2\omega$
As insect moves along a diameter, the effective mass and hence moment of inertia (I) first decreases then increases so from principle of conservation of angular momentum, angular speed ω first increases then decreases.

26. (a) Given,
Force, $F = (20t - 5t^2)$
Radius, $r = 2m$
Torque, $T = r\,f = I\alpha$
$\Rightarrow 2(20t - 5t^2) = 10\alpha$
$\therefore \alpha = 4t - t^2$

$$\Rightarrow \frac{d\omega}{dt} = 4t - t^2$$

$$\Rightarrow \int_0^{\omega} d\omega = \int_0^{t} \left(4t - t^2\right) dt$$

$$\Rightarrow \omega = 2t^2 - \frac{t^3}{3} \text{ (as } \omega = 0 \text{ at } t = 0, 6\text{s)}$$

$$\int_0^{\theta} d\theta = \int_0^{6} \left(2t^2 - \frac{t^3}{3}\right) dt$$

$\Rightarrow \theta = 36$ rad
$\Rightarrow 2\pi n = 36$

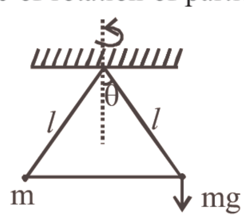
$$\Rightarrow n = \frac{36}{2\pi} < 6$$

27. (c)

From conservation of angular momentum about any fix point on the surface,
$mr^2\omega_0 = 2mr^2\omega$

$$\Rightarrow \omega = \omega_0/2 \Rightarrow v = \frac{\omega_0 r}{2} \quad [\because v = r\omega]$$

28. (c) Torque working on the bob of mass m is, $\tau = mg \times \ell \sin\theta$. (Direction parallel to plane of rotation of particle)

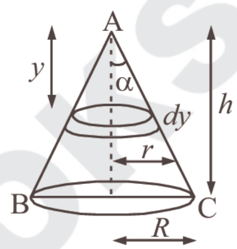


As τ is perpendicular to $\vec{L}$, direction of L changes but magnitude remains same.

29. (d) Let density of cone = ρ.

$$\text{Centre of mass, } ycm = \frac{\int y\,dm}{\int dm}$$

$$= \frac{\int_0^h y\pi r^2 dy\rho}{\frac{1}{3}\pi R^2 h\rho} = \frac{\int_0^h r^2 y\,dy}{\frac{1}{3}R^2 h} \quad ...(i)$$



For a cone, we know that

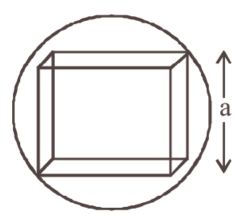
$$\frac{r}{R} = \frac{y}{n} \quad \therefore r = \frac{y}{n}R$$

$$ycm = \frac{\int_0^h 3y^3 dy}{h^3} = \frac{3\left[\frac{y^4}{4}\right]_0^h}{h^3} = \frac{3}{4}h$$

30. (a) Here $a = \frac{2}{\sqrt{3}}R$

$$\text{Now, } \frac{M}{M'} = \frac{\frac{4}{3}\pi R^3}{a^3}$$

$$= \frac{\frac{4}{3}\pi R^3}{\left(\frac{2}{\sqrt{3}}R\right)^3} = \frac{\sqrt{3}}{2}\pi.$$

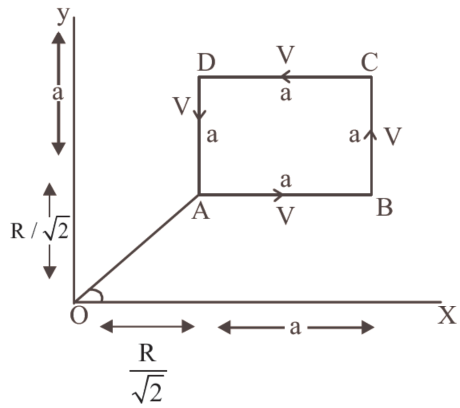


$$M' = \frac{2M}{\sqrt{3}\pi}$$

Moment of inertia of the cube about the given axis, $I = \frac{M'a^2}{6}$

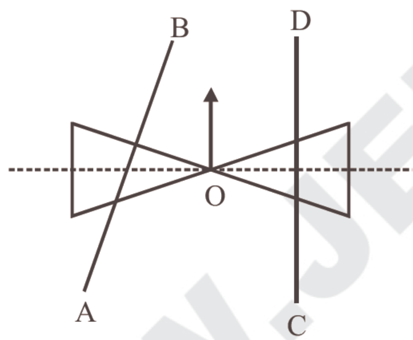
$$= \frac{\frac{2M}{\sqrt{3}\pi} \times \left(\frac{2}{\sqrt{3}}R\right)^2}{6} = \frac{4MR^2}{9\sqrt{3}\pi}$$

31. (a) We know that $|L| = mvr_{\perp}$



In none of the cases, the perpendicular distance $r_\perp$ is $\left(\frac{R}{\sqrt{2}}+a\right)$

32. **(c)** As shown in the diagram, the normal reaction of AB on roller will shift towards O.
This will lead to tending of the system of cones to turn left.



33. **(c)** As we know, moment of inertia of a solid cylinder about an axis which is perpendicular bisector

$$I=\frac{mR^2}{4}+\frac{ml^2}{12}$$

$$I=\frac{m}{4}\left[R^2+\frac{l^2}{3}\right]$$

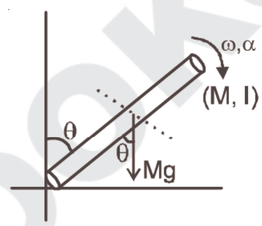
$$=\frac{m}{4}\left[\frac{V}{\pi l}+\frac{l^2}{3}\right] \Rightarrow \frac{dl}{dl}=\frac{m}{4}\left[\frac{-V}{\pi l^2}+\frac{2l}{3}\right]=0$$

$$\frac{V}{\pi l^2}=\frac{2l}{3} \quad \Rightarrow V=\frac{2\pi l^3}{3}$$

$$\pi R^2 l=\frac{2\pi l^3}{3} \Rightarrow \frac{l^2}{R^2}=\frac{3}{2} \text{ or, } \frac{l}{R}=\sqrt{\frac{3}{2}}$$

34. **(c)** Torque at angle θ

$$\tau=Mg\sin\theta\cdot\frac{l}{2}$$



Also $\tau=I\alpha$

$$\therefore \quad I\alpha=Mg\sin\theta\frac{l}{2}$$

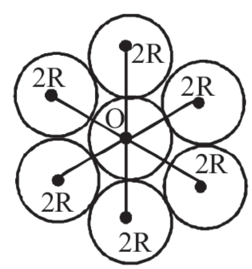
$$\frac{Ml^2}{3}\cdot\alpha=Mg\sin\theta\frac{l}{2} \quad \left[\because I_{rod}=\frac{Ml^2}{3}\right]$$

$$\Rightarrow \frac{l\alpha}{3}=g\frac{\sin\theta}{2} \quad \therefore \quad \alpha=\frac{3g\sin\theta}{2l}$$

35. **(d)** Using parallel axes theorem, moment of inertia about 'O'

$I_o=I_{cm}+md^2$

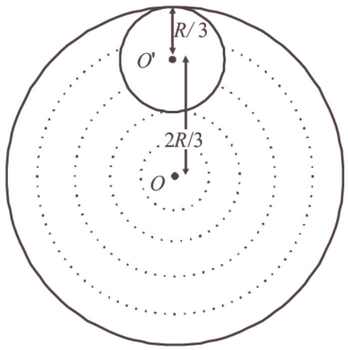
$$=\frac{7MR^2}{2}+6(M\times(2R)^2)=\frac{55MR^2}{2}$$



Again, moment of inertia about point P,
$I_p=I_o+md^2$

$$=\frac{55MR^2}{2}+7M(3R)^2=\frac{181}{2}MR^2$$

36. **(a)** Let σ be the mass per unit area.



The total mass of the disc
$= \sigma \times \pi R^2 = 9M$

The mass of the circular disc cut

$$= \sigma \times \pi\left(\frac{R}{3}\right)^2 = \sigma \times \frac{\pi R^2}{9} = M$$

Let us consider the above system as a complete disc of mass $9M$ and a negative mass M super imposed on it.

Moment of inertia (I_1) of the complete disc $= \frac{1}{2}9MR^2$ about an axis passing through O and perpendicular to the plane of the disc.

M.I. of the cut out portion about an axis passing through O' and perpendicular to the plane of disc

$$= \frac{1}{2} \times M \times \left(\frac{R}{3}\right)^2$$

$\therefore$ *M.I.* (I_2) of the cut out portion about an axis passing through O and perpendicular to the plane of disc

$$= \left[\frac{1}{2} \times M \times \left(\frac{R}{3}\right)^2 + M \times \left(\frac{2R}{3}\right)^2\right]$$

[Using perpendicular axis theorem]

$\therefore$ The total *M.I.* of the system about an axis passing through O and perpendicular to the plane of the disc is

$I = I_1 + I_2$

$$= \frac{1}{2}9MR^2 - \left[\frac{1}{2} \times M \times \left(\frac{R}{3}\right)^2 + M \times \left(\frac{2R}{3}\right)^2\right]$$

$$= \frac{9MR^2}{2} - \frac{9MR^2}{18} = \frac{(9-1)MR^2}{2} = 4MR^2$$

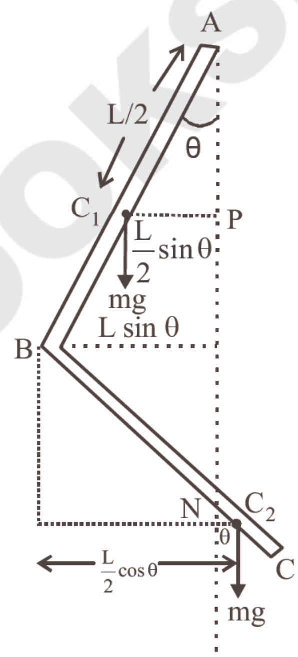
37. **(d)** Given that, the rod is of uniform mass density and AB = BC

Let mass of one rod is m.

Balancing torque about hinge point.

mg (C_1P) = mg (C_2N)

$$mg\left(\frac{L}{2}\sin\theta\right) = mg\left(\frac{L}{2}\cos\theta - L\sin\theta\right)$$



$$\Rightarrow \frac{3}{2}mgL\sin\theta = \frac{mgL}{2}\cos\theta \Rightarrow \frac{\sin\theta}{\cos\theta} = \frac{1}{3}$$

or, $\tan\theta = \frac{1}{3}$

38. **(c)** Distance of c.m from m/2

$$= \frac{\frac{m}{2} \times 0 + m \times \ell}{\frac{m}{2} + m} = \frac{2\ell}{3}$$

$$I_{cm} = \frac{m}{2}\left(\frac{2\ell}{3}\right)^2 + m\left(\frac{\ell}{3}\right)^2 = \frac{1}{3}m\ell^2$$

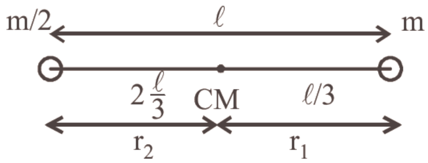
At the mean position

$$\frac{1}{2}I_{cm}\omega^2 = \frac{1}{2}k\theta_0^2$$

$$\therefore \omega^2 = \frac{k}{I_{cm}}\theta_0^2$$

$$\omega^2 = \frac{3k}{m\ell^2}\theta_0^2$$

As we know, $\omega = \sqrt{\frac{k}{I_{cm}}}$



Tension in the rod when it passes through the mean position,

$$= m\omega^2\frac{\ell}{3} = m\left[\frac{3k}{m\ell^2}\theta_0^2\right]\frac{\ell}{3} = \frac{k\theta_0^2}{\ell}$$

39. (d) Work done by torque is responsible for change in kinetic energy.

$$\therefore \tau = \frac{dE}{d\theta} \quad \therefore I\alpha = 2K\theta \therefore \alpha = \frac{2k\theta}{I}$$

40. (c) $mgh = \frac{1}{2}mv_{cm}^2 + \frac{1}{2}I_{cm}\omega^2$

$$= \frac{1}{2}mv_{cm}^2 + \frac{1}{2}I_{cm}\left(\frac{v_{cm}}{R}\right)^2$$

$$= \frac{1}{2}\left(m + \frac{I_{cm}}{R^2}\right)v_{cm}^2 = \frac{1m}{2}\left[1 + \frac{K^2}{R^2}\right]$$

$$\therefore \eta \propto 1 + \frac{K^2}{R^2}$$

For ring : $h \propto 2$ ($\therefore$ K = R)

For solid cylisnder, $h \propto \frac{3}{2}$ $\left(\because K = \frac{R}{\sqrt{2}}\right)$

For solid sphere, $h \propto \frac{7}{5}$ $\left(\because K = \sqrt{\frac{2}{5}}R\right)$

Ratio of heights $2 : \frac{3}{2} : \frac{7}{5} \Rightarrow 20 : 15 : 14$

41. (a) When the bob covered a distance 'h'

Using $mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$

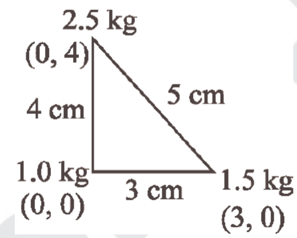
$$= \frac{1}{2}m(\omega r)^2 + \frac{1}{2}\times\frac{mr^2}{2}\times\omega^2$$

($\because v = \omega r$ no slipping)

$$\Rightarrow mgh = \frac{3}{4}m\omega^2 r^2$$

$$\Rightarrow \omega = \sqrt{\frac{4gh}{3r^2}} = \frac{1}{r}\sqrt{\frac{4gh}{3}}$$

42. (d)



$$X_{cm} = \frac{m_1x_1 + m_2x_2 + m_3x_3}{m_1 + m_2 + m_3}$$

$$X_{cm} = \frac{1\times 0 + 1.5\times 3 + 2.5\times 0}{1 + 1.5 + 2.5} = \frac{1.5\times 3}{5}$$

$= 0.9cm$

$$Y_{cm} = \frac{m_1y_1 + m_2y_2 + m_3y_3}{m_1 + m_2 + m_3}$$

$$Y_{cm} = \frac{1\times 0 + 1.5\times 0 + 2.5\times 4}{1 + 1.5 + 2.5}$$

$$= \frac{2.5\times 4}{5} = 2cm$$

Hence, centre of mass of system is at point (0.9, 2)

43. (c) Moment inertia of the rod passing through a point away from the centre of the rod $\frac{\ell}{4}$

$I = Ig + m\ell^2$

$$\Rightarrow I = \frac{MI^2}{12} + M\times\left(\frac{I^2}{16}\right) = \frac{7MI^2}{48}$$

Using $I = MK^2 = \frac{7MI^2}{48}$ (K = radius of gyration)

$$\Rightarrow K = \sqrt{\frac{7}{48}}I$$

7 Gravitation

1. The kinetic energy needed to project a body of mass m from the earth surface (radius R) to infinity is **[2002]**
   (a) $mgR/2$ (b) $2mgR$
   (c) mgR (d) $mgR/4$.

2. If suddenly the gravitational force of attraction between Earth and a satellite revolving around it becomes zero, then the satellite will **[2002]**
   (a) continue to move in its orbit with same velocity
   (b) move tangentially to the original orbit in the same velocity
   (c) become stationary in its orbit
   (d) move towards the earth

3. Energy required to move a body of mass m from an orbit of radius $2R$ to $3R$ is **[2002]**
   (a) $GMm/12R^2$ (b) $GMm/3R^2$
   (c) $GMm/8R$ (d) $GMm/6R$.

4. The escape velocity of a body depends upon mass as **[2002]**
   (a) m^0 (b) m^1
   (c) m^2 (d) m^3

5. The time period of a satellite of earth is 5 hours. If the separation between the earth and the satellite is increased to 4 times the previous value, the new time period will become **[2003]**
   (a) 10 hours (b) 80 hours
   (c) 40 hours (d) 20 hours

6. Two spherical bodies of mass M and $5M$ & radii R & $2R$ respectively are released in free space with initial separation between their centres equal to $12\,R$. If they attract each other due to gravitational force only, then the distance covered by the smaller body just before collision is **[2003]**
   (a) $2.5\,R$ (b) $4.5\,R$
   (c) $7.5\,R$ (d) $1.5\,R$

7. The escape velocity for a body projected vertically upwards from the surface of earth is 11 km/s. If the body is projected at an angle of 45°with the vertical, the escape velocity will be **[2003]**
   (a) $11\sqrt{2}$ km/s (b) 22 km/s
   (c) 11 km/s (d) $\frac{11}{\sqrt{2}}$ km/s

8. A satellite of mass m revolves around the earth of radius R at a height x from its surface. If g is the acceleration due to gravity on the surface of the earth, the orbital speed of the satellite is **[2004]**
   (a) $\frac{gR^2}{R+x}$ (b) $\frac{gR}{R-x}$
   (c) gx (d) $\left(\frac{gR^2}{R+x}\right)^{1/2}$

9. The time period of an earth satellite in circular orbit is independent of **[2004]**
   (a) both the mass and radius of the orbit
   (b) radius of its orbit
   (c) the mass of the satellite
   (d) neither the mass of the satellite nor the radius of its orbit.

10. If 'g' is the acceleration due to gravity on the earth's surface, the gain in the potential energy of an object of mass 'm' raised from the surface of the earth to a height equal to the radius 'R' of the earth is **[2004]**
    (a) $\frac{1}{4}mgR$ (b) $\frac{1}{2}mgR$
    (c) $2\,mgR$ (d) mgR

11. Suppose the gravitational force varies inversely as the nth power of distance. Then the time period of a planet in circular orbit of radius 'R' around the sun will be proportional to **[2004]**
    (a) R^n (b) $R^{\left(\frac{n-1}{2}\right)}$
    (c) $R^{\left(\frac{n+1}{2}\right)}$ (d) $R^{\left(\frac{n-2}{2}\right)}$

12. The change in the value of 'g' at a height 'h' above the surface of the earth is the same as at a depth 'd' below the surface of earth. When both 'd' and 'h' are much smaller than the radius of earth, then which one of the following is correct? **[2005]**

(a) $d=\frac{3h}{2}$ (b) $d=\frac{h}{2}$
(c) $d=h$ (d) $d=2h$

13. A particle of mass 10 g is kept on the surface of a uniform sphere of mass 100 kg and radius 10 cm. Find the work to be done against the gravitational force between them to take the particle far away from the sphere **[2005]**

(you may take $G=6.67\times 10^{-11}\ \mathrm{Nm}^2/\mathrm{kg}^2$)

(a) 3.33×10^{-10} J (b) 13.34×10^{-10} J
(c) 6.67×10^{-10} J (d) 6.67×10^{-9} J

14. Average density of the earth **[2005]**
(a) is a complex function of g
(b) does not depend on g
(c) is inversely proportional to g
(d) is directly proportional to g

15. A planet in a distant solar system is 10 times more massive than the earth and its radius is 10 times smaller. Given that the escape velocity from the earth is 11 km s^{-1}, the escape velocity from the surface of the planet would be **[2008]**
(a) 1.1 km s^{-1} (b) 11 km s^{-1}
(c) 110 km s^{-1} (d) 0.11 km s^{-1}

16. This question contains Statement-1 and Statement-2. Of the four choices given after the statements, choose the one that best describes the two statements. **[2008]**
Statement-1 : For a mass M kept at the centre of a cube of side 'a', the flux of gravitational field passing through its sides $4\pi GM$. and
Statement-2: If the direction of a field due to a point source is radial and its dependence on the distance 'r' from the source is given as $\frac{1}{r^2}$, its flux through a closed surface depends only on the strength of the source enclosed by the surface and not on the size or shape of the surface.
(a) Statement -1 is false, Statement-2 is true
(b) Statement -1 is true, Statement-2 is true; Statement -2 is a correct explanation for Statement-1
(c) Statement -1 is true, Statement-2 is true; Statement -2 is not a correct explanation for Statement-1
(d) Statement -1 is true, Statement-2 is false

17. The height at which the acceleration due to gravity becomes $\frac{g}{9}$ (where g = the acceleration due to gravity on the surface of the earth) in terms of R, the radius of the earth, is **[2009]**

(a) $\frac{R}{\sqrt{2}}$ (b) $R/2$
(c) $\sqrt{2}R$ (d) 2R

18. Two bodies of masses m and $4\,m$ are placed at a distance r. The gravitational potential at a point on the line joining them where the gravitational field is zero is: **[2011]**

(a) $-\frac{4Gm}{r}$ (b) $-\frac{6Gm}{r}$
(c) $-\frac{9Gm}{r}$ (d) zero

19. Two particles of equal mass 'm' go around a circle of radius R under the action of their mutual gravitational attraction. The speed of each particle with respect to their centre of mass is **[2011 RS]**

(a) $\sqrt{\frac{Gm}{4R}}$ (b) $\sqrt{\frac{Gm}{3R}}$
(c) $\sqrt{\frac{Gm}{2R}}$ (d) $\sqrt{\frac{Gm}{R}}$

20. The mass of a spaceship is 1000 kg. It is to be launched from the earth's surface out into free space. The value of g and R (radius of earth) are 10 m/s^2 and 6400 km respectively. The required energy for this work will be **[2012]**
(a) 6.4×10^{11} Joules (b) 6.4×10^{8} Joules
(c) 6.4×10^{9} Joules (d) 6.4×10^{10} Joules

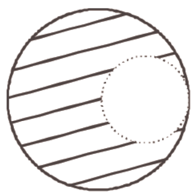
21. What is the minimum energy required to launch a satellite of mass m from the surface of a planet of mass M and radius R in a circular orbit at an altitude of 2R? **[2013]**

(a) $\frac{5\mathrm{GmM}}{6\mathrm{R}}$ (b) $\frac{2\mathrm{GmM}}{3\mathrm{R}}$
(c) $\frac{\mathrm{GmM}}{2\mathrm{R}}$ (d) $\frac{\mathrm{GmM}}{2\mathrm{R}}$

22. Four particles, each of mass M and equidistant from each other, move along a circle of radius R under the action of their mutual gravitational attraction. The speed of each particle is: **[2014]**

(a) $\sqrt{\frac{\mathrm{GM}}{\mathrm{R}}}$ (b) $\sqrt{2\sqrt{2}\frac{\mathrm{GM}}{\mathrm{R}}}$
(c) $\sqrt{\frac{\mathrm{GM}}{\mathrm{R}}\left(1+2\sqrt{2}\right)}$ (d) $\frac{1}{2}\sqrt{\frac{\mathrm{GM}}{\mathrm{R}}\left(1+2\sqrt{2}\right)}$

23. From a solid sphere of mass M and radius R, a spherical portion of radius R/2 is removed, as shown in the figure. Taking gravitational potential V = 0 at r = ∞, the potential at the centre of the cavity thus formed is : **[2015]**
(*G = gravitational constant*)

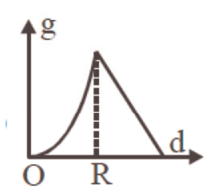


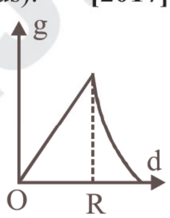
(a) $\frac{-2GM}{3R}$ (b) $\frac{-2GM}{R}$
(c) $\frac{-GM}{2R}$ (d) $\frac{-GM}{R}$

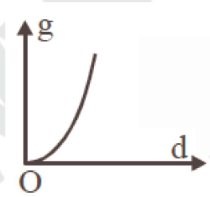
24. A satellite is revolving in a circular orbit at a height 'h' from the earth's surface (radius of earth R; h << R). The minimum increase in its orbital velocity required, so that the satellite could escape from the earth's gravitational field, is close to : (Neglect the effect of atmosphere.) **[2016]**

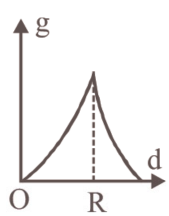
(a) $\sqrt{gR/2}$ (b) $\sqrt{gR}\left(\sqrt{2}-1\right)$
(c) $\sqrt{2gR}$ (d) $\sqrt{gR}$

25. The variation of acceleration due to gravity g with distance *d* from centre of the earth is best represented by (R = Earth's radius): **[2017]**

(a)


(b)


(c)


(d)


26. A particle is moving with a uniform speed in a circular orbit of radius R in a central force inversely proportional to the n[th] power of R. If the period of rotation of the particle is T, then: **[2018]**

(a) $T \propto R^{3/2}$ for any n. (b) $T \propto R^{n/2+1}$
(c) $T \propto R^{(n+1)/2}$ (d) $T \propto R^{n/2}$

27. If the angular momentum of a planet of mass m, moving around the Sun in a circular orbit is L, about the center of the Sun, its areal velocity is: **[2019]**

(a) $\frac{L}{m}$ (b) $\frac{4L}{m}$ (c) $\frac{L}{2m}$ (d) $\frac{2L}{m}$

28. A solid sphere of mass 'M' and radius '*a*' is surrounded by a uniform concentric spherical shell of thickness 2*a* and mass 2M. The gravitational field at distance '3*a*' from the centre will be: **[2019]**

(a) $\frac{2GM}{9a^2}$ (b) $\frac{GM}{9a^2}$
(c) $\frac{GM}{3a^2}$ (d) $\frac{2GM}{3a^2}$

29. A satellite of mass *m* is launched vertically upwards with an initial speed *u* from the surface of the earth. After it reaches height *R* (*R* = radius of the earth), it ejects a rocket of mass so that subsequently the satellite moves in a circular orbit. The kinetic energy of the rocket is (*G* is the gravitational constant; *M* is the mass of the earth): **[2020]**

(a) $\frac{m}{20}\left(u^2+\frac{113}{200}\frac{GM}{R}\right)$

(b) $5m\left(u^2-\frac{119}{200}\frac{GM}{R}\right)$

(c) $\frac{3m}{8}\left(u+\sqrt{\frac{5GM}{6R}}\right)^2$

(d) $\frac{m}{20}\left(u-\sqrt{\frac{2GM}{3R}}\right)^2$

Answer Key

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
(c)	(b)	(d)	(a)	(c)	(c)	(c)	(d)	(c)	(b)	(c)	(d)	(c)	(d)	(c)
16	17	18	19	20	21	22	23	24	25	26	27	28	29	
(b)	(d)	(c)	(a)	(d)	(a)	(d)	(d)	(b)	(b)	(c)	(c)	(c)	(b)	

Solutions

1. **(c)** $K.E = \frac{1}{2} m v_e^2$

Here v_e = escape velocity is independent of mass of the body

Escape velocity, $v_e = \sqrt{2gR}$

Substituting value of v_e in above equation we get

$K.E = \frac{1}{2} m \times 2gR = mgR$

2. **(b)** Due to inertia of motion it will move tangentially to the original orbit with the same velocity.

3. **(d)** Gravitational potential energy of mass m in an orbit of radius R

$u = -\frac{GMm}{R}$

Energy required = potential energy at 3R – potential energy a 2R

$= \frac{-GMm}{3R} - \left(\frac{-GMm}{2R}\right)$

$= \frac{-GMm}{3R} + \frac{GMm}{2R}$

$= \frac{-2GMm + 3GMm}{6R} = \frac{GMm}{6R}$

4. **(a)** Escape velocity, $v_e = \sqrt{2gR} = \sqrt{\frac{2GM}{R}}$

$\Rightarrow v_e \propto m^0$

Where M, R are the mass and radius of the planet respectively. Clearly, escape velocity is independent of mass of the body

5. **(c)** According to Kepler's law of periods $T^2 \propto R^3$

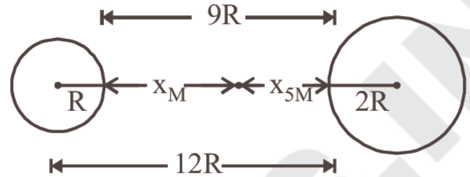
$\therefore \left(\frac{T_2}{T_1}\right)^2 = \left(\frac{R_2}{R_1}\right)^3$

$\Rightarrow T_2 = T_1\left(\frac{R_2}{R_1}\right)^{3/2} = 5 \times \left[\frac{4R}{R}\right]^{3/2}$

$= 5 \times 2^3 = 40$ hours

6. **(c)** We know that

Force = mass × acceleration.



The gravitational force acting on both the masses is the same.

$F_1 = F_2$

$ma_1 = ma_2$

$\Rightarrow \frac{9M}{95M} = \frac{5M}{M} = 5$

$\Rightarrow \frac{9M}{95M} = \frac{1}{5}$

Let t be the time taken for the two masses to collide and x_{5M}, x_M be the distance travelled by the mass $5M$ and M respectively.

For mass 5*M*

$u = 0,$

$S = ut + \frac{1}{2}at^2$

$\therefore \ x_{5M} = \frac{1}{2}a_{5M}t^2$ ....(ii)

For mass *M*

$u = 0, s = x_M, t = t, \ a = a_M$

$\therefore \ s = ut + \frac{1}{2}at^2$

$\Rightarrow x_M = \frac{1}{2}a_M t^2$ ...(iii)

Dividing (ii) by (iii)

$\frac{x_{5M}}{x_M} = \frac{\frac{1}{2}a_{5M}t^2}{\frac{1}{2}a_M t^2} = \frac{a_{5M}}{a_M} = \frac{1}{5}$ [From (i)]

$\therefore \ 5x_{5M} = x_M$ ....(iv)

From the figure it is clear that

$x_{5M} + x_M = 9R$ ....(v)

Where O is the point where the two spheres collide.

From (iv) and (v)

$\frac{x_M}{5} + x_M = 9R$

$\therefore \quad 6x_M = 45R$

$\therefore \quad x_M = \frac{45}{6}R = 7.5R$

7. (c) $v_e = \sqrt{2gR}$

Clearly escape velocity does not depend on the angle at which the body is projected.

8. (d) Gravitational force provides the necessary centripetal force.

$\therefore$ Centripetal force on a satellite = Gravitational force

$$\therefore \quad \frac{mv^2}{(R+x)} = \frac{GmM}{(R+x)^2} \text{ also } g = \frac{GM}{R^2}$$

$$\therefore \frac{mv^2}{(R+x)} = m\left(\frac{GM}{R^2}\right)\frac{R^2}{(R+x)^2}\frac{n!}{r!(n-r)!}$$

$$\therefore \frac{mv^2}{(R+x)} = mg\frac{R^2}{(R+x)^2}$$

$$\therefore v^2 = \frac{gR^2}{R+x} \Rightarrow v = \left(\frac{gR^2}{R+x}\right)^{1/2}$$

9. (c) Time period of satellite is given by

$$T = 2\pi\sqrt{\frac{(R+h)^3}{GM}}$$

Where $R + h$ = radius of orbit of satellite
M = mass of earth.

Time period is independent of mass of satellite.

10. (b) On earth's surface potential energy,

$$U = \frac{GmM}{R}$$

At a height R from the earth's surface, $P.E.$

of system $= -\frac{GmM}{2R}$

$$\therefore \Delta U = \frac{-GmM}{2R} + \frac{GmM}{R};$$

$$\Rightarrow \Delta U = \frac{GmM}{2R}$$

Now $\frac{GM}{R^2} = g; \therefore \frac{GM}{R} = gR$

$$\therefore \Delta U = \frac{1}{2}mgR$$

11. (c) Gravitational force, $F = KR^{-n}$

This force provides the centripetal force $MR\omega^2$ to the planet at height h above earth's surface.

$\therefore \quad F = KR^{-n} = MR\omega^2$

$\Rightarrow \quad \omega^2 = KR^{-(n+1)}$

$$\Rightarrow \quad \omega = KR^{\frac{-(n+1)}{2}}$$

$$\frac{2\pi}{T} \propto R^{\frac{-(n+1)}{2}}$$

$$\therefore T \propto R^{\frac{+(n+1)}{2}}$$

12. (d) Value of g with altitude is,

$$g_h = g\left[1 - \frac{2h}{R}\right];$$

Value of g at depth d below earth's surface,

$$g_d = g\left[1 - \frac{d}{R}\right]$$

Equating g_h and g_d, we get $d = 2h$

13. (c) Initial P.E. $U_i = -\frac{GMm}{R}$

When the particle is far away from the sphere, the P.E. of the system is zero.

$\therefore \quad U_f = 0$

$$W = \Delta U = U_f - U_i = 0 - \left[\frac{-GMm}{R}\right]$$

$$W = \frac{6.67 \times 10^{-11} \times 100}{0.1} \times \frac{10}{1000}$$

$= 6.67 \times 10^{-10}$ J

14. (d) Value of g on earth's surface,

$$g = \frac{GM}{R^2} = \frac{G\rho \times V}{R^2}$$

$$\Rightarrow \quad g = \frac{G \times \rho \times \frac{4}{3}\pi R^3}{R^2}$$

$g = \frac{4}{3}\rho\pi G.R$ where $\rho \rightarrow$ average density

$$\rho = \left(\frac{3g}{4\pi GR}\right)$$

$\Rightarrow \rho$ is directly proportional to g.

15. **(c)** Escape velocity on earth,

$$v_e = \sqrt{\frac{2GM_e}{R_e}} = 11\text{ km } s^{-1}$$

$$\therefore \quad \frac{(v_e)_p}{(v_e)_e} = \frac{\sqrt{\frac{2GM_p}{R_p}}}{\sqrt{\frac{2GM_e}{\mathrm{R_e}}}} = \sqrt{\frac{M_p}{M_e} \times \frac{\mathrm{R_e}}{R_p}}$$

$$= \sqrt{\frac{10M_e}{M_e} \times \frac{\mathrm{R_e}}{\mathrm{R_e}/10}} = 10$$

$$\therefore (v_e)_p = 10 \times (v_e)_e = 10 \times 11 = 110\ km/s$$

16. **(b)** Gravitational field, $E = -\frac{GM}{r^2}$

Flux, $\phi = \int \overrightarrow{E_g} \cdot \vec{dS} = |E \cdot 4\pi r^2| = -4\pi GM$

where, M = mass enclosed in the closed surface

This relationship is valid when $|\vec{E}_g| \propto \frac{1}{r^2}$.

17. **(d)** On earth's surface $g = \frac{GM}{R^2}$

At height above earth's surface

$$g_h = \frac{GM}{(R+h)^2}$$

$$\therefore \frac{g_n}{g} = \frac{R^2}{(R+h)^2}$$

$$\Rightarrow \frac{g/9}{g} = \left[\frac{R}{R+h}\right]^2$$

$$\Rightarrow \frac{R}{R+h} = \frac{1}{3}$$

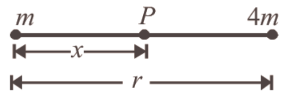
$$\therefore \quad h = 2R$$

18. **(c)** Let P be the point where gravitational field is zero.

$$\therefore \frac{Gm}{x^2} = \frac{4Gm}{(r-x)^2}$$

$$\Rightarrow \frac{1}{x} = \frac{2}{r-x} \quad \Rightarrow r - x = 2x$$

$$\Rightarrow x = \frac{r}{3}$$

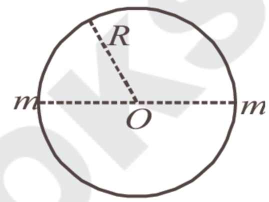


Gravitational potential at P,

$$V = -\frac{Gm}{\frac{r}{3}} - \frac{4Gm}{\frac{2r}{3}} = -\frac{9Gm}{r}$$

19. **(a)** As two masses revolve about the common centre of mass O.

$\therefore$ Mutual gravitational attraction = centripetal force



$$\frac{Gm^2}{(2R)^2} = m\omega^2 R$$

$$\Rightarrow \quad \frac{Gm}{4R^3} = \omega^2$$

$$\Rightarrow \quad \omega = \sqrt{\frac{Gm}{4R^3}}$$

If the velocity of the two particles with respect to the centre of gravity is v then $v = \omega R$

$$v = \sqrt{\frac{Gm}{4R^3}} \times R = \sqrt{\frac{Gm}{4R}}$$

20. **(d)** The work done to launch the spaceship

$$W = -\int_R^\infty \vec{F} \cdot \vec{dr} = -\int_R^\infty \frac{GMm}{r^2} dr$$

$$W = +\frac{GMm}{R} \qquad \ldots\text{(i)}$$

The force of attraction of the earth on the spaceship, when it was on the earth's surface

$$F = \frac{GMm}{R^2}$$

$$\Rightarrow \quad mg = \frac{GMm}{R^2} \Rightarrow g = \frac{GM}{R^2} \qquad \ldots\text{(ii)}$$

Substituting the value of g in (i) we get

$$W = \frac{gR^2 m}{R}$$

$\Rightarrow \quad W = mgR$

$\Rightarrow W = 1000 \times 10 \times 6400 \times 10^3$

$= 6.4 \times 10^{10}$ Joule

21. (a) As we know,

Gravitational potential energy $= \dfrac{-GMm}{r}$

and orbital velocity, $v_0 = \sqrt{GM/R+h}$

$$E_f = \frac{1}{2}mv_0^2 - \frac{GMm}{3R} = \frac{1}{2}m\frac{GM}{3R} - \frac{GMm}{3R}$$

$$= \frac{GMm}{3R}\left(\frac{1}{2}-1\right) = \frac{-GMm}{6R}$$

$$E_i = \frac{-GMm}{R} + K$$

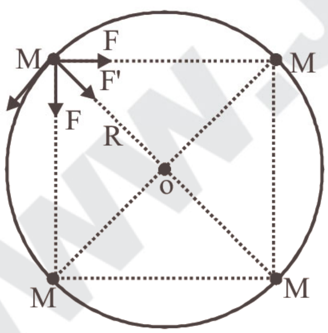
$E_i = E_f$

Therefore minimum required energy,

$$K = \frac{5GMm}{6R}$$

22. (d) $2F\cos 45° + F' = \dfrac{Mv^2}{R}$ (From figure)

Where $F = \dfrac{GM^2}{(\sqrt{2}R)^2}$ and $F' = \dfrac{GM^2}{4R^2}$



$$\Rightarrow \quad \frac{2 \times GM^2}{\sqrt{2}(R\sqrt{2})^2} + \frac{GM^2}{4R^2} = \frac{Mv^2}{R}$$

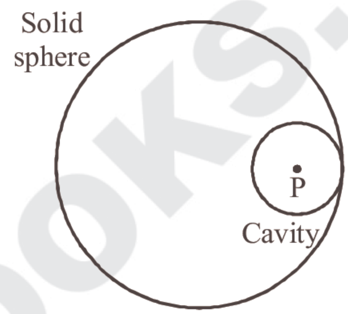
$$\Rightarrow \quad \frac{GM^2}{R}\left[\frac{1}{4} + \frac{1}{\sqrt{2}}\right] = Mv^2$$

$$\therefore \; v = \sqrt{\frac{GM}{R}\left(\frac{\sqrt{2}+4}{4\sqrt{2}}\right)} = \frac{1}{2}\sqrt{\frac{GM}{R}(1+2\sqrt{2})}$$

23. (d) Due to complete solid sphere, potential at point P

$$V_{sphere} = \frac{-GM}{2R^3}\left[3R^2 - \left(\frac{R}{2}\right)^2\right]$$

$$= \frac{-GM}{2R^3}\left(\frac{11R^2}{4}\right) = -11\frac{GM}{8R}$$



Due to cavity part potential at point P

$$V_{cavity} = -\frac{3}{2}\frac{\frac{GM}{8}}{\frac{R}{2}} = -\frac{3GM}{8R}$$

So potential at the centre of cavity

$= V_{sphere} - V_{cavity}$

$$= -\frac{11GM}{8R} - \left(-\frac{3}{8}\frac{GM}{R}\right) = \frac{-GM}{R}$$

24. (b) For h << R, the orbital velocity is $\sqrt{gR}$

Escape velocity $= \sqrt{2gR}$

$\therefore$ The minimum increase in its orbital velocity

$= \sqrt{2gR} - \sqrt{gR} = \sqrt{gR}\,(\sqrt{2}-1)$

25. (b) Variation of acceleration due to gravity, g with distance 'd' from centre of the earth

If $d < R, g = \dfrac{Gm}{R^2} \cdot d \quad i.e., g \propto d$ (straight line)

If $\quad d = R, g_s = \dfrac{Gm}{R^2}$

If $\quad d > R, g = \dfrac{Gm}{d^2} \qquad i.e., g \propto \dfrac{1}{d^2}$

26. (c) $m\omega^2 R = \text{Force} \propto \frac{1}{R^n}$

$(\text{Force} = \frac{mv^2}{R})$

$\Rightarrow \quad \omega^2 \propto \frac{1}{R^{n+1}} \quad \Rightarrow \quad \omega \propto \frac{1}{R^{\frac{n+1}{2}}}$

Time period $T = \frac{2\pi}{\omega}$

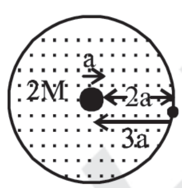
Time period, $T \propto R^{\frac{n+1}{2}}$

27. (c) Areal velocity $= \frac{\pi R^2}{T} = \frac{\pi R^2}{(2\pi R/v)} = \frac{vR}{2}$

$\therefore \quad \frac{dA}{dt} = \frac{R}{2} \times \frac{L}{mR} \quad [\because L = mvR]$

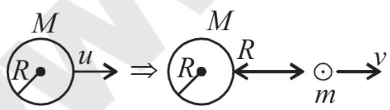
$\therefore \quad \frac{dA}{dt} = \frac{1}{2}\frac{L}{m}$

28. (c) $E_P = \frac{GM}{(3a)^2} + \frac{G(2M)}{(3a)^2} = \frac{GM}{3a^2}$



For a part on the surface of a spherical uniform charge distribution the whole mass acts as a point mass kept at the centre.

29. (b)



$\frac{1}{2}mu^2 + \frac{-GMm}{R} = \frac{1}{2}mv^2 + \frac{-GMm}{2R}$

$\Rightarrow \quad \frac{1}{2}m(v^2 - u^2) = \frac{-GMm}{2R}$

$\Rightarrow \quad V = \sqrt{V = u^2 - \frac{GM}{R}} \quad$ ...(i)

$v_0 = \sqrt{\frac{GM}{2R}} \quad \therefore \; v_{rad} = \frac{m \times v}{\left(\frac{m}{10}\right)} = 10\,v$

Ejecting a rocket of mass $\frac{m}{10}$

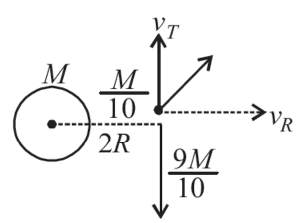
$\therefore \; \frac{9m}{10} \times \sqrt{\frac{GM}{2R}} = \frac{m}{10} \times v_\tau \Rightarrow V_\tau^2 = 81\frac{GM}{2R}$

Kinetic energy of rocket,

$KE_{rocket} = \frac{1}{2}\frac{M}{10}\left(V_T^2 + V_r^2\right)$

$= \frac{1}{2} \times \frac{m}{10} \times \left((u^2 - \frac{GM}{R})100 + 81\frac{GM}{R} \right)$

$= \frac{m}{20} \times 100\left(u^2 - \frac{GM}{R} + \frac{81}{200}\frac{GM}{R} \right)$



$= 5m\left(u^2 - \frac{119}{200}\frac{GM}{R} \right)$

8 Mechanical Properties of Solids

1. A spring of force constant 800 N/m has an extension of 5 cm. The work done in extending it from 5 cm to 15 cm is **[2002]**
(a) 16 J (b) 8 J (c) 32 J (d) 24 J

2. A wire fixed at the upper end stretches by length ℓ by applying a force F. The work done in stretching is **[2004]**
(a) $2F\ell$ (b) $F\ell$ (c) $\frac{F}{2\ell}$ (d) $\frac{F\ell}{2}$

3. If 'S' is stress and 'Y' is young's modulus of material of a wire, the energy stored in the wire per unit volume is **[2005]**
(a) $\frac{S^2}{2Y}$ (b) $2S^2Y$ (c) $\frac{S}{2Y}$ (d) $\frac{2Y}{S^2}$

4. A wire elongates by l mm when a load W is hanged from it. If the wire goes over a pulley and two weights W each are hung at the two ends, the elongation of the wire will be (in mm) **[2006]**
(a) l (b) $2l$ (c) zero (d) $l/2$

5. Two wires are made of the same material and have the same volume. However wire 1 has cross-sectional area A and wire 2 has cross-sectional area $3A$. If the length of wire 1 increases by Δx on applying force F, how much force is needed to stretch wire 2 by the same amount? **[2009]**
(a) $4F$ (b) $6F$ (c) $9F$ (d) F

6. The pressure that has to be applied to the ends of a steel wire of length 10 cm to keep its length constant when its temperature is raised by 100ºC is:
(For steel Young's modulus is 2×10^{11} Nm^{-2} and coefficient of thermal expansion is 1.1×10^{-5}K^{-1}) **[2014]**
(a) 2.2×10^8 Pa (b) 2.2×10^9 Pa
(c) 2.2×10^7 Pa (d) 2.2×10^6 Pa

7. A solid sphere of radius r made of a soft material of bulk modulus K is surrounded by a liquid in a cylindrical container. A massless piston of area a floats on the surface of the liquid, covering entire cross-section of cylindrical container. When a mass m is placed on the surface of the piston to compress the liquid, the fractional decrement in the radius of the sphere $\left(\frac{dr}{r}\right)$, is: **[2018]**
(a) $\frac{\text{Ka}}{\text{mg}}$ (b) $\frac{\text{Ka}}{3\text{mg}}$ (c) $\frac{\text{mg}}{3\text{Ka}}$ (d) $\frac{\text{mg}}{\text{Ka}}$

Answer Key

1	2	3	4	5	6	7								
(b)	(d)	(a)	(a)	(c)	(a)	(c)								

Solutions

1. **(b)** Given, Force constant, $k = 800$ N/m
Initial extension, $x_1 = 5$cm
Final extension, $x_2 = 15$ cm
The work done is stored as elastic potential energy which is given by

$$W = \frac{1}{2} \times R\left(x_2^2 - x_1^2\right)$$

$$= \frac{800}{2}\left[(0.15)^2 - (0.05)^2\right]$$

$= 400\ [(0.15 + 0.05)(0.15 - 0.05)]$
$= 400 \times 0.2 \times 0.1 = 8$ J

2. **(d)** Let A and L be the area and length of the wire.

Work done by constant force in displacing the wire by a distance ℓ.
= change in potential energy

$$=\frac{1}{2}\times \text{stress} \times \text{strain} \times \text{volume}$$

$$=\frac{1}{2}\times\frac{F}{A}\times\frac{\ell}{L}\times A\times L=\frac{F\ell}{2}$$

3. (a) Energy stored in the wire per unit volume,

$$E=\frac{1}{2}\times \text{stress}\times\text{strain} \qquad ...(i)$$

We know that,

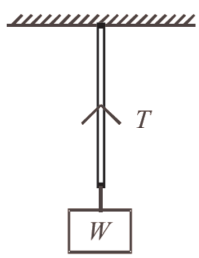
$$Y=\frac{\text{stress}}{\text{strain}}$$

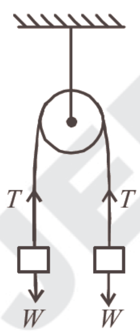
$$\Rightarrow \text{ strain}=\frac{\text{stress}}{Y}$$

On substituting the expression of strain in equation (i) we get

$$E=\frac{1}{2}\times\text{stress}\times\frac{\text{stress}}{Y}=\frac{1}{2}\cdot\frac{S^2}{Y}$$

4. (a) Case (i)





At equilibrium, $T=W$

Young's modules, $Y=\dfrac{W/A}{\ell/L}$ .....(1)

Elongation, $\ell=\dfrac{W}{A}\times\dfrac{L}{Y}$

Case (ii) At equilibrium $T=W$

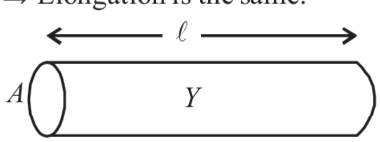
$\therefore$ Young's moduls, $Y=\dfrac{W/A}{\dfrac{\ell/2}{L/2}}$

$$\Rightarrow \quad Y=\frac{W/A}{\ell/L}$$

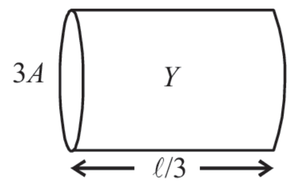
$$\Rightarrow \quad \ell=\frac{W}{A}\times\frac{L}{Y}$$

$\Rightarrow$ Elongation is the same.

5. (c)



Wire (1)



Wire (2)

For wire 1
Length, $L_1=1$
Area, $A_1=A$
For wire 2
Length, $L_2=\dfrac{\ell}{3}$
Area, $A_2=3A$
As the wires are made of same material, so they will have same young's modulus.
For wire 1,

$$Y=\frac{F/A}{\Delta x/\ell} \qquad ...(i)$$

For wire 2 ,

$$Y=\frac{F'/3A}{\Delta x/(\ell/3)} \qquad ...(ii)$$

From (i) and (ii) we get,

$$\frac{F}{A}\times\frac{\ell}{\Delta x}=\frac{F'}{3A}\times\frac{\ell}{3\Delta x}\Rightarrow F'=9F$$

6. (a) Young's modulus Y = $\dfrac{\text{stress}}{\text{strain}}$

stress = Y × strain
Stress in steel wire = Applied pressure
Pressure = stress = Y × strain

Strain = $\dfrac{\Delta L}{L}=\alpha\,\Delta T$

(As length is constant)

$=2\times10^{11}\times1.1\times10^{-5}\times100=2.2\times10^{8}$ Pa

7. (c) Bulk modulus, K = $\dfrac{\text{volumetric stress}}{\text{volumetric strain}}$

$$K=\frac{mg}{a\left(\dfrac{dV}{V}\right)}$$

$$\Rightarrow \quad \frac{dV}{V}=\frac{mg}{Ka} \qquad ....(i)$$

volume of sphere, $V=\dfrac{4}{3}\pi R^3$

Fractional change in volume $\dfrac{dV}{V}=\dfrac{3dr}{r}$ ...(ii)

Using eq. (i) & (ii) $\dfrac{3dr}{r}=\dfrac{mg}{Ka}$

$$\therefore \quad \frac{dr}{r}=\frac{mg}{3Ka}$$

(fractional decrement in radius)

9 Mechanical Properties of Fluids

1. A cylinder of height 20 m is completely filled with water. The velocity of efflux of water (in ms^{-1}) through a small hole on the side wall of the cylinder near its bottom is **[2002]**
   (a) 10 (b) 20
   (c) 25.5 (d) 5

2. Spherical balls of radius 'R' are falling in a viscous fluid of viscosity 'η' with a velocity 'v'. The retarding viscous force acting on the spherical ball is **[2004]**
   (a) inversely proportional to both radius 'R' and velocity 'v'
   (b) directly proportional to both radius 'R' and velocity 'v'
   (c) directly proportional to 'R' but inversely proportional to 'v'
   (d) inversely proportional to 'R' but directly proportional to velocity 'v'

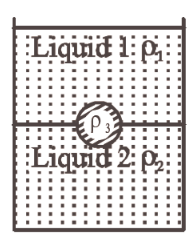
3. If two soap bubbles of different radii are connected by a tube **[2004]**
   (a) air flows from the smaller bubble to the bigger
   (b) air flows from bigger bubble to the smaller bubble till the sizes are interchanged
   (c) air flows from the bigger bubble to the smaller bubble till the sizes become equal
   (d) there is no flow of air.

4. A 20 cm long capillary tube is dipped in water. The water rises up to 8 cm. If the entire arrangement is put in a freely falling elevator the length of water column in the capillary tube will be **[2005]**
   (a) 10 cm (b) 8 cm
   (c) 20 cm (d) 4 cm

5. If the terminal speed of a sphere of gold (density = 19.5 kg/m^3) is 0.2 m/s in a viscous liquid (density = 1.5 kg/m^3), find the terminal speed of a sphere of silver (density = 10.5 kg/m^3) of the same size in the same liquid **[2006]**
   (a) 0.4 m/s (b) 0.133 m/s
   (c) 0.1 m/s (d) 0.2 m/s

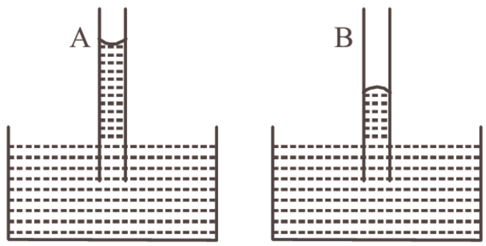
6. A spherical solid ball of volume V is made of a material of density ρ_1. It is falling through a liquid of density ρ_1 ($\rho_2 < \rho_1$). Assume that the liquid applies a viscous force on the ball that is proportional to the square of its speed v, i.e., $F_{viscous} = -kv^2$ ($k > 0$). The terminal speed of the ball is **[2008]**
   (a) $\sqrt{\dfrac{Vg(\rho_1-\rho_2)}{k}}$ (b) $\dfrac{Vg\rho_1}{k}$
   (c) $\sqrt{\dfrac{Vg\rho_1}{k}}$ (d) $\dfrac{Vg(\rho_1-\rho_2)}{k}$

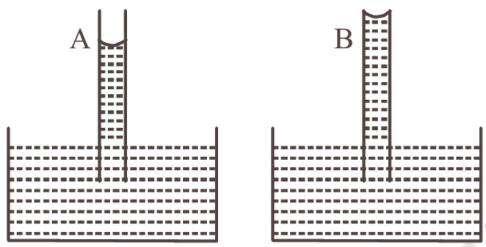
7. A jar is filled with two non-mixing liquids 1 and 2 having densities ρ_1 and, ρ_2 respectively. A solid ball, made of a material of density ρ_3, is dropped in the jar. It comes to equilibrium in the position shown in the figure. Which of the following is true for ρ_1, ρ_2 and ρ_3? **[2008]**

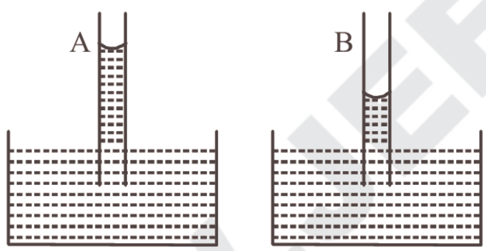


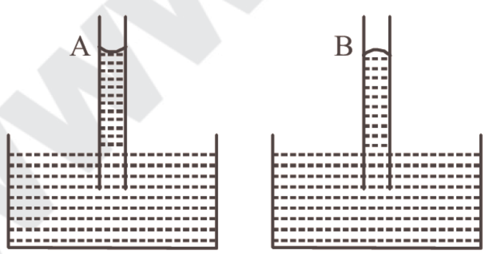
   (a) $\rho_3 < \rho_1 < \rho_2$ (b) $\rho_1 > \rho_3 > \rho_2$
   (c) $\rho_1 < \rho_2 < \rho_3$ (d) $\rho_1 < \rho_3 < \rho_2$

8. A capillary tube (A) is dipped in water. Another identical tube (B) is dipped in a soap-water solution. Which of the following shows the relative nature of the liquid columns in the two tubes? **[2008]**

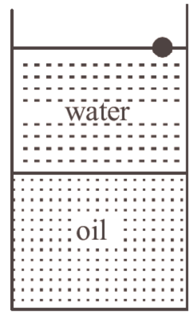
(a)


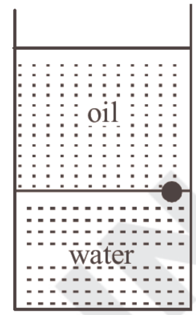
(b)


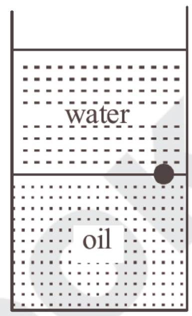
(c)


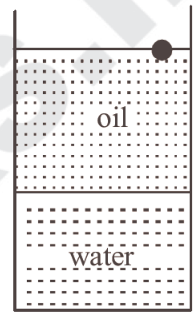
(d)


9. A ball is made of a material of density ρ where $\rho_{oil} < \rho < \rho_{water}$ with ρ_{oil} and ρ_{water} representing the densities of oil and water, respectively. The oil and water are immiscible. If the above ball is in equilibrium in a mixture of this oil and water, which of the following pictures represents its equilibrium position ? **[2010]**

(a)


(b)


(c)


(d)


10. Work done in increasing the size of a soap bubble from a radius of 3 cm to 5 cm is nearly (Surface tension of soap solution = 0.03 Nm^{-1}) **[2011]**

(a) 0.2π mJ (b) 2π mJ
(c) 0.4π mJ (d) 4π mJ

11. Water is flowing continuously from a tap having an internal diameter 8×10^{-3} m. The water velocity as it leaves the tap is 0.4 ms^{-1}. The diameter of the water stream at a distance 2×10^{-1} m below the tap is close to: **[2011]**

(a) 7.5×10^{-3} m (b) 9.6×10^{-3} m
(c) 3.6×10^{-3} m (d) 5.0×10^{-3} m

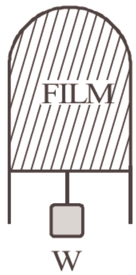
12. Two mercury drops (each of radius 'r') merge to form bigger drop. The surface energy of the bigger drop, if T is the surface tension, is : **[2011 RS]**

(a) $4\pi r^2 T$ (b) $2\pi r^2 T$
(c) $2^{8/3}\pi r^2 T$ (d) $2^{5/3}\pi r^2 T$

13. If a ball of steel (density $\rho = 7.8$ g cm^{-3}) attains a terminal velocity of 10 cm s^{-1} when falling in water (Coefficient of viscosity $\eta_{water} = 8.5 \times 10^{-4}$ Pa.s), then, its terminal velocity in glycerine ($\rho = 1.2$ g cm^{-3}, $\eta = 13.2$ Pa.s) would be, nearly **[2011 RS]**

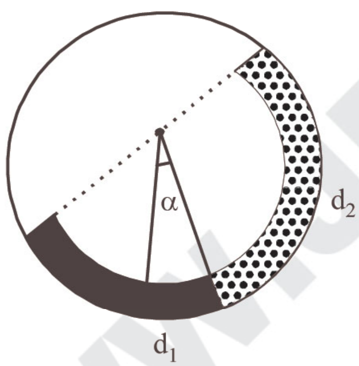
(a) 6.25×10^{-4} cm s^{-1} (b) 6.45×10^{-4} cm s^{-1}
(c) 1.5×10^{-5} cm s^{-1} (d) 1.6×10^{-5} cm s^{-1}

14. A thin liquid film formed between a U-shaped wire and a light slider supports a weight of 1.5×10^{-2} N (see figure). The length of the slider is 30 cm and its weight is negligible. The surface tension of the liquid film is **[2012]**



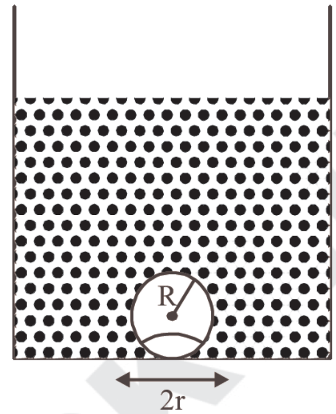
(a) $0.0125\ Nm^{-1}$ (b) $0.1\ Nm^{-1}$
(c) $0.05\ Nm^{-1}$ (d) $0.025\ Nm^{-1}$

15. There is a circular tube in a vertical plane. Two liquids which do not mix and of densities d_1 and d_2 are filled in the tube. Each liquid subtends 90º angle at centre. Radius joining their interface makes an angle α with vertical. Ratio $\frac{d_1}{d_2}$ is: **[2014]**



(a) $\frac{1+\sin\alpha}{1-\sin\alpha}$ (b) $\frac{1+\cos\alpha}{1-\cos\alpha}$
(c) $\frac{1+\tan\alpha}{1-\tan\alpha}$ (d) $\frac{1+\sin\alpha}{1-\cos\alpha}$

16. On heating water, bubbles being formed at the bottom of the vessel detach and rise. Take the bubbles to be spheres of radius R and making a circular contact of radius r with the bottom of the vessel. If r ≪ R and the surface tension of water is T, value of r just before bubbles detach is: (density of water is ρ_w) **[2014]**



(a) $R^2\sqrt{\frac{2\rho_W g}{3T}}$ (b) $R^2\sqrt{\frac{\rho_W g}{6T}}$
(c) $R^2\sqrt{\frac{\rho_W g}{T}}$ (d) $R^2\sqrt{\frac{3\rho_W g}{T}}$

17. An open glass tube is immersed in mercury in such a way that a length of 8 cm extends above the mercury level. The open end of the tube is then closed and sealed and the tube is raised vertically up by additional 46 cm. What will be length of the air column above mercury in the tube now?
(Atmospheric pressure = 76 cm of Hg) **[2014]**
(a) 16 cm (b) 22 cm
(c) 38 cm (d) 6 cm

18. A man grows into a giant such that his linear dimensions increase by a factor of 9. Assuming that his density remains same, the stress in the leg will change by a factor of **[2017]**

(a) 81 (b) $\frac{1}{81}$ (c) 9 (d) $\frac{1}{9}$

19. If 'M' is the mass of water that rises in a capillary tube of radius 'r', then mass of water which will rise in a capillary tube of radius '2r' is : **[2019]**

(a) M (b) $\frac{M}{2}$ (c) 4 M (d) 2 M

Answer Key

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
(b)	(b)	(a)	(c)	(c)	(a)	(d)	(c)	(b)	(c)	(c)	(c)	(a)	(d)	(c)
16	17	18	19											
(a)	(a)	(c)	(d)											

Solutions

1. **(b)** Given, Height of cylinder, h=20 cm
Acceleration due to gravity, g=10 ms^{-2}
Velocity of efflux

$v = \sqrt{2gh}$

Where h is the height of the free surface of liquid from th e hole

$\Rightarrow \; v = \sqrt{2 \times 10 \times 20} = 20 \text{m/s}$

2. **(b)** From Stoke's law, force of viscosity acting on a spherical body is

$F = 6\pi\eta rv$

hence F is directly proportional to radius & velocity.

3. **(a)** Let pressure outside be P_0 and r and R be the radius of smaller bubble and bigger bubble respectively.

$\therefore$ Pressure P_1 For smaller bubble $= P_0 + \frac{2T}{r}$

P_2 For bigger bubble $= P_0 + \frac{2T}{R} \quad (R > r)$

$\therefore P_1 > P_2$

hence air moves from smaller bubble to bigger bubble.

4. **(c)** Water fills the tube entirely in gravityless condition i.e., 20 cm.

5. **(c)** Given,
Density of gold, $\rho_G = 19.5 \text{ kg/m}^3$
Density of silver, $\rho_5 = 10.5 \text{kg/m}^3$
Density of liquid, $\sigma = 1.5 \text{kg/m}^3$

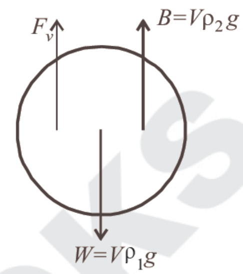
Terminal velocity, $v_T = \frac{2r^2(\rho - \sigma)g}{9\eta}$

$\therefore \frac{v_{T_2}}{0.2} = \frac{(10.5 - 1.5)}{(19.5 - 1.5)}$

$\Rightarrow v_{T_2} = 0.2 \times \frac{9}{18}$

$\therefore v_{T_2} = 0.1 \text{ m/s}$

6. **(a)** When the ball attains terminal velocity
Weight of the ball = Buoyant force + Viscous force



$\therefore \; V\rho_1 g = V\rho_2 g + kv_t^2 \Rightarrow Vg(\rho_1 - \rho_2)g = kv_t^2$

$\Rightarrow v_t = \sqrt{\frac{Vg(\rho_1 - \rho_2)}{k}}$

7. **(d)** As liquid 1 floats over liquid 2. The lighter liquid floats over heavier liquid. So,

$\rho_1 < \rho_2$

Also $\rho_3 < \rho_2$ because the ball of density ρ_3 does not sink to the bottom of the jar.
Also $\rho_3 > \rho_1$ otherwise the ball would have floated in liquid 1. we conclude that
$\rho_1 < \rho_3 < \rho_2$.

8. **(c)** In case of water, the meniscus shape is concave upwards. From ascent formula

$h = \frac{2\sigma \cos\theta}{r\rho g}$

The surface tension (σ) of soap solution is less than water. Therefore height of capillary rise for soap solution should be less as compared to water. As in the case of water, the meniscus shape of soap solution is also concave upwards.

9. **(b)** Oil will float on water so, (b) or (d) is the correct option. But density of ball is more than that of oil,, hence it will sink in oil.

10. **(c)** Work done = increase in surface area × surface tension

$\Rightarrow \; W = 2T4\pi[(5^2) - (3)^2] \times 10^{-4}$
$= 2 \times 0.03 \times 4\pi \, [25 - 9] \times 10^{-4} \text{ J}$
$= 0.4\pi \times 10^{-3} \text{ J} = 0.4\pi \text{ mJ}$

11. (c) Using Bernoulli's theorem, for horizontal flow

$$P_0+\frac{1}{2}\rho v_1^2+\rho gh=P_0+\frac{1}{2}\rho v_2^2+0$$

$$v_2=\sqrt{v_1^2+2gh}=\sqrt{0.16+2\times10\times0.2}=2.03\,\text{m/s}$$

According to equation of continuity

$$A_2v_2=A_1v_1$$

$$\pi\frac{D_2^2}{4}\times v_2=\pi\frac{D_1^2}{4}v_1$$

$$\Rightarrow D_2=D_1\sqrt{\frac{v_1}{v_2}}=3.55\times10^{-3}\,\text{m}$$

12. (c) As volume remains constant

$\therefore$ Sum of volumes of 2 smaller drops
= Volume of the bigger drop

$$2.\frac{4}{3}\pi r^3=\frac{4}{3}\pi R^3 \Rightarrow R=2^{1/3}\,r$$

Surface energy = Surface tension × Surface area = $T.4\pi R^2$

$$=T4\pi 2^{2/3}r^2=T.2^{8/3}\pi r^2.$$

13. (a) When the ball attains terminal velocity
Weight of the ball = viscous force + buoyant force

$$\therefore V\rho g=6\pi\eta rv+V\rho_\ell g$$

$$\Rightarrow Vg(\rho-\rho_\ell)=6\pi\eta rv$$

Also $Vg(\rho-\rho_i')=6\pi\eta' rv'$

$$\therefore v'\eta'=\frac{(\rho-\rho_\ell')}{(\rho-\rho_\ell)}\times v\eta$$

$$\Rightarrow v'=\frac{(\rho-\rho_\ell')}{(\rho-\rho_\ell)}\times\frac{v\eta}{\eta'}$$

$$=\frac{(7.8-1.2)}{(7.8-1)}\times\frac{10\times8.5\times10^{-4}}{13.2}$$

$$\therefore v'=6.25\times10^{-4}\text{ cm/s}$$

14. (d) Let T is the force due to surface tension per unit length, then

$F=2lT$

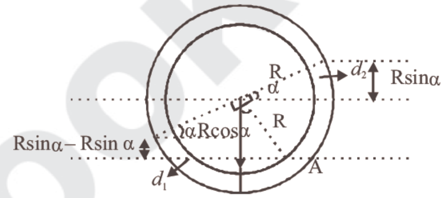
l = length of the slider.

At equilibrium, $F=W$

$\therefore 2Tl=mg$

$$\Rightarrow T=\frac{mg}{2l}=\frac{1.5\times10^{-2}}{2\times30\times10^{-2}}=\frac{1.5}{60}=0.025\,\text{Nm}^{-1}$$

15. (c) Pressure at interface A must be same from both the sides to be in equilibrium.

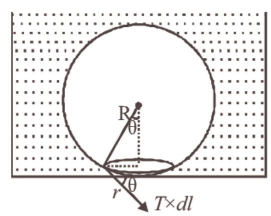


$$\therefore (R\cos\alpha+R\sin\alpha)d_2g$$

$$=(R\cos\alpha-R\sin\alpha)d_1g$$

$$\Rightarrow \frac{d_1}{d_2}=\frac{\cos\alpha+\sin\alpha}{\cos\alpha-\sin\alpha}=\frac{1+\tan\alpha}{1-\tan\alpha}$$

16. (a) When the bubble gets detached,
Buoyant force = force due to surface tension



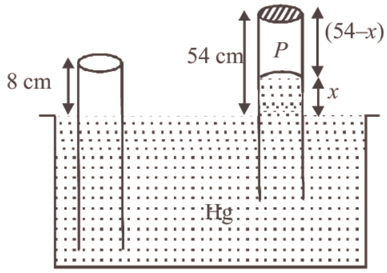
Force due to excess pressure = upthrust

Access pressure in air bubble = $\frac{2T}{R}$

$$\frac{2T}{R}(\pi r^2)=\frac{4\pi R^3}{3T}\rho_w g$$

$$\Rightarrow r^2=\frac{2R^4\rho_w g}{3T} \Rightarrow r=R^2\sqrt{\frac{2\rho_w g}{3T}}$$

17. (a)



Length of the air column above mercury in the tube is,

$P + x = P_0$

$\Rightarrow \quad P = (76 - x)$

$\Rightarrow \quad 8 \times A \times 76 = (76 - x) \times A \times (54 - x)$

$\therefore \quad x = 38$

Thus, length of air column

$= 54 - 38 = 16$ cm.

18. (c) As linear dimension increases by a factor of 9

$\therefore \quad \dfrac{v_f}{v_i} = 9^3$

$\because$ Density remains same

So, mass $\propto$ Volume

$$\frac{m_f}{m_i} = 9^3 \quad \Rightarrow \quad \frac{(Area)_f}{(Area)_i} = 9^2$$

$$\text{Stress }(\sigma) = \frac{force}{area} = \frac{(mass) \times g}{area}$$

$$\frac{\sigma_2}{\sigma_1} = \left(\frac{m_f}{m_i}\right)\left(\frac{A_i}{A_f}\right) = \frac{9^3}{9^2} = 9$$

19. (d) We have, $h = \dfrac{2T\cos\theta}{r\rho g}$

Mass of the water in the capillary

$$m = \rho V = \rho \cdot \pi r^2 h = \rho \times \pi r^2 \times \frac{2T\cos\theta}{r\rho g}$$

$\Rightarrow m \propto r$

$\therefore \dfrac{m_1}{m_2} = \dfrac{r}{2r}$

or, $m_2 = 2m_1 = 2m$

10 Thermal Properties of Matter

1. Heat given to a body which raises its temperature by 1°C is **[2002]**
   (a) water equivalent
   (b) thermal capacity
   (c) specific heat
   (d) temperature gradient

2. Infrared radiation is detected by **[2002]**
   (a) spectrometer (b) pyrometer
   (c) nanometer (d) photometer

3. Which of the following is more close to a black body? **[2002]**
   (a) black board paint
   (b) green leaves
   (c) black holes
   (d) red roses

4. If mass-energy equivalence is taken into account, when water is cooled to form ice, the mass of water should **[2002]**
   (a) increase
   (b) remain unchanged
   (c) decrease
   (d) first increase then decrease

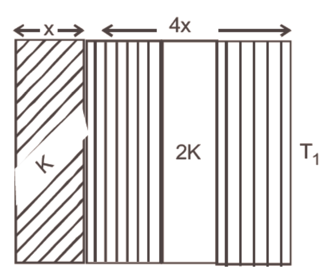
5. Two spheres of the same material have radii 1 m and 4 m and temperatures 4000 K and 2000 K respectively. The ratio of the energy radiated per second by the first sphere to that by the second is **[2002]**
   (a) 1 : 1 (b) 16 : 1 (c) 4 : 1 (d) 1 : 9.

6. The earth radiates in the infra-red region of the spectrum. The spectrum is correctly given by **[2003]**
   (a) Rayleigh Jeans law
   (b) Planck's law of radiation
   (c) Stefan's law of radiation
   (d) Wien's law

7. According to Newton's law of cooling, the rate of cooling of a body is proportional to $(\Delta\theta)^n$, where $\Delta\theta$ is the difference of the temperature of the body and the surroundings, and n is equal to **[2003]**
   (a) two (b) three
   (c) four (d) one

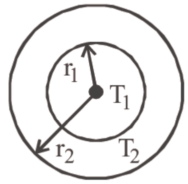
8. If the temperature of the sun were to increase from T to $2T$ and its radius from R to $2R$, then the ratio of the radiant energy received on earth to what it was previously will be **[2004]**
   (a) 32 (b) 16 (c) 4 (d) 64

9. The temperature of the two outer surfaces of a composite slab, consisting of two materials having coefficients of thermal conductivity K and $2K$ and thickness x and $4x$, respectively, are T_2 and $T_1 (T_2 > T_1)$. The rate of heat transfer through the slab, in a steady state is $\left(\frac{A(T_2 - T_1)K}{x}\right) f$, with f equal to **[2004]**



   (a) $\frac{2}{3}$ (b) $\frac{1}{2}$ (c) 1 (d) $\frac{1}{3}$

10. The figure shows a system of two concentric spheres of radii r_1 and r_2 are kept at temperatures T_1 and T_2, respectively. The radial rate of flow of heat in a substance between the two concentric spheres is proportional to **[2005]**



(a) $\ell n\left(\frac{r_2}{r_1}\right)$ (b) $\frac{(r_2-r_1)}{(r_1 r_2)}$

(c) (r_2-r_1) (d) $\frac{r_1 r_2}{(r_2-r_1)}$

11. Assuming the Sun to be a spherical body of radius R at a temperature of TK, evaluate the total radiant powerd incident of Earth at a distance r from the Sun **[2006]**

(a) $4\pi r_0^2 R^2 \sigma \frac{T^4}{r^2}$ (b) $\pi r_0^2 R^2 \sigma \frac{T^4}{r^2}$

(c) $r_0^2 R^2 \sigma \frac{T^4}{4\pi r^2}$ (d) $R^2 \sigma \frac{T^4}{r^2}$

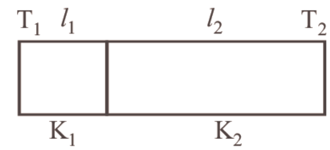
where r_0 is the radius of the Earth and σ is Stefan's constant.

12. Two rigid boxes containing different ideal gases are placed on a table. Box A contains one mole of nitrogen at temperature T_0, while Box contains one mole of helium at temperature $\left(\frac{7}{3}\right)T_0$. The boxes are then put into thermal contact with each other, and heat flows between them until the gases reach a common final temperature (ignore the heat capacity of boxes). Then, the final temperature of the gases, T_f in terms of T_0 is **[2006]**

(a) $T_f = \frac{3}{7}T_0$ (b) $T_f = \frac{7}{3}T_0$

(c) $T_f = \frac{3}{2}T_0$ (d) $T_f = \frac{5}{2}T_0$

13. One end of a thermally insulated rod is kept at a temperature T_1 and the other at T_2. The rod is composed of two sections of length l_1 and l_2 and thermal conductivities K_1 and K_2 respectively. The temperature at the interface of the two section is **[2007]**



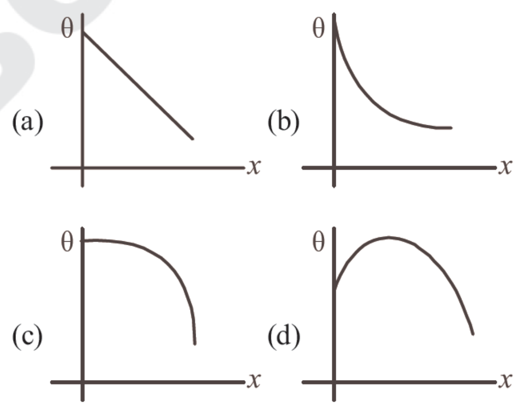
(a) $\frac{(K_1 l_1 T_1 + K_2 l_2 T_2)}{(K_1 l_1 + K_2 l_2)}$

(b) $\frac{(K_2 l_2 T_1 + K_1 l_1 T_2)}{(K_1 l_1 + K_2 l_2)}$

(c) $\frac{(K_2 l_1 T_1 + K_1 l_2 T_2)}{(K_2 l_1 + K_1 l_2)}$

(d) $\frac{(K_1 l_2 T_1 + K_2 l_1 T_2)}{(K_1 l_2 + K_2 l_1)}$

14. A long metallic bar is carrying heat from one of its ends to the other end under steady–state. The variation of temperature θ along the length x of the bar from its hot end is best described by which of the following figures? **[2009]**



15. 100g of water is heated from 30°C to 50°C. Ignoring the slight expansion of the water, the change in its internal energy is (specific heat of water is 4184 J/kg/K): **[2011]**

(a) 8.4 kJ (b) 84 kJ

(c) 2.1 kJ (d) 4.2 kJ

16. The specific heat capacity of a metal at low temperature (T) is given as

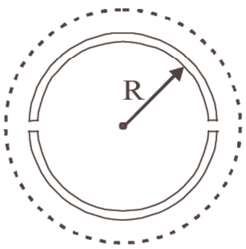
$$C_p\,(kJK^{-1}\text{kg}^{-1}) = 32\left(\frac{T}{400}\right)^3$$

A 100 gram vessel of this metal is to be cooled from 20ºK to 4ºK by a special refrigerator operating at room temperature (27°C). The amount of work required to cool the vessel is

[2011 RS]

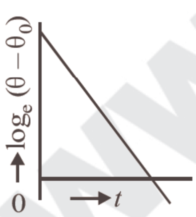
(a) greater than 0.148 kJ
(b) between 0.148 kJ and 0.028 kJ
(c) less than 0.028 kJ
(d) equal to 0.002 kJ

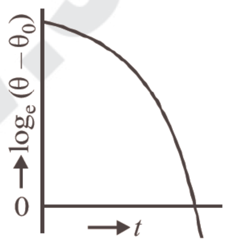
17. A wooden wheel of radius R is made of two semicircular part (see figure). The two parts are held together by a ring made of a metal strip of cross sectional area S and length L. L is slightly less than $2\pi R$. To fit the ring on the wheel, it is heated so that its temperature rises by ΔT and it just steps over the wheel. As it cools down to surrounding temperature, it presses the semicircular parts together. If the coefficient of linear expansion of the metal is α, and its Young's modulus is Y, the force that one part of the wheel applies on the other part is : **[2012]**

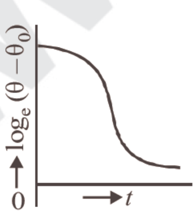


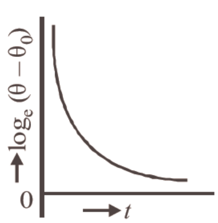
(a) $2\pi SY\alpha\Delta T$ (b) $SY\alpha\Delta T$
(c) $\pi SY\alpha\Delta T$ (d) $2SY\alpha\Delta T$

18. A liquid in a beaker has temperature $\theta(t)$ at time t and θ_0 is temperature of surroundings, then according to Newton's law of cooling the correct graph between $\log_e(\theta - \theta_0)$ and t is : **[2012]**

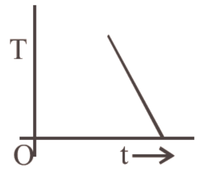
(a) 

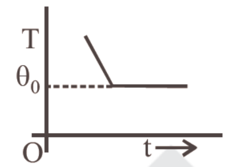
(b) 

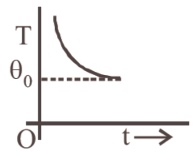
(c) 

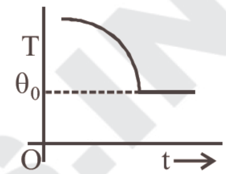
(d) 

19. If a piece of metal is heated to temperature θ and then allowed to cool in a room which is at temperature θ_0, the graph between the temperature T of the metal and time t will be closest to **[2013]**

(a) 

(b) 

(c) 

(d) 

20. Three rods of Copper, Brass and Steel are welded together to form a Y shaped structure. Area of cross - section of each rod = 4 cm^2. End of copper rod is maintained at 100ºC where as ends of brass and steel are kept at 0ºC. Lengths of the copper, brass and steel rods are 46, 13 and 12 cms respectively. The rods are thermally insulated from surroundings excepts at ends. Thermal conductivities of copper, brass and steel are 0.92, 0.26 and 0.12 CGS units respectively. Rate of heat flow through copper rod is: **[2014]**

(a) 1.2 cal/s (b) 2.4 cal/s
(c) 4.8 cal/s (d) 6.0 cal/s

21. Consider a spherical shell of radius R at temperature T. The black body radiation inside it can be considered as an ideal gas of photons with internal energy per unit volume $u = \frac{U}{V} \propto T^4$ and pressure $p = \frac{1}{3}\left(\frac{U}{V}\right)$. If the shell now undergoes an adiabatic expansion the relation between T and R is : **[2015]**

(a) $T \propto \frac{1}{R}$ (b) $T \propto \frac{1}{R^3}$
(c) $T \propto e^{-R}$ (d) $T \propto e^{-3R}$

22. A pendulum clock loses 12 s a day if the temperature is 40°C and gains 4 s a day if the temperature is 20°C. The temperature at which the clock will show correct time, and the co-efficient of linear expansion (α) of the metal of the pendulum shaft are respectively : **[2016]**

(a) 30°C; $\alpha = 1.85 \times 10^{-3}$/°C
(b) 55°C; $\alpha = 1.85 \times 10^{-2}$/°C
(c) 25°C; $\alpha = 1.85 \times 10^{-5}$/°C
(d) 60°C; $\alpha = 1.85 \times 10^{-4}$/°C

23. A copper ball of mass 100 gm is at a temperature T. It is dropped in a copper calorimeter of mass 100 gm, filled with 170 gm of water at room temperature. Subsequently, the temperature of the system is found to be 75°C. T is given by (Given : room temperature = 30° C, specific heat of copper = 0.1 cal/gm°C **[2017]**

(a) 1250°C (b) 825°C

(c) 800°C (d) 885° C

24. An external pressure P is applied on a cube at 0°C so that it is equally compressed from all sides. K is the bulk modulus of the material of the cube and α is its coefficient of linear expansion. Suppose we want to bring the cube to its original size by heating. The temperature should be raised by : **[2017]**

(a) $\frac{3\alpha}{PK}$ (b) $3PK\alpha$

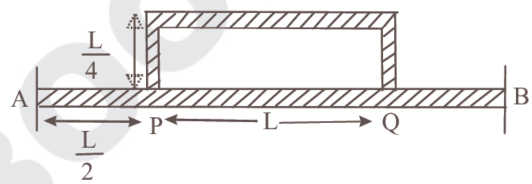
(c) $\frac{P}{3\alpha K}$ (d) $\frac{P}{\alpha K}$

25. A rod, of length L at room temperature and uniform area of cross section A, is made of a metal having coefficient of linear expansion α/°C. It is observed that an external compressive force F, is applied on each of its ends, prevents any change in the length of the rod, when its temperature rises by ΔT K. Young's modulus, Y, for this metal is: **[2019]**

(a) $\frac{F}{A\alpha\Delta T}$ (b) $\frac{F}{A\alpha(\Delta T-273)}$

(c) $\frac{F}{2A\alpha\Delta T}$ (d) $\frac{2F}{A\alpha\Delta T}$

26. Temperature difference of 120°C is maintained between two ends of a uniform rod AB of length 2L. Another bent rod PQ, of same cross-section as AB and length , is connected across AB (See figure). In steady state, temperature difference between P and Q will be close to: **[2019]**



(a) 45°C (b) 75°C (c) 60°C (d) 35°C

27. A non-isotropic solid metal cube has coefficients of linear expansion as: $5 \infty 10^{-5}$/°C along the *x*-axis and 5×10^{-6}/°C along the *y* and the *z*-axis. If the coefficient of volume expansion of the solid is $C \times 10^{-6}$/°C then the value of C is ____ . **[2020]**

Answer Key

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
(b)	(b)	(a)	(c)	(a)	(d)	(d)	(d)	(d)	(d)	(b)	(c)	(d)	(a)	(a)
16	17	18	19	20	21	22	23	24	25	26	27			
(d)	(d)	(a)	(c)	(c)	(a)	(c)	(d)	(c)	(a)	(a)	60.00			

Solutions

1. **(b)** Heat required for raising the temperature of a body through 1°C is called its thermal capacity.

2. **(b)** Pyrometer is used to detect infra-red radiation.

3. **(a)** Black body is one which absorb all incident radiation. Black board paint is quite approximately equal to black bodies.

4. **(c)** When water is cooled at 0°C to form ice, energy is released from water in the form of heat. As energy is equivalent to mass, therefore, when water is cooled to ice, its mass decreases.

5. **(a)** From stefan's law, the energy radiated per second is given by $E = e\sigma T^4 A$
Here, T = temperature of the body
A = surface area of the body

For same material e is same. σ is stefan's constant

Let T_1 and T_2 be the temperature of two spheres. A_1 and A_2 be the area of two spheres.

$$\therefore \quad \frac{E_1}{E_2} = \frac{T_1^4 A_1}{T_2^4 A_2} = \frac{T_1^4 4\pi r_1^2}{T_2^4 4\pi r_2^2}$$

$$= \frac{(4000)^4 \times 1^2}{(2000)^4 \times 4^2} = \frac{1}{1}$$

6. **(d)** Wein's law correctly explains the spectrum

7. **(d)** From Newton's law of cooling

$$-\frac{dQ}{dt} \propto (\Delta\theta)$$

8. **(d)** From stefan's law, energy radiated by sun per second

$E = \sigma A T^4$;

$\therefore A \propto R^2$

$\therefore E \propto R^2 T^4$

$$\therefore \frac{E_2}{E_1} = \frac{R_2^2 T_2^4}{R_1^2 T_1^4}$$

put $R_2 = 2R, R_1 = R$; $T_2 = 2T, T_1 = T$

$$\Rightarrow \frac{E_2}{E_1} = \frac{(2R)^2 (2T)^4}{R^2 T^4} = 64$$

9. **(d)** The thermal resistance is given by

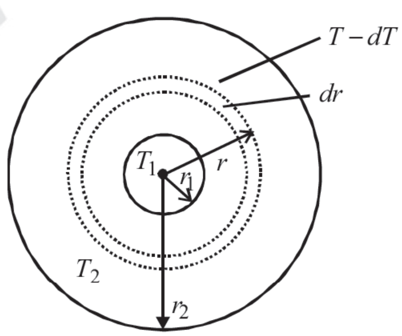
$$\frac{x}{KA} + \frac{4x}{2KA} = \frac{x}{KA} + \frac{2x}{KA} = \frac{3x}{KA}$$

Amount of heat flow per second,

$$\frac{dQ}{dt} = \frac{\Delta T}{\frac{3x}{KA}} = \frac{(T_2 - T_1)KA}{3x}$$

$$= \frac{1}{3}\left\{\frac{A(T_2 - T_1)K}{x}\right\} \quad \therefore f = \frac{1}{3}$$

10. **(d)**



Consider a thin concentric shell of thickness (dr) and of radius (r) and let the temperature of inner and outer surfaces of this shell be T and ($T - dT$) respectively. The radial rate of flow of heat through this elementary shell will be

$$\frac{dQ}{dt} = \frac{KA[(T - dT) - T]}{dr} = \frac{-KAdT}{dr}$$

$$= -4\pi K r^2 \frac{dT}{dr} \qquad (\because A = 4\pi r^2)$$

Since the area of the surface through which heat will flow is not constant. Integrating both sides between the limits of radii and temperatures of the two shells, we get

$$\left(\frac{dQ}{dt}\right)\int_{r_1}^{r_2} \frac{1}{r^2} dr = -4\pi K \int_{T_1}^{T_2} dT$$

$$\left(\frac{dQ}{dt}\right)\int_{r_1}^{r_2} r^{-2} dr = -4\pi K \int_{T_1}^{T_2} dT$$

$$\frac{dQ}{dt}\left[\frac{1}{r_1} - \frac{1}{r_2}\right] = -4\pi K\left[T_2 - T_1\right]$$

$$\text{or} \quad \frac{dQ}{dt} = \frac{-4\pi K r_1 r_2 (T_2 - T_1)}{(r_2 - r_1)}$$

$$\therefore \quad \frac{dQ}{dt} \propto \frac{r_1 r_2}{(r_2 - r_1)}$$

11. **(b)** From stefan's law, total power radiated by Sun, $E = \sigma T^4 \times 4\pi R^2$

The intensity of power Per unit area incident on earth's surface

$$= \frac{\sigma T^4 \times 4\pi R^2}{4\pi r^2}$$

Total power received by Earth

$$E' = \frac{E}{4\pi r^2} \times \text{Cross – Section area of earth}$$

$$\text{facing the sun} = \frac{\sigma T^4 R^2}{r^2}(\pi r_0^2)$$

12. (c) When two gases are mixed to gether then Heat lost by He gas = Heat gained by N_2 gas

$$n_1 C_{v_1} \Delta T_1 = n_2 C_{v_2} \Delta T_2$$

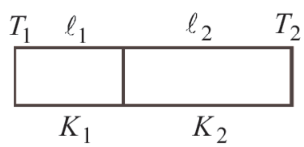
$$\frac{3}{2} R\left[\frac{7}{3} T_0 - T_f\right] = \frac{5}{2} R\left[T_f - T_0\right]$$

$$7T_0 - 3T_f = 5T_f - 5T_0$$

$$\Rightarrow 12T_0 = 8T_f \Rightarrow T_f = \frac{12}{8} T_0$$

$$\Rightarrow T_f = \frac{3}{2} T_0.$$

13. (d) Let T be the temperature of the interface. In the steady state, $Q_1 = Q_2$



$$\therefore \frac{K_1 A(T_1 - T)}{\ell_1} = \frac{K_2 A(T - T_2)}{\ell_2},$$

where A is the area of cross-section.

$$\Rightarrow \quad K_1 A(T_1 - T)\ell_2 = K_2 A(T - T_2)\ell_1$$

$$\Rightarrow \quad K_1 T_1 \ell_2 - K_1 T \ell_2 = K_2 T \ell_1 - K_2 T_2 \ell_1$$

$$\Rightarrow \quad (K_2 \ell_1 + K_1 \ell_2) T = K_1 T_1 \ell_2 + K_2 T_2 \ell_1$$

$$\Rightarrow \quad T = \frac{K_1 T_1 \ell_2 + K_2 T_2 \ell_1}{K_2 \ell_1 + K_1 \ell_2}$$

$$= \frac{K_1 \ell_2 T_1 + K_2 \ell_1 T_2}{K_1 \ell_2 + K_2 \ell_1}.$$

14. (a) Let Q be the temperature at a distance x from hot end of bar. Let Q is the temperature of hot end.
The heat flow rate is given by

$$\frac{dQ}{dt} = \frac{kA(\theta_1 - \theta)}{x}$$

$$\Rightarrow \theta_1 - \theta = \frac{x}{kA} \frac{dQ}{dt} \Rightarrow \theta = \theta_1 - \frac{x}{kA} \frac{dQ}{dt}$$

Thus, the graph of Q versus x is a straight line with a positive intercept and a negative slope. The above equation can be graphically represented by option (a).

15. (a) $\Delta U = \Delta Q = mc\Delta T$

$$= \frac{100}{1000} \times 4184\,(50 - 30) \approx 8.4 \text{ kJ}$$

16. (d) Required work = energy released

Here, $Q = \int mc\, dT$

$$= \int_{20}^{4} 0.1 \times 32 \times \left(\frac{T^3}{400^3}\right) dT$$

$$= \int_{20}^{4} \frac{3.2}{64 \times 10^6} T^3\, dT$$

$$= 5 \times 10^{-8} \int_{20}^{4} T^3\, dT$$

$$= 0.002 kJ$$

Therefore, required work = 0.002 kJ

17. (d) The Young modulus is given as

$$\text{Y} = \frac{\text{stress}}{\text{strain}} = \frac{\text{F/S}}{\Delta\text{L/L}}$$

Here, $\Delta L = 2\pi \Delta R\ L = 2\pi R$

$$\text{Y} = \frac{\text{F}}{\text{S}2\pi\Delta\text{R}} \times 2\pi\text{R}$$

$$\Rightarrow \quad \text{Y} = \frac{\text{FR}}{\text{S}.\Delta\text{R}} \qquad \text{...(i)}$$

The coefficient of linear expansion

$$\alpha = \frac{\Delta R}{R\Delta T}$$

$$\Rightarrow \quad \frac{\Delta R}{R} = \alpha.\Delta T$$

$$\Rightarrow \frac{R}{\Delta R} = \frac{1}{\alpha \Delta T} \qquad \text{...(ii)}$$

From equation (i) and (ii)

$$Y = \frac{F}{S.\alpha\Delta T} \Rightarrow F = Y.S.\alpha\Delta T$$

$\therefore$ The ring is pressing the wheel from both sides, Thus

$$F_{\text{net}} = 2\text{F} = 2\text{Y}S\alpha\Delta T$$

18. (a) According to newton's law of cooling

$$\frac{d\theta}{dt} = -k(\theta - \theta_0)$$

$$\Rightarrow \frac{d\theta}{(\theta-\theta_0)} = -kdt$$

$$\Rightarrow \int_{\theta_0}^{\theta} \frac{d\theta}{(\theta-\theta_0)} = -k\int_{\theta}^{t} dt$$

$$\Rightarrow \log(\theta-\theta_0) = -kt + c$$

Which represents an equation of straight line.

Thus the option (a) is correct.

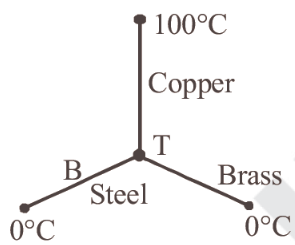
19. **(c)** According to Newton's law of cooling, the temperature goes on decreasing with time non-linearly.

20. **(c)** Rate of heat flow is given by,

$$Q = \frac{KA(\theta_1-\theta_2)}{l}$$

Where, K = coefficient of thermal conductivity

l = length of rod and A = area of cross-section of rod



If the junction temperature is T, then

$Q_{Copper} = Q_{Brass} + Q_{Steel}$

$$\frac{0.92\times 4(100-T)}{46} = \frac{0.26\times 4\times(T-0)}{13} + \frac{0.12\times 4\times(T-0)}{12}$$

$\Rightarrow$ $200 - 2T = 2T + T$

$\Rightarrow$ $T = 40°C$

$$\therefore \quad Q_{Copper} = \frac{0.92\times 4\times 60}{46} = 4.8 \text{ cal/s}$$

21. **(a)** As, $P = \frac{1}{3}\left(\frac{U}{V}\right)$

But $\frac{U}{V} = KT^4$

So, $P = \frac{1}{3}KT^4$

or $\frac{uRT}{V} = \frac{1}{3}KT^4$ [As PV = u RT]

$$\frac{4}{3}pR^3T^3 = \text{constant}$$

Therefore, $T \propto \frac{1}{R}$

22. **(c)** Time lost/gained per day $= \frac{1}{2}\alpha\,\Delta\theta\times 86400$ second

$$12 = \frac{1}{2}\alpha(40-\theta)\times 86400 \quad .... \text{(i)}$$

$$4 = \frac{1}{2}\alpha(\theta-20)\times 86400 \quad ....\text{(ii)}$$

On dividing we get, $3 = \frac{40-\theta}{\theta-20}$

$3\theta - 60 = 40 - \theta$

$4\theta = 100 \Rightarrow \theta = 25°C$

Substituting the value of θ in (i) we get

$$\frac{12}{86400} = \frac{1}{2}\alpha\,(15)$$

$$\Rightarrow \alpha = \frac{24}{86400\times 15}$$

$$\Rightarrow \alpha = 185\times 10^{-5}\ °C$$

23. **(d)** According to principle of calorimetry,

Heat lost = Heat gain

$100\times 0.1(T-75) = 100\times 0.1\times 45 + 170\times 1\times 45$

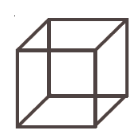
$10\,T - 750 = 450 + 7650 = 8100$

$\Rightarrow T - 75 = 810$

$T = 885°C$

24. **(c)** As we know, Bulk modulus

$$K = \frac{\Delta P}{\left(\frac{-\Delta V}{V}\right)} \quad \Rightarrow \frac{\Delta V}{V} = \frac{P}{K}$$



$\gamma = 3\,\alpha$

$V = V_0(1+\gamma\Delta t)$

$$\frac{\Delta V}{V_0} = \gamma\Delta t$$

$$\therefore \quad \frac{P}{K} = \gamma \Delta t \Rightarrow \Delta t = \frac{P}{\gamma K} = \frac{P}{3\alpha K}$$

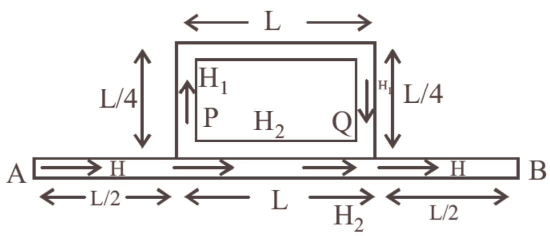
25. (a) Young's modulus $Y = \frac{\text{stress}}{\text{strain}} = \frac{F/A}{(\Delta\ell/\ell)}$

Using, coefficient of linear expansion,

$$\frac{\Delta\ell}{\ell} = \alpha\Delta T$$

$$\therefore Y = \frac{F}{A(\alpha\Delta T)}$$

26. (a)



At P,
$H = H_1 + H_{2s}$

$$\frac{kA(T_A - T_P)}{L/2} = \frac{kA(T_P - T_Q)}{3L/2} + \frac{kA(T_P - T_Q)}{L}$$

$$\therefore 2(T_A - T_P) = \frac{2}{3}(T_P - T_Q) + (T_P - T_Q)$$

$$\therefore 2(T_A - T_P) = \frac{5}{3}(T_P - T_Q) \quad \ldots(i)$$

At, Q
$H_1 + H_2 = H$

$$\therefore \frac{kA(T_P - T_Q)}{3L/2} + \frac{kA(T_P - T_Q)}{L} = \frac{kA(T_Q - T_B)}{L/2}$$

$$\therefore 2(T_Q - T_P) = \frac{5}{3}(T_P - T_Q) \quad \ldots(ii)$$

From (i) & (ii)
$2(T_A - T_P) + 2(T_Q - T_B)$

$$= \frac{10}{3}(T_P - T_Q)$$

$$T_A - T_B = \frac{8}{3}(T_P - T_Q)$$

$$\therefore T_P - T_Q = \frac{3}{8} \times 120 = 45°C$$

27. (60.00) Volume, $V = Ibh$

$$\therefore \gamma = \frac{\Delta V}{V} = \frac{\Delta\ell}{\ell} + \frac{\Delta b}{b} + \frac{\Delta h}{h}$$

(γ = coefficient of volume expansion)

$\Rightarrow \gamma = 5 \times 10^{-5} + 5 \times 10^{-6} + 5 \times 10^{-6}$
$= 60 \times 10^{-6}/°C$

$\Rightarrow$ Value of $C = 60.00$

11 Thermodynamics

1. Which statement is incorrect? **[2002]**
   (a) All reversible cycles have same efficiency
   (b) Reversible cycle has more efficiency than an irreversible one
   (c) Carnot cycle is a reversible one
   (d) Carnot cycle has the maximum efficiency in all cycles

2. Even Carnot engine cannot give 100% efficiency because we cannot **[2002]**
   (a) prevent radiation
   (b) find ideal sources
   (c) reach absolute zero temperature
   (d) eliminate friction

3. "Heat cannot by itself flow from a body at lower temperature to a body at higher temperature" is a statement or consequence of **[2003]**
   (a) second law of thermodynamics
   (b) conservation of momentum
   (c) conservation of mass
   (d) first law of thermodynamics

4. During an adiabatic process, the pressure of a gas is found to be proportional to the cube of its absolute temperature. The ratio C_P/C_V for the gas is **[2003]**
   (a) $\frac{4}{3}$ (b) 2 (c) $\frac{5}{3}$ (d) $\frac{3}{2}$

5. Which of the following parameters does not characterize the thermodynamic state of matter?
   (a) Temperature (b) Pressure **[2003]**
   (c) Work (d) Volume

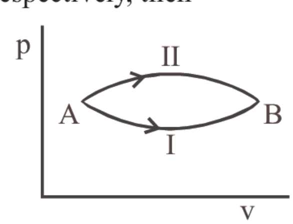
6. A Carnot engine takes 3×10^6 cal of heat from a reservoir at 627°C, and gives it to a sink at 27°C. The work done by the engine is **[2003]**
   (a) 4.2×10^6 J (b) 8.4×10^6 J
   (c) 16.8×10^6 J (d) zero

7. Which of the following statements is **correct** for any thermodynamic system? **[2004]**
   (a) The change in entropy can never be zero
   (b) Internal energy and entropy are state functions
   (c) The internal energy changes in all processes
   (d) The work done in an adiabatic process is always zero.

8. Two thermally insulated vessels 1 and 2 are filled with air at temperatures (T_1, T_2), volume (V_1, V_2), and pressure (P_1, P_2) respectively. If the valve joining the two vessels is opened, the temperature inside the vessel at equilibrium will be **[2004]**
   (a) $T_1T_2(P_1V_1 + P_2V_2)/(P_1V_1T_2 + P_2V_2T_1)$
   (b) $(T_1 + T_2)/2$
   (c) $T_1 + T_2$
   (d) $T_1T_2(P_1V_1 + P_2V_2)/(P_1V_1T_1 + P_2V_2T_2)$

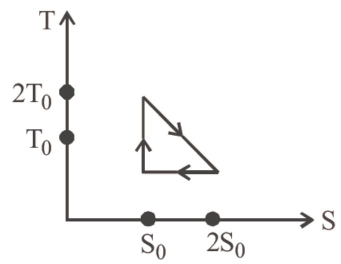
9. Which of the following is **incorrect** regarding the first law of thermodynamics? **[2005]**
   (a) It is a restatement of the principle of conservation of energy
   (b) It is not applicable to any cyclic process
   (c) It introduces the concept of the entropy
   (d) It introduces the concept of the internal energy

10. A system goes from A to B via two processes I and II as shown in figure. If ΔU_1 and ΔU_2 are the changes in internal energies in the processes I and II respectively, then **[2005]**



(a) relation between ΔU_1 and ΔU_2 can not be determined
(b) $\Delta U_1 = \Delta U_2$
(c) $\Delta U_2 = \Delta U_1 \, \Delta U_2 < \Delta U_1$
(d) $\Delta U_2 > \Delta U_1$

11. The temperature-entropy diagram of a reversible engine cycle is given in the figure. Its efficiency is **[2005]**



(a) $\frac{1}{4}$ (b) $\frac{1}{2}$ (c) $\frac{2}{3}$ (d) $\frac{1}{3}$

12. The work of 146 kJ is performed in order to compress one kilo mole of gas adiabatically and in this process the temperature of the gas increases by 7°C. The gas is ($R = 8.3 \text{ J mol}^{-1} \text{ K}^{-1}$) **[2006]**
(a) diatomic
(b) triatomic
(c) a mixture of monoatomic and diatomic
(d) monoatomic

13. When a system is taken from state i to state f along the path iaf, it is found that $Q = 50$ cal and $W = 20$ cal. Along the path ibf $Q = 36$ cal. W along the path ibf is **[2007]**

(a) 14 cal (b) 6 cal
(c) 16 cal (d) 66 cal

14. A Carnot engine, having an efficiency of $\eta = 1/10$ as heat engine, is used as a refrigerator. If the work done on the system is 10 J, the amount of energy absorbed from the reservoir at lower temperature is **[2007]**
(a) 100 J (b) 99 J (c) 90 J (d) 1 J

15. An insulated container of gas has two chambers separated by an insulating partition. One of the chambers has volume V_1 and contains ideal gas at pressure P_1 and temperature T_1. The other chamber has volume V_2 and contains ideal gas at pressure P_2 and temperature T_2. If the partition is removed without doing any work on the gas, the final equilibrium temperature of the gas in the container will be **[2008]**

(a) $\dfrac{T_1T_2(P_1V_1 + P_2V_2)}{P_1V_1T_2 + P_2V_2T_1}$

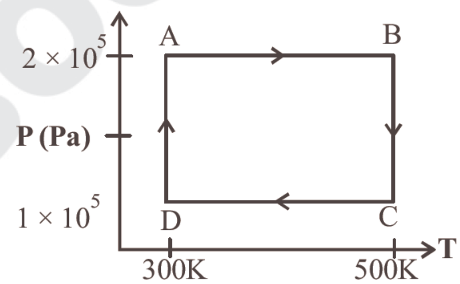
(b) $\dfrac{P_1V_1T_1 + P_2V_2T_2}{P_1V_1 + P_2V_2}$

(c) $\dfrac{P_1V_1T_2 + P_2V_2T_1}{P_1V_1 + P_2V_2}$

(d) $\dfrac{T_1T_2(P_1V_1 + P_2V_2)}{P_1V_1T_1 + P_2V_2T_2}$

Directions for questions 16 to 18 : *Questions are based on the following paragraph.*

Two moles of helium gas are taken over the cycle *ABCDA*, as shown in the *P*-*T* diagram. **[2009]**



16. Assuming the gas to be ideal the work done on the gas in taking it from A to B is
(a) 300 R (b) 400 R
(c) 500 R (d) 200 R

17. The work done on the gas in taking it from D to A is
(a) + 414 R (b) – 690 R
(c) + 690 R (d) – 414 R

18. The net work done on the gas in the cycle *ABCDA* is
(a) 276 R (b) 1076 R
(c) 1904 R (d) zero

19. A diatomic ideal gas is used in a Carnot engine as the working substance. If during the adiabatic expansion part of the cycle the volume of the gas increases from V to 32 V, the efficiency of the engine is **[2010]**
(a) 0.5 (b) 0.75 (c) 0.99 (d) 0.25

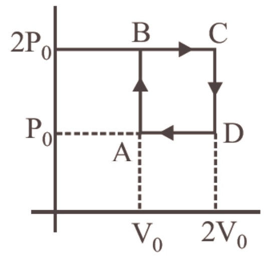
20. A Carnot engine operating between temperatures T_1 and T_2 has efficiency $\frac{1}{6}$. When

T_2 is lowered by 62 K its efficiency increases to $\frac{1}{3}$. Then T_1 and T_2 are, respectively: **[2011]**

(a) 372 K and 310 K (b) 330 K and 268 K
(c) 310 K and 248 K (d) 372 K and 310 K

21. Helium gas goes through a cycle ABCDA (consisting of two isochoric and isobaric lines) as shown in figure. The efficiency of this cycle is nearly : (Assume the gas to be close to ideal gas) **[2012]**

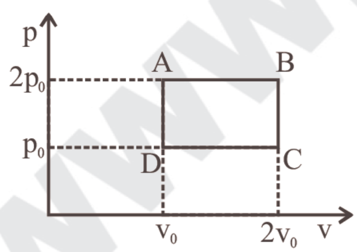
(a) 15.4 % (b) 9.1 % (c) 10.5% (d) 12.5%



22. A Carnot engine, whose efficiency is 40%, takes in heat from a source maintained at a temperature of 500K. It is desired to have an engine of efficiency 60%. Then, the intake temperature for the same exhaust (sink) temperature must be : **[2012]**

(a) efficiency of Carnot engine cannot be made larger than 50%
(b) 1200 K
(c) 750 K
(d) 600 K

23.

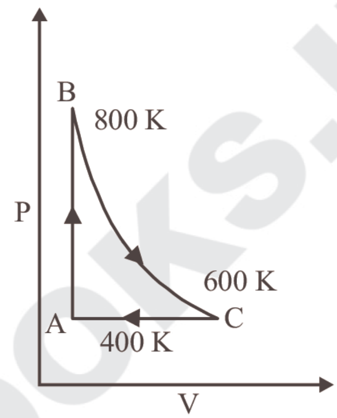


The above p-v diagram represents the thermodynamic cycle of an engine, operating with an ideal monatomic gas. The amount of heat, extracted from the source in a single cycle is **[2013]**

(a) P_0V_0 (b) $\left(\frac{13}{2}\right)P_0V_0$

(c) $\left(\frac{11}{2}\right)P_0V_0$ (d) $4P_0V_0$

24. One mole of a diatomic ideal gas undergoes a cyclic process ABC as shown in figure. The process BC is adiabatic. The temperatures at A, B and C are 400 K, 800 K and 600 K respectively. Choose the correct statement: **[2014]**



(a) The change in internal energy in whole cyclic process is 250 R.
(b) The change in internal energy in the process CA is 700 R.
(c) The change in internal energy in the process AB is -350 R.
(d) The change in internal energy in the process BC is – 500 R.

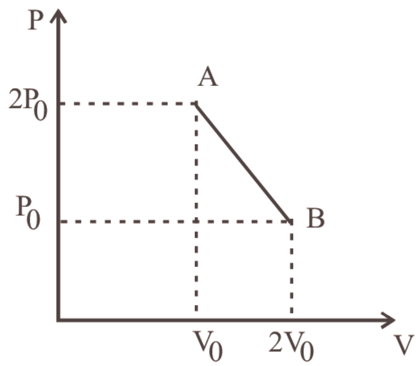
25. A solid body of constant heat capacity 1 J/°C is being heated by keeping it in contact with reservoirs in two ways : **[2015]**

(i) Sequentially keeping in contact with 2 reservoirs such that each reservoir supplies same amount of heat.
(ii) Sequentially keeping in contact with 8 reservoirs such that each reservoir supplies same amount of heat.

In both the cases body is brought from initial temperature 100°C to final temperature 200°C. Entropy change of the body in the two cases respectively is :

(a) *ln2, 2ln2* (b) *2ln2, 8ln2*
(c) *ln2, 4ln2* (d) *ln2, ln2*

26. 'n' moles of an ideal gas undergoes a process $A \rightarrow B$ as shown in the figure. The maximum temperature of the gas during the process will be : **[2016]**



(a) $\frac{9P_0V_0}{2nR}$ (b) $\frac{9P_0V_0}{nR}$

(c) $\frac{9P_0V_0}{4nR}$ (d) $\frac{3P_0V_0}{2nR}$

27. An ideal gas undergoes a quasi static, reversible process in which its molar heat capacity C remains constant. If during this process the relation of pressure P and volume V is given by PV^n = constant, then n is given by (Here C_P and C_V are molar specific heat at constant pressure and constant volume, respectively) : **[2016]**

(a) $n = \frac{C_P - C}{C - C_V}$ (b) $n = \frac{C - C_V}{C - C_P}$

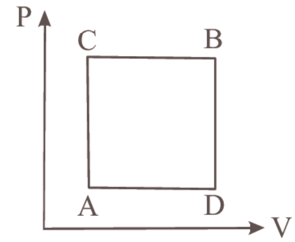
(c) $n = \frac{C_P}{C_V}$ (d) $n = \frac{C - C_P}{C - C_V}$

28. Two moles of an ideal monoatomic gas occupies a volume V at 27°C. The gas expands adiabatically to a volume 2 V. Calculate (a) the final temperature of the gas and (b) change in its internal energy. **[2019]**

(a) (a) 189 K (b) 2.7 kJ

(b) (a)195 K (b) –2.7 kJ

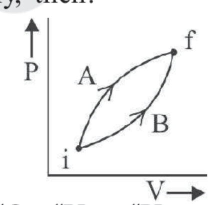
29. A gas can be taken from A to B via two different processes ACB and ADB. **[2019]**



When path ACB is used 60 J of heat flows into the system and 30J of work is done by the system. If path ADB is used work done by the system is 10 J. The heat Flow into the system in path ADB is :

(a) 40 J (b) 80 J (c) 100 J (d) 20 J

30. Following figure shows two processes A and B for a gas. If “Q_A and “Q_B are the amount of heat absorbed by the system in two cases, and “U_A and “U_B are changes in internal energies, respectively, then: **[2019]**



(a) “Q_A < “Q_B, “U_A < “U_B

(b) “Q_A > “Q_B, “U_A > “U_B

(c) “Q_A > “Q_B, “U_A = “U_B

(d) “Q_A = “Q_B; “U_A = “U_B

31. A litre of dry air at STP expands adiabatically to a volume of 3 litres. If γ = 1.40, the work done by air is: **[2020]**

($3^{1.4}$ = 4.6555) [Take air to be an ideal gas]

(a) 60.7 J (b) 90.5 J

(c) 100.8 J (d) 48 J

32. A Carnot engine operates between two reservoirs of temperatures 900 K and 300 K. The engine performs 1200 J of work per cycle. The heat energy (in J) delivered by the engine to the low temperature reservoir, in a cycle, is .... **[2020]**

Answer Key

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
(a)	(c)	(a)	(d)	(c)	(b)	(b)	(a)	(b, c)	(b)	(d)	(a)	(b)	(c)	(a)
16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
(b)	(a)	(a)	(b)	(d)	(a)	(c)	(b)	(d)	(d)	(c)	(d)	(c)	(a)	(c)
31	32													
(b)	(600)													

Solutions

1. **(a)** All reversible engines have same efficiencies if they are working for the same temperature of source and sink. If the temperatures are different, the efficiency is different.

2. **(c)** In Carnot's cycle we assume frictionless piston, absolute insulation and ideal source and sink (reservoirs). The efficiency of carnot's cycle $\eta = 1 - \frac{T_2}{T_1}$
The efficiency of carnot engine will be 100% when its sink (T_2) is at 0 K.
The temperature of 0 K (absolute zero) cannot be realised in practice so, efficiency is never 100%.

3. **(a)** This is a consequence of second law of thermodynamics

4. **(d)** Given, $P \propto T^3 \Rightarrow PT^{-3}$ = constant ....(i)
For an adiabatic process,
$P^{1-\gamma} \quad T^{\gamma}$ = constant
$$\Rightarrow PT^{\frac{\gamma}{1-\gamma}} = \text{constt.} \quad ....(ii)$$
From (i) and (ii) we get $\frac{\gamma}{1-\gamma} = -3$
$$\Rightarrow \gamma = -3 + 3\gamma \Rightarrow \gamma = \frac{3}{2}$$

5. **(c)** Work is not a state function. The remaining three parameters are state function.

6. **(b)** Here, $T_1 = 627 + 273 = 900$ K
$T_2 = 27 + 273 = 300$ K
Efficiency, $\eta = 1 - \frac{T_2}{T_1}$
$$= 1 - \frac{300}{900} = 1 - \frac{1}{3} = \frac{2}{3}$$
But $\eta = \frac{W}{Q}$
$$\therefore \frac{W}{Q} = \frac{2}{3} \Rightarrow W = \frac{2}{3} \times Q = \frac{2}{3} \times 3 \times 10^6$$
$= 2 \times 10^6$ cal
$= 2 \times 10^6 \times 4.2$ J $= 8.4 \times 10^6$ J

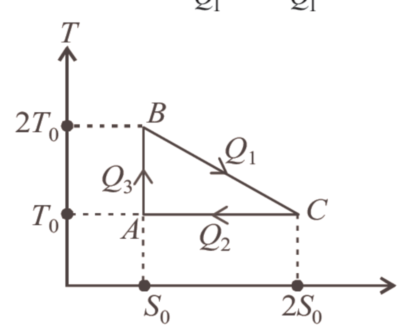
7. **(b)** Internal energy and entropy are state function, they are independent of path taken.

8. **(a)** Here $Q = 0$ and $W = 0$.
Therefore, from first law of thermodynamics
$\Delta U = Q + W = 0$
$\therefore$ Internal energy of first vessel + Internal energy of second vessel = Internal energy of combined vessel
$$n_1 C_v\, T_1 + n_2\, C_v\, T_2 = (n_1 + n_2) C_v\, T$$
$$\therefore \quad T = \frac{n_1 T_1 + n_2 T_2}{n_1 + n_2}$$
For first vessle, $n_1 = \frac{P_1 V_1}{RT_1}$
and For second vessel $n_2 = \frac{P_2 V_2}{RT_2}$
$$\therefore \quad T = \frac{\frac{P_1 V_1}{RT_1} \times T_1 + \frac{P_2 V_2}{RT_2} \times T_2}{\frac{P_1 V_1}{RT_1} + \frac{P_2 V_2}{RT_2}}$$
$$= \frac{T_1 T_2\, (P_1 V_1 + P_2 V_2)}{P_1 V_1 T_2 + P_2 V_2 T_1}$$

9. **(b, c)** First law is applicable to a cyclic process. Concept of entropy is introduced by the second law of thermodynamics.

10. **(b)** Change in internal energy is independent of path taken by the process. It only depends on initial and final states *i.e.*,
$$\Delta U_1 = \Delta U_2$$

11. **(d)** Q_1 = area under BC = $T_0 S_0 + \frac{1}{2} T_0 S_0$
Q_2 = area under AC = $T_0(2S_0 - S_0) = T_0 S_0$
and $Q_3 = 0$
Efficiency, $\eta = \frac{W}{Q_1} = \frac{Q_1 - Q_2}{Q_1}$



$$= 1 - \frac{Q_2}{Q_1} = 1 - \frac{T_0 S_0}{\frac{3}{2} T_0 S_0} = \frac{1}{3}$$

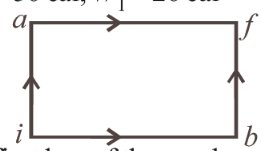
12. **(a)** Work done in adiabatic compression is given by
$$W = \frac{nR\Delta T}{1-\gamma}$$

$$\Rightarrow -146000 = \frac{1000 \times 8.3 \times 7}{1-\gamma}$$

or $1-\gamma = -\frac{58.1}{146} \Rightarrow \gamma = 1 + \frac{58.1}{146} = 1.4$

Hence the gas is diatomic.

13. (b) For path *iaf*,
$Q_1 = 50$ cal, $W_1 = 20$ cal



By first law of thermodynamics,
$\Delta U = Q_1 - W_1 = 50 - 20 = 30$ cal.
For path *ibf*
$Q_2 = 36$ cal
$W_2 = ?$
$\Delta U_{ibf} = Q_2 - W_2$
Since, the change in internal energy does not depend on the path, therefore $\Delta U_{iaf} = \Delta U_{ibf}$

$\Delta U_{iaf} = \Delta U_{ibf}$

$\Rightarrow \quad 30 = Q_2 - W_2$

$\Rightarrow \quad W_2 = 36 - 30 = 6$ cal.

14. (c) The efficiency (η) of a Carnot engine and the coefficient of performance (β) of a refrigerator are related as

$$\beta = \frac{1-\eta}{\eta}$$

Also, $\beta = \frac{Q_2}{W}$

$$\therefore \beta = \frac{1-n}{n} = \frac{Q_2}{W}$$

$$\therefore \beta = \frac{1-\frac{1}{10}}{\left(\frac{1}{10}\right)} = \frac{Q_2}{W}.$$

is independent of path taken by the process.

$$\Rightarrow 9 = \frac{Q_2}{10}$$

$\Rightarrow Q_2 = 90$ J.

15. (a) Here $Q = 0$ and $W = 0$. Therefore from first law of thermodynamics $\Delta U = Q + W = 0$
Internal energy of first vessle + Internal energy of second vessel = Internal energy of combined vessel

$$n_1 C_v T_1 + n_2 C_v T_2 = (n_1 + n_2) C_v T$$

$$\therefore T = \frac{n_1 T_1 + n_2 T_2}{n_1 + n_2}$$

For first vessel $n_1 = \frac{P_1 V_1}{RT_1}$ and for second vessle

$$n_2 = \frac{P_2 V_2}{RT_2}$$

$$\therefore T = \frac{\frac{P_1 V_1}{RT_1} \times T_1 + \frac{P_2 V_2}{RT_2} \times T_2}{\frac{P_1 V_1}{RT_1} + \frac{P_2 V_2}{RT_2}}$$

$$= \frac{T_1 T_2 (P_1 V_1 + P_2 V_2)}{P_1 V_1 T_2 + P_2 V_2 T_1}$$

16. (b) The process $A \to B$ is isobaric.
$\therefore$ work done $W_{AB} = nR(T_2 - T_1)$
$= 2R(500 - 300) = 400R$

17. (a) The process D to A is isothermal as temperature is constant.

Work done, $W_{DA} = 2.303 nRT \log_{10} \frac{P_D}{P_A}$

$= 2.303 \times 2\ R \times 300$

$\log_{10} \frac{1 \times 10^5}{2 \times 10^5} - 414$R.

Therefore, work done on the gas is $+414\ R$.

18. (a) The net work in the cycle *ABCDA* is

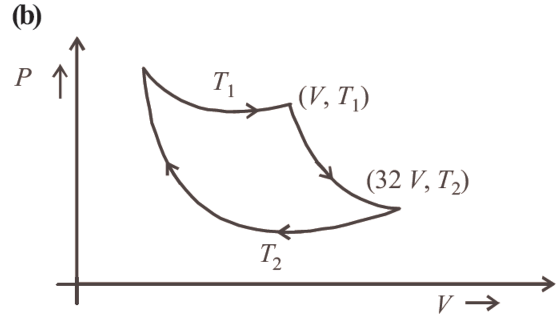
$$W = W_{AB} + W_{BC} + W_{CD} + W_{DA}$$

$$= 400R + 2.303 nRT \log \frac{P_B}{P_C} + (-400R) - 414R$$

$$= 2.303 \times 2R \times 500 \log \frac{2 \times 10^5}{1 \times 10^5} - 414\,R$$

$$= 693.2\,R - 414\,R \quad = 279.2\,R$$

19. (b)



For adiabatic expansion $T_1 V_1^{\gamma-1} = T_2 V_2^{\gamma-1}$

$$\Rightarrow T_1 V^{g-1} = T_2 (32V)^{g-1}$$

$$\Rightarrow \frac{T_1}{T_2} = (32)^{\gamma-1}$$

For diatomic gas, $\gamma = \frac{7}{5}$

$\therefore \gamma - 1 = \frac{2}{5}$

$\therefore \frac{T_1}{T_2} = (32)^{\frac{2}{5}} \Rightarrow T_1 = 4T_2$

Now, efficiency $= 1 - \frac{T_2}{T_1}$

$= 1 - \frac{T_2}{4T_2} = 1 - \frac{1}{4} = \frac{3}{4} = 0.75.$

20. (d) Efficiency of engine

$\eta_1 = 1 - \frac{T_2}{T_1} = \frac{1}{6}$

$\Rightarrow \frac{T_2}{T_1} = \frac{5}{6}$ ....(i)

When T_2 is lowered by 62K, then

Again, $\eta_2 = 1 - \frac{T_2 - 62}{T_1}$

$= 1 - \frac{T_2}{T_1} + \frac{62}{T_1} = \frac{1}{3}$ ....(ii)

Solving (i) and (ii), we get,

$T_1 = 372$ K and $T_2 = \frac{5}{6} \times 372 = 310$ K

21. (a) The efficiency

$\eta = \frac{\text{output work}}{\text{heat given to the system}}$

Heat given in going A to B, $Q_{AB} = nCv\Delta T$

$= n\frac{3}{2}R\Delta T = \frac{3}{2}V_0\Delta P = \frac{3}{2} P_0V_0$

$W_i = \frac{n}{2}(P_0V_0) + \frac{n}{2}(2P_0V_0) + 2P_0V_0$

Heat given in going B to $C = nCp\Delta T$

$= n\left(\frac{5}{2}R\right)\Delta T = \frac{5}{2}(2P_0)\Delta V$

$= 5P_0V_0$

and W_0 = area under PV diagram P_0V_0

$\eta = \frac{W}{Q} = \frac{P_0V_0}{\frac{13}{2}P_0V_0} = \frac{2}{13}$

Efficiency in %

$\eta = \frac{2}{13} \times 100 = \frac{200}{13} \simeq 15.4\%$

22. (c) The efficiency of the carnot's heat engine is given as

$\eta = \left(1 - \frac{T_2}{T_1}\right) \times 100$

When efficiency is 40%,

$T_1 = 500$ K; $\eta = 40$

$40 = \left(1 - \frac{T_2}{500}\right) \times 100$

$\Rightarrow \frac{40}{100} = 1 - \frac{T_2}{500}$

$\Rightarrow \frac{T_2}{500} = \frac{60}{100} \Rightarrow T_2 = 300$ K

When efficiency is 60%, then

$\frac{60}{100} = \left(1 - \frac{300}{T_2}\right) \Rightarrow \frac{300}{T_2} = \frac{40}{100}$

$\Rightarrow T_2 = \frac{100 \times 300}{40} \Rightarrow T_2 = 750$ K

23. (b) Heat is extracted from the source in path DA and AB is

$\Delta Q = \frac{3}{2}R\left(\frac{P_0V_0}{R}\right) + \frac{5}{2}R\left(\frac{2P_0V_0}{R}\right)$

$\Rightarrow \frac{3}{2}P_0V_0 + \frac{5}{2}2P_0V_0 = \left(\frac{13}{2}\right)P_0V_0$

24. (d) In cyclic process, change in total internal energy is zero.

$\Delta U_{cyclic} = 0$

$\Delta U_{BC} = nC_v\Delta T = 1 \times \frac{5R}{2}\Delta T$

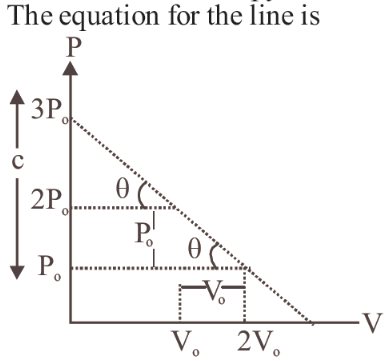
Where, C_v = molar specific heat at constant volume.

For BC, $\Delta T = -200$ K

$\therefore \Delta U_{BC} = -500R$

25. (d) The entropy change of the body in the two cases is same as entropy is a state function.

26. (c) The equation for the line is



$P = \frac{-P_0}{V_0}V + 3P$

$\left[\text{slope} = \frac{-P_0}{V_0}, c = 3P_0\right]$

$PV_0 + P_0V = 3P_0V_0$ ...(i)

But $pV = nRT$

$\therefore P = \frac{nRT}{V}$ ...(ii)

From (i) & (ii) $\frac{nRT}{V}V_0 + P_0V = 3P_0V_0$

$\therefore nRT\,V_0 + P_0V^2 = 3P_0V_0V$ ...(iii)

For temperature to be maximum $\frac{dT}{dV} = 0$

Differentiating e.q. (iii) by 'V' we get

$nRV_0\frac{dT}{dV} + P_0(2V) = 3P_0V_0$

$\therefore nRV_0\frac{dT}{dV} = 3P_0V_0 - 2P_0V$

$\frac{dT}{dV} = \frac{3P_0V_0 - 2P_0V}{nRV_0} = 0$

$V = \frac{3V_0}{2} \quad \therefore \quad P = \frac{3P_0}{2}$ [From (i)]

$\therefore T_{max} = \frac{9P_0V_0}{4nR}$ [From (iii)]

27. (d) For a polytropic process

$C = C_v + \frac{R}{1-n} \quad \therefore \quad C - C_v = \frac{R}{1-n}$

$\therefore \; 1 - n = \frac{R}{C - C_v} \quad \therefore \; 1 - \frac{R}{C - C_v} = n$

$\therefore \; n = \frac{C - C_v - R}{C - C_v} = \frac{C - C_v - C_p + C_v}{C - C_v}$

$= \frac{C - C_p}{C - C_v} \; (\because C_p - C^{v=R})$

28. (c) In an adiabatic process

$TV^{\gamma-1} = \text{Constant}$

or, $T_1V_1^{\gamma-1} = T_2V_2^{\gamma-1}$

For monoatomic gas $\gamma = \frac{5}{3}$

$(300)V^{2/3} = T_2(2V)^{2/3}$

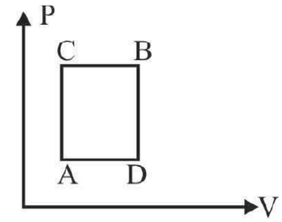
$\Rightarrow \; T_2 = \frac{300}{(2)^{2/3}}$

$T_2 = 189$ K (final temperature)

Change in internal energy $\Delta U = n\frac{f}{2}R\;\Delta T$

$= 2\left(\frac{3}{2}\right)\left(\frac{25}{3}\right)(-111) = -2.7\,\text{kJ}$

29. (a)



$\Delta Q_{ACB} = \Delta W_{ACB} + \Delta U_{ACB}$

$\Rightarrow 60\text{ J} = 30\text{ J} + \Delta U_{ACB}$

$\Rightarrow U_{ACB} = 30\text{ J}$

$\Delta Q_{ADB} = \Delta U_{ADB} + \Delta W_{ADB}$

$= 10\text{ J} + 30\text{ J} = 40\text{ J}$

$[\because \Delta U_{ADB} = \Delta U_{ACB} = 30\text{ J}]$

30. (c) Internal energy depends only on initial and final state

So, $\Delta U_A = \Delta U_B$

Also $\Delta Q = \Delta U + \Delta W$

$\therefore W_A > W_B \Rightarrow \Delta Q_A > \Delta Q_B$

[Area under P-V graph gives the work done.]

31. (b) Given, $V_1 = 1$ litre, $P_1 = 1$ atm

$V_2 = 3$ litre, $\gamma = 1.40$,

Using, $PV^r = \text{constant} \Rightarrow P_1V_1^{\gamma} = P_2V_2^{\gamma}$

$\Rightarrow \; P_2 = P_1 \times \left(\frac{1}{3}\right)^{1.4} = \frac{1}{4.6555}\text{ atm}$

$\therefore$ Work done, $W = \frac{P_1V_1 - P_2V_2}{\gamma - 1}$

$= \frac{\left(1 \times 1 - \frac{1}{4.6555} \times 3\right)1.01325 \times 10^5 \times 10^{-3}}{0.4}$

$= 90.1\,J$

Closest value of $W = 90.5\,J$

32. (600.00) Given: $T_1 = 900\,K$, $T_2 = 300K$, $W = 1200\,J$

Using, $1 - \frac{T_2}{T_1} = \frac{W}{Q_1}$

$\Rightarrow 1 - \frac{300}{900} = \frac{1200}{Q_1}$

$\Rightarrow \frac{2}{3} = \frac{1200}{Q_1} \Rightarrow Q_1 = 1800$

Therefore heat energy delivered by the engine to the low temperature reservoir, $Q_2 = Q_1 - W = 1800 - 1200 = 600.00\,J$

12 Kinetic Theory

1. Cooking gas containers are kept in a lorry moving with uniform speed. The temperature of the gas molecules inside will **[2002]**
(a) increase
(b) decrease
(c) remain same
(d) decrease for some, while increase for others

2. At what temperature is the r.m.s velocity of a hydrogen molecule equal to that of an oxygen molecule at 47°C? **[2002]**
(a) 80 K (b) –73 K
(c) 3 K (d) 20 K.

3. A gaseous mixture consists of 16 g of helium and 16 g of oxygen. The ratio $\frac{C_p}{C_v}$ of the mixture is **[2005]**
(a) 1.62 (b) 1.59 (c) 1.54 (d) 1.4

4. The speed of sound in oxygen (O_2) at a certain temperature is 460 ms^{-1}. The speed of sound in helium (He) at the same temperature will be (assume both gases to be ideal) **[2008]**
(a) 1421 ms^{-1} (b) 500 ms^{-1}
(c) 650 ms^{-1} (d) 330 ms^{-1}

5. One kg of a diatomic gas is at a pressure of $8 \times 10^4 N/m^2$. The density of the gas is $4 kg/m^3$. What is the energy of the gas due to its thermal motion? **[2009]**
(a) 5×10^4 J (b) 6×10^4 J
(c) 7×10^4 J (d) 3×10^4 J

6. A thermally insulated vessel contains an ideal gas of molecular mass M and ratio of specific heats γ. It is moving with speed v and it's suddenly brought to rest. Assuming no heat is lost to the surroundings, its temperature increases by: **[2011]**
(a) $\frac{(\gamma-1)}{2\gamma R}Mv^2K$ (b) $\frac{\gamma M^2 v}{2R}K$
(c) $\frac{(\gamma-1)}{2R}Mv^2K$ (d) $\frac{(\gamma-1)}{2(\gamma+1)R}Mv^2K$

7. Consider an ideal gas confined in an isolated closed chamber. As the gas undergoes an adiabatic expansion, the average time of collision between molecules increases as V^q, where V is the volume of the gas. The value of q is : $\left(\gamma = \frac{C_p}{C_v}\right)$ **[2015]**
(a) $\frac{\gamma+1}{2}$ (b) $\frac{\gamma-1}{2}$
(c) $\frac{3\gamma+5}{6}$ (d) $\frac{3\gamma-5}{6}$

8. The temperature of an open room of volume 30 m^3 increases from 17°C to 27°C due to sunshine. The atmospheric pressure in the room remains 1×10^5 Pa. If n_i and n_f are the number of molecules in the room before and after heating, then $n_f - n_i$ will be : **[2017]**
(a) 2.5×10^{25} (b) -2.5×10^{25}
(c) -1.61×10^{23} (d) 1.38×10^{23}

9. C_p and C_v are specific heats at constant pressure and constant volume respectively. It is observed that **[2017]**
$C_p - C_v = a$ for hydrogen gas
$C_p - C_v = b$ for nitrogen gas
The correct relation between a and b is :
(a) $a = 14\,b$ (b) $a = 28\,b$
(c) $a = \frac{1}{14}b$ (d) $a = b$

10. The mass of a hydrogen molecule is 3.32×10^{-27} kg. If 10^{23} hydrogen molecules strike, per second, a fixed wall of area 2 cm^2 at an angle of 45° to the normal, and rebound elastically with a speed of 10^3 m/s, then the pressure on the wall is nearly: **[2018]**

(a) 2.35×10^3 N/m^2 (b) 4.70×10^3 N/m^2
(c) 2.35×10^2 N/m^2 (c) 4.70×10^2 N/m^2

11. A mixture of 2 moles of helium gas (atomic mass = 4u), and 1 mole of argon gas (atomic mass = 40u) is kept at 300 K in a container. The ratio of their rms speeds **[2019]**

$\left[\dfrac{V_{rms}(\text{helium})}{V_{rms}(\text{argon})}\right]$ is close to :

(a) 3.16 (b) 0.32 (c) 0.45 (d) 2.24

12. For a given gas at 1 atm pressure, rms speed of the molecules is 200 m/s at 127°C. At 2 atm pressure and at 227°C, the rms speed of the molecules will be: **[2019]**

(a) 100 m/s (b) $80\sqrt{5}$ m/s
(c) $100\sqrt{5}$ m/s (d) 80 m/s

13. An HCl molecule has rotational, translational and vibrational motions. If the rms velocity of HCl molecules in its gaseous phase is $\bar{v}$, m is its mass and k_B is Boltzmann constant, then its temperature will be: **[2019]**

(a) $\dfrac{m\bar{v}^2}{6k_B}$ (b) $\dfrac{m\bar{v}^2}{3k_B}$

(c) $\dfrac{m\bar{v}^2}{7k_B}$ (d) $\dfrac{m\bar{v}^2}{5k_B}$

14. Two moles of an ideal gas with $\dfrac{C_p}{C_V} = \dfrac{5}{3}$ are mixed with 3 moles of another ideal gas with $\dfrac{C_p}{C_V} = \dfrac{4}{3}$.

The value of $\dfrac{C_p}{C_V}$ for the mixture is: **[2020]**

(a) 1. 45 (b) 1.50 (c) 1.47 (d) 1.42

Answer Key

1	2	3	4	5	6	7	8	9	10	11	12	13	14	
(c)	(d)	(a)	(a)	(a)	(c)	(a)	(b)	(a)	(a)	(a)	(c)	(a)	(d)	

Solutions

1. **(c)** The centre of mass of gas molecules also moves with lorry with uniform speed. As there is no relative motion of gas molecule. So, kinetic energy and hence temperature remain same.

2. **(d)** RMS velocity of a gas molecule is given by

$$V_{rms} = \sqrt{\frac{3RT}{M}}$$

Let T be the temperature at which the velocity of hydrogen molecule is equal to the velocity of oxygen molecule.

$$\therefore \sqrt{\frac{3RT}{2}} = \sqrt{\frac{3R\times(273+47)}{32}}$$

$\Rightarrow T = 20\text{K}$

3. **(a)** For mixture of gas specific heat at constant volume

$$C_v = \frac{n_1 C_{v_1} + n_2 C_{v_2}}{n_1 + n_2}$$

No. of moles of helium,

$$n_1 = \frac{m_{He}}{M_{He}}$$

$$= \frac{16}{4} = 4$$

Number of moles of oxygen,

$$n_2 = \frac{16}{32} = \frac{1}{2}$$

$$\therefore C_v = \frac{4\times\frac{3}{2}R + \frac{1}{2}\times\frac{5}{2}R}{\left(4+\frac{1}{2}\right)} = \frac{6R + \frac{5}{4}R}{\frac{9}{2}}$$

$$= \frac{29R \times 2}{9 \times 4} = \frac{29R}{18} \text{ and}$$

Specific heat at constant pressure

$$C_p = \frac{n_1 C_{p_1} + n_2 C_{p_2}}{(n_1 + n_2)} = \frac{4 \times \frac{5R}{2} + \frac{1}{2} \times \frac{7R}{2}}{\left(4 + \frac{1}{2}\right)}$$

$$= \frac{10R + \frac{7}{4}R}{\frac{9}{2}} = \frac{47R}{18}$$

$$\therefore \frac{C_p}{C_v} = \frac{47R}{18} \times \frac{18}{29R} = 1.62$$

4. (a) The speed of sound in a gas is given by

$$v = \sqrt{\frac{\gamma RT}{M}}$$

$$\therefore v \propto \sqrt{\frac{\gamma}{M}} \quad \text{[As } R \text{ and } T \text{ is constant]}$$

$$\therefore \frac{v_{O_2}}{v_{He}} = \sqrt{\frac{\gamma_{O_2}}{M_{O_2}} \times \frac{M_{He}}{\gamma_{He}}}$$

$$= \sqrt{\frac{1.4}{32} \times \frac{4}{1.67}} = 0.3237$$

$$\therefore v_{He} = \frac{v_{O_2}}{0.3237} = \frac{460}{0.3237} = 1421 \text{ m/s}$$

5. (a) Given, mass = 1 kg
Density = 4 kg m^{-3}

$$\text{Volume} = \frac{\text{mass}}{\text{density}} = \frac{1}{4} \text{ m}^3$$

Internal energy of the diatomic gas

$$= \frac{5}{2}PV = \frac{5}{2} \times 8 \times 10^4 \times \frac{1}{4} = 5 \times 10^4 J$$

Alternatively:

$$\text{K.E} = \frac{5}{2}nRT = \frac{5}{2}\frac{m}{M}RT = \frac{5}{2}\frac{m}{M} \times \frac{PM}{d}$$

$$[\because PM = dRT]$$

$$= \frac{5}{2}\frac{mP}{d} = \frac{5}{2} \times \frac{1 \times 8 \times 10^4}{4} = 5 \times 10^4 \text{ J}$$

6. (c) As, work done is zero.
So, loss in kinetic energy = heat gain by the gas

$$\frac{1}{2}mv^2 = nC_v\Delta T = n\frac{R}{\gamma - 1}\Delta T$$

$$\frac{1}{2}mv^2 = \frac{m}{M}\frac{R}{\gamma - 1}\Delta T$$

$$\therefore \Delta T = \frac{Mv^2(\gamma - 1)}{2R}K$$

7. (a) $\tau = \dfrac{1}{\sqrt{2}\pi d^2 \left(\frac{N}{V}\right)\sqrt{\frac{3RT}{M}}}$

$$\tau \propto \frac{V}{\sqrt{T}}$$

As, $TV^{\gamma-1} = K$ So, $\tau \propto V^{(\gamma+1)/2}$

Therefore, $q = \dfrac{\gamma + 1}{2}$

8. (b) Given: Temperature $T_i = 17 + 273 = 290\,K$
Temperature $T_f = 27 + 273 = 300\,K$
Atmospheric pressure, $P_0 = 1 \times 10^5\ Pa$
Volume of room, $V_0 = 30\ m^3$
Difference in number of molecules, $n_f - n_i = ?$
Using ideal gas equation, $PV = nRT(N_0)$,
N_0 = Avogadro's number

$$\Rightarrow n = \frac{PV}{RT}(N_0)$$

$$\therefore n_f - n_i = \frac{P_0 V_0}{R}\left(\frac{1}{T_f} - \frac{1}{T_i}\right) N_0$$

$$= \frac{1 \times 10^5 \times 30}{8.314} \times 6.023 \times 10^{23}\left(\frac{1}{300} - \frac{1}{290}\right)$$

$$= -2.5 \times 10^{25}$$

9. (a) As we know, $C_p - C_v = R$ where C_p and C_v are molar specific heat capacities

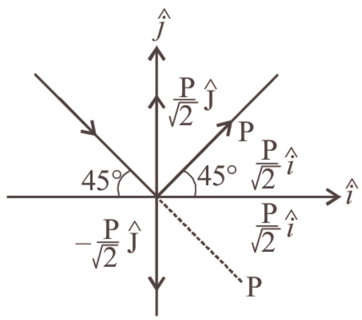
or, $C_p - C_v = \dfrac{R}{M}$

For hydrogen ($M = 2$) $C_p - C_v = a = \dfrac{R}{2}$

For nitrogen ($M = 28$) $C_p - C_v = b = \dfrac{R}{28}$

$$\therefore \frac{a}{b} = 14 \quad \text{or,} \quad a = 14b$$

10. **(a)** Change in momentum



$$\Delta P = \frac{P}{\sqrt{2}}\hat{j} + \frac{P}{\sqrt{2}}\hat{j} + \frac{P}{\sqrt{2}}\hat{i} - \frac{P}{\sqrt{2}}\hat{i}$$

$$\Delta P = \frac{2P}{\sqrt{2}}\hat{j} = I_H \text{ molecule}$$

$$\Rightarrow \ I_{wall} = -\frac{2P}{\sqrt{2}}\hat{j}$$

Pressure, P

$$= \frac{F}{A} = \frac{\sqrt{2}P}{A}n \quad (\because n = \text{no. of particles})$$

$$= \frac{\sqrt{2} \times 3.32 \times 10^{-27} \times 10^{3} \times 10^{23}}{2 \times 10^{-4}}$$

$= 2.35 \times 10^3 N/m^2$

11. **(a)** Using $V_{rms} = \sqrt{\frac{\gamma RT}{M}}$ we have

$$\frac{V_{rms}(He)}{V_{rms}(Ar)} = \sqrt{\frac{M_{Ar}}{M_{He}}} = \sqrt{\frac{40}{4}}$$

$= 3.16$

12. **(c)** $V_{rms} = \sqrt{\frac{3RT}{M}}$

$$\frac{v_1}{v_2} = \sqrt{\frac{T_1}{T_2}} = \sqrt{\frac{(273+127)}{(273+227)}}$$

$$= \sqrt{\frac{400}{500}} = \sqrt{\frac{4}{5}} = \frac{2}{\sqrt{5}}$$

$$\therefore \ v_2 = \frac{\sqrt{5}}{2}v_1 = \frac{\sqrt{5}}{2} \times 200 = 100\sqrt{5} \text{ m/s.}$$

13. **(a)** In this case the total degree of freedom is 6.

According to law of equipartition of energy,

$$\frac{1}{2}m\bar{v}^{2} = 6\left(\frac{1}{2}k_B T\right)$$

$$\therefore \ \frac{1}{2}m\bar{v}^{2} = 3k_B T$$

$$\text{or, } T = \frac{m\bar{v}^{2}}{6k_B}$$

14. **(d)** Using, $\gamma_{mixture} = \frac{n_1 C_{p_1} + n_2 C_{p_2}}{n_1 C_{v_1} + n_2 C_{v_2}}$

$$\Rightarrow \ \frac{n_1}{\gamma_1 - 1} + \frac{n_2}{\gamma_2 - 1} = \frac{n_1 + n_2}{\gamma_m - 1}$$

$$\Rightarrow \ \frac{3}{\frac{4}{3} - 1} + \frac{2}{\frac{5}{3} - 1} = \frac{5}{\gamma_m - 1}$$

$$\Rightarrow \ \frac{9}{1} + \frac{2 \times 3}{2} = \frac{5}{\gamma_m - 1}$$

$$\Rightarrow \ \gamma_m - 1 = \frac{5}{12}$$

$$\Rightarrow \ \gamma_m = \frac{17}{12} = 1.42$$

13 Oscillations

1. In a simple harmonic oscillator, at the mean position **[2002]**
   (a) kinetic energy is minimum, potential energy is maximum
   (b) both kinetic and potential energies are maximum
   (c) kinetic energy is maximum, potential energy is minimum
   (d) both kinetic and potential energies are minimum

2. If a spring has time period T, and is cut into n equal parts, then the time period of each part will be **[2002]**
   (a) $T\sqrt{n}$ (b) $T/\sqrt{n}$
   (c) nT (d) T

3. A child swinging on a swing in sitting position, stands up, then the time period if the swing will
   (a) increase **[2002]**
   (b) decrease
   (c) remains same
   (d) increases if the child is long and decreases if the child is short

4. A mass M is suspended from a spring of negligible mass. The spring is pulled a little and then released so that the mass executes SHM of time period T. If the mass is increased by m, the time period becomes $\frac{5T}{3}$. Then the ratio of $\frac{m}{M}$ is **[2003]**
   (a) $\frac{3}{5}$ (b) $\frac{25}{9}$
   (c) $\frac{16}{9}$ (d) $\frac{5}{3}$

5. Two particles A and B of equal masses are suspended from two massless springs of spring constants k_1 and k_2, respectively. If the maximum velocities, during oscillation, are equal, the ratio of amplitude of A and B is **[2003]**
   (a) $\sqrt{\frac{k_1}{k_2}}$ (b) $\frac{k_2}{k_1}$
   (c) $\sqrt{\frac{k_2}{k_1}}$ (d) $\frac{k_1}{k_2}$

6. The length of a simple pendulum executing simple harmonic motion is increased by 21%. The percentage increase in the time period of the pendulum of increased length is **[2003]**
   (a) 11% (b) 21%
   (c) 42% (d) 10%

7. The displacement of a particle varies according to the relation $x = 4(\cos \pi t + \sin \pi t)$. The amplitude of the particle is **[2003]**
   (a) -4 (b) 4
   (c) $4\sqrt{2}$ (d) 8

8. A body executes simple harmonic motion. The potential energy (P.E), the kinetic energy (K.E) and total energy (T.E) are measured as a function of displacement x. Which of the following statements is true ? **[2003]**
   (a) K.E. is maximum when $x = 0$
   (b) T.E is zero when $x = 0$
   (c) K.E is maximum when x is maximum
   (d) P.E is maximum when $x = 0$

9. The bob of a simple pendulum executes simple harmonic motion in water with a period t, while the period of oscillation of the bob is t_0 in air. Neglecting frictional force of water and given that the density of the bob is $(4/3) \times 1000$ kg/m^3. Which relationship between t and t_0 is true?
   (a) $t = 2t_0$ (b) $t = t_0/2$ **[2004]**
   (c) $t = t_0$ (d) $t = 4t_0$

10. A particle at the end of a spring executes S.H.M with a period t_1. while the corresponding period for another spring is t_2. If the period of oscillation with the two springs in series is T then **[2004]**

(a) $T^{-1} = t_1^{-1} + t_2^{-1}$ (b) $T^2 = t_1^2 + t_2^2$

(c) $T = t_1 + t_2$ (d) $T^{-2} = t_1^{-2} + t_2^{-2}$

11. The total energy of a particle, executing simple harmonic motion is **[2004]**

(a) independent of x (b) $\propto x^2$

(c) $\propto x$ (d) $\propto x^{1/2}$

where x is the displacement from the mean position.

12. A particle of mass m is attached to a spring (of spring constant k) and has a natural angular frequency ω_0. An external force $F(t)$ proportional to $\cos\omega t(\omega \neq \omega_0)$ is applied to the oscillator. The displacement of the oscillator will be proportional to **[2004]**

(a) $\dfrac{1}{m(\omega_0^2 + \omega^2)}$ (b) $\dfrac{1}{m(\omega_0^2 - \omega^2)}$

(c) $\dfrac{m}{\omega_0^2 - \omega^2}$ (d) $\dfrac{m}{(\omega_0^2 + \omega^2)}$

13. In forced oscillation of a particle the amplitude is maximum for a frequency ω_1 of the force while the energy is maximum for a frequency ω_2 of the force; then **[2004]**

(a) $\omega_1 < \omega_2$ when damping is small and $\omega_1 > \omega_2$ when damping is large

(b) $\omega_1 > \omega_2$

(c) $\omega_1 = \omega_2$

(d) $\omega_1 < \omega_2$

14. Two simple harmonic motions are represented by the equations $y_1 = 0.1 \sin\left(100\pi t + \dfrac{\pi}{3}\right)$ and $y_2 = 0.1 \cos \pi t$. The phase difference of the velocity of particle 1 with respect to the velocity of particle 2 is **[2005]**

(a) $\dfrac{\pi}{3}$ (b) $\dfrac{-\pi}{6}$

(c) $\dfrac{\pi}{6}$ (d) $\dfrac{-\pi}{3}$

15. The function $\sin^2(\omega t)$ represents **[2005]**

(a) a periodic, but not simple harmonic motion with a period $\dfrac{\pi}{\omega}$

(b) a periodic, but not simple harmonic motion with a period $\dfrac{2\pi}{\omega}$

(c) a simple harmonic motion with a period $\dfrac{\pi}{\omega}$

(d) a simple harmonic motion with a period $\dfrac{2\pi}{\omega}$

16. The bob of a simple pendulum is a spherical hollow ball filled with water. A plugged hole near the bottom of the oscillating bob gets suddenly unplugged. During observation, till water is coming out, the time period of oscillation would **[2005]**

(a) first decrease and then increase to the original value

(b) first increase and then decrease to the original value

(c) increase towards a saturation value

(d) remain unchanged

17. If a simple harmonic motion is represented by $\dfrac{d^2x}{dt^2} + \alpha x = 0$, its time period is **[2005]**

(a) $\dfrac{2\pi}{\sqrt{\alpha}}$ (b) $\dfrac{2\pi}{\alpha}$

(c) $2\pi\sqrt{\alpha}$ (d) $2\pi\alpha$

18. The maximum velocity of a particle, executing simple harmonic motion with an amplitude 7 mm, is 4.4 m/s. The period of oscillation is **[2006]**

(a) 0.01 s (b) 10 s

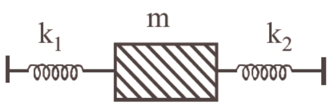
(c) 0.1 s (d) 100 s

19. Starting from the origin a body oscillates simple harmonically with a period of 2 s. After what time will its kinetic energy be 75% of the total energy? **[2006]**

(a) $\dfrac{1}{6}$s (b) $\dfrac{1}{4}$s

(c) $\dfrac{1}{3}$s (d) $\dfrac{1}{12}$s

20. Two springs, of force constants k_1 and k_2 are connected to a mass m as shown. The frequency of oscillation of the mass is f. If both k_1 and k_2 are made four times their original values, the frequency of oscillation becomes **[2007]**



(a) $2f$ (b) $f/2$
(c) $f/4$ (d) $4f$

21. A particle of mass m executes simple harmonic motion with amplitude a and frequency ν. The average kinetic energy during its motion from the position of equilibrium to the end is **[2007]**

(a) $2\pi^2 ma^2\nu^2$ (b) $\pi^2 ma^2\nu^2$
(c) $\frac{1}{4}ma^2\nu^2$ (d) $4\pi^2 ma^2\nu^2$

22. The displacement of an object attached to a spring and executing simple harmonic motion is given by $x = 2 \times 10^{-2} \cos \pi t$ metre. The time at which the maximum speed first occurs is **[2007]**

(a) 0.25 s (b) 0.5 s
(c) 0.75 s (d) 0.125 s

23. A point mass oscillates along the x-axis according to the law $x = x_0 \cos(\omega t - \pi/4)$. If the acceleration of the particle is written as $a = A\cos(\omega t + \delta)$,then **[2007]**

(a) $A = x_0\omega^2,\ \delta = 3\pi/4$
(b) $A = x_0, \delta = -\pi/4$
(c) $A = x_0\omega^2,\ \delta = \pi/4$
(d) $A = x_0\omega^2,\ \delta = -\pi/4$

24. If x, v and a denote the displacement, the velocity and the acceleration of a particle executing simple harmonic motion of time period T, then, which of the following does not change with time? **[2009]**

(a) aT/x (b) $aT + 2\pi v$
(c) aT/v (d) $a^2T^2 + 4\pi^2v^2$

25. Two particles are executing simple harmonic motion of the same amplitude A and frequency ω along the x-axis. Their mean position is separated by distance $X_0 (X_0 > A)$. If the maximum separation between them is $(X_0 + A)$, the phase difference between their motion is: **[2011]**

(a) $\frac{\pi}{3}$ (b) $\frac{\pi}{4}$
(c) $\frac{\pi}{6}$ (d) $\frac{\pi}{2}$

26. A mass M, attached to a horizontal spring, executes S.H.M. with amplitude A_1. When the mass M passes through its mean position then a smaller mass m is placed over it and both of them move together with amplitude A_2. The ratio of $\left(\frac{A_1}{A_2}\right)$ is: **[2011]**

(a) $\frac{M+m}{M}$ (b) $\left(\frac{M}{M+m}\right)^{\frac{1}{2}}$
(c) $\left(\frac{M+m}{M}\right)^{\frac{1}{2}}$ (d) $\frac{M}{M+m}$

27. A wooden cube (density of wood 'd') of side 'ℓ' floats in a liquid of density 'ρ' with its upper and lower surfaces horizontal. If the cube is pushed slightly down and released, it performs simple harmonic motion of period 'T' **[2011 RS]**

(a) $2\pi\sqrt{\frac{\ell d}{\rho g}}$ (b) $2\pi\sqrt{\frac{\ell \rho}{dg}}$
(c) $2\pi\sqrt{\frac{\ell d}{(\rho - d)g}}$ (d) $2\pi\sqrt{\frac{\ell \rho}{(\rho - d)g}}$

28. If a simple pendulum has significant amplitude (up to a factor of 1/e of original) only in the period between $t = 0$s to $t = \tau$ s, then τ may be called the average life of the pendulum. When the spherical bob of the pendulum suffers a retardation (due to viscous drag) proportional to its velocity with b as the constant of proportionality, the average life time of the pendulum in second is (assuming damping is small) **[2012]**

(a) $\frac{0.693}{b}$ (b) b (c) $\frac{1}{b}$ (d) $\frac{2}{b}$

29. This question has Statement 1 and Statement 2. Of the four choices given after the Statements, choose the one that best describes the two Statements.
If two springs S_1 and S_2 of force constants k_1 and k_2 respectively, are stretched by the same force, it is found that more work is done on spring S_1 than on spring S_2.
Statement 1 : If stretched by the same amount work done on S_1
Statement 2 : $k_1 < k_2$ **[2012]**

(a) Statement 1 is false, Statement 2 is true.
(b) Statement 1 is true, Statement 2 is false.
(c) Statement 1 is true, Statement 2 is true, Statement 2 is the correct explanation for Statement 1
(d) Statement 1 is true, Statement 2 is true, Statement 2 is not the correct explanation for Statement 1

30. The amplitude of a damped oscillator decreases to 0.9 times its original magnitude in 5s. In another 10s it will decrease to α times its original magnitude, where α equals **[2013]**

(a) 0.7 (b) 0.81
(c) 0.729 (d) 0.6

31. An ideal gas enclosed in a vertical cylindrical container supports a freely moving piston of mass M. The piston and the cylinder have equal cross sectional area A. When the piston is in equilibrium, the volume of the gas is V_0 and its pressure is P_0. The piston is slightly displaced from the equilibrium position and released. Assuming that the system is completely isolated from its surrounding, the piston executes a simple harmonic motion with frequency **[2013]**

(a) $\frac{1}{2\pi}\frac{A\gamma P_0}{V_0 M}$ (b) $\frac{1}{2\pi}\frac{V_0 M P_0}{A^2\gamma}$

(c) $\frac{1}{2\pi}\sqrt{\frac{A^2\gamma P_0}{MV_0}}$ (d) $\frac{1}{2\pi}\sqrt{\frac{MV_0}{A\gamma P_0}}$

32. A particle moves with simple harmonic motion in a straight line. In first τs, after starting from rest it travels a distance a, and in next τs it travels 2a, in same direction, then: **[2014]**

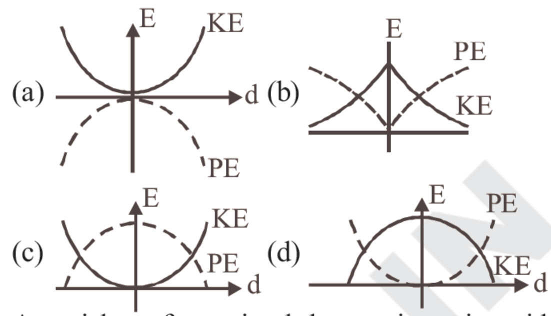
(a) amplitude of motion is 3a
(b) time period of oscillations is 8τ
(c) amplitude of motion is 4a
(d) time period of oscillations is 6τ

33. A pendulum made of a uniform wire of cross sectional area A has time period T. When an additional mass M is added to its bob, the time period changes to T_M. If the Young's modulus of the material of the wire is Y then $\frac{1}{Y}$ is equal to :
(g = *gravitational acceleration*) **[2015]**

(a) $\left[1-\left(\frac{T_M}{T}\right)^2\right]\frac{A}{Mg}$ (b) $\left[1-\left(\frac{T}{T_M}\right)^2\right]\frac{A}{Mg}$

(c) $\left[\left(\frac{T_M}{T}\right)^2-1\right]\frac{A}{Mg}$ (d) $\left[\left(\frac{T_M}{T}\right)^2-1\right]\frac{Mg}{A}$

34. For a simple pendulum, a graph is plotted between its kinetic energy (KE) and potential energy (PE) against its displacement d. Which one of the following represents these correctly? (*graphs are schematic and not drawn to scale*) **[2015]**

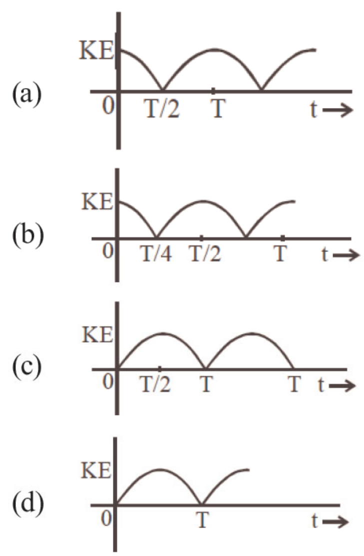


35. A particle performs simple harmonic motion with amplitude A. Its speed is trebled at the instant that it is at a distance $\frac{2A}{3}$ from equilibrium position. The new amplitude of the motion is : **[2016]**

(a) $A\sqrt{3}$ (b) $\frac{7A}{3}$

(c) $\frac{A}{3}\sqrt{41}$ (d) 3A

36. A particle is executing simple harmonic motion with a time period T. At time t = 0, it is at its position of equilibrium. The kinetic energy-time graph of the particle will look like: **[2017]**



37. A silver atom in a solid oscillates in simple harmonic motion in some direction with a frequency of 10^{12}/sec. What is the force constant of the bonds connecting one atom with the other? (Mole wt. of silver = 108 and Avagadro number = 6.02×10^{23} gm mole^{-1}) **[2018]**

(a) 6.4 N/m (b) 7.1 N/m
(c) 2.2 N/m (d) 5.5 N/m

38. A simple pendulum oscillating in air has period T. The bob of the pendulum is completely immersed in a non-viscous liquid. The density of the liquid is $\frac{1}{16}$th of the material of the bob. If the bob is inside liquid all the time, its period of oscillation in this liquid is : **[2019]**

(a) $2T\sqrt{\frac{1}{10}}$ (b) $2T\sqrt{\frac{1}{14}}$

(c) $4T\sqrt{\frac{1}{15}}$ (d) $4T\sqrt{\frac{1}{14}}$

Answer Key

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
(c)	(b)	(b)	(c)	(c)	(d)	(c)	(a)	(a)	(b)	(a)	(b)	(c)	(b)	(a)
16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
(b)	(a)	(a)	(a)	(a)	(b)	(b)	(a)	(a)	(a)	(c)	(a)	(d)	(b)	(c)
31	32	33	34	35	36	37	38							
(c)	(d)	(c)	(d)	(b)	(b)	(b)	(c)							

Solutions

1. **(c)** The kinetic energy (K. E.) of particle in SHM is given by,

$$K.E = \frac{1}{2}k(A^2 - x^2);$$

Potential energy of particle in SHM is

$$U = \frac{1}{2}kx^2$$

Where A = amplitude and $k = m\omega^2$

x = displacement from the mean position

At the mean position $x = 0$

$$\therefore K.E. = \frac{1}{2}kA^2 = \text{Maximum}$$

and U = 0

2. **(b)** Let k be the spring constant of the original spring.

Time period T = $2\pi\sqrt{\frac{m}{k}}$ where m is the mas s of oscillating body.

When the spring is cut into n equal parts, the spring constant of one part becomes nk. Therefore the new time period,

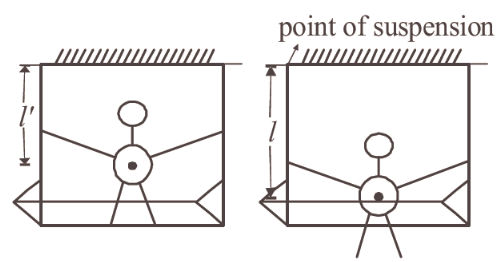
$$T' = 2\pi\sqrt{\frac{m}{nk}} = \frac{T}{\sqrt{n}}$$

3. **(b)** The time period $T = 2\pi\sqrt{\frac{\ell}{g}}$ where ℓ = distance between the point of suspension and the centre of mass of the child.

As the child stands up, her centre of mass is raised. The distance between point of suspension and centre of mass decreases ie length ℓ decreases.

$$\therefore \ell' < \ell$$

$$\therefore T' < T \text{ i.e., the period decreases.}$$



Case (ii) child standing Case (i) child sitting

4. **(c)** With mass M, the time period of the spring.

$$T = 2\pi\sqrt{\frac{M}{k}}$$

With mass $M + m$, the time period becomes,

$$T' = 2\pi\sqrt{\frac{M+m}{k}} = \frac{5T}{3}$$

$$\therefore 2\pi\sqrt{\frac{M+m}{k}} = \frac{5}{3}\times 2\pi\sqrt{\frac{M}{k}}$$

$$\Rightarrow \quad M + m = \frac{25}{9} \times M$$

$$\Rightarrow 1 + \frac{m}{M} = \frac{25}{9}$$

$$\Rightarrow \frac{m}{M} = \frac{25}{9} - 1 = \frac{16}{9}$$

5. **(c)** Maximum velocity during SHM, $V_{max} = A\omega$
But $k = m\omega^2$

$$\therefore \quad \omega = \sqrt{\frac{k}{m}}$$

$\therefore$ Maximum velocity of the body in SHM

$$= A\sqrt{\frac{k}{m}}$$

As maximum velocities are equal

$$\therefore A_1\sqrt{\frac{k_1}{m}} = A_2\sqrt{\frac{k_2}{m}}$$

$$\Rightarrow A_1\sqrt{k_1} = A_2\sqrt{k_2}$$

$$\Rightarrow \frac{A_1}{A_2} = \sqrt{\frac{k_2}{k_1}}$$

6. **(d)** Time period, $T = 2\pi\sqrt{\frac{\ell}{g}}$

New length, $\ell' = \ell + 21\%$ of ℓ

$\ell' = \ell + 0.21\,\ell$

$\Rightarrow \quad \ell' = 1.21\,\ell$

$$T' = 2\pi\sqrt{\frac{1.21\ell}{g}}$$

$$\%\text{ increase in length} = \frac{T' - T}{T} \times 100$$

$$= \frac{\sqrt{1.21\ell} - \sqrt{\ell}}{\sqrt{\ell}} \times 100 = \left(\sqrt{1.21} - \sqrt{1}\right) \times 100$$

$$= (1.1 - 1) \times 100 = 10\%$$

7. **(c)** Displacement, $x = 4(\cos\pi t + \sin\pi t)$

$$= \sqrt{2} \times 4\left(\frac{\sin\pi t}{\sqrt{2}} + \frac{\cos\pi t}{\sqrt{2}}\right)$$

$$= 4\sqrt{2}(\sin\pi\, t\cos 45° + \cos\pi\, t\sin 45°)$$

$$x = 4\sqrt{2}\sin(\pi\, t + 45°)$$

On comparing it with standard equation $x = A\sin(\omega t + \phi)$

we get $A = 4\sqrt{2}$

8. **(a)** K.E. of simple harmonic motion

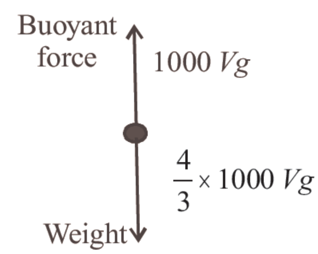
$$= \frac{1}{2}m\omega^2(a^2 - x^2)$$

When x = 0,

$$\text{K.E} = \frac{1}{2}m\omega^2(a^2 - 0)^2\frac{1}{2}m\omega^2a^2 = \text{maximum}$$

9. **(a)** Time period, $t = 2\pi\sqrt{\frac{\ell}{g_{eff}}}$;

In air, $t_0 = 2\pi\sqrt{\frac{\ell}{g}}$



$$\text{Net force} = \left(\frac{4}{3} - 1\right) \times 1000\,Vg = \frac{1000}{3}Vg$$

$$g_{eff} = \frac{1000\,Vg}{3 \times \frac{4}{3} \times 1000V} = \frac{g}{4}$$

$$\therefore t = 2\pi\sqrt{\frac{\ell}{g/4}} = 2 \times 2\pi\frac{\ell}{g}$$

$$t = 2t_0$$

10. **(b)** Time period for first spring, $t_1 = 2\pi\sqrt{\frac{m}{k_1}}$,

Time period for second spring, $t_2 = 2\pi\sqrt{\frac{m}{k_2}}$

Force constant of the series combination

$$k_{\text{eff}} = \frac{k_1 k_2}{k_l + k_2}$$

$\therefore$ Time period of oscillation for series combination $T = 2\pi\sqrt{\frac{m(k_l + k_2)}{k_1 k_2}}$

$$\therefore T = 2\pi\sqrt{\frac{m}{k_2} + \frac{m}{k_1}} = 2\pi\sqrt{\frac{t_2^2}{(2\pi)^2} + \frac{t_1^2}{(2\pi)^2}}$$

$$\Rightarrow T^2 = t_1^2 + t_2^2$$

where x is the displacement from the mean position

11. **(a)** At any instant the total energy in SHM is

$\frac{1}{2}kA_0^2 =$ constant,

where $A_0 =$ amplitude

$k =$ spring constant

hence total energy is independent of x.

12. **(b)** Equation of displacement in forced oscillation is given by

$$y = \frac{F_0}{m\,({\omega_0}^2 - \omega^2)^2}$$

$$= \frac{F_0}{m({\omega_0}^2 - \omega^2)}$$

Here damping effect is considered to be zero

$$\therefore x \propto \frac{1}{m({\omega_0}^2 - \omega^2)}$$

13. **(c)** As energy $\propto$ (Amplitude)^2, the maximum for both of them occurs at the same frequency and this is only possible in case of resonance.

In resonance state $\omega_1 = \omega_2$

14. **(b)** Velocity of particle 1,

$$v_1 = \frac{dy_1}{dt} = 0.1 \times 100\pi \cos\left(100\pi t + \frac{\pi}{3}\right)$$

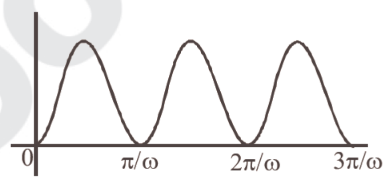
Velocity of particle 2,

$$v_2 = \frac{dy_2}{dt} = -0.1\pi \sin \pi t = 0.1\pi \cos\left(\pi t + \frac{\pi}{2}\right)$$

$\therefore$ Phase difference of velocity of particle 1 with respect to the velocity of particle 2 is

$$= \phi_1 - \phi_2 = \frac{\pi}{3} - \frac{\pi}{2} = \frac{2\pi - 3\pi}{6} = -\frac{\pi}{6}$$

15. **(a)** Clearly $\sin 2\omega t$ is a periodic function with period $\frac{\pi}{\omega}$



For SHM $\frac{d^2y}{dt^2} \propto -y$

$$y = \sin^2 \omega t = \frac{1 - \cos 2\omega t}{2}$$

$$= \frac{1}{2} - \frac{1}{2}\cos 2\omega t$$

$$v = \frac{dy}{dt} = \frac{1}{2} \times 2\omega \sin 2\omega t = 2\omega \sin \omega t \cos \omega t$$

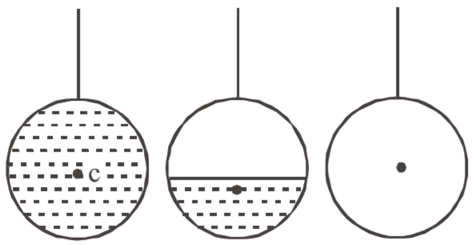
$$= \omega \sin 2\omega t$$

Acceleration, $a = \frac{d^2y}{dt^2} = 2\omega^2 \cos 2\omega t$ which is not proportional to $-y$. Hence, it is not in SHM.

16. **(b)** When plugged hole near the bottom of the oscillating bob gets suddenly unplugged, centre of mass of combination of liquid and hollow portion (at position ℓ), first goes down (to $\ell + \Delta\ell$) and when total water is drained out, centre of mass regain its original position (to ℓ),

Time period, $T = 2\pi\sqrt{\frac{\ell}{g}}$

$\therefore$ 'T' first increases and then decreases to original value.



17. (a) The equation of SHM, $\frac{d^2x}{dt^2} + \alpha x = 0$

$$\frac{d^2x}{dt^2} = -\alpha x = -\omega^2 x$$

$$\Rightarrow \omega = \sqrt{\alpha}$$

$$\Rightarrow T = \frac{2\pi}{\omega} = \frac{2\pi}{\sqrt{\alpha}}$$

18. (a) Maximum velocity,

$v_{max} = a\omega$

Here, a = amplitude of SHM

ω = angular velocity of SHM

$$v_{max} = a \times \frac{2\pi}{T} \therefore \left(\because \omega = \frac{2\pi}{T}\right)$$

$$\Rightarrow T = \frac{2\pi a}{v_{max}} = \frac{2 \times 3.14 \times 7 \times 10^{-3}}{4.4} \approx 0.01\ s$$

19. (a) K.E. of a body undergoing SHM is given by,

$$K.E. = \frac{1}{2}ma^2\omega^2 \cos^2 \omega t$$

Here, a = amplitude of SHM

ω = angular velocity of SHM

Total energy in S.H.M $= \frac{1}{2}ma^2\omega^2$

Given K.E. = 75% T.E.

$$\frac{1}{2}ma^2\omega^2 \cos^2 \omega t = \frac{75}{100} \times \frac{1}{2}ma^2\omega^2$$

$$\Rightarrow 0.75 = \cos^2 \omega t \Rightarrow \omega t = \frac{\pi}{6}$$

$$\Rightarrow t = \frac{\pi}{6 \times \omega} \Rightarrow t = \frac{\pi \times 2}{6 \times 2\pi} \Rightarrow t = \frac{1}{6}\ s$$

20. (a) The two springs are in parallel.

$\therefore$ Effective spring constant,

$k = k_1 + k_2$

Initial frequency of oscillation is given by

$$v = \frac{1}{2p}\sqrt{\frac{k_1 + k_2}{m}} \quad ....(i)$$

When both k_1 and k_2 are made four times their original values, the new frequency is given by

$$v' = \frac{1}{2\pi}\sqrt{\frac{4k_1 + 4k_2}{m}}$$

$$= \frac{1}{2\pi}\sqrt{\frac{4(k_1 + 4k_2)}{m}} = 2\left(\frac{1}{2\pi}\sqrt{\frac{k_1 + k_2}{m}}\right) = 2v$$

21. (b) The kinetic energy of a particle executing S.H.M. at any instant t is given by

$$K = \frac{1}{2}ma^2\omega^2 \sin^2\omega t$$

where, m = mass of particle

a = amplitude

ω = angular frequency

t = time

The average value of $\sin^2\omega t$ over a cycle is $\frac{1}{2}$.

$$\therefore KE = \frac{1}{2}m\omega^2a^2\left(\frac{1}{2}\right) \quad \left(\because \langle \sin^2\theta \rangle = \frac{1}{2}\right)$$

$$= \frac{1}{4}m\omega^2a^2 = \frac{1}{4}ma^2(2\pi\nu)^2 \quad (\because \omega = 2\pi\nu)$$

or, $\langle K \rangle = \pi^2 ma^2\nu^2$

22. (b) Here, Displacement, $x = 2 \times 10^{-2} \cos \pi t$

Velocity is given by

$$v = \frac{dx}{dt} = 2 \times 10^{-2}\ \pi \sin \pi t$$

For the first time, the when velocity becomes maximum,

$\sin \pi t = 1$

$\Rightarrow \sin \pi t = \sin \frac{\pi}{2}$

$\Rightarrow \pi t = \frac{\pi}{2}$ or, $t = \frac{1}{2} = 0.5$ sec.

23. **(a)** Given,

Displacement, $x = x_0 \cos(\omega t - \pi/4)$

$\therefore$ Velocity, $v = \frac{dx}{dt} = -x_0 \omega \sin\left(\omega t - \frac{\pi}{4}\right)$

Acceleration,

$$a = \frac{dv}{dt} = -x_0\omega^2 \cos\left(\omega t - \frac{\pi}{4}\right)$$

$$= x_0\omega^2 \cos\left[\pi + \left(\omega t - \frac{\pi}{4}\right)\right]$$

$$= x_0\omega^2 \cos\left(\omega t + \frac{3\pi}{4}\right) \quad ...(1)$$

Acceleration, $a = A\cos(\omega t + \delta)$ ...(2)

Comparing the two equations, we get

$A = x_0\omega^2$ and $\delta = \frac{3\pi}{4}$.

24. **(a)** For an SHM, the acceleration

$a = -\omega^2 x$ where ω^2 is a constant.

$$a = \frac{-4\pi^2 x}{T^2} \Rightarrow \frac{aT}{x} = \frac{-4\pi^2}{T}$$

The time period T is also constant.

Therefore, $\frac{aT}{x}$ is a constant.

25. **(a)** Let, $x_1 = A \sin \omega t$ and $x_2 = A\sin(\omega t + \phi)$

$$x_2 - x_1 = 2A\cos\left(\omega t + \frac{\phi}{2}\right)\sin\frac{\phi}{2}$$

The above equation is SHM with amplitude

$2A \sin \frac{\phi}{2}$

$\therefore 2A\sin\frac{\phi}{2} = A$

$\Rightarrow \sin\frac{\phi}{2} = \frac{1}{2} \Rightarrow \phi = \frac{\pi}{3}$

26. **(c)** At mean position, F net = 0

Therefore, by principal of conseruation of linear momentum.

$\therefore Mv_1 = (M+m)v_2$

Mw, a, $= (M+m)$ w$_2$ a_2

$$MA_1\sqrt{\frac{k}{M}} = (M+m)A_2\sqrt{\frac{k}{m+M}}$$

$\therefore \left(V = A\sqrt{\frac{k}{M}}\right)$

$\Rightarrow A_1\sqrt{M} = A_2\sqrt{M+m}$

$\Rightarrow \frac{A_1}{A_2} = \sqrt{\frac{m+M}{M}}$

27. **(a)** Let the cube be at a depth x from the equilibrium position.

Force acting on the cube = up thrust on the portion of length x.

$F = -\rho\ell^2 xg$ [$\therefore$ mass density X volume] ....(i)

Clearly $F \propto -x$, Hence it is a SHM.

Equation of SHM is $F = -kx$ ....(ii)

Comparing equation (i) and (ii) we have

$k = \rho\ell^2 g$

Now, Time period, T = $2\pi\sqrt{\frac{m}{k}}$

$$\Rightarrow T = 2\pi\sqrt{\frac{\ell^3 d}{\rho\ell^2 g}}$$

$$= 2\pi\sqrt{\frac{\ell d}{\rho g}}$$

Comparing the above equation with $a = -\omega^2 x$, we get

$$\therefore \quad \omega = \sqrt{\frac{\rho g}{d\ell}} \Rightarrow T = 2\pi\sqrt{\frac{\ell d}{\rho g}}$$

28. **(d)** The equation of motion for the pendulum, for damped harmonic motion

$F = -kx - bv$

$\Rightarrow ma + kx + bv = 0$

$$\Rightarrow m\frac{d^2x}{dt^2} + kx + b\frac{dx}{dt} = 0$$

$$\Rightarrow \frac{d^2x}{dt^2}+\frac{k}{m}x+\frac{b}{m}\frac{dx}{dt}=0$$

$$\Rightarrow \frac{d^2x}{dt^2}+\frac{b}{m}\frac{dx}{dt}+\frac{k}{m}x=0 \quad \ldots (1)$$

Let $x=e^{\lambda t}$ is the solution of the equation (1)

$$\frac{dx}{dt}=\lambda e^{\lambda t} \Rightarrow \frac{d^2x}{dt^2}=\lambda^2 e^{\lambda t}$$

Substituting in the equation (1)

$$\lambda^2 e^{\lambda t}+\frac{b}{m}\lambda e^{\lambda t}+\frac{k}{m}e^{\lambda t}=0$$

$$\lambda^2+\frac{b}{m}\lambda+\frac{k}{m}=0$$

$$\lambda=\frac{-\frac{b}{m}\pm\sqrt{\frac{b^2}{m^2}-4\frac{k}{m}}}{2}=\frac{-b\pm\sqrt{b^2-4km}}{2m}$$

Solving the equation (1) for x, we have

$$x=e^{\frac{-b}{2m}t}$$

$$\omega=\sqrt{\omega_0{}^2-\lambda^2} \text{ where } \omega_0=\frac{k}{m},\ \lambda=\frac{+b}{2}$$

The average life $=\frac{1}{\lambda}=\frac{2}{b}$

29. **(b)** Work done, $w=\frac{1}{2}kx^2$

Work done by spring S_1, $w_1=\frac{1}{2}k_1x^2$

Work done by spring S_2, $w_2=\frac{1}{2}k_2x^2$

Since $w_1>w_2$ Thus $(k_1>k_2)$

30. **(c)** $\because A=A_0e^{-\frac{bt}{2m}}$

(where, A_0 = maximum amplitude)

According to the questions, after 5 second,

$$0.9A_0=A_0e^{-\frac{b(5)}{2m}} \quad \ldots (i)$$

After 10 more second,

$$A=A_0e^{-\frac{b(15)}{2m}} \quad \ldots (ii)$$

From eq^{n}s (i) and (ii)

$A=0.729\,A_0 \therefore \alpha=0.729$

31. **(c)** $\frac{Mg}{A}=P_0$

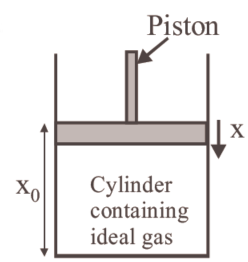
$P_0V_0{}^{\gamma}=PV^{\gamma}$

$Mg=P_0A \quad \ldots (1)$

Let piston is displaced by distance x

$P_0Ax_0{}^{\gamma}=PA(x_0-x)^{\gamma}$

$$P=\frac{P_0x_0^{\gamma}}{(x_0-x)^{\gamma}}$$



$$Mg-\left(\frac{P_0x_0^{\gamma}}{(x_0-x)^{\gamma}}\right)A=F_{\text{restoring}}$$

$$P_0A\left(1-\frac{x_0^{\gamma}}{(x_0-x)^{\gamma}}\right)=F_{\text{restoring}}$$

$[x_0-x\approx x_0]$

$$F=-\frac{\gamma P_0Ax}{x_0}$$

$\therefore$ Frequency with which piston executes SHM.

$$f=\frac{1}{2\pi}\sqrt{\frac{\gamma P_0A}{x_0M}}=\frac{1}{2\pi}\sqrt{\frac{\gamma P_0A^2}{MV_0}}$$

32. **(d)** In simple harmonic motion, starting from rest,

At $t=0$, $x=A$

$x=A\cos\omega t \quad \ldots\ldots (i)$

When $t = \tau$, $x = A - a$

When $t = 2\tau$, $x = A - 3a$

From equation (i)

$A - a = A\cos\omega\tau$ ......(ii)

$A - 3a = A\cos 2\omega\tau$ ....(iii)

As $\cos 2\omega\tau = 2\cos^2\omega\tau - 1$...(iv)

From equation (ii), (iii) and (iv)

$$\frac{A-3a}{A} = 2\left(\frac{A-a}{A}\right)^2 - 1$$

$$\Rightarrow \frac{A-3a}{A} = \frac{2A^2 + 2a^2 - 4Aa - A^2}{A^2}$$

$\Rightarrow A^2 - 3aA = A^2 + 2a^2 - 4Aa$

$\Rightarrow 2a^2 = aA$

$\Rightarrow A = 2a$

$$\Rightarrow \frac{a}{A} = \frac{1}{2}$$

Now, $A - a = A\cos\omega\tau$

$$\Rightarrow \cos\omega\tau = \frac{A-a}{A}$$

$$\Rightarrow \cos\omega\tau = \frac{1}{2} \quad \text{or} \quad \frac{2\pi}{T}\tau = \frac{\pi}{3}$$

$\Rightarrow T = 6\tau$

33. **(c)** As we know, time period, $T = 2p\sqrt{\frac{\ell}{g}}$

When additional mass M is added then

$$T_M = 2p\sqrt{\frac{\ell + D\ell}{g}}$$

$$\frac{T_M}{T} = \sqrt{\frac{\ell + \Delta\ell}{\ell}}$$

$$\Rightarrow \left(\frac{T_M}{T}\right)^2 = \frac{\ell + \Delta\ell}{\ell}$$

$$\text{or, } \left(\frac{T_M}{T}\right)^2 = 1 + \frac{Mg}{AY}$$

$$\left[\because \Delta\ell = \frac{Mg\ell}{AY}\right]$$

$$\therefore \frac{1}{Y} = \left[\left(\frac{T_M}{T}\right)^2 - 1\right]\frac{A}{Mg}$$

34. **(d)** $K.E = \frac{1}{2}k(A^2 - d^2)$

and P.E. $= \frac{1}{2}kd^2$

At mean position $d = 0$. At extreme positions $d = A$

35. **(b)** We know that $V = \omega\sqrt{A^2 - x^2}$

Initially $V = \omega\sqrt{A^2 - \left(\frac{2A}{3}\right)^2}$

Finally $3V = \omega\sqrt{A'^2 - \left(\frac{2A}{3}\right)^2}$

Where A'= final amplitude (Given at $x = \frac{2A}{3}$, velocity to trebled)

On dividing we get

$$\frac{3}{1} = \frac{\sqrt{A'^2 - \left(\frac{2A}{3}\right)^2}}{\sqrt{A^2 - \left(\frac{2A}{3}\right)^2}}$$

$$9\left[A^2 - \frac{4A^2}{9}\right] = A'^2 - \frac{4A^2}{9}$$

$$\therefore A' = \frac{7A}{3}$$

36. **(b)** For a particle executing SHM

At mean position; $t = 0$, $\omega t = 0$, $y = 0$, $V = V_{max} = a\omega$

$$\therefore \quad K.E. = KE_{max} = \frac{1}{2}m\omega^2 a^2$$

At extreme position : $t = \frac{T}{4}$, $\omega t = \frac{\pi}{2}$, $y = A$, $V = V_{min} = 0$

$\therefore \quad K.E. = KE_{min} = 0$

Kinetic energy in SHM, $KE = \frac{1}{2}m\omega^2(a^2 - y^2)$

$= \frac{1}{2}m\omega^2 a^2 \cos^2 \omega t$

Hence graph (b) correctly depicts kinetic energy time graph.

37. **(b)** As we know, frequency in SHM

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = 10^{12}$$

where m = mass of one atom
Mass of one atom of silver,

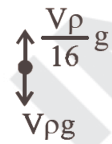
$$= \frac{108}{\left(6.02\times10^{23}\right)}\times10^{-3}\,\text{kg}$$

$$\frac{1}{2\pi}\sqrt{\frac{k}{108\times10^{-3}}\times6.02\times10^{23}} = 10^{12}$$

Solving we get, spring constant,
K = 7.1N/m

38. **(c)** $T = 2\pi\sqrt{\frac{l}{g}}$

$$V\rho\, g_{eff} = V\rho g - \frac{V\rho}{16}g$$



$$g_{eff} = \left(g - \frac{g}{16}\right) = \frac{15g}{16}$$

Now

$$T' = 2\pi\sqrt{\frac{l}{g_{eff}}} = 2\pi\sqrt{\frac{l}{\frac{15g}{16}}} = \frac{4}{\sqrt{15}}T$$

14 Waves

1. Length of a string tied to two rigid supports is 40 cm. Maximum length (wavelength in cm) of a stationary wave produced on it is **[2002]**
(a) 20 (b) 80 (c) 40 (d) 120

2. Tube *A* has both ends open while tube *B* has one end closed, otherwise they are identical. The ratio of fundamental frequency of tube *A* and *B* is **[2002]**
(a) 1 : 2 (b) 1 : 4 (c) 2 : 1 (d) 4 : 1.

3. A tuning fork arrangement (pair) produces 4 beats/sec with one fork of frequency 288 cps. A little wax is placed on the unknown fork and it then produces 2 beats/sec. The frequency of the unknown fork is **[2002]**
(a) 286 cps (b) 292 cps
(c) 294 cps (d) 288 cps

4. A wave $y = a\sin(\omega t - kx)$ on a string meets with another wave producing a node at $x = 0$. Then the equation of the unknown wave is **[2002]**
(a) $y = a\sin(\omega t + kx)$
(b) $y = -a\sin(\omega t + kx)$
(c) $y = a\sin(\omega t - kx)$
(d) $y = -a\sin(\omega t - kx)$

5. When temperature increases, the frequency of a tuning fork **[2002]**
(a) increases
(b) decreases
(c) remains same
(d) increases or decreases depending on the material

6. The displacement y of a wave travelling in the x -direction is given by
$$y = 10^{-4}\sin\left(600t - 2x + \frac{\pi}{3}\right)\text{metres}$$
where x is expressed in metres and t in seconds. The speed of the wave - motion, in ms^{-1} , is **[2003]**
(a) 300 (b) 600 (c) 1200 (d) 200

7. A metal wire of linear mass density of 9.8 g/m is stretched with a tension of 10 kg-wt between two rigid supports 1 metre apart. The wire passes at its middle point between the poles of a permanent magnet, and it vibrates in resonance when carrying an alternating current of frequency n. The frequency n of the alternating source is **[2003]**
(a) 50 Hz (b) 100 Hz
(c) 200Hz (d) 25Hz

8. A tuning fork of known frequency 256 Hz makes 5 beats per second with the vibrating string of a piano. The beat frequency decreases to 2 beats per second when the tension in the piano string is slightly increased. The frequency of the piano string before increasing the tension was **[2003]**
(a) $(256+2)$ Hz (b) $(256-2)$ Hz
(c) $(256-5)$ Hz (d) $(256+5)$ Hz

9. The displacement y of a particle in a medium can be expressed as,
$y = 10^{-6}\sin\left(100t + 20x + \frac{\pi}{4}\right)m$ where t is in second and x in meter. The speed of the wave is **[2004]**
(a) 20 m/s (b) 5 m/s
(c) 2000 m/s (d) 5π m/s

10. When two tuning forks are sounded simultaneously, 4 beats per second are heard. Now, some tape is attached on the prong of the fork 2. When the tuning forks are sounded again, 6 beats per second are heard. If the frequency of fork 1 is 200 Hz, then what was the original frequency of fork 2? **[2005]**
(a) 202 Hz (b) 200 Hz
(c) 204 Hz (d) 196 Hz

11. An observer moves towards a stationary source of sound, with a velocity one-fifth of the velocity of sound. What is the percentage increase in the apparent frequency ? **[2005]**

(a) 0.5% (b) zero
(c) 20% (d) 5%

12. A whistle producing sound waves of frequencies 9500 HZ and above is approaching a stationary person with speed v ms^{-1}. The velocity of sound in air is 300 ms^{-1}. If the person can hear frequencies upto a maximum of 10,000 HZ, the maximum value of v upto which he can hear whistle is **[2006]**

(a) $15\sqrt{2}\ ms^{-1}$ (b) $\frac{15}{\sqrt{2}}\ ms^{-1}$
(c) $15\ ms^{-1}$ (d) $30\ ms^{-1}$

13. A string is stretched between fixed points separated by 75.0 cm. It is observed to have resonant frequencies of 420 Hz and 315 Hz. There are no other resonant frequencies between these two. Then, the lowest resonant frequency for this string is **[2006]**

(a) 105 Hz (b) 1.05 Hz
(c) 1050 Hz (d) 10.5 Hz

14. A sound absorber attenuates the sound level by 20 dB. The intensity decreases by a factor of **[2007]**

(a) 100 (b) 1000
(c) 10000 (d) 10

15. While measuring the speed of sound by performing a resonance column experiment, a student gets the first resonance condition at a column length of 18 cm during winter. Repeating the same experiment during summer, she measures the column length to be x cm for the second resonance. Then **[2008]**

(a) $18 > x$ (b) $x > 54$
(c) $54 > x > 36$ (d) $36 > x > 18$

16. A wave travelling along the x-axis is described by the equation $y(x, t) = 0.005 \cos (\alpha x - \beta t)$. If the wavelength and the time period of the wave are 0.08 m and 2.0s, respectively, then α and β in appropriate units are **[2008]**

(a) $\alpha = 25.00\pi, \beta = \pi$

(b) $\alpha = \frac{0.08}{\pi}, \beta = \frac{2.0}{\pi}$

(c) $\alpha = \frac{0.04}{\pi}, \beta = \frac{1.0}{\pi}$

(d) $\alpha = 12.50\pi, \beta = \frac{\pi}{2.0}$

17. Three sound waves of equal amplitudes have frequencies $(\nu - 1), \nu, (\nu + 1)$. They superpose to give beats. The number of beats produced per second will be : **[2009]**

(a) 3 (b) 2 (c) 1 (d) 4

18. A motor cycle starts from rest and accelerates along a straight path at $2 m/s^2$. At the starting point of the motor cycle there is a stationary electric siren. How far has the motor cycle gone when the driver hears the frequency of the siren at 94% of its value when the motor cycle was at rest? (Speed of sound = 330 ms^{-1}) **[2009]**

(a) 98 m (b) 147 m
(c) 196 m (d) 49 m

19. The equation of a wave on a string of linear mass density 0.04 kg m^{-1} is given by

$$y = 0.02(m) \sin\left[2\pi\left(\frac{t}{0.04(s)} - \frac{x}{0.50(m)}\right)\right].$$

The tension in the string is **[2010]**

(a) 4.0 N (b) 12.5 N
(c) 0.5 N (d) 6.25 N

20. The transverse displacement $y(x, t)$ of a wave on a string is given by $y(x,t) = e^{-\left(ax^2 + bt^2 + 2\sqrt{ab}\,xt\right)}$.

This represents a: **[2011]**

(a) wave moving in $-x$ direction with speed $\sqrt{\frac{b}{a}}$

(b) standing wave of frequency $\sqrt{b}$

(c) standing wave of frequency $\frac{1}{\sqrt{b}}$

(d) wave moving in $+x$ direction speed $\sqrt{\frac{a}{b}}$

21. A travelling wave represented by $y = A \sin(\omega t - kx)$ is superimposed on another wave represented by $y = A \sin(\omega t + kx)$. The resultant is **[2011 RS]**

(a) A wave travelling along + x direction
(b) A wave travelling along – x direction
(c) A standing wave having nodes at

$$x = \frac{n\lambda}{2}, n = 0, 1, 2....$$

(d) A standing wave having nodes at

$$x = \left(n + \frac{1}{2}\right)\frac{\lambda}{2};\ n = 0, 1, 2....$$

22. **Statement - 1 :** Two longitudinal waves given by equations : $y_1(x,t) = 2a\sin(\omega t - kx)$ and $y_2(x,t) = a\sin(2\omega t - 2kx)$ will have equal intensity. **[2011 RS]**

Statement - 2 : Intensity of waves of given frequency in same medium is proportional to square of amplitude only.

(a) Statement-1 is true, statement-2 is false.

(b) Statement-1 is true, statement-2 is true, statement-2 is the correct explanation of statement-1

(c) Statement-1 is true, statement-2 is true, statement-2 is not the correct explanation of statement-1

(d) Statement-1 is false, statement-2 is true.

23. A cylindrical tube, open at both ends, has a fundamental frequency f in air. The tube is dipped vertically in water so that half of it is in water. The fundamental frequency of the air-column is now : **[2012]**

(a) f (b) $f/2$ (c) $3f/4$ (d) $2f$

24. A sonometer wire of length 1.5 m is made of steel. The tension in it produces an elastic strain of 1%. What is the fundamental frequency of steel if density and elasticity of steel are 7.7×10^3 kg/m^3 and 2.2×10^{11} N/m^2 respectively? **[2013]**

(a) 188.5 Hz (b) 178.2 Hz
(c) 200.5 Hz (d) 770 Hz

25. A pipe of length 85 cm is closed from one end. Find the number of possible natural oscillations of air column in the pipe whose frequencies lie below 1250 Hz. The velocity of sound in air is 340 m/s. **[2014]**

(a) 12 (b) 8 (c) 6 (d) 4

26. A train is moving on a straight track with speed 20 ms^{-1}. It is blowing its whistle at the frequency of 1000 Hz. The percentage change in the frequency heard by a person standing near the track as the train passes him is (speed of sound = 320 ms^{-1}) close to : **[2015]**

(a) 18% (b) 24% (c) 6% (d) 12%

27. A pipe open at both ends has a fundamental frequency f in air. The pipe is dipped vertically in water so that half of it is in water. The fundamental frequency of the air column is now : **[2016]**

(a) 2f (b) f (c) $\frac{f}{2}$ (d) $\frac{3f}{4}$

28. A uniform string of length 20 m is suspended from a rigid support. A short wave pulse is introduced at its lowest end. It starts moving up the string. The time taken to reach the supports is : (take g = 10 ms^{-2}) **[2016]**

(a) $2\sqrt{2}$s (b) $\sqrt{2}$ s
(c) $2\pi\sqrt{2}$ s (d) 2 s

29. A granite rod of 60 cm length is clamped at its middle point and is set into longitudinal vibrations. The density of granite is 2.7×10^3 kg/m^3 and its Young's modulus is 9.27×10^{10} Pa. What will be the fundamental frequency of the longitudinal vibrations? **[2018]**

(a) 5 kHz (b) 2.5 kHz
(c) 10 kHz (d) 7.5 kHz

30. A heavy ball of mass M is suspended from the ceiling of a car by a light string of mass m (m<<M). When the car is at rest, the speeed of transverse waves in the string is 60 ms^{-1}. when the car has acceleration a, the wave-speed increases to 60.5 ms^{-1}. The value of a, in terms of gravitational acceleration g, is closest to: **[2019]**

(a) $\frac{g}{30}$ (b) $\frac{g}{5}$ (c) $\frac{g}{10}$ (d) $\frac{g}{20}$

31. The pressure wave,
$P = 0.01\sin[1000t - 3x]$ Nm^{-2}, corresponds to the sound produced by a vibrating blade on a day when atmospheric temperature is 0°C. On some other day when temperature is T, the speed of sound produced by the same blade and at the same frequency is found to be 336 ms^{-1}. Approximate value of T is : **[2019]**

(a) 4°C (b) 11°C (c) 12°C (d) 15°C

32. A string is clamped at both the ends and it is vibrating in its 4th harmonic. The equation of the stationary wave is Y = 0.3 sin(0.157x) cos(200πt). The length of the string is: (All quantities are in SI units.) **[2019]**

(a) 20 m (b) 80 m (c) 40 m (d) 60 m

33. Speed of a transverse wave on a straight wire (mass 6.0 g, length 60 cm and area of cross-section 1.0 mm^2) is 90 ms^{-1}. If the Young's modulus of wire is 16×10^{11} Nm^{-2} the extension of wire over its natural length is: **[2020]**

(a) 0.03 mm (b) 0.02 mm
(c) 0.04 mm (d) 0.01 mm

Answer Key

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
(b)	(c)	(b)	(b)	(b)	(a)	(a)	(c)	(b)	(d)	(c)	(c)	(a)	(a)	(b)
16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
(a)	(b)	(a)	(d)	(a)	(d)	(a)	(a)	(b)	(c)	(d)	(b)	(a)	(a)	(b)
31	32	33												
(a)	(b)	(a)												

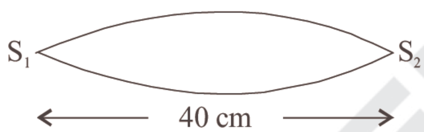
Solutions

1. (b) As the string vibrates in one segment, So this will be fundamental mode of vibration as shown in the figure.

$$\therefore \frac{\lambda}{2} = L$$

$$\Rightarrow \frac{\lambda}{2} = 40$$

$$\therefore \lambda = 80\,\text{cm}$$



2. (c) The fundamental frequency for tube B closed at one end is given by

$$\upsilon_B = \frac{v}{4\ell} \qquad \left[\because \ell = \frac{\lambda}{4}\right]$$

Where ℓ = length of the tube and v is the velocity of sound in air.

The fundamental frequency for tube A open with both ends is given by

$$\upsilon_A = \frac{v}{2\ell} \qquad \left[\because \ell = \frac{\lambda}{2}\right]$$

$$\therefore \frac{\upsilon_A}{\upsilon_B} = \frac{v}{2\ell} \times \frac{4\ell}{v} = \frac{2}{1}$$

3. (b) Frequency of unknown fork = known frequency ± Beat frequency = 288 + 4 cps or 288 – 4 cps i.e. 292 cps or 284 cps. When a little wax is placed on the unknown fork, it produces 2 beats/sec. When a little wax is placed on the unknown fork, its frequency decreases and simultaneously the beat frequency decreases confirming that the frequency of the unknown fork is 292 cps.

NOTE Had the frequency of unknown fork been 284 cps, then on placing wax its frequency would have decreased thereby increasing the gap between its frequency and the frequency of known fork. This would produce high beat frequency.

4. (b) To form a node there should be superposition of this wave with the reflected wave. The reflected wave should travel in opposite direction with a phase change of π. The equation of the reflected wave will be

$$y = a\sin(\omega t + kx + \pi)$$

$$\Rightarrow y = -a\sin(\omega t + kx)$$

5. (b) The frequency of a tuning fork is given by

$$f = \frac{m^2 k}{4\sqrt{3}\,\pi\ell^2}\sqrt{\frac{Y}{\rho}}$$

As temperature increases, the length or dimension of the prongs increases and also young's modulus increases therefore f decreases.

6. (a) $y = 10^{-4}\sin\left(600t - 2x + \frac{\pi}{3}\right)$

On comparing with standard equation

$$y = A\sin(\omega t - kx + \phi)$$

we get $\omega = 600$; k=2

Velocity of wave

$$v = \frac{\omega}{k} = \frac{600}{2} = 300\ \text{ms}^{-1}$$

7. (a) The fundamental frequency of vibration of the string is given by

$$n = \frac{v}{2\ell} = \frac{1}{2\ell}\sqrt{\frac{T}{\mu}}$$

$$= \frac{1}{2 \times 1}\sqrt{\frac{10 \times 9.8}{9.8 \times 10^{-3}}} = 50\,\text{Hz}$$

As the string is vibrating in resonance to a.c of frequency n, therefore both the frequencies are same.

8. **(c)** It is given that tuning fork of frequency 256 Hz makes 5 beats/second with the vibrating string of a piano. Therefore, possible frequency of the piano are (256 ± 5) Hz. i.e., either 261Hz or 251 Hz. When the tension in the piano string increases, its frequency will increases. As the original frequency was 261Hz, the beat frequency should decreases, we can conclude that the frequency of piano string is 251Hz

9. **(b)** Given, $y = 10^{-6}\sin\left(100t + 20x + \frac{\pi}{4}\right) m$

Comparing it with standard equation, we get

$\omega = 100$ and $k = 20$

$$v = \frac{\omega}{k} = \frac{100}{20} = 5\,\text{m/s}$$

10. **(d)** Frequency of fork 1, no = 200 Hz

No. of beats heard when fork 2 is sounded with fork 1 = $\Delta n = 4$

Now on loading (attaching tape) on unknown fork, the mass of tuning fork increases, So the beat frequency increases (from 4 to 6 in this case) then the frequency of the unknown fork 2 is given by,

$n = n_0 - \Delta n = 200 - 4 = 196$ Hz

11. **(c)** Apparent frequency

$$n' = n\left[\frac{v + v_0}{v}\right] = n\left[\frac{v + \frac{v}{5}}{v}\right] = n\left[\frac{6}{5}\right]$$

$$\frac{n'}{n} = \frac{6}{5}$$

The percentage increase in apparent frequency $\frac{n' - n}{n} = \frac{6 - 5}{5} \times 100 = 20\%$

12. **(c)** Apparent frequency $\nu' = \nu\left[\frac{v}{v - v_s}\right]$

$$\Rightarrow 10000 = 9500\left[\frac{300}{300 - v}\right]$$

$$\Rightarrow 300 - v = 300 \times 0.95$$

$$\Rightarrow v = 300 - 285 = 15\ \text{ms}^{-1}$$

13. **(a)** It is given that 315 Hz and 420 Hz are two resonant frequencies, let these be n^{th} and $(n + 1)^{th}$ harmonies, then we have

$$\frac{nv}{2\ell} = 315$$

and $(n+1)\frac{v}{2\ell} = 420$

$$\Rightarrow \frac{n+1}{n} = \frac{420}{315}$$

$$\Rightarrow n = 3$$

Hence $3 \times \frac{v}{2\ell} = 315$

$$\Rightarrow \frac{v}{2\ell} = 105\ Hz$$

The lowest resonant frequency is when n = 1

Therefore lowest resonant frequency = 105 Hz.

14. **(a)** Loudness of sound. $L_1 = 10\log\left(\frac{I_1}{I_0}\right)$;

$$L_2 = 10\log\left(\frac{I_2}{I_0}\right)$$

$$\therefore L_1 - L_2 = 10\log\left(\frac{I_1}{I_0}\right) - 10\log\left(\frac{I_2}{I_0}\right)$$

or, $\Delta L = 10\log\left(\frac{I_1}{I_0} \times \frac{I_0}{I_2}\right)$

or, $\Delta L = 10\log\left(\frac{I_1}{I_2}\right)$

The sound level attenuated by 20 dB ie

$L_1 - L_2 = 20$ dB

or, $20 = 10\log\left(\frac{I_1}{I_2}\right)$ or, $2 = \log\left(\frac{I_1}{I_2}\right)$

or, $\frac{I_1}{I_2} = 10^2$ or, $I_2 = \frac{I_1}{100}$.

$\Rightarrow$ Intensity decreases by a factor 100.

15. (b) Fundamental frequency for first resonant length

$$\nu = \frac{v}{4\ell_1} = \frac{v}{4\times 18} \text{ (in winter)}$$

Fundamental frequency for second resonant length

$$\nu' = \frac{3v'}{4\ell_2} = \frac{3v'}{4x} \text{ (in summer)}$$

According to questions,

$$\therefore \quad \frac{v}{4\times 18} = \frac{3v'}{4\times x}$$

$$\therefore \quad x = 3\times 18\times\frac{v'}{v}$$

$$\therefore \quad x = 54\times\frac{v'}{v}\text{cm}$$

$v' > v$ because velocity of light is greater in summer as compared to winter $(v \propto \sqrt{T})$

$$\therefore \quad x > 54\,\text{cm}$$

16 (a) Given,
Wavelength, $l = 0.08$m
Time period, T = 2.05

$y(x, t) = 0.005 \cos(\alpha x - \beta t)$ (Given)

Comparing it with the standard equation of wave

$y(x, t) = a \cos(kx - \omega t)$ we get

$$k = \alpha = \frac{2\pi}{l} \text{ and } \omega = \beta = \frac{2\pi}{T}$$

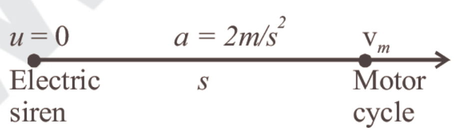
$$\therefore \quad \alpha = \frac{2\pi}{0.08} = 25\pi \quad \text{and} \quad \beta = \frac{2\pi}{2} = \pi$$

17. (b) Maximum number of beats

= Maximum frequency – Minimum frequency

$= (\nu + 1) - (\nu - 1) = 2$ Beats per second

18. (a)



Let the motorcycle has travelled a distances, its velocity at that point

$$v_m^2 - u^2 = 2as$$

$$\therefore v_m^2 = 2\times 2\times s$$

$$\therefore \quad v_m = 2\sqrt{s}$$

The observed frequency will be

$$\nu' = \nu\left[\frac{v - v_m}{v}\right]$$

$$0.94\nu = \nu\left[\frac{330 - 2\sqrt{s}}{330}\right]$$

$$\Rightarrow s = 98.01\ m$$

19. (d) $y = 0.02(m)\sin\left[2\pi\left(\frac{t}{0.04(s)}\right) - \frac{x}{0.50(m)}\right]$

Comparing it with the standard wave equation

$y = a\sin(\omega t - kx)$

we get

$$\omega = \frac{2\pi}{0.04} \text{ rad s}^{-1}$$

and $k = \dfrac{2\pi}{0.50}$

Wave velocity, $v = \dfrac{w}{k}$

$$\Rightarrow v = \frac{2\pi / 0.04}{2\pi / 0.5} = 12.5\ m/s$$

Velocity on a string is given by

$$v = \sqrt{\frac{T}{\mu}}$$

$$\therefore \ T = v^2 \times \mu = (12.5)^2 \times 0.04 = 6.25\ \text{N}$$

20. (a) Given

$$y(x, t) = e^{\left(-ax^2 + bt^2 + 2\sqrt{ab}\,xt\right)}$$

$$= e^{-[(\sqrt{a}x)^2 + (\sqrt{b}t)^2 + 2\sqrt{a}x.\sqrt{b}t]}$$

$$= e^{-(\sqrt{a}x + \sqrt{b}t)^2}$$

$$= e^{-\left(x + \sqrt{\frac{b}{a}}t\right)^2}$$

It is a function of type $y = f(x + vt)$

$\therefore y(x, t)$ represents wave travelling along –ve x direction

$$\Rightarrow \text{Speed of wave} = \frac{w}{k} = \sqrt{\frac{b}{a}}$$

21. (d)

$$y = A\sin(\omega t - kx) + A\sin(\omega t + kx)$$

$$y = 2A\sin\omega t\cos kx$$

This is an equation of standing wave. For position of nodes

$$\cos kx = 0$$

$$\Rightarrow \frac{2\pi}{\lambda}.x = (2n+1)\frac{\pi}{2}$$

$$\Rightarrow x = \frac{(2n+1)\lambda}{4}, n = 0,1,2,3,..........$$

22. (a) Intensity of a wave

$$I = \frac{1}{2} pw^2 A^2 v$$

Since, $I \propto A^2\omega^2$

$\therefore \quad I_1 \propto (2a)^2 \omega^2$

and $I_2 \propto a^2 (2\omega)^2$

$I_1 = I_2$

In the same medium, p and v are same.
Intensity depends on amplitude and frequency.

23. (a) Initially for open organ pipe, fundamental frequency

$$\nu_0 = \frac{v}{2l_0} \quad ...(i)$$

where l_0 is the length of the tube
v = speed of sound
But when it is half dipped in water, it becomes closed organ pipe of length $\frac{\ell_0}{2}$.
Fundamental frequency of closed organ pipe

$$\nu_c = \frac{v}{4l_c} \quad ...(ii)$$

New length, $l_c = \frac{l_0}{2}$

Thus $\nu_c = \frac{v}{4l_0/2} \Rightarrow \nu_c = \frac{v}{2l} \quad ...(iii)$

From equations (i) and (iii)

$\nu_0 = \nu_c$

Thus, $\nu_c = f \quad (\because \nu_0 = f$ is given)

24. (b) Fundamental frequency,

$$f = \frac{v}{2\ell} = \frac{1}{2\ell}\sqrt{\frac{T}{\mu}} = \frac{1}{2\ell}\sqrt{\frac{T}{A\rho}}$$

$$\left[\because v = \sqrt{\frac{T}{\mu}} \text{ and } \mu = \frac{m}{\ell}\right]$$

Also, $Y = \frac{T\ell}{A\Delta\ell} \Rightarrow \frac{T}{A} = \frac{Y\Delta\ell}{\ell}$

$$\Rightarrow f = \frac{1}{2\ell}\sqrt{\frac{y\Delta\ell}{\ell\rho}} \quad ....(i)$$

$\ell = 1.5$ m, $\frac{\Delta\ell}{\ell} = 0.01$,
$\rho = 7.7 \times 10^3$ kg/m^3 (given)
$y = 2.2 \times 10^{11}$ N/m^2 (given)

Putting the value of $\ell, \frac{\Delta\ell}{\ell}$, ρ and y in eq^n.

(i) we get,

$$f = \sqrt{\frac{2}{7}} \times \frac{10^3}{3} \text{ or } f \approx 178.2 \text{ Hz}$$

25. (c) Length of pipe = 85 cm = 0.85m
Frequency of oscillations of air column in closed organ pipe is given by,

$$f = \frac{(2n-1)\upsilon}{4L}$$

$$f = \frac{(2n-1)\upsilon}{4L} \le 1250$$

$$\Rightarrow \frac{(2n-1)\times 340}{0.85 \times 4} \le 1250$$

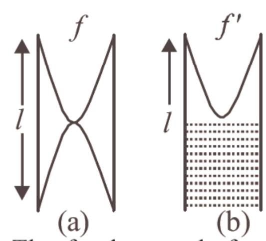
$\Rightarrow \quad 2n - 1 \le 12.5 \approx 6$

26. (d) $f_1 = f\left[\frac{v}{v - v_s}\right] = f \times \frac{320}{300} Hz$

$$f_2 = f\left[\frac{v}{v + v_s}\right] = f \times \frac{320}{340} Hz$$

$$\left(\frac{f_2}{f_1} - 1\right) \times 100 = \left(\frac{300}{340} - 1\right) \times 100 \simeq 12\%$$

27. (b)



The fundamental frequency in case (a) is

$$f = \frac{v}{2\ell}$$

The fundamental frequency in case (b) is

$$f' = \frac{v}{4(\ell/2)} = \frac{v}{2\ell} = f$$

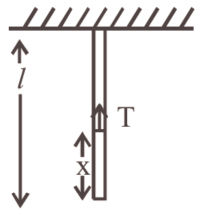
28. (a) We know that velocity in string is given by

$$v = \sqrt{\frac{T}{\mu}} \quad ...(i)$$

where $\mu = \frac{m}{l} = \frac{\text{mass of string}}{\text{length of string}}$

The tension $T=\frac{m}{\ell}\times x\times g$ ..(ii)

From (1) and (2)



$$\frac{dx}{dt}=\sqrt{gx}$$

$x^{-1/2}dx=\sqrt{g}\,dt$

$\therefore \int_0^{\ell} x^{-1/2}dx-\sqrt{g}\int_0^{\ell} dt$

$\Rightarrow 2\sqrt{l}$

$=\sqrt{g}\times t \quad \therefore\ t=2\sqrt{\frac{\ell}{g}}=2\sqrt{\frac{20}{10}}=2\sqrt{2}$

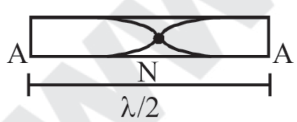
29. (a) In solids, Velocity of wave

$$V=\sqrt{\frac{Y}{\rho}}=\sqrt{\frac{9.27\times10^{10}}{2.7\times10^{3}}}$$

$v=5.85\times10^{3}$ m/sec

Since rod is clamped at middle fundamental wave shape is as follow

$\frac{\lambda}{2}=L \ \Rightarrow\ \lambda=2L$



$\lambda=1.2$m ($\because$ L = 60 cm = 0.6m (given)

Using $v=f\lambda$

$\Rightarrow \quad f=\frac{v}{\lambda}=\frac{5.85\times10^{3}}{1.2}$

$=4.88\times10^{3}$ Hz $\simeq 5$ KHz

30. (b) Wave speed $V=\sqrt{\frac{T}{\mu}}$

when car is at rest a = 0

$\therefore 60=\sqrt{\frac{Mg}{\mu}}$

Similarly when the car is moving with acceleration a,

$$60.5=\sqrt{\frac{M\left(g^{2}+a^{2}\right)^{1/2}}{\mu}}$$

on solving we get

$a=\frac{g}{\sqrt{30}}$ [which is closest to g/5]

31. (a) On comparing with $P=P_0\sin(\omega t-kx)$, we have

$\omega=1000$ rad/s, $K=3\ m^{-1}$

$\therefore v_0=\frac{\omega}{k}=\frac{1000}{3}=333.3$m/s

$\frac{v_1}{v_2}=\sqrt{\frac{T_1}{T_2}}$

or, $\frac{333.3}{336}=\sqrt{\frac{273+0}{273+t}} \ \therefore\ t=4°C$

32. (b) Given, $y=0.3\sin(0.157x)\cos(200\pi t)$

So, $k=0.157$ and $\omega=200\pi=2\pi f$

$\therefore f=100$ Hz and $v=\frac{\omega}{k}=\frac{200\pi}{0.157}=4000$m/s

Now, using $f=\frac{nv}{2l}=\frac{4v}{2l}=\frac{2v}{l}$ [here n = 4]

$\therefore l=\frac{2v}{f}=\frac{2\times4000}{100}=80m$

33. (a) Given, $l=60$ cm, $m=6$ g, $A=1$ mm^2, $v=90$ m/s and $Y=16\times10^{11}$ Nm^{-2}

Using, $v=\sqrt{\frac{T}{m}\times l}\Rightarrow T=\frac{mv^{2}}{I}$

Again from, $Y=\frac{T}{A}\Delta L/L_0$

$\Delta L=\frac{Tl}{YA}=\frac{mv^{2}\times l}{l(YA)}$

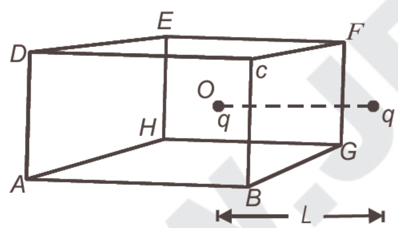
$=\frac{6\times10^{-3}\times90^{2}}{16\times10^{11}\times10^{-6}}=3\times10^{-4}m$

$=0.03$ mm

15 Electric Charges and Fields

1. If a charge q is placed at the centre of the line joining two equal charges Q such that the system is in equilibrium then the value of q is **[2002]**
   (a) $Q/2$ (b) $-Q/2$
   (c) $Q/4$ (d) $-Q/4$

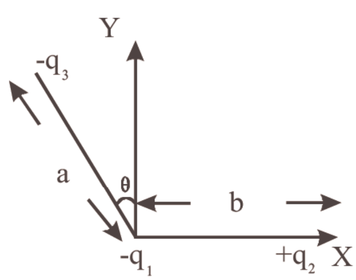
2. A charged particle q is placed at the centre O of cube of length L ($A\,B\,C\,D\,E\,F\,G\,H$). Another same charge q is placed at a distance L from O. Then the electric flux through $ABCD$ is **[2002]**



   (a) $q\,/4\pi\epsilon_0 L$ (b) zero
   (c) $q/2\pi\epsilon_0 L$ (d) $q/3\pi\epsilon_0 L$

3. If the electric flux entering and leaving an enclosed surface respectively is ϕ_1 and ϕ_2, the electric charge inside the surface will be **[2003]**
   (a) $(\phi_2 - \phi_1)\varepsilon_o$ (b) $(\phi_1 + \phi_2)/\varepsilon_o$
   (c) $(\phi_2 - \phi_1)/\varepsilon_o$ (d) $(\phi_1 + \phi_2)\varepsilon_o$

4. Three charges $-q_1$, $+q_2$ and $-q_3$ are place as shown in the figure. The x-component of the force on $-q_1$ is proportional to **[2003]**



   (a) $\dfrac{q_2}{b^2} - \dfrac{q_3}{a^2}\cos\theta$ (b) $\dfrac{q_2}{b^2} + \dfrac{q_3}{a^2}\sin\theta$
   (c) $\dfrac{q_2}{b^2} + \dfrac{q_3}{a^2}\cos\theta$ (d) $\dfrac{q_2}{b^2} - \dfrac{q_3}{a^2}\sin\theta$

5. Two spherical conductors B and C having equal radii and carrying equal charges on them repel each other with a force F when kept apart at some distance. A third spherical conductor having same radius as that B but **uncharged** is brought in contact with B, then brought in contact with C and finally removed away from both. The new force of repulsion between B and C is **[2004]**
   (a) $F/8$ (b) $3\,F/4$
   (c) $F/4$ (d) $3\,F/8$

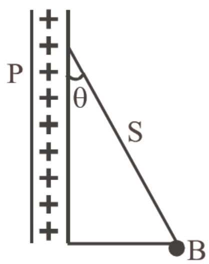
6. Four charges equal to $-Q$ are placed at the four corners of a square and a charge q is at its centre. If the system is in equilibrium the value of q is **[2004]**
   (a) $-\dfrac{Q}{2}(1+2\sqrt{2})$ (b) $\dfrac{Q}{4}(1+2\sqrt{2})$
   (c) $-\dfrac{Q}{4}(1+2\sqrt{2})$ (d) $\dfrac{Q}{2}(1+2\sqrt{2})$

7. A charged oil drop is suspended in a uniform field of 3×10^4 v/m so that it neither falls nor rises. The charge on the drop will be (Take the mass of the charge = 9.9×10^{-15} kg and $g = 10$ m/s^2) **[2004]**
   (a) 1.6×10^{-18} C (b) 3.2×10^{-18} C
   (c) 3.3×10^{-18} C (d) 4.8×10^{-18} C

8. Two point charges $+8q$ and $-2q$ are located at $x = 0$ and $x = L$ respectively. The location of a point on the x axis at which the net electric field due to these two point charges is zero is **[2005]**

(a) $\frac{L}{4}$ (b) $2L$
(c) $4L$ (d) $8L$

9. A charged ball B hangs from a silk thread S, which makes an angle θ with a large charged conducting sheet P, as shown in the figure. The surface charge density σ of the sheet is proportional to **[2005]**



(a) $\cot\theta$ (b) $\cos\theta$
(c) $\tan\theta$ (d) $\sin\theta$

10. An electric dipole is placed at an angle of 30° to a non-uniform electric field. The dipole will experience **[2006]**

(a) a translational force only in the direction of the field
(b) a translational force only in a direction normal to the direction of the field
(c) a torque as well as a translational force
(d) a torque only

11. Two spherical conductors A and B of radii 1 mm and 2 mm are separated by a distance of 5 cm and are uniformly charged. If the spheres are connected by a conducting wire then in equilibrium condition, the ratio of the magnitude of the electric fields at the surfaces of spheres A and B is **[2006]**

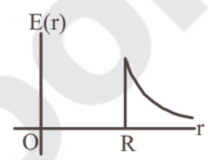
(a) 4 : 1 (b) 1 : 2
(c) 2 : 1 (d) 1 : 4

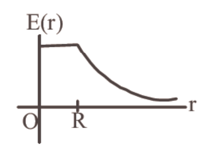
12. If g_E and g_M are the accelerations due to gravity on the surfaces of the earth and the moon respectively and if Millikan's oil drop experiment could be performed on the two surfaces, one will find the ratio **[2007]**

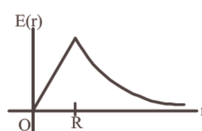
$\frac{\text{electronic charge on the moon}}{\text{electronic charge on the earth}}$ to be

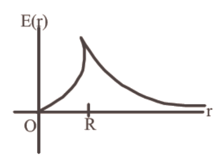
(a) g_M/g_E (b) 1
(c) 0 (d) g_E/g_M

13. A thin spherical shell of radus R has charge Q spread uniformly over its surface. Which of the following graphs most closely represents the electric field $E(r)$ produced by the shell in the range $0 \le r < \infty$, where r is the distance from the centre of the shell? **[2008]**

(a) 

(b) 

(c) 

(d) 

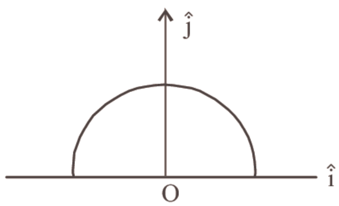
14. A charge Q is placed at each of the opposite corners of a square. A charge q is placed at each of the other two corners. If the net electrical force on Q is zero, then Q/q equals: **[2009]**

(a) -1 (b) 1
(c) $-\frac{1}{\sqrt{2}}$ (d) $-2\sqrt{2}$

15. Let $\rho(r) = \frac{Q}{\pi R^4} r$ be the charge density distribution for a solid sphere of radius R and total charge Q. For a point 'P' inside the sphere at distance r_1 from the centre of the sphere, the magnitude of electric field is : **[2009]**

(a) $\frac{Q}{4\pi \in_0 r_1^2}$ (b) $\frac{Q r_1^2}{4\pi \in_0 R^4}$
(c) $\frac{Q r_1^2}{3\pi \in_0 R^4}$ (d) 0

16. A thin semi-circular ring of radius r has a positive charge q distributed uniformly over it. The net field $\vec{E}$ at the centre O is **[2010]**



(a) $\dfrac{q}{4\pi^2\varepsilon_0 r^2}\hat{j}$ (b) $-\dfrac{q}{4\pi^2\varepsilon_0 r^2}\hat{j}$

(c) $-\dfrac{q}{2\pi^2\varepsilon_0 r^2}\hat{j}$ (d) $\dfrac{q}{2\pi^2\varepsilon_0 r^2}\hat{j}$

17. Let there be a spherically symmetric charge distribution with charge density varying as $\rho(r)=\rho_0\left(\dfrac{5}{4}-\dfrac{r}{R}\right)$ upto $r = R$, and $\rho(r) = 0$ for $r > R$, where r is the distance from the origin. The electric field at a distance $r(r < R)$ from the origin is given by **[2010]**

(a) $\dfrac{\rho_0 r}{4\varepsilon_0}\left(\dfrac{5}{3}-\dfrac{r}{R}\right)$ (b) $\dfrac{4\pi\rho_0 r}{3\varepsilon_0}\left(\dfrac{5}{3}-\dfrac{r}{R}\right)$

(c) $\dfrac{\rho_0 r}{4\varepsilon_0}\left(\dfrac{5}{4}-\dfrac{r}{R}\right)$ (d) $\dfrac{\rho_0 r}{3\varepsilon_0}\left(\dfrac{5}{4}-\dfrac{r}{R}\right)$

18. Two identical charged spheres suspended from a common point by two massless strings of length l are initially a distance $d(d << l)$ apart because of their mutual repulsion. The charge begins to leak from both the spheres at a constant rate. As a result charges approach each other with a velocity v. Then as a function of distance x between them, **[2011]**

(a) $v \propto x^{-1}$ (b) $v \propto x^{1/2}$

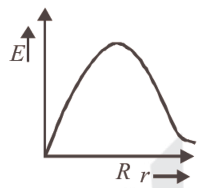
(c) $v \propto x$ (d) $v \propto x^{-1/2}$

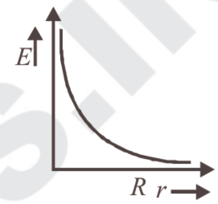
19. The potential inside a charged spherical ball is given by $\phi = ar^2 + b$ where r is the distance from the centre and a, b are constants. Then the charge density inside the ball is: **[2011]**

(a) $-6a\varepsilon_0 r$ (b) $-24\pi a\varepsilon_0$

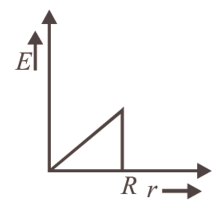
(c) $-6a\varepsilon_0$ (d) $-24\pi a\varepsilon_0 r$

20. In a uniformly charged sphere of total charge Q and radius R, the electric field E is plotted as function of distance from the centre, The graph which would correspond to the above will be: **[2012]**

(a) 

(b) 

(c) 

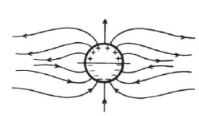
(d) 

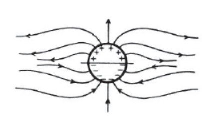
21. Two charges, each equal to q, are kept at $x = -a$ and $x = a$ on the x-axis. A particle of mass m and charge $q_0 = \dfrac{q}{2}$ is placed at the origin. If charge q_0 is given a small displacement $(y << a)$ along the y-axis, the net force acting on the particle is proportional to **[2013]**

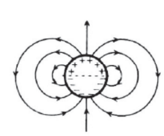
(a) y (b) $-y$

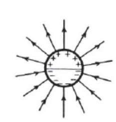
(c) $\dfrac{1}{y}$ (d) $-\dfrac{1}{y}$

22. A long cylindrical shell carries positive surface charge σ in the upper half and negative surface charge $-\sigma$ in the lower half. The electric field lines around the cylinder will look like figure given in : (*figures are schematic and not drawn to scale*) **[2015]**

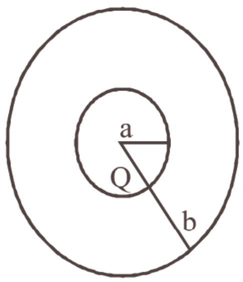
(a) 

(b) 

(c) 

(d) 

23. The region between two concentric spheres of radii 'a' and 'b', respectively (see figure), have volume charge density $\rho = \frac{A}{r}$, where A is a constant and r is the distance from the centre. At the centre of the spheres is a point charge Q. The value of A such that the electric field in the region between the spheres will be constant, is: **[2016]**



(a) $\frac{2Q}{\pi(a^2-b^2)}$ (b) $\frac{2Q}{\pi a^2}$

(c) $\frac{Q}{2\pi a^2}$ (d) $\frac{Q}{2\pi(b^2-a^2)}$

24. An electric dipole has a fixed dipole moment $\vec{p}$, which makes angle θ with respect to x-axis. When subjected to an electric field $\vec{E}_1 = E\hat{i}$, it experiences a torque $\vec{T}_1 = \tau\hat{i}$. When subjected to another electric field $\vec{E}_2 = \sqrt{3}E_1\hat{j}$ it experiences torque $\vec{T}_2 = -\vec{T}_1$. The angle θ is: **[2017]**

(a) 60° (b) 90°
(c) 30° (d) 45°

25. Three charges $+Q$, q, $+Q$ are placed respectively, at distance, $d/2$ and d from the origin, on the x-axis. If the net force experienced by $+Q$, placed at $x = 0$, is zero, then value of q is: **[2012]**

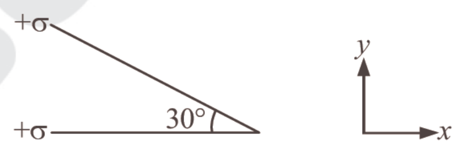
(a) $-Q/4$ (b) $+Q/2$
(c) $+Q/4$ (d) $-Q/2$

26. For a uniformly charged ring of radius R, the electric field on its axis has the largest magnitude at a distance h from its centre. Then value of h is: **[2019]**

(a) $\frac{R}{\sqrt{5}}$ (b) $\frac{R}{\sqrt{2}}$

(c) R (d) $R\sqrt{2}$

27. Two infinite planes each with uniform surface charge density $+\sigma$ are kept in such a way that the angle between them is 30°. The electric field in the region shown between them is given by: **[2020]**



(a) $\frac{\sigma}{2\epsilon_0}\left[(1+\sqrt{3})\hat{y} - \frac{\hat{x}}{2}\right]$

(b) $\frac{\sigma}{\epsilon_0}\left[\left(1+\frac{\sqrt{3}}{2}\right)\hat{y} + \frac{\hat{x}}{2}\right]$

(c) $\frac{\sigma}{2\epsilon_0}\left[\left(1+\sqrt{3}\right)\hat{y} + \frac{\hat{x}}{2}\right]$

(d) $\frac{\sigma}{2\epsilon_0}\left[\left(1-\frac{\sqrt{3}}{2}\right)\hat{y} - \frac{\hat{x}}{2}\right]$

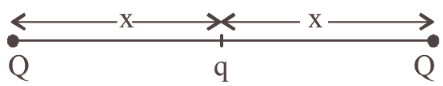
Answer Key

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
(d)	(none)	(a)	(b)	(d)	(b)	(c)	(b)	(c)	(c)	(c)	(b)	(a)	(d)	(b)
16	17	18	19	20	21	22	23	24	25	26	27			
(c)	(a)	(d)	(c)	(c)	(a)	(c)	(c)	(a)	(a)	(b)	(d)			

Solutions

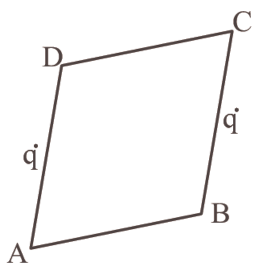
1. (d) At equilibrium net force is zero,

$$\therefore\ k\frac{Q\times Q}{(2x)^2}+k\frac{Qq}{x^2}=0$$



$$\Rightarrow\ q=-\frac{Q}{4}$$

2. (None) Electric flux due to charge placed outside is zero. But for the charge inside the cube, flux due to each face is $\frac{1}{6}\left[\frac{q}{\epsilon_0}\right]$ which is not in option.



3. (a) The electric flux ϕ_1 entering an enclosed surface is taken as negative and the electric flux ϕ_2 leaving the surface is taken as positive, by convention. Therefore the net flux leaving the enclosed surface,

$\phi=\phi_2-\phi_1$

According to Gauss theorem

$$\phi=\frac{q}{\epsilon_0}$$

$$\Rightarrow q=\epsilon_0\phi=\epsilon_0(\phi_2-\phi_1)$$

4. (b) Force applied by charge q_2 on q_1

$$F_{12}=k\frac{q_1q_2}{b^2}$$

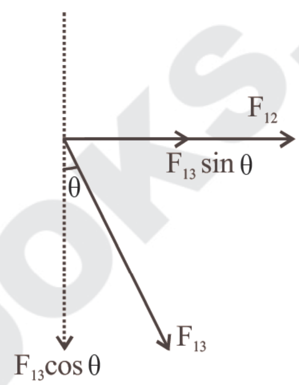
Force applied by charge q_3 on q_1

$$F_{13}=k\frac{q_1q_3}{a^2}$$

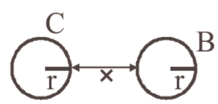
The X-component of net force (F_x) on q_1 is $F_{12}+F_{13}\sin\theta$

$$\therefore\ F_x=k\frac{q_1q_2}{b^2}+k\frac{q_1q_2}{a^2}\sin\theta$$

$$\therefore\ F_x\propto\frac{q_2}{b^2}+\frac{q_3}{a^2}\sin\theta$$



5. (d)



Initial force, $F=K\frac{Q_BQ_C}{x^2}$

x is distance between the spheres. When third spherical conductor comes in contact with B charge on B is halved *i.e.*, $\frac{Q}{2}$ and charge on third sphere becomes $\frac{Q}{2}$. Now it is touched to C, charge then equally distributes themselves to make potential same, hence charge on C becomes $\left(Q+\frac{Q}{2}\right)\frac{1}{2}=\frac{3Q}{4}$.

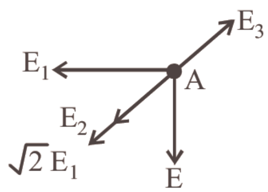
$$\therefore\ F_{new}=k\frac{Q'_CQ'_B}{x^2}=k\frac{\left(\frac{3Q}{4}\right)\left(\frac{Q}{2}\right)}{x^2}=k\frac{3}{8}\frac{Q^2}{x^2}$$

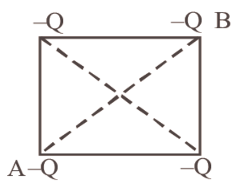
or $F_{new}=\frac{3}{8}F$

6. **(b)** For the system to be equilibrium, net field at A should be zero

$$\sqrt{2}\,E_1 + E_2 = E_3$$

$$\therefore \frac{kQ\times\sqrt{2}}{a^2} + \frac{kQ}{(\sqrt{2}\,a)^2} = \frac{kq}{\left(\frac{a}{\sqrt{2}}\right)^2}$$





$$\Rightarrow \frac{Q\sqrt{2}}{1} + \frac{Q}{2} = 2q \;\Rightarrow q = \frac{Q}{4}(2\sqrt{2}+1)$$

7. **(c)** Given, Electric field, $E = 3 \times 10^4$
Mass of the drop, $m = 9.9 \times 10^{-15}$ kg
At equilibrium, coulomb force on drop balances weight of drop.
$qE = mg$

$$\Rightarrow q = \frac{mg}{E}$$

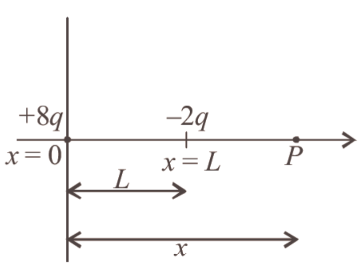
$$\Rightarrow q = \frac{9.9\times10^{-15}\times10}{3\times10^4} = 3.3\times10^{-18}\text{C}$$

8. **(b)** At $P \dfrac{-K2q}{(x-L)^2} + \dfrac{K8q}{x^2} = 0$

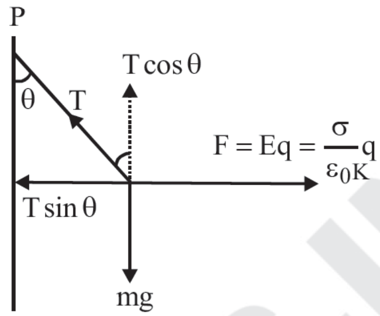
$$\Rightarrow \frac{1}{(x-L)^2} = \frac{4}{x^2}$$

or $\dfrac{1}{x-L} = \dfrac{2}{x}$

$\Rightarrow x = 2x - 2L$ or $x = 2L$



9. **(c)**

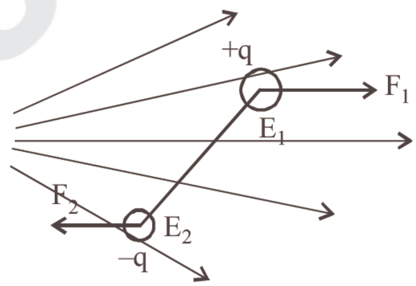


$T\sin\theta = qE$ .... (i)
$T\cos\theta = mg$ .... (ii)
Dividing (i) by (ii),

$$\tan\theta = \frac{qE}{mg} = \frac{q}{mg}\left(\frac{\sigma}{\varepsilon_0 K}\right)\frac{\sigma q}{\varepsilon_0 K.mg}$$

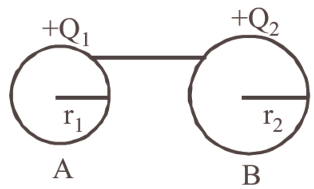
$\therefore \sigma \propto \tan\theta$

10. **(c)**



As the dipole is placed in non-uniform field, so the force acting on the dipole will not cancel each other. This will result in a force as well as torque.

11. **(c)**



When the two conducting spheres are connected by a conducting wire, charge will flow from one to other till both acquire same potential.
$\therefore$ After connection, $V_1 = V_2$

$$\Rightarrow k\frac{Q_1}{r_1} = k\frac{Q_2}{r_2} \Rightarrow \frac{Q_1}{r_1} = \frac{Q_2}{r_2}$$

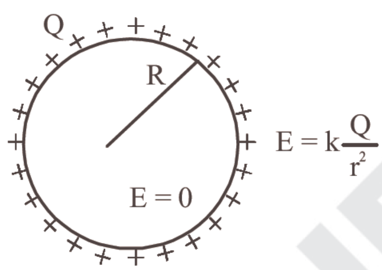
The ratio of electric fields

$$\frac{E_1}{E_2}=\frac{k\frac{Q_1}{r_1^2}}{k\frac{Q_2}{r_2^2}}\Rightarrow\frac{E_1}{E_2}=\frac{Q_1}{r_1^2}\times\frac{r_2^2}{Q_2}$$

$$\Rightarrow\frac{E_1}{E_2}=\frac{r_1\times r_2^2}{r_1^2\times r_2}\Rightarrow\frac{E_1}{E_2}=\frac{r_2}{r_1}=\frac{2}{1}$$

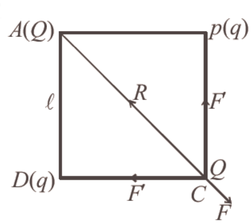
12. (b) It is obvious that by charge conservaiton law, electronic charge does not depend on acceleration due to gravity as it is a universal constant.
So, electronic charge on earth
= electronic charge on moon
$\therefore$ Required ratio = 1.

13. (a) The electric field inside a thin spherical shell of radius R has charge Q spread uniformly over its surface is zero.



Outside the shell the electric field is $E=k\frac{Q}{r^2}$. These characteristics are represented by graph (a).

14. (d) Let F be the force between Q and Q. The force between q and Q should be attractive for net force on Q to be zero. Let F' be the force between Q and q. The resultant of F' and F' is R. For equilibrium



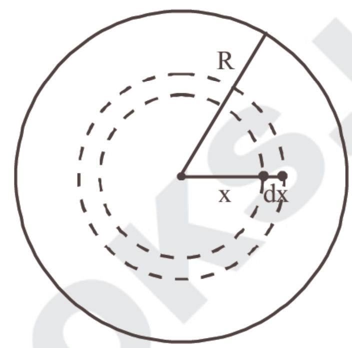
Net force on Q at C is zero.

$$\therefore\ \vec{R}+\vec{F}=0\quad\Rightarrow\quad\sqrt{2}\,F'=-F$$

$$\Rightarrow\quad\sqrt{2}\times k\frac{Qq}{\ell^2}=-k\frac{Q^2}{(\sqrt{2}\,\ell)^2}$$

$$\Rightarrow\quad\frac{Q}{q}=-2\sqrt{2}$$

15. (b)



Let us consider a spherical shell of thickness dx and radius x. The area of this spherical shell = $4\pi x^2$.
The volume of this spherical shell = $4\pi x^2 dx$.
The charge enclosed within shell

$$dq=\left[\frac{Qx}{\pi R^4}\right][4\pi x^2 dx]=\frac{4Q}{R^4}x^3dx$$

The charge enclosed in a sphere of radius r_1 can be calculated by

$$Q=\int dq$$

$$=\frac{4Q}{R^4}\int_0^{r_1}x^3dx=\frac{4Q}{R^4}\left[\frac{x^4}{4}\right]_0^{r_1}=\frac{Q}{R^4}r_1^4$$

$\therefore$ The electric field at point P inside the sphere at a distance r_1 from the centre of the sphere is

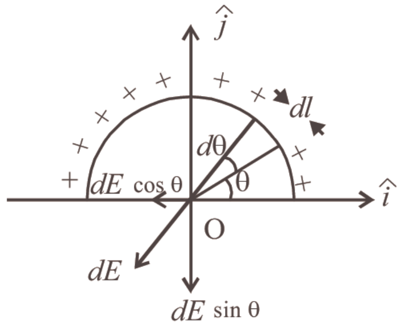
$$E=\frac{1}{4\pi E}\frac{Q}{r_1^2}$$

$$\Rightarrow E=\frac{1}{4\pi\in_0}\frac{\left[\frac{Q}{R^4}r_1^4\right]}{r_1^2}$$

$$=\frac{1}{4\pi\in_0}\frac{Q}{R^4}r_1^2$$

16. **(c)** Let us consider a differential element dl subtending at angle dQ at the centre Q as shown in the figure. Linear charge density

$$\lambda = \frac{q}{Qr}$$



Charge on the element, $dq = \left(\frac{q}{\pi r}\right) dl$

$$= \frac{q}{\pi r}(r d\theta) \qquad (\because dl = r d\theta)$$

$$= \left(\frac{q}{\pi}\right) d\theta$$

Electric field at the center O due to dq is

$$dE = \frac{1}{4\pi \in_0} \cdot \frac{dq}{r^2} = \frac{1}{4\pi \in_0} \cdot \frac{q}{\pi r^2} d\theta$$

Resolving dE into two rectangular component, we find the component $dE\cos\theta$ will be counter balanced by another element on left portion. Hence resultant field at O is the resultant of the component $dE\sin\theta$ only.

$$\therefore\ E = \int dE\sin\theta = \int_0^{\pi} \frac{q}{4\pi^2 r^2 \in_0}\sin\theta d\theta$$

$$= \frac{q}{4\pi^2 r^2 \in_0}\left[-\cos\theta\right]_0^{\pi}$$

$$= \frac{q}{4\pi^2 r^2 \in_0}(+1+1) = \frac{q}{2\pi^2 r^2 \in_0}$$

The direction of E is towards negative y-axis.

$$\therefore\ \vec{E} = -\frac{q}{2\pi^2 r^2 \in_0}\hat{j}$$

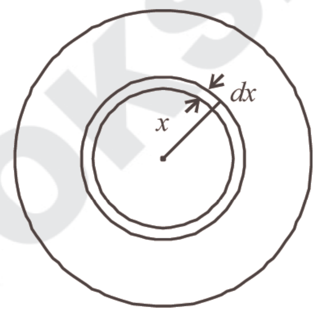
17. **(a)** Let us consider a spherical shell of radius x and thickness dx.

Due to shpherically symmetric charge distribution, the chrge on the spherical surface of radius x is

$$dq = dV\rho \cdot 4\pi x^2 dx = \rho_0\left(\frac{5}{4} - \frac{x}{R}\right) \cdot 4\pi x^2 dx$$

$\therefore$ Total charge in the spherical region from centre to r $(r < R)$ is

$$q = \int dq = 4\pi\rho_0 \int_0^r \left(\frac{5}{4} - \frac{x}{R}\right) x^2 dx$$



$$= 4\pi\rho_0\left[\frac{5}{4}\cdot\frac{r^3}{3} - \frac{1}{R}\cdot\frac{r^4}{4}\right] = \pi\rho_0 r^3\left(\frac{5}{3} - \frac{r}{R}\right)$$

$\therefore$ Electric field intensity at a point on this spherical surface

$$E = \frac{1}{4\pi \in_0}\cdot\frac{q}{r^2}$$

$$= \frac{1}{4\pi \in_0}\cdot\frac{\pi\rho_0 r^3}{r^2}\left(\frac{5}{3} - \frac{r}{R}\right) = \frac{\rho_0 r}{4 \in_0}\left(\frac{5}{3} - \frac{r}{R}\right)$$

18. **(d)** From figure

$$T\cos\theta = \text{mg} \qquad \text{....(i)}$$

$$T\sin\theta = F_e \qquad \text{....(ii)}$$

Dividing equation (ii) by (i), we get

$$\Rightarrow \frac{\sin\theta}{\cos\theta} = \frac{F_e}{mg}$$

$$\Rightarrow F_e = \text{mg}\tan\theta$$

$$\Rightarrow \frac{kq^2}{x^2} = \text{mg}\tan\theta$$

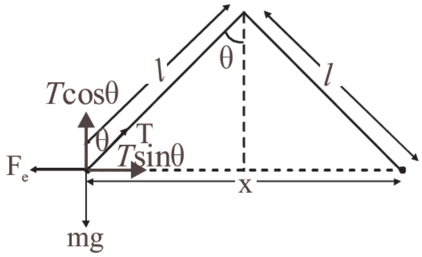
$$\Rightarrow q^2 = \frac{x^2 mg\tan\theta}{k}$$

Since θ is small

$$\therefore \tan\theta \approx \sin\theta = \frac{x}{2l}$$

$\therefore\ q^2 = \frac{x^3 mg}{2kl}$

$\Rightarrow q^2 \propto x^{3/2}$



$\Rightarrow \frac{dq}{dt} \alpha \frac{3}{2}\sqrt{x}\frac{dx}{dt} = \frac{3}{2}\sqrt{x}V$

Since $\frac{dq}{dt} = \text{const.}$

$\Rightarrow v \propto x^{-1/2}$ $[\because q^2 \propto x^3]$

19. **(c)** Potential inside a charged spherical ball, $\phi = ar^2 + b$

Electric field, $E = -\frac{d\phi}{dr} = -2ar$ ....(i)

By Gauss's theorem

$E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}$ ....(ii)

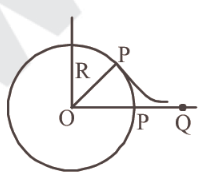
From (i) and (ii),

$q = -8\,\pi\varepsilon_0 ar^3$

$\Rightarrow dq = -24\pi\varepsilon_0 ar^2\, dr$

Charge density, $\rho = \frac{dq}{4\pi r^2 dr} = -6\varepsilon_0 a$

20. **(c)** Electric field inside the charged sphere



$\vec{E}_{in} = 0$ …(i)

$\vec{E}$ on the surface of the charged sphere

$\vec{E}_s = \frac{1}{4\pi\in_0}\frac{q}{R^2}$ *i.e.*, $\vec{E}_s \propto \frac{1}{R^2}\hat{n}$ …(ii)

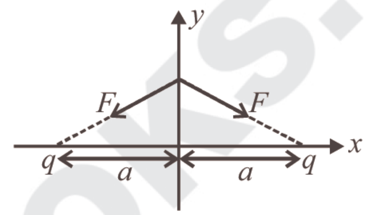
$\vec{E}$ on any point away from the uniformly charged sphere is given

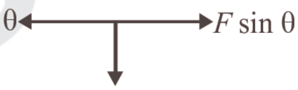
$\vec{E} = \frac{1}{4\pi\in_0}\frac{q}{r^2}\hat{n}$

$\vec{E} \propto \frac{1}{r^2}\hat{n}$ …(iii)

$\because R$ is the radius of the sphere, which is constant, thus $\vec{E}$ is maximum and constant at the surface of the sphere. But decreases on moving away from the surface of the uniformally charged sphere.

21. **(a)**





$\Rightarrow F_{\text{net}} = 2F\cos\theta$

$F_{net} = \frac{2kq\left(\frac{q}{2}\right)}{\left(\sqrt{y^2+a^2}\right)^2} \cdot \frac{y}{\sqrt{y^2+a^2}}$

$F_{net} = \frac{2kq\left(\frac{q}{2}\right)y}{(y^2+a^2)^{3/2}}$ $(\because y << a)$

$\Rightarrow \frac{kq^2 y}{a^3}$

So, $F \propto y$

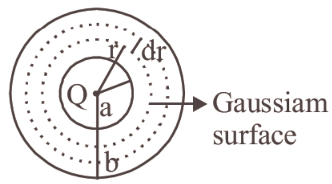
22. **(c)** Field lines originate perpendicular from positive charge and terminate perpendicular at negative charge. Further this system can be treated as an electric dipole.

23. **(c)** Applying Gauss's law

$\oint_S \vec{E}\cdot\vec{ds} = \frac{Q}{\in_0}$

$\therefore E \times 4\pi r^2 = \frac{Q + 2\pi Ar^2 - 2\pi Aa^2}{\in_0}$

$\rho = \frac{dr}{dV}$



$Q = \rho 4\pi r^2$

$$Q = \int_a^r \frac{A}{r} 4\pi r^2 dr = 2\pi A[r^2 - a^2]$$

$$E = \frac{1}{4\pi \epsilon_0}\left[\frac{Q - 2\pi Aa^2}{r^2} + 2\pi A\right]$$

For E to be independent of 'r'

$Q - 2\pi Aa^2 = 0$

$$\therefore\ A = \frac{Q}{2\pi a^2}$$

24. (a) T = PE sin θ Torque experienced by the dipole in an electric field, $\vec{T} = \vec{P} \times \vec{E}$

$\vec{p} = p\cos\theta\ \hat{i} + p\sin\theta\ \hat{j}$

$\vec{E}_1 = E\vec{i}$

$\vec{T}_1 = \vec{p} \times \vec{E}_1 = (p\cos\theta\,\hat{i} + p\sin\theta\,\hat{j}) \times E(\hat{i})$

$\tau\hat{k} = pE\sin\theta(-\hat{k})$ ...(i)

$\vec{E}_2 = \sqrt{3}E_1\hat{j}$

$\vec{T}_2 = p\cos\theta\hat{i} + p\sin\theta\hat{j}) \times \sqrt{3}E_1\hat{j}$

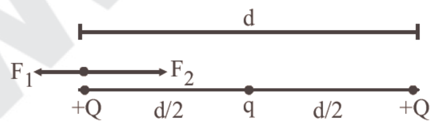
$\tau\hat{k} = \sqrt{3}\,pE_1\cos\theta\hat{k}$ ...(ii)

From eqns. (*i*) and (*ii*)

$pE\sin\theta = \sqrt{3}\,pE\cos\theta$

$\tan\theta = \sqrt{3} \quad \therefore \quad \theta = 60°$

25. (a)



Force due to charge $+Q$,

$$F_1 = \frac{KQQ}{d^2}$$

Force due to charge q,

$$F_2 = \frac{KQq}{\left(\frac{d}{2}\right)^2}$$

For equilibrium,

$\vec{F_1} + \vec{F_2} = 0$

$$\frac{kQQ}{d^2} + \frac{kQq}{(d/2)^2} = 0 \quad \therefore \quad q = -\frac{Q}{4}$$

26. (b) Electric field on the axis of a ring of radius R at a distance h from the centre,

$$E = \frac{kQh}{\left(h^2 + R^2\right)^{3/2}}$$

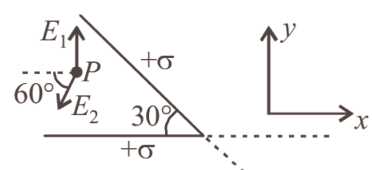
Condition: for maximum electric field

$$\frac{dE}{dh} = 0$$

$$\Rightarrow \frac{d}{dh}\left[\frac{kQh}{\left(R^2 + h^2\right)^{3/2}}\right] = 0$$

On solving we get, $h = \dfrac{R}{\sqrt{2}}$

27. (d)



From figure,

$\vec{E}_1 = \dfrac{\sigma}{2\varepsilon_0}\hat{y}$ and

$$\vec{E}_2 = \frac{\sigma}{2\varepsilon_0}(-\cos 60°\hat{x} - \sin 60°\hat{y})$$

$$= \frac{\sigma}{2\varepsilon_0}\left(-\frac{1}{2}\hat{x} - \frac{\sqrt{3}}{2}\hat{y}\right)$$

Electric field in the region shown in figure (*P*)

$$\vec{E}_P = \vec{E}_1 + \vec{E}_2 = \frac{\sigma}{2\varepsilon_0}\left[-\frac{1}{2}\hat{x} + \left(1 - \frac{\sqrt{3}}{2}\right)\hat{y}\right]$$

or, $$\vec{E}_P = \frac{\sigma}{2\varepsilon_0}\left[\left(1 - \frac{\sqrt{3}}{2}\right)\hat{y} - \frac{\hat{x}}{2}\right]$$

16 Electrostatic Potential and Capacitance

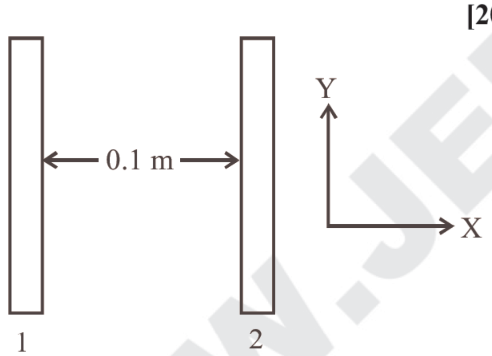
1. On moving a charge of 20 coulomb by 2 cm, 2 J of work is done, then the potential difference between the points is **[2002]**
   (a) 0.1 V (b) 8 V
   (c) 2 V (d) 0.5 V
2. If there are n capacitors in parallel connected to V volt source, then the energy stored is equal to **[2002]**
   (a) CV (b) $\frac{1}{2}nCV^2$
   (c) CV^2 (d) $\frac{1}{2n}CV^2$
3. Capacitance (in F) of a spherical conductor with radius 1 m is **[2002]**
   (a) 1.1×10^{-10} (b) 10^{-6}
   (c) 9×10^{-9} (d) 10^{-3}
4. A sheet of aluminium foil of negligible thickness is introduced between the plates of a capacitor. The capacitance of the capacitor **[2003]**
   (a) decreases
   (b) remains unchanged
   (c) becomes infinite
   (d) increases
5. A thin spherical conducting shell of radius R has a charge q. Another charge Q is placed at the centre of the shell. The electrostatic potential at a point P, a distance $\frac{R}{2}$ from the centre of the shell is **[2003]**
   (a) $\frac{2Q}{4\pi\varepsilon_o R}$ (b) $\frac{2Q}{4\pi\varepsilon_o R}-\frac{2q}{4\pi\varepsilon_o R}$
   (c) $\frac{2Q}{4\pi\varepsilon_o R}+\frac{q}{4\pi\varepsilon_o R}$ (d) $\frac{(q+Q)2}{4\pi\varepsilon_o R}$
6. The work done in placing a charge of 8×10^{-18} coulomb on a condenser of capacity 100 micro-farad is **[2003]**
   (a) 16×10^{-32} joule
   (b) 3.1×10^{-26} joule
   (c) 4×10^{-10} joule
   (d) 32×10^{-32} joule
7. Two thin wire rings each having a radius R are placed at a distance d apart with their axes coinciding. The charges on the two rings are $+q$ and $-q$. The potential difference between the centres of the two rings is **[2005]**
   (a) $\frac{q}{2\pi\epsilon_0}\left[\frac{1}{R}-\frac{1}{\sqrt{R^2+d^2}}\right]$
   (b) $\frac{qR}{4\pi\epsilon_0 d^2}$
   (c) $\frac{q}{4\pi\epsilon_0}\left[\frac{1}{R}-\frac{1}{\sqrt{R^2+d^2}}\right]$
   (d) zero
8. A parallel plate capacitor is made by stacking n equally spaced plates connected alternatively. If the capacitance between any two adjacent plates is 'C' then the resultant capacitance is **[2005]**
   (a) $(n+1)C$ (b) $(n-1)C$
   (c) nC (d) C

9. A fully charged capacitor has a capacitance 'C'. It is discharged through a small coil of resistance wire embedded in a thermally insulated block of specific heat capacity 's' and mass 'm'. If the temperature of the block is raised by 'ΔT', the potential difference 'V' across the capacitance is **[2005]**

(a) $\frac{mC\Delta T}{s}$ (b) $\sqrt{\frac{2mC\Delta T}{s}}$

(c) $\sqrt{\frac{2ms\Delta T}{C}}$ (d) $\frac{ms\Delta T}{C}$

10. Two insulating plates are both uniformly charged in such a way that the potential difference between them is $V_2 - V_1 = 20$ V. (i.e., plate 2 is at a higher potential). The plates are separated by $d = 0.1$ m and can be treated as infinitely large. An electron is released from rest on the inner surface of plate 1. What is its speed when it hits plate 2? ($e = 1.6 \times 10^{-19}$ C, $m_e = 9.11 \times 10^{-31}$ kg) **[2006]**



(a) 2.65×10^6 m/s (b) 7.02×10^{12} m/s

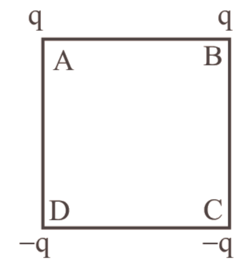
(c) 1.87×10^6 m/s (d) 32×10^{-19} m/s

11. An electric charge $10^{-3}\,\mu$C is placed at the origin (0, 0) of $X - Y$ co-ordinate system. Two points A and B are situated at $(\sqrt{2}, \sqrt{2})$ and (2, 0) respectively. The potential difference between the points A and B will be **[2007]**

(a) 4.5 volts (b) 9 volts

(c) Zero (d) 2 volt

12. Charges are placed on the vertices of a square as shown. Let $\vec{E}$ be the electric field and V the potential at the centre. If the charges on A and B are interchanged with those on D and C respectively, then **[2007]**



(a) $\vec{E}$ changes, V remains unchanged

(b) $\vec{E}$ remains unchanged, V changes

(c) both $\vec{E}$ and V change

(d) $\vec{E}$ and V remain unchanged

13. The potential at a point x (measured in μm) due to some charges situated on the x-axis is given by $V(x) = 20/(x^2 - 4)$ volt. The electric field E at $x = 4\ \mu$m is given by **[2007]**

(a) (10/9) volt/μm and in the +ve x direction

(b) (5/3) volt/μm and in the –ve x direction

(c) (5/3) volt/μm and in the +ve x direction

(d) (10/9) volt/μm and in the –ve x direction

14. A parallel plate condenser with a dielectric of dielectric constant K between the plates has a capacity C and is charged to a potential V volt. The dielectric slab is slowly removed from between the plates and then reinserted. The net work done by the system in this process is **[2007]**

(a) zero (b) $\frac{1}{2}(K-1)\,CV^2$

(c) $\frac{CV^2(K-1)}{K}$ (d) $(K-1)\,CV^2$

15. A parallel plate capacitor with air between the plates has capacitance of 9 pF. The separation between its plates is 'd'. The space between the plates is now filled with two dielectrics. One of the dielectrics has dielectric constant $k_1 = 3$ and thickness $\frac{d}{3}$ while the other one has dielectric constant $k_2 = 6$ and thickness $\frac{2d}{3}$. Capacitance of the capacitor is now **[2008]**

(a) 1.8 pF (b) 45 pF

(c) 40.5 pF (d) 20.25 pF

16. Two points P and Q are maintained at the potentials of 10 V and – 4 V, respectively. The work done in moving 100 electrons from P to Q is: **[2009]**

(a) 9.60×10^{-17} J (b) -2.24×10^{-16} J

(c) 2.24×10^{-16} J (d) -9.60×10^{-17} J

17. Two positive charges of magnitude 'q' are placed, at the ends of a side (side 1) of a square of side '$2a$'. Two negative charges of the same magnitude are kept at the other corners. Starting from rest, if a charge Q moves from the middle of side 1 to the centre of square, its kinetic energy at the centre of square is **[2011 RS]**

(a) zero

(b) $\frac{1}{4\pi\varepsilon_0}\frac{2qQ}{a}\left(1+\frac{1}{\sqrt{5}}\right)$

(c) $\frac{1}{4\pi\varepsilon_0}\frac{2qQ}{a}\left(1-\frac{2}{\sqrt{5}}\right)$

(d) $\frac{1}{4\pi\varepsilon_0}\frac{2qQ}{a}\left(1-\frac{1}{\sqrt{5}}\right)$

18. This questions has statement-1 and statement-2. Of the four choices given after the statements, choose the one that best describes the two statements.

An insulating solid sphere of radius R has a uniformly positive charge density ρ. As a result of this uniform charge distribution there is a finite value of electric potential at the centre of the sphere, at the surface of the sphere and also at a point outside the sphere. The electric potential at infinite is zero. **[2012]**

Statement -1 When a charge q is taken from the centre to the surface of the sphere its potential energy changes by $\frac{q\rho}{3\varepsilon_0}$.

Statement -2 The electric field at a distance r $(r<R)$ from the centre of the sphere is $\frac{\rho r}{3\varepsilon_0}$.

(a) Statement 1 is true, Statement 2 is true; Statement 2 is not the correct explanation of statement 1.

(b) Statement 1 is true Statement 2 is false.

(c) Statement 1 is false Statement 2 is true.

(d) Statement 1 is true, Statement 2 is true, Statement 2 is the correct explanation of Statement 1

19. Assume that an electric field $\vec{E} = 30x^2\hat{i}$ exists in space. Then the potential difference $V_A - V_O$, where V_O is the potential at the origin and V_A the potential at x = 2 m is: **[2014]**

(a) 120 J/C (b) –120 J/C

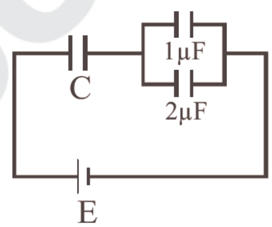
(c) –80 J/C (d) 80 J/C

20. A parallel plate capacitor is made of two circular plates separated by a distance 5 mm and with a dielectric of dialectric constant 2.2 between them. When the electric field in the dielectric is 3×10^4 V/m the charge density of the positive plate will be close to: **[2014]**

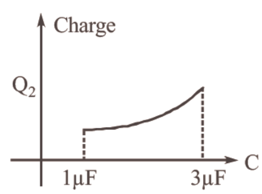
(a) $6\times10^{-7}\,C/m^2$ (b) $3\times10^{-7}\,C/m^2$

(c) $3\times10^{4}\,C/m^2$ (d) $6\times10^{4}\,C/m^2$

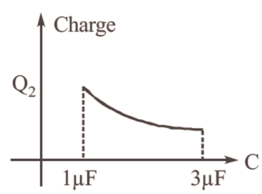
21. In the given circuit, charge Q_2 on the 2μF capacitor changes as C is varied from 1μF to 3μF. Q_2 as a function of 'C' is given properly by: (*figures are drawn schematically and are not to scale*) **[2015]**



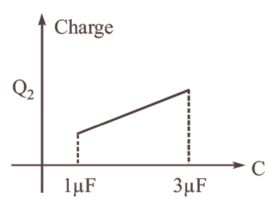
(a)



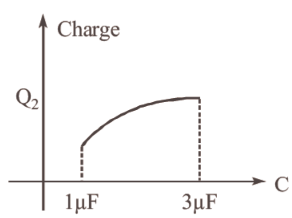
(b)



(c)



(d)



22. A uniformly charged solid sphere of radius R has potential V_0 (measured with respect to ∞) on its surface. For this sphere the equipotential surfaces with potentials $\frac{3V_0}{2}, \frac{5V_0}{4}, \frac{3V_0}{4}$ and $\frac{V_0}{4}$ have radius R_1, R_2, R_3 and R_4 respectively. Then **[2015]**

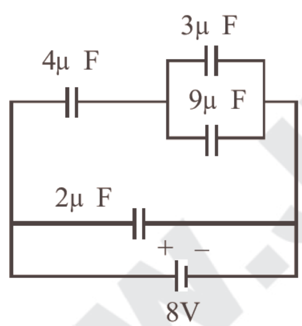
(a) $R_1 = 0$ and $R_2 < (R_4 - R_3)$

(b) $2R < R_4$

(c) $R_1 = 0$ and $R_2 > (R_4 - R_3)$

(d) $R_1 \neq 0$ and $(R_2 - R_1) > (R_4 - R_3)$

23. A combination of capacitors is set up as shown in the figure. The magnitude of the electric field, due to a point charge Q (having a charge equal to the sum of the charges on the 4 μF and 9 μF capacitors), at a point distance 30 m from it, would equal : **[2016]**



(a) 420 N/C (b) 480 N/C
(c) 240 N/C (d) 360 N/C

24. A capacitance of 2μF is required in an electrical circuit across a potential difference of 1.0 kV. A large number of 1μF capacitors are available which can withstand a potential difference of not more than 300 V. The minimum number of capacitors required to achieve this is **[2017]**

(a) 24 (b) 32 (c) 2 (d) 16

25. Three concentric metal shells A, B and C of respective radii a, b and c $(a < b < c)$ have surface charge densities $+\sigma$, $-\sigma$ and $+\sigma$ respectively. The potential of shell B is: **[2018]**

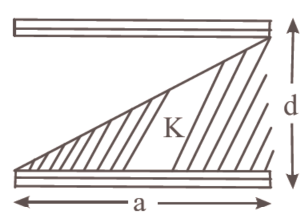
(a) $\frac{\sigma}{\epsilon_0}\left[\frac{a^2-b^2}{a}+c\right]$ (b) $\frac{\sigma}{\epsilon_0}\left[\frac{a^2-b^2}{b}+c\right]$

(c) $\frac{\sigma}{\epsilon_0}\left[\frac{b^2-c^2}{b}+a\right]$ (d) $\frac{\sigma}{\epsilon_0}\left[\frac{b^2-c^2}{c}+a\right]$

26. A parallel plate capacitor of capacitance 90 pF is connected to a battery of emf 20V. If a dielectric material of dielectric constant $k = \frac{5}{3}$ is inserted between the plates, the magnitude of the induced charge will be: **[2018]**

(a) 1.2 n C (b) 0.3 n C
(c) 2.4 n C (d) 0.9 n C

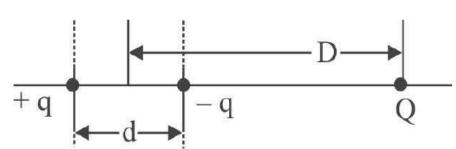
27. A parallel plate capacitor is made of two square plates of side 'a', separated by a distance d $(d << a)$. The lower triangular portion is filled with a dielectric of dielectric constant K, as shown in the figure. Capacitance of this capacitor is: **[2019]**



(a) $\frac{K \in_0 a^2}{2d\,(K+1)}$ (b) $\frac{K \in_0 a^2}{d(K-1)}$ In K

(c) $\frac{K \in_0 a^2}{d}$ In K (d) $\frac{1}{2}\frac{K \in_0 a^2}{d}$

28. A system of three charges are placed as shown in the figure: **[2019]**



If $D >> d$, the potential energy of the system is best given by

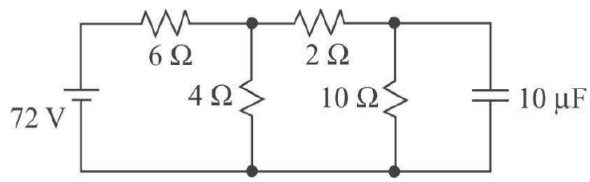
(a) $\frac{1}{4\pi\in_0}\left[\frac{-q^2}{d}\frac{-qQd}{2D^2}\right]$

(b) $\frac{1}{4\pi\in_0}\left[\frac{-q^2}{d}+\frac{2qQd}{D^2}\right]$

(c) $\frac{1}{4\pi\in_0}\left[+\frac{q^2}{d}+\frac{qQd}{D^2}\right]$

(d) $\frac{1}{4\pi\in_0}\left[-\frac{q^2}{d}-\frac{qQd}{D^2}\right]$

29. Determine the charge on the capacitor in the following circuit: **[2019]**

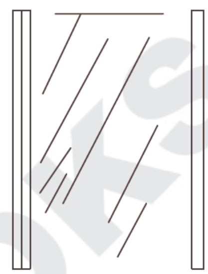


(a) 60 μC (b) 2 μC
(c) 10 μC (d) 200 μC

30. A capacitor with capacitance 5 μF is charged to 5 μC. If the plates are pulled apart to reduce the capacitance to 2 μF, how much work is done? **[2020]**

(a) 6.25×10^{-6} J (b) 3.75×10^{-6} J
(c) 2.16×10^{-6} J (d) 2.55×10^{-6} J

31. A parallel plate capacitor has plates of area A separated by distance 'd' between them. It is filled with a dielectric which has a dielectric constant that varies as $k(x) = K(1 + \alpha x)$ where 'x' is the distance measured from one of the plates. If $(\alpha d) << 1$, the total capacitance of the system is best given by the expression: **[2020]**



(a) $\frac{AK\in_0}{d}\left(1+\frac{\alpha d}{2}\right)$

(b) $\frac{A\in_0 K}{d}\left(1+\left(\frac{\alpha d}{2}\right)^2\right)$

(c) $\frac{A\in_0 K}{d}\left(1+\frac{\alpha^2 d^2}{2}\right)$

(d) $\frac{AK\in_0}{d}(1+\alpha d)$

Answer Key

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
(a)	(b)	(a)	(b)	(c)	(d)	(a)	(b)	(c)	(a)	(c)	(a)	(a)	(a)	(c)
16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
(c)	(d)	(c)	(c)	(a)	(d)	(a)	(a)	(b)	(b)	(a)	(b)	(d)	(d)	(b)
31														
(a)														

Solutions

1. **(a)** By using

$W = q(V_B - V_A)$

$$\therefore V_B - V_A = \frac{2\,\text{J}}{20\,\text{C}} = 0.1\,\text{J/C} = 0.1\,\text{V}$$

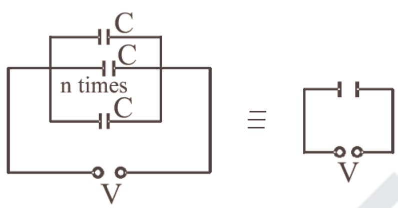
2. **(b)** In parallel, equivalent capacitance of n capacitor of capacitance C

$C' = nC$

Energy stored in this capacitor

$$E = \frac{1}{2}C^1V^2$$

$$\Rightarrow E = \frac{1}{2}(nC)V^2 = \frac{1}{2}nCV^2$$



Alternatively

Each capacitor has a potential difference of V between the plates.

So, energy stored in each capacitor

$$= \frac{1}{2}CV^2.$$

$\therefore$ Energy stored in n capacitor

$$= \left[\frac{1}{2}CV^2\right] \times n$$

3. **(a)** Capacitance of spherical conductor = $4\pi E_0 R$

Here, R is radius of conductor

$$\therefore\ C = 4\pi \in_0 R = \frac{1}{9\times 10^9}\times 1 = 1.1\times 10^{-10} F$$

4. **(b)** The capacitance without aluminium foil is

$$C = \frac{\varepsilon_0 A}{d}$$

Here, d is distance between the plates of a capacitor

A = Area of plates of capacitor

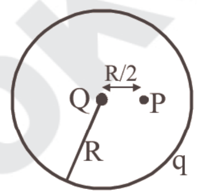
When an aluminium foil of thickness t is introduced between the plates.

$$\text{Capacitance, } C' = \frac{\varepsilon_0 A}{d-t}$$

If thickness of foil is negligible 50 $d - t \sim d$.

Hence, $C = C'$.

5. **(c)** Electric potential due to charge Q at point P is



$$V_1 = \frac{1}{4\pi\varepsilon_o}\frac{Q}{R/2} = \frac{1}{4\pi\varepsilon_o}\frac{2Q}{R}$$

Electric potential due to charge q inside the shell is

$$V_2 = \frac{1}{4\pi\varepsilon_o}\frac{q}{R}$$

$\therefore$ The net electric potential at point P is

$$V = V_1 + V_2 = \frac{1}{4\pi\varepsilon_o}\frac{2Q}{R} + \frac{1}{4\pi\varepsilon_o}\frac{q}{R}$$

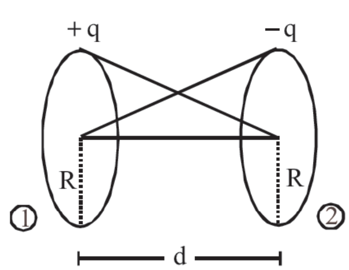
6. **(d)** The work done is stored in the form of potential energy which is given by

$$U = \frac{1}{2}\frac{Q^2}{C}$$

$$\therefore\ U = \frac{1}{2}\times\frac{\left(8\times 10^{-18}\right)^2}{100\times 10^{-6}}$$

$= 32 \times 10^{-32}$ J

7. **(a)**



Potential at the center of ring of charge $+q$ = potential due to iteself + potential due to other ring of charge $-q$.

$$\Rightarrow V_1 = \frac{1}{4\pi\varepsilon_0}\left[\frac{q}{R} - \frac{q}{\sqrt{R^2+d^2}}\right]$$

Potential at the centre of ring of charge $-q$ = potential due to itself + potential due to other ring of charge $+q$.

$$\Rightarrow V_2 = \frac{1}{4\pi\varepsilon_0}\left[\frac{-q}{R} + \frac{q}{\sqrt{R^2+d^2}}\right]$$

$$\Delta V = V_1 - V_2$$

$$= \frac{1}{4\pi\varepsilon_0}\left[\frac{q}{R} + \frac{q}{R} - \frac{q}{\sqrt{R^2+d^2}} - \frac{q}{\sqrt{R^2+d^2}}\right]$$

$$= \frac{1}{2\pi\varepsilon_0}\left[\frac{q}{R} - \frac{q}{\sqrt{R^2+d^2}}\right]$$

8. **(b)** As n plates are joined alternately positive plate of all $(n-1)$ capacitor are connected to one point and negative plate of all $(n-1)$ capacitors are connected to other point. It means $(n-1)$ capacitors joined in parallel.

$\therefore$ Resultant capacitance $= (n-1)C$

9. **(c)** Applying conservation of energy,

Electric potential energy of capacitor = heat absorbed

$$\frac{1}{2}CV^2 = m.s\,\Delta t;\ \ V = \sqrt{\frac{2m.s.\Delta t}{C}}$$

10. **(a)** Gain in kinetic energy = work done by potential difference

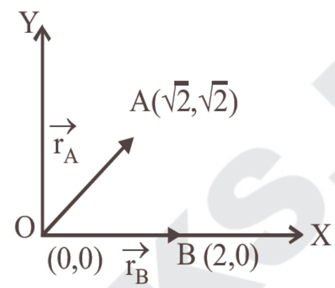
$$eV = \frac{1}{2}mv^2$$

$$\Rightarrow v = \sqrt{\frac{2eV}{m}}$$

$$= \sqrt{\frac{2\times1.6\times10^{-19}\times20}{9.1\times10^{-31}}}$$

$$= 2.65 \times 10^6 \text{ m/s}$$

11. **(c)**



The distance of point $A\left(\sqrt{2},\sqrt{2}\right)$ from the origin,

$$r_A = \sqrt{(\sqrt{2})^2 + (\sqrt{2})^2} = \sqrt{4} = 2 \text{ units.}$$

The distance of point $B(2, 0)$ from the origin,

$$r_B = \sqrt{(2)^2 + (0)^2} = 2 \text{ units.}$$

Now, potential at A, due to charge $\theta = 10^{-3}\mu C$

$$V_A = \frac{1}{4\pi \in_0}\cdot\frac{Q}{(r_A)}$$

Potential at B, due to charge $Q = 10^{-3}\ QC$

$$V_B = \frac{1}{4\pi \in_0}\cdot\frac{Q}{(r_B)}$$

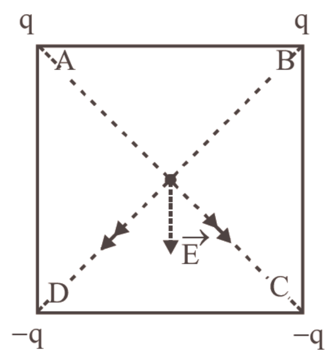
$\therefore$ Potential difference between the points A and B is given by

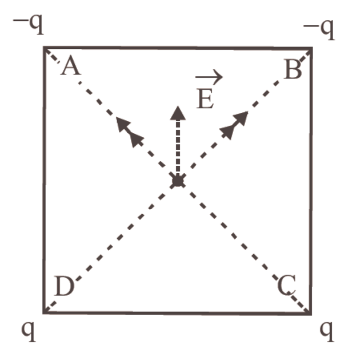
$$V_A - V_B = \frac{1}{4\pi\in_0}\cdot\frac{10^{-3}}{r_A} - \frac{1}{4\pi\in_0}\cdot\frac{10^{-3}}{r_B}$$

$$= \frac{10^{-3}}{4\pi\in_0}\left(\frac{1}{r_A} - \frac{1}{r_B}\right) = \frac{10^{-3}}{4\pi\in_0}\left(\frac{1}{2} - \frac{1}{2}\right)$$

$$= \frac{Q}{4\pi\in_0}\times 0 = 0.$$

12. **(a)** As shown in the figure, the resultant electric fields before and after interchanging the charges will have the same magnitude, but opposite directions. As potential is a scalar quantity, So the potential will be same in both cases.





13. **(a)** Given, potential $V(x) = \dfrac{20}{x^2 - 4}$ volt

Electric field $E = -\dfrac{dV}{dx} = -\dfrac{d}{dx}\left(\dfrac{20}{x^2-4}\right)$

$\Rightarrow E = +\dfrac{40x}{(x^2-4)^2}$

At $x = 4\ \mu\text{m}$,

$E = +\dfrac{40 \times 4}{(4^2-4)^2} = +\dfrac{160}{144} = +\dfrac{10}{9}$ volt / μm.

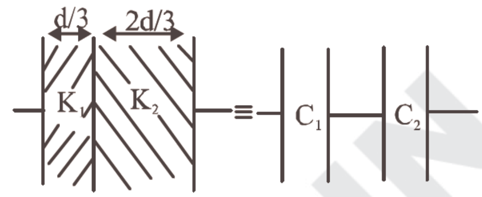
Positive sign indicates that $\vec{E}$ is in +ve x-direction.

14. **(a)** The potential energy of a charged capacitor is given by $U = \dfrac{Q^2}{2C}$.

When a dielectric slab is introduced between the plates the energy is given by $\dfrac{Q^2}{2KC}$, where K is the dielectric constant.

Again, when the dielectric slab is removed slowly its energy increases to initial potential energy. Thus, work done is zero.

15. **(c)**



The capacitance with air between the plates

$C = \dfrac{\varepsilon_0 A}{d} = 9\text{pF}$

On introducing two dielectric between the plates, the given capacitance is equal to two capacitances connected in series where

$C_1 = \dfrac{k_1 \in_0 A}{d/3} = \dfrac{3 \in_0 A}{d/3}$

$= \dfrac{3 \times 3 \in_0 A}{d} = \dfrac{9 \in_0 A}{d}$

and

$C_2 = \dfrac{k_2 \in_0 A}{2d/3} = \dfrac{3k_2 \in_0 A}{2d}$

$= \dfrac{3 \times 6 \in_0 A}{2d} = \dfrac{9 \in_0 A}{d}$

The equivalent capacitance C_{eq} is

$\dfrac{1}{C_{eq}} = \dfrac{1}{C_1} + \dfrac{1}{C_2}$

$= \dfrac{d}{9 \in_0 A} + \dfrac{d}{9 \in_0 A} = \dfrac{2d}{9 \in_0 A}$

$\therefore C_{eq} = \dfrac{9}{2} \dfrac{\in_0 A}{d} = \dfrac{9}{2} \times 9\ pF = 40.5\,pF$

16. **(c)** Work done, $W_{PQ} = q(V_Q - V_P)$

$= (-100 \times 1.6 \times 10^{-19})(-4 - 10)$

$= +2.24 \times 10^{-16}\text{J}$

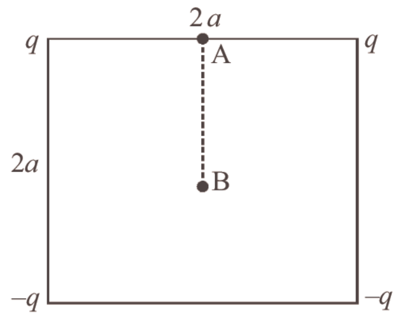
17. **(d)** Initial potential of the charge,

$V_A = \dfrac{2kq}{a} - \dfrac{2kq}{a\sqrt{5}}$

$\Rightarrow \; V_A = \frac{1}{4\pi E}\frac{2q}{a}\left(1-\frac{1}{\sqrt{5}}\right)$

(Here potential due to each $q = \frac{kq}{a}$ and

potential due to each $-q = \frac{-kq}{a\sqrt{5}}$)



Final potential of the charge

$V_B = 0$

($\because$ Point B is equidistant from all the four charges)

$\therefore$ Using work energy theorem,

$(W_{AB})_{\text{electric}} = Q(V_A - V_B)$

$= \frac{2qQ}{4\pi E_0 a}\left[1-\frac{1}{\sqrt{5}}\right]$

$= \left(\frac{1}{4\pi\varepsilon_0}\right)\frac{2Qq}{a}\left[1-\frac{1}{\sqrt{5}}\right]$

18. **(c)** The potential energy at the centre of the sphere

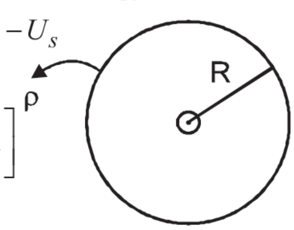
$U_c = \frac{3}{2}\frac{KQq}{R}$

The potential energy at the surface of the sphere

$U_s = \frac{KqQ}{R}$

Now change in the energy

$\Delta U = U_c - U_s$



$= \frac{KQq}{R}\left[\frac{3}{2}-1\right]$

$= \frac{KQq}{2R}$

Where $Q = \rho.V = \rho.\frac{4}{3}\pi R^3$

$\Delta U = \frac{2K}{3}\frac{\pi R^3 \rho q}{R}$

$\Delta U = \frac{2}{3}\times\frac{1}{4\pi \in_0}\frac{\pi R^3 \rho q}{R}$

$\Delta U = \frac{R^2 \rho q}{6 \in_0}$

Using Gauss's law

$\int \vec{E}\cdot\vec{dA} = \frac{q_{en}}{E_0} = \frac{\beta\times\frac{4}{3}\pi R^3}{E_0}$

$\Rightarrow \int E dA(\cos\theta) = \frac{\beta\times 4\pi R^3}{3E_0}$

$\Rightarrow E(4\pi R^2) = \beta\times\frac{4}{3}\pi R^3\times\frac{1}{E_0}$

$\Rightarrow E = \frac{\beta r}{3E_0}\,(r < R)$

19. **(c)** Potential difference between any two points in an electric field is given by,

$dV = -\vec{E}\cdot\vec{dx}$

$\int_{V_O}^{V_A} dV = -\int_0^2 30x^2\,dx$

$V_A - V_O = -[10x^3]_0^2 = -80 J/C$

20. **(a)** Electric field in presence of dielectric between the two plates of a parallel plate capacitor is given by,

$E = \frac{\sigma}{K\varepsilon_0}$

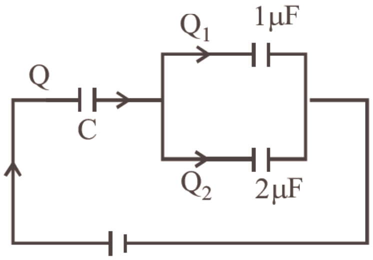
Then, charge density

$\sigma = K\varepsilon_0 E$

$= 2.2\times 8.85\times 10^{-12}\times 3\times 10^4$

$\approx 6\times 10^{-7}\ \text{C/m}^2$

21. (d)

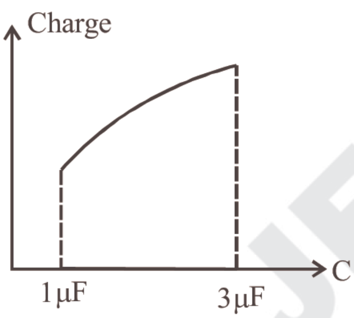


From figure, $Q_2 = \frac{2}{2+1}Q = \frac{2}{3}Q$

$$Q = E\left(\frac{C \times 3}{C+3}\right)$$

$$\therefore Q_2 = \frac{2}{3}\left(\frac{3CE}{C+3}\right) = \frac{2CE}{C+3}$$

Therefore graph *d* correctly dipicts.



22. (a) We know, $V_0 = \frac{Kq}{R} = V$ surface

Now, $V_i = \frac{Kq}{2R^3}(3R^2 - r^2)$ [For $r < R$]

At the centre of sphere r = 0. Here

$V = \frac{3}{2}V_0$

Now, $\frac{5}{4}\frac{Kq}{R} = \frac{Kq}{2R^3}(3R^2 - r^2)$

$\Rightarrow R_2 = \frac{R}{\sqrt{2}}$

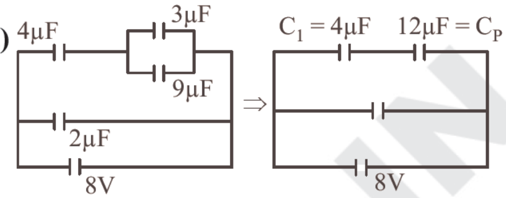
$\frac{3}{4}\frac{Kq}{R} = \frac{Kq}{R^3}$

$\frac{1}{4}\frac{Kq}{R} = \frac{Kq}{R_4}$

$R_4 = 4R$

Also, $R_1 = 0$ and $R_2 < (R_4 - R_3)$

23. (a)



Charge on C_1 is $q_1 = \left[\left(\frac{12}{4+12}\right) \times 8\right] \times 4 =$ 24μC

The voltage across C_P is $V_P = \frac{4}{4+12} \times 8 = 2V$

$\therefore$ Voltage across 9μF is also 2V

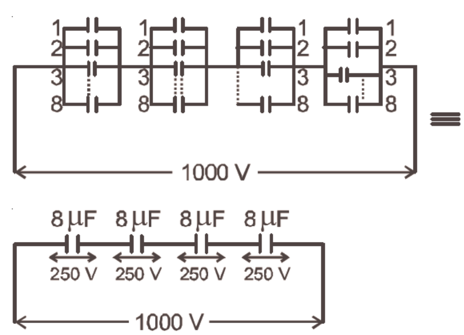
$\therefore$ Charge on 9μF capacitor = 9 × 2 = 18μC

$\therefore$ Total charge on 4 μF and 9μF = 42μC

$$\therefore E = \frac{KQ}{r^2} = 9 \times 10^9 \times \frac{42 \times 10^{-6}}{30 \times 30}$$

$$= 420\ NC^{-1}$$

24. (b) To get a capacitance of 2μF arrangement of capacitors of capacitance 1μF as shown in figure 8 capacitors of 1μF in parallel with four such branches in series i.e., 32 such capacitors are required.



$$\frac{1}{C_{eq}} = \frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8} \quad \therefore C_{eq} = 2\ \mu F$$

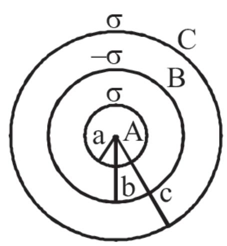
25. (b) Potential outside the shell,

$$V_{outside} = \frac{KQ}{r}$$

where r is distance of point from the centre of shell

Potential inside the shell, $V_{inside} = \dfrac{KQ}{R}$

where 'R" is radius of the shell



$$V_B = \frac{Kq_A}{r_b} + \frac{Kq_B}{r_b} + \frac{Kq_C}{r_c}$$

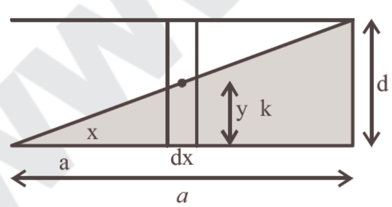
$$V_B = \frac{1}{4\pi\in_0}\left[\frac{\sigma 4\pi a^2}{b} - \frac{\sigma 4\pi b^2}{b} + \frac{\sigma 4\pi c^2}{c}\right]$$

$$V_B = \frac{\sigma}{\in_0}\left[\frac{a^2 - b^2}{b} + c\right]$$

26. (a) Charge on Capacitor, $Q_i = CV$
After inserting dielectric of dielectric constant $= K$ $Q_f = (kC)\,V$
Induced charges on dielectric
$Q_{ind} = Q_f - Q_i = KCV - CV$

$$(K-1)CV = \left(\frac{5}{3} - 1\right) \times 90\,\text{pF} \times 2\text{V} = 1.2\text{nc}$$

27. (b)



From figure, $\dfrac{y}{x} = \dfrac{d}{a} \Rightarrow y = \dfrac{d}{a}x$

$$dy = \frac{d}{a}(dx) \Rightarrow \frac{1}{dC} = \frac{y}{K\varepsilon_0 adx} + \frac{(d-y)}{\varepsilon_0 adx}$$

$$\frac{1}{dC} = \frac{y}{\varepsilon_0 adx}\left(\frac{y}{k} + d - y\right)$$

$$\int dC = \int \frac{\varepsilon_0 adx}{\frac{y}{k} + d - y}$$

$$\text{or, } C = \varepsilon_0 a \cdot \frac{a}{d}\int_0^d \frac{dy}{d + y\left(\frac{1}{k} - 1\right)} \quad \left[\because dy = \frac{d}{a}dx\right]$$

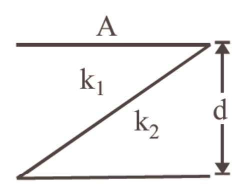
$$= \frac{\varepsilon_0 a^2}{\left(\frac{1}{k} - 1\right)d}\left[\ell n\left(d + y\left(\frac{1}{k} - 1\right)\right)\right]_0^d$$

$$= \frac{k \in_0 a^2}{(1-k)d}\ell n\left(\frac{d + d\left(\frac{1}{k} - 1\right)}{d}\right)$$

$$= \frac{k \in_0 a^2}{(1-k)d}\ell n\left(\frac{1}{k}\right) = \frac{k \in_0 a^2 \ell nk}{(k-1)d}$$

Alternatively remember

$$C = \frac{k_1 k_3 A\,\varepsilon_0}{d(k_1 - k_2)}\log_e \frac{k_1}{k_2}$$



Here $k_1 = 1$, $k_2 = k$, $A = a^2$

$$\therefore \quad C = \frac{k\,a^2 \varepsilon_0}{d(1-k)}\log_e \frac{1}{k} = \frac{ka^2\varepsilon_0}{d(k-1)}\log_e k$$

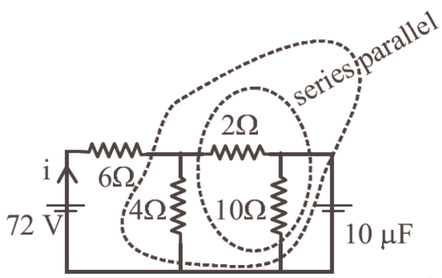
28. (d) $$U = \frac{1}{4\pi\in_0}\left[\frac{q(-q)}{d} + \frac{qQ}{\left(D + \frac{d}{2}\right)} + \frac{(-q)Q}{\left(D - \frac{d}{2}\right)}\right]$$

$$= \frac{1}{4\pi\varepsilon_0}\left[\frac{-q^2}{d} + \frac{qQ\left(D - \frac{d}{2}\right) - qQ\left(D + \frac{d}{2}\right)}{D^2 - \frac{d^2}{4}}\right]$$

$$= \frac{1}{4\pi \in_0}\left[-\frac{q^2}{d} - \frac{qQd}{D^2}\right], \because \frac{d^2}{4} << D$$

29. (d) At steady state, there is no current in capacitor. 2Ω and 10Ω are in series. There equivalent resistance is 12Ω. This 12Ω is in parallel with 4Ω and there combined resistance is $12 \times 4/(12+4)$. This resistance is in series with 6Ω. Therefore, current drawn from battery

$$i = \frac{V}{R} = \left(\frac{72}{6 + \frac{12\times 4}{12+4}}\right) = 8A$$



Current in 10Ω resistor

$$i' = \left(\frac{4}{4+12}\right)8 = 2A$$

Pd across capacitor, $V = i'R = 2 \times 10 = 20V$

$\therefore$ Charge on the capacitor, $q = CV$
$= 10 \times 20 = 200\ \mu C$.

30. (b)

$$W = U_f - U_i = \frac{q^2}{2}\left(\frac{1}{C_f} - \frac{1}{C_i}\right)\left(\because U = \frac{q^2}{2C}\right)$$

$$= \frac{(5\times 10^{-6})^2}{2}\left(\frac{1}{2} - \frac{1}{5}\right)\times 10^6$$

$= 3.75 \times 10^{-6}$J

31. (a) Given, $K(x) = K(1+\alpha x)$

Capacitance of element, $C_{el} = \frac{K\varepsilon_0 A}{dx}$

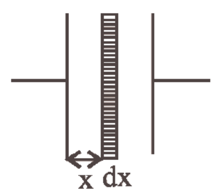
$$\Rightarrow C_{el} = \frac{\varepsilon_0 K(1+\alpha x)A}{dx}$$

$$\therefore \int d\left(\frac{1}{C}\right) = \frac{1}{C_{el}} = \int_0^d \left(\frac{dx}{\varepsilon_0 KA(1+\alpha x)}\right)$$

$$\Rightarrow \frac{1}{C} = \frac{1}{\varepsilon_0 KA\alpha}[ln(1+\alpha x)]_0^d$$

$$\Rightarrow \frac{1}{C} = \frac{1}{\varepsilon_0 KA\alpha}\ln(1+\alpha d)[\alpha d << 1]$$

$$= \frac{1}{\varepsilon_0 KA\alpha}\left[\alpha d - \frac{\alpha^2 d^2}{2}\right]$$



$$= \frac{1}{\varepsilon_0 KA}\left[1 - \frac{\alpha d}{2}\right]$$

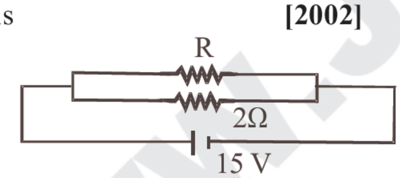
$$\therefore C = \frac{\varepsilon_0 KA}{d\left(1 - \frac{\alpha d}{2}\right)} \Rightarrow C = \frac{\varepsilon_0 KA}{d}\left(1 + \frac{\alpha d}{2}\right)$$

17 Current Electricity

1. If an ammeter is to be used in place of a voltmeter, then we must connect with the ammeter a **[2002]**
(a) low resistance in parallel
(b) high resistance in parallel
(c) high resistance in series
(d) low resistance in series.

2. A wire when connected to 220 V mains supply has power dissipation P_1. Now the wire is cut into two equal pieces which are connected in parallel to the same supply. Power dissipation in this case is P_2. Then $P_2 : P_1$ is **[2002]**
(a) 1 (b) 4
(c) 2 (d) 3

3. If in the circuit, power dissipation is 150 W, then R is **[2002]**



(a) 2Ω (b) 6Ω
(c) 5Ω (d) 4Ω

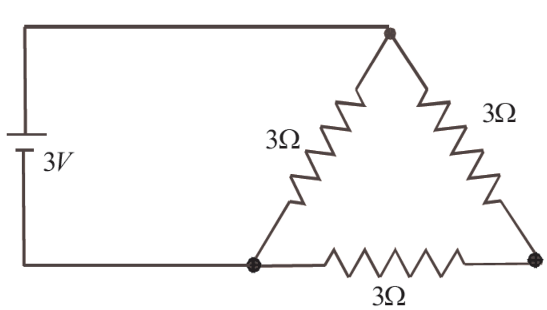
4. The mass of product liberated on anode in an electrochemical cell depends on **[2002]**
(a) $(It)^{1/2}$ (b) It
(c) I/t (d) I^2t
(where t is the time period for which the current is passed).

5. The length of a wire of a potentiometer is 100 cm, and the e. m.f. of its standard cell is E volt. It is employed to measure the e.m.f. of a battery whose internal resistance is 0.5Ω. If the balance point is obtained at $\ell = 30$ cm from the positive end, the e.m.f. of the battery is **[2003]**
(a) $\frac{30E}{100.5}$ (b) $\frac{30E}{(100-0.5)}$
(c) $\frac{30(E-0.5i)}{100}$ (d) $\frac{30E}{100}$
where i is the current in the potentiometer wire.

6. The thermo e.m.f. of a thermo-couple is 25μV/°C at room temperature. A galvanometer of 40 ohm resistance, capable of detecting current as low as 10^{-5} A, is connected with the thermo couple. The smallest temperature difference that can be detected by this system is **[2003]**
(a) 16°C (b) 12°C
(c) 8°C (d) 20°C

7. The negative Zn pole of a Daniell cell, sending a constant current through a circuit, decreases in mass by 0.13g in 30 minutes. If the electeochemical equivalent of Zn and Cu are 32.5 and 31.5 respectively, the increase in the mass of the positive Cu pole in this time is **[2003]**
(a) 0.180 g (b) 0.141g
(c) 0.126 g (d) 0.242 g

9. A 3 volt battery with negligible internal resistance is connected in a circuit as shown in the figure. The current I, in the circuit will be **[2003]**



(a) 1 A (b) 1.5 A
(c) 2 A (d) 1/3 A

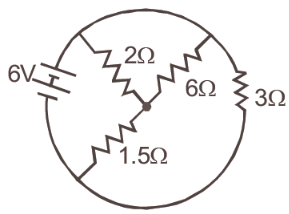
10. A 220 volt, 1000 watt bulb is connected across a 110 volt mains supply. The power consumed will be **[2003]**

(a) 750 watt (b) 500 watt
(c) 250 watt (d) 1000 watt

11. The length of a given cylindrical wire is increased by 100%. Due to the consequent decrease in diameter the change in the resistance of the wire will be **[2003]**

(a) 200% (b) 100%
(c) 50% (d) 300%

12. The total current supplied to the circuit by the battery is **[2004]**



(a) 4 A (b) 2 A
(c) 1 A (d) 6 A

13. The resistance of the series combination of two resistances is S. when they are joined in parallel the total resistance is P. If $S = nP$ then the minimum possible value of n is **[2004]**

(a) 2 (b) 3
(c) 4 (d) 1

14. An electric current is passed through a circuit containing two wires of the same material, connected in parallel. If the lengths and radii arein the ratio of $\frac{4}{3}$ and $\frac{2}{3}$, then the ratio of the current passing through the wires will be **[2004]**

(a) 8/9 (b) 1/3
(c) 3 (d) 2

15. In a meter bridge experiment null point is obtained at 20 cm. from one end of the wire when resistance X is balanced against another resistance Y. If $X < Y$, then where will be the new position of the null point from the same end, if one decides to balance a resistance of $4X$ against Y **[2004]**

(a) 40 cm (b) 80 cm
(c) 50 cm (d) 70 cm

16. The thermistors are usually made of **[2004]**

(a) metal oxides with high temperature coefficient of resistivity
(b) metals with high temperature coefficient of resistivity
(c) metals with low temperature coefficient of resistivity
(d) semiconducting materials having low temperature coefficient of resistivity

17. Time taken by a 836 W heater to heat one litre of water from 10°C to 40°C is **[2004]**

(a) 150 s (b) 100 s
(c) 50 s (d) 200 s

18. The thermo emf of a thermocouple varies with the temperature θ of the hot junction as $E = a\theta + b\theta^2$ in volts where the ratio a/b is 700°C. If the cold junction is kept at 0°C, then the neutral temperature is **[2004]**

(a) 1400°C
(b) 350°C
(c) 700°C
(d) No neutral temperature is possible for this termocouple.

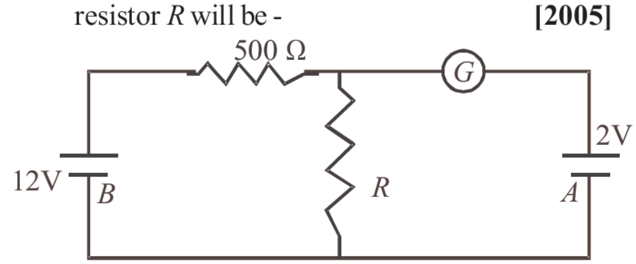
19. The electrochemical equivalent of a metal is 3.35109^{-7} kg per Coulomb. The mass of the metal liberated at the cathode when a 3A current is passed for 2 seconds will be **[2004]**

(a) 6.6×10^{57}kg (b) 9.9×10^{-7} kg
(c) 19.8×10^{-7} kg (d) 1.1×10^{-7} kg

21. A heater coil is cut into two equal parts and only one part is now used in the heater. The heat generated will now be **[2005]**

(a) four times (b) doubled
(c) halved (d) one fourth

22. In the circuit , the galvanometer G shows zero deflection. If the batteries A and B have negligible internal resistance, the value of the resistor R will be - **[2005]**



(a) 100Ω (b) 200Ω
(c) 1000Ω (d) 500Ω

24. Two sources of equal emf are connected to an external resistance R. The internal resistance of the two sources are R_1 and R_2 $(R_1 > R_1)$. If the potential difference across the source having internal resistance R_2 is zero, then **[2005]**

(a) $R = R_2 - R_1$
(b) $R = R_2 \times (R_1 + R_2)/(R_2 - R_1)$
(c) $R = R_1 R_2/(R_2 - R_1)$
(d) $R = R_1 R_2/(R_1 - R_2)$

25. Two voltameters, one of copper and another of silver, are joined in parallel. When a total charge q flows through the voltameters, equal amount of metals are deposited. If the electrochemical equivalents of copper and silver are Z_1 and Z_2 respectively the charge which flows through the silver voltameter is **[2005]**

(a) $\dfrac{q}{1+\dfrac{Z_2}{Z_1}}$ (b) $\dfrac{q}{1+\dfrac{Z_1}{Z_2}}$

(c) $q\dfrac{Z_2}{Z_1}$ (d) $q\dfrac{Z_1}{Z_2}$

26. In a potentiometer experiment the balancing with a cell is at length 240 cm. On shunting the cell with a resistance of 2Ω, the balancing length becomes 120 cm. The internal resistance of the cell is **[2005]**

(a) 0.5Ω (b) 1Ω
(c) 2Ω (d) 4Ω

27. The resistance of hot tungsten filament is about 10 times the cold resistance. What will be the resistance of 100 W and 200 V lamp when not in use ? **[2005]**

(a) 20 Ω (b) 40 Ω
(c) 200Ω (d) 400Ω

28. An energy source will supply a constant current into the load if its internal resistance is **[2005]**

(a) very large as compared to the load resistance
(b) equal to the resistance of the load
(c) non-zero but less than the resistance of the load
(d) zero

29. The Kirchhoff's first law $(\Sigma i = 0)$ and second law $(\Sigma iR = \Sigma E)$, where the symbols have their usual meanings, are respectively based on **[2006]**

(a) conservation of charge, conservation of momentum
(b) conservation of energy, conservation of charge
(c) conservation of momentum, conservation of charge
(d) conservation of charge, conservatrion of energy

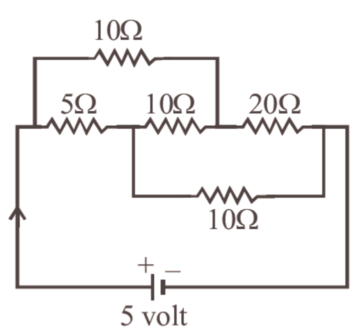
30. A material 'B' has twice the specific resistance of 'A'. A circular wire made of 'B' has twice the diameter of a wire made of 'A'. then for the two wires to have the same resistance, the ratio l_B/l_A of their respective lengths must be **[2006]**

(a) 1 (b) $\dfrac{1}{2}$
(c) $\dfrac{1}{4}$ (d) 2

31. A thermocouple is made from two metals, Antimony and Bismuth. If one junction of the couple is kept hot and the other is kept cold, then, an electric current will **[2006]**

(a) flow from Antimony to Bismuth at the hot junction
(b) flow from Bismuth to Antimony at the cold junction
(c) now flow through the thermocouple
(d) flow from Antimony to Bismuth at the cold junction

32. The current I drawn from the 5 volt source will be **[2006]**



(a) 0.33 A (b) 0.5 A
(c) 0.67 A (d) 0.17 A

33. The resistance of a bulb filmanet is 100Ω at a temperature of 100°C. If its temperature coefficient of resistance be 0.005 per °C, its resistance will become 200 Ω at a temperature of **[2006]**

(a) 300°C (b) 400°C
(c) 500°C (d) 200°C

34. In a Wheatstone's bridge, three resistances P, Q and R connected in the three arms and the fourth arm is formed by two resistances S_1 and S_2 connected in parallel. The condition for the bridge to be balanced will be **[2006]**

(a) $\frac{P}{Q}=\frac{2R}{S_1+S_2}$ (b) $\frac{P}{Q}=\frac{R(S_1+S_2)}{S_1S_2}$

(c) $\frac{P}{Q}=\frac{R(S_1+S_2)}{2S_1S_2}$ (d) $\frac{P}{Q}=\frac{R}{S_1+S_2}$

35. An electric bulb is rated 220 volt - 100 watt. The power consumed by it when operated on 110 volt will be **[2006]**

(a) 75 watt (b) 40 watt
(c) 25 watt (d) 50 watt

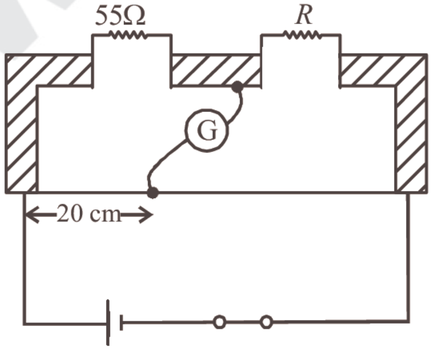
36. A battery is used to charge a parallel plate capacitor till the potential difference between the plates becomes equal to the electromotive force of the battery. The ratio of the energy stored in the capacitor and the work done by the battery will be **[2007]**

(a) 1/2 (b) 1
(c) 2 (d) 1/4

37. The resistance of a wire is 5 ohm at 50°C and 6 ohm at 100°C. The resistance of the wire at 0°C will be **[2007]**

(a) 3 ohm (b) 2 ohm
(c) 1 ohm (d) 4 ohm

38. Shown in the figure below is a meter-bridge set up with null deflection in the galvanometer.



The value of the unknown resistor R is **[2008]**

(a) 13.75 Ω (b) 220 Ω
(c) 110 Ω (d) 55 Ω

DIRECTIONS : Question No. 39 and 40 are based on the following paragraph.

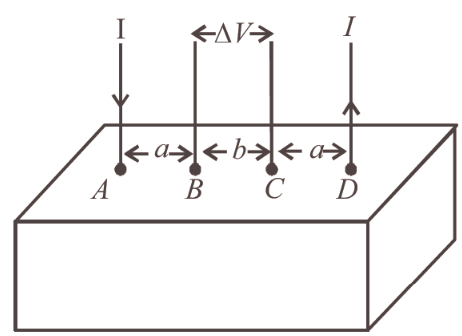
Consider a block of conducting material of resistivity 'ρ' shown in the figure. Current 'I' enters at 'A' and leaves from 'D'. We apply superposition principle to find voltage 'ΔV' developed between 'B' and 'C'. The calculation is done in the following steps:

(i) Take current 'I' entering from 'A' and assume it to spread over a hemispherical surface in the block.

(ii) Calculate field E(r) at distance 'r' from A by using Ohm's law E = ρ j, where j is the current per unit area at 'r'.

(iii) From the 'r' dependence of E(r), obtain the potential V(r) at r.

(iv) Repeat (i), (ii) and (iii) for current 'I' leaving 'D' and superpose results for 'A' and 'D'.



39. ΔV measured between B and C is **[2008]**

(a) $\frac{\rho I}{\pi a}-\frac{\rho I}{\pi(a+b)}$ (b) $\frac{\rho I}{a}-\frac{\rho I}{(a+b)}$

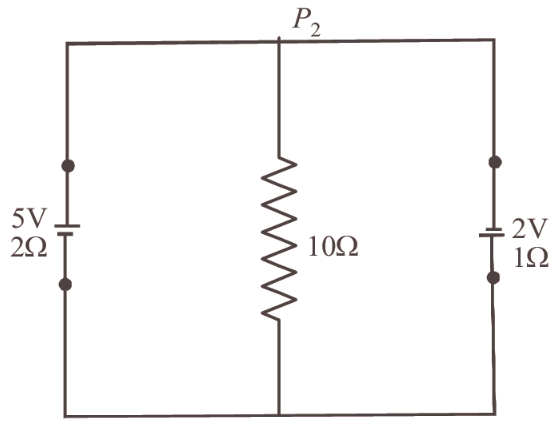
(c) $\frac{\rho I}{2\pi a}-\frac{\rho I}{2\pi(a+b)}$ (d) $\frac{\rho I}{2\pi(a-b)}$

40. For current entering at A, the electric field at a distance 'r' from A is **[2008]**

(a) $\frac{\rho I}{8\pi r^2}$ (b) $\frac{\rho I}{r^2}$

(c) $\frac{\rho I}{2\pi r^2}$ (d) $\frac{\rho I}{4\pi r^2}$

41. A 5V battery with internal resistance 2Ω and a 2V battery with internal resistance 1Ω are connected to a 10Ω resistor as shown in the figure. **[2008]**



The current in the 10Ω resistor is

(a) 0.27 A P_2 to P_1 (b) 0.03 A P_1 to P_2
(c) 0.03 A P_2 to P_1 (d) 0.27 A P_1 to P_2

42. Let C be the capacitance of a capacitor discharging through a resistor R. Suppose t_1 is the time taken for the energy stored in the capacitor to reduce to half its initial value and t_2 is the time taken for the charge to reduce to one-fourth its initial value. Then the ratio t_1/t_2 will be **[2010]**

(a) 1 (b) $\frac{1}{2}$
(c) $\frac{1}{4}$ (d) 2

43. Two conductors have the same resistance at 0°C but their temperature coefficients of resistance are α_1 and α_2. The respective temperature coefficients of their series and parallel combinations are nearly **[2010]**

(a) $\frac{\alpha_1+\alpha_2}{2}, \alpha_1+\alpha_2$

(b) $\alpha_1+\alpha_2, \frac{\alpha_1+\alpha_2}{2}$

(c) $\alpha_1+\alpha_2, \frac{\alpha_1\alpha_2}{\alpha_1+\alpha_2}$

(d) $\frac{\alpha_1+\alpha_2}{2}, \frac{\alpha_1+\alpha_2}{2}$

44. If a wire is stretched to make it 0.1% longer, its resistance will : **[2011]**

(a) increase by 0.2%
(b) decrease by 0.2%
(c) decrease by 0.05%
(d) increase by 0.05%

45. If 400 Ω of resistance is made by adding four 100 Ω resistances of tolerance 5%, then the tolerance of the combination is **[2011 RS]**

(a) 5% (b) 10%
(c) 15% (b) 20%

46. The current in the primary circuit of a potentiometer is 0.2 A. The specific resistance and cross-section of the potentiometer wire are 4×10^{-7} ohm metre and 8×10^{-7} m^2, respectively. The potential gradient will be equal to **[2011 RS]**

(a) 1 V/m (b) 0.5 V/m
(c) 0.1 V/m (d) 0.2 V/m

47. Two electric bulbs rated 25W – 220 V and 100W – 220V are connected in series to a 440 V supply. Which of the bulbs will fuse? **[2012]**

(a) Both (b) 100 W
(c) 25 W (d) Neither

48. The supply voltage to room is 120V. The resistance of the lead wires is 6Ω. A 60 W bulb is already switched on. What is the decrease of voltage across the bulb, when a 240 W heater is switched on in parallel to the bulb? **[2013]**

(a) zero (b) 2.9 Volt
(c) 13.3 Volt (d) 10.04 Volt

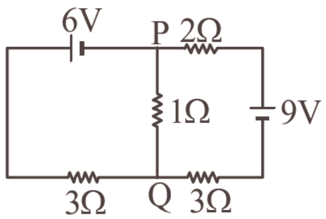
50. In a large building, there are 15 bulbs of 40 W, 5 bulbs of 100 W, 5 fans of 80 W and 1 heater of 1 kW. The voltage of electric mains is 220 V. The minimum capacity of the main fuse of the building will be: **[2014]**

(a) 8 A (b) 10 A
(c) 12 A (d) 14 A

51. When 5V potential difference is applied across a wire of length 0.1 m, the drift speed of electrons is 2.5×10^{-4} ms^{-1}. If the electron density in the wire is 8×10^{28} m^{-3}, the resistivity of the material is close to: **[2015]**

(a) 1.6×10^{-6} Ωm (b) 1.6×10^{-5} Ωm
(c) 1.6×10^{-8} Ωm (d) 1.6×10^{-7} Ωm

52. In the circuit shown, the current in the 1Ω resistor is : **[2015]**



(a) 0.13 A, from Q to P
(b) 0.13 A, from P to Q
(c) 1.3A from P to Q
(d) 0A

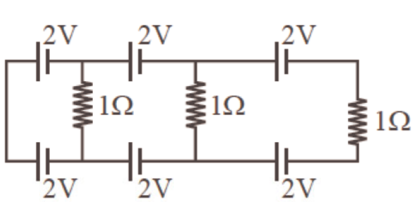
53. The temperature dependence of resistances of Cu and undoped Si in the temperature range 300-400 K, is best described by : **[2016]**

(a) Linear increase for Cu, exponential decrease of Si.
(b) Linear decrease for Cu, linear decrease for Si.
(c) Linear increase for Cu, linear increase for Si.
(d) Linear increase for Cu, exponential increase for Si.

54. Which of the following statements is false ? **[2017]**

(a) A rheostat can be used as a potential divider
(b) Kirchhoff's second law represents energy conservation
(c) Wheatstone bridge is the most sensitive when all the four resistances are of the same order of magnitude
(d) In a balanced wheatstone bridge if the cell and the galvanometer are exchanged, the null point is disturbed.

55.



In the above circuit the current in each resistance is **[2017]**

(a) 0.5A (b) 0 A
(c) 1 A (d) 0.25 A

56. Two batteries with e.m.f. 12 V and 13 V are connected in parallel across a load resistor of 10Ω. The internal resistances of the two batteries are 1Ω and 2Ω respectively. The voltage across the load lies between: **[2018]**

(a) 11.6 V and 11.7 V (b) 11.5 V and 11.6 V
(c) 11.4 V and 11.5 V (d) 11.7 V and 11.8 V

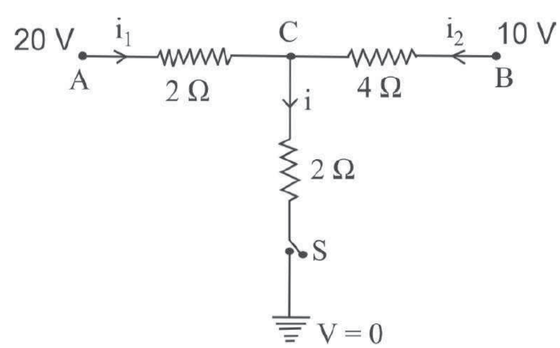
57. In a potentiometer experiment, it is found that no current passes through the galvanometer when the terminals of the cell are connected across 52 cm of the potentiometer wire. If the cell is shunted by a resistance of 5 Ω, a balance is found when the cell is connected across 40 cm of the wire. Find the internal resistance of the cell. **[2018]**

(a) 1 Ω (b) 1.5 Ω
(c) 2 Ω (d) 2.5 Ω

58. On interchanging the resistances, the balance point of a meter bridge shifts to the left by 10 cm. The resistance of their series combination is 1kΩ. How much was the resistance on the left slot before interchanging the resistances?

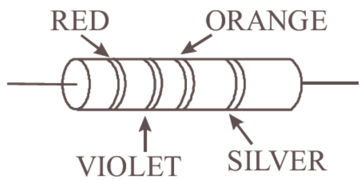
(a) 990 Ω (b) 505 Ω **[2018]**
(c) 550 Ω (d) 910 Ω

59. When the switch S, in the circuit shown, is closed then the valued of current i will be: **[2019]**



(a) 3A (b) 5A (c) 4A (d) 2A

60. A resistance is shown in the figure. Its value and tolerance are given respectively by: **[2019]**



(a) 270 Ω, 10% (b) 27 kΩ, 10%
(c) 27 kΩ, 20% (d) 270 Ω, 5%

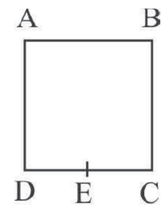
61. A copper wire is stretched to make it 0.5% longer. The percentage change in its electrical resistance if its volume remains unchanged is: **[2019]**

(a) 2.0% (b) 2.5%
(c) 1.0% (d) 0.5%

62. Drift speed of electrons, when 1.5 A of current flows in a copper wire of cross section 5 mm^2, is v. If the electron density in copper is 9×10^{28}/m^3 the value of v in mm/s close to (Take charge of electron to be = 1.6×10^{-19}C) **[2019]**

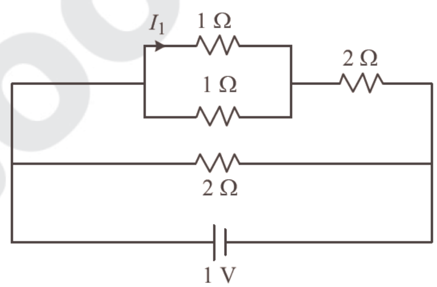
(a) 0.02 (b) 3
(c) 2 (d) 0.2

63. A wire of resistance R is bent to form a square ABCD as shown in the figure. The effective resistance between E and C is: (E is mid-point of arm CD) **[2019]**



(a) R (b) $\frac{7}{64}R$
(c) $\frac{3}{4}R$ (d) $\frac{1}{16}R$

64. The current I_1(in A) flowing through 1 Ω resistor in the following circuit is: **[2020]**



(a) 0.4 (b) 0.5
(c) 0.2 (d) 0.25

Answer Key

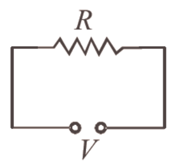
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
(c)	(b)	(b)	(b)	(d)	(a)	(c)	(d)	(b)	(c)	(d)	(a)	(c)	(b)	(c)
16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
(a)	(a)	(d)	(c)	(b)	(b)	(a)	(c)	(a)	(a)	(c)	(b)	(d)	(d)	(d)
31	32	33	34	35	36	37	38	39	40	41	42	43	44	45
(d)	(b)	(b)	(b)	(c)	(a)	(d)	(b)	(a)	(c)	(c)	(c)	(d)	(a)	(a)
46	47	48	49	50	51	52	53	54	55	56	57	58	59	60
(c)	(c)	(d)	(d)	(c)	(b)	(a)	(a)	(d)	(b)	(b)	(b)	(c)	(b)	(b)
61	62	63	64											
(c)	(a)	(b)	(c)											

Solutions

1. **(c)** To use an ammeter in place of voltmeter, we must connect a high resistance in series with the ammeter. Connecting high resistance in series makes its resistance much higher.

2. **(b)** **Case 1** Initial power dissipation,

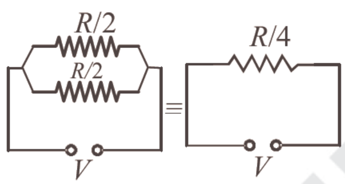
$$P_1 = \frac{V^2}{R}$$



Case 2

When wire is cut into two equal pieces, the resistance of each piece is $\frac{R}{2}$. When they are connected in parallel

Equivalent resistance, $R_{eq} = \frac{R/2}{2} = \frac{R}{4}$



Power dissipated,

$$P_2 = \frac{V^2}{R/4} = 4\left(\frac{V^2}{R}\right) = 4P_1$$

3. **(b)** The equivalent resistance of parallel combination of 2Ω and R is

$$R_{eq} = \frac{2 \times R}{2+R}$$

$\therefore$ Power dissipation $P = \frac{V^2}{R_{eq}}$

$$\therefore 150 = \frac{(15)^2}{R_{eq}}$$

$$\Rightarrow 150 = \frac{225 \times (R+2)}{2R}$$

$$\Rightarrow \frac{2R}{2+R} = \frac{3}{2}$$

$$\Rightarrow 4R = 6 + 3R$$

$$\Rightarrow R = 6\Omega$$

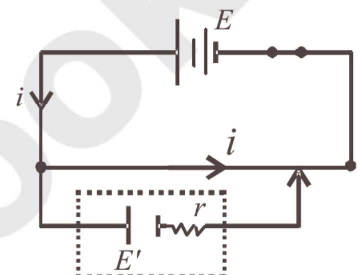
4. **(b)** From the Faraday's first law of electrolysis $m = ZIt \Rightarrow m \propto It$

5. **(d)** From the principle of potentiometer, $V \propto l$

If a cell of emF E is employed in the circuit between the ends of potentiometer wire of length L, then

$$\frac{V}{E} = \frac{l}{L};$$

$$\Rightarrow \text{V} = \frac{\text{E}l}{\text{L}} = \frac{30\text{E}}{100}$$



NOTE In this arrangement, the internal resistance of the battery E does not play any role as current is not passing through the battery.

6. **(a)** Let the smallest temperature difference be θ°C that can be detected by the thermocouple, then

Thermo emf $= (25 \times 10^{-6})\,\theta$

Let I is the smallest current which can be detected by the galvanometer of resistance R.

Potential difference across galvanometer

$IR = 10^{-5} \times 40$

$\therefore\ 10^{-5} \times 40 = 25 \times 10^{-6} \times \theta$

$\Rightarrow \theta = 16$°C.

7. **(c)** According to Faraday's first law of electrolysis

$m = Z \times I \times t$

When I and t is same, $m \propto Z$

$$\therefore\ \frac{m_{\text{Cu}}}{m_{\text{Zn}}} = \frac{Z_{\text{Cu}}}{Z_{\text{Zn}}}$$

$$\Rightarrow m_{\text{Cu}} = \frac{Z_{\text{Cu}}}{Z_{\text{Zn}}} \times m_{\text{Zn}}$$

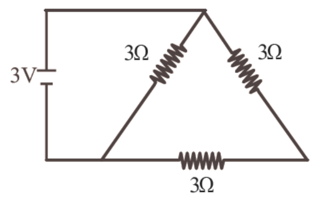
$$\Rightarrow m_{Cu} = \frac{31.5}{32.5} \times 0.13 = 0.126 \text{ g}$$

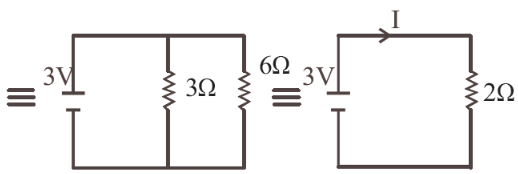
9. **(b)** In the given circuit, resistance of 3Ω is in parallel with series combination of two 3Ω resistance.

$$R_p = \frac{3\times 6}{3+6} = \frac{18}{9} = 2\Omega$$

Using ohm's law $V = IR$

$$\Rightarrow I = \frac{V}{R} = \frac{3}{2} = 1.5A$$





10. **(c)** We know that resistance,

$$R = \frac{V_{\text{rated}}^2}{P_{\text{rated}}} = \frac{(220)^2}{1000} = 48.4\ \Omega$$

When this bulb is connected to 110 volt mains supply we get

$$P = \frac{V^2}{R} = \frac{(110)^2}{48.4} = 250\text{W}$$

11. **(d)** Since volume of wire remains unchanged on increasing length, hence

$A \times \ell == A' \times \ell'$

$\Rightarrow \ell' = 2\ell$

$$\therefore\ A' = \frac{A\times\ell}{\ell'} = \frac{A\times\ell}{2\ell} = \frac{A}{2}$$

Percentage change in resistance

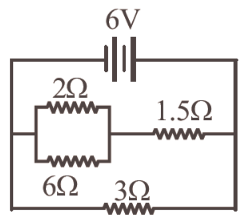
$$= \frac{R_f - R_i}{R_i}\times 100 = \frac{\rho\frac{\ell'}{A'} - \beta\frac{\ell}{A}}{\rho\frac{\ell}{A}}\times 100$$

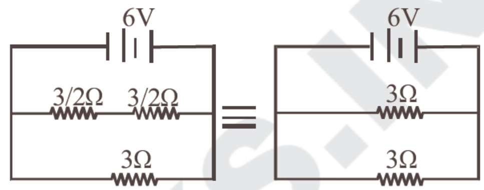
$$= \left[\left(\frac{\ell'}{A'}\times\frac{A}{\ell}\right) - 1\right]\times 100$$

$$= \left[\left(\frac{2\ell}{A/2}\times\frac{A}{\ell}\right) - 1\right]\times 100 = (4-1)\times 100$$

$= 300\%$

12. **(a)**





hence $R_{eq} = 3/2$; $\quad \therefore I = \frac{6}{3/2} = 4\text{A}$

13. **(c)** Let R_1 and R_2 be the two given resistances

Resistance of the series combination,

$S = R_1 + R_2$

Resistance of the parallel combination,

$$P = \frac{R_1 R_2}{R_1 + R_2}$$

As per question $S = nP$

$$\Rightarrow R_1 + R_2 = \frac{n(R_1 R_2)}{(R_1 + R_2)}$$

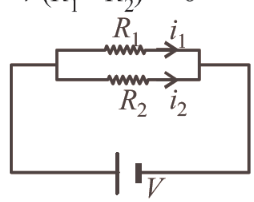
$\Rightarrow (R_1 + R_2)^2 = nR_1R_2$

Minimum value of n is 4 for that

$(R_1 + R_2)^2 = 4R_1R_2$

$\Rightarrow (R_1 - R_2)^2 = 0$

14. **(b)**



Given,

$$\frac{\ell_1}{\ell_2} = \frac{4}{3} \text{ and } \frac{r_1}{r_2} = \frac{2}{3}$$

$$R_1 = \frac{\rho\ell_1}{\pi r_1^2};\ R_2 = \frac{\rho\ell_2}{\pi r_2^2}$$

When wires are in parallel to the circuit potential difference across each wire is same

$i_1R_1 = i_2R_2$

$$\therefore\ \frac{i_1}{i_2} = \frac{R_2}{R_1} = \frac{\rho\ell_2}{\pi r_2^2}\times\frac{\pi r_1^2}{\rho r_1} = \frac{\ell_2}{\ell_1}\times\frac{r_1^2}{r_2^2}$$

$$= \frac{3}{4}\times\frac{4}{9} = \frac{1}{3}$$

15. (c) From the balanced wheat stone bridge

$$\frac{R_1}{R_2}=\frac{\ell_1}{\ell_2}$$

where $\ell_2 = 100-\ell_1$

In the first case $\frac{X}{Y}=\frac{20}{80}$

$Y=4X$

In the second case

$$\frac{4X}{Y}=\frac{\ell}{100-\ell}$$

$$\Rightarrow \frac{4X}{4X}=\frac{\ell}{100-\ell}$$

$\Rightarrow \ell=50$

16. (a) Thermistors are usually made of metaloxides with high temperature coefficient of resistivity.

17. (a) Heat supplied in time t for heating 1L water from 10°C to 40°C

$\Delta Q = mC_p \times \Delta T$

$= 1\times 4180\times(40-10) = 4180\times 30$

But $\Delta Q = P\times t = 836\times t$

$$\Rightarrow t=\frac{4180\times 30}{836}=150\text{s}$$

18. (d) Given $E = a\theta + b\theta^2$

$$\Rightarrow \frac{dE}{d\theta}=a+2b\theta$$

At neutral temperature

$$\theta=\theta_n : \frac{dE}{d\theta}=0$$

$$\Rightarrow \theta_n=\frac{-a}{2b}=-350$$

$$\Rightarrow \frac{d^2E}{d\theta^2}=2b$$

hence no θ is possible for E to be maximum no neutral temperature is possible.

19. (c) From the Faraday's first law of electrolysis,

$m = Zit$

$\Rightarrow m = 3.3\times10^{-7}\times 3\times 2$

$= 19.8\times10^{-7}$ kg

21. (b) Heat generated,

$$H=\frac{V^2t}{R}$$

After cutting equal length of heater coil will become half.

As $R\propto \ell$

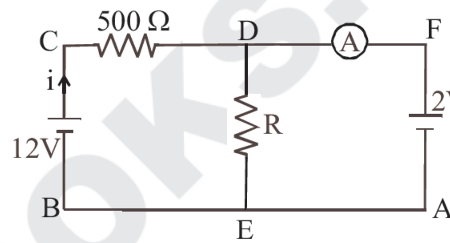
Resistance of half the coil $=\frac{R}{2}$

$$H'=\frac{V^2t}{\frac{R}{2}}$$

$=2H$

$\therefore$ As R reduces to half, 'H' will be doubled.

22. (a)



Applying Kirchhoff's law for loop BCDFAEBC

$12-2=(500\Omega)i$

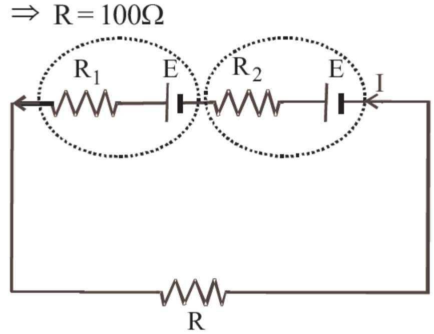
$$\Rightarrow i=\frac{10}{500}=\frac{1}{50}$$

For loop $BCDEB$ $i=\frac{12}{500+\text{R}}=\frac{1}{50}$

$\Rightarrow 500+\text{R}=600$

$\Rightarrow \text{R}=100\Omega$

24. (a)



Let E be the emf of each source of current

Current in the circuit $I=\frac{2E}{R+R_1+R_2}$

Potential difference across cell having internal resistance R_2

$V=E-iR_2=0$

$$E-\frac{2E}{R+R_1+R_2}\cdot R_2=0$$

$\Rightarrow \text{R}+\text{R}_1+\text{R}_2-2\text{R}_2=0$

$\Rightarrow \text{R}+\text{R}_1-\text{R}_2=0$

$\Rightarrow \text{R}=\text{R}_2-\text{R}_1$

25. (a) From Faraday's first law of electrolysis, mass deposited

$m = Zq$

$$\Rightarrow Z \propto \frac{1}{q} \Rightarrow \frac{Z_1}{Z_2} = \frac{q_2}{q_1} \qquad \text{.... (i)}$$

Also $q = q_1 + q_2$ .... (ii)

$$\Rightarrow \frac{q}{q_2} = \frac{q_1}{q_2} + 1$$

(Dividing (ii) by q_2)

$$\Rightarrow q_2 = \frac{q}{1 + \frac{q_1}{q_2}} \qquad \text{.... (iii)}$$

From equation (i) and (iii),

$$q_2 = \frac{q}{1 + \frac{Z_2}{Z_1}}$$

26. (c) Initial balancing length, $\ell_1 = 240$ cm New balancing length, $\ell_2 = 120$ cm.

The internal resistance of the cell,

$$r = \left(\frac{\ell_1 - \ell_2}{\ell_2}\right) \times R = \frac{240 - 120}{120} \times 2 = 2\Omega$$

27. (b) Power, $P = Vi = \frac{V^2}{R}$

$\therefore$ Resistance of tungsten filament when in use

$$R_{\text{hot}} = \frac{V^2}{P} = \frac{200 \times 200}{100} = 400\,\Omega$$

Resistance when not in use *i.e.*, cold resistance

$$R_{\text{cold}} = \frac{400}{10} = 40\,\Omega$$

28. (d) Current is given by

$$I = \frac{E}{R + r},$$

If internal resistance (r) is zero,

$$I = \frac{E}{R} = \text{constant.}$$

Thus, energy source will supply a constant current if its internal resistance is zero.

29. (d) **NOTE** Kirchhoff's first law is based on conservation of charge and Kirchhoff's second law is based on conservation of energy.

30. (d) Let d_A and d_B are the diameter of wire A and B respectively.

Let ρ_B and ρ_A be the resistivity of wire A and B. We have given

$\rho_B = 2\rho_A$

$d_B = 2d_A$

If both resistances are equal

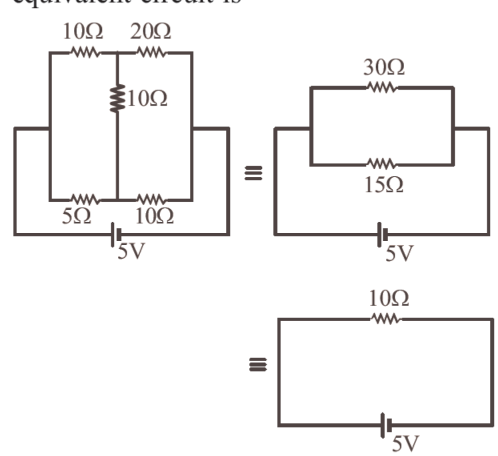
$R_B = R_A$

$$\Rightarrow \frac{\rho_B \ell_B}{A_B} = \frac{\rho_A \ell_A}{A_A}$$

$$\therefore \frac{\ell_B}{\ell_A} = \frac{\rho_A}{\rho_B} \times \frac{d_B^2}{d_A^2} = \frac{\rho_A}{2\rho_A} \times \frac{4d_A^2}{d_A^2} = 2$$

31. (d) At cold junction, current flows from Antimony to Bismuth because current flows from metal occurring later in the series to metal occurring earlier in the thermoelectric series. In thermoelectric series, Bismuth comes earlier than Antimony so at cold junction, current. Flow from Antimony to Bismuth.

32. (b) The network of resistors is a balanced wheatstone bridge. Hence, no current will flow through centre resistor. The equivalent circuit is



$$R_{eq} = \frac{15 \times 30}{15 + 30} = 10\ \Omega$$

$$\Rightarrow \quad I = \frac{V}{R} = \frac{5}{10} = 0.5\ \text{A}$$

33. **(b)** Let resistance of bulb filament be R_0 at 0°C using $R = R_0(1 + \alpha\,\Delta t)$ we have

$R_1 = R_0[1 + \alpha \times 100] = 100$ ....(1)

$R_2 = R_0[1 + \alpha \times T] = 200$ ....(2)

On dividing we get

$$\frac{200}{100} = \frac{1+\alpha T}{1+100\alpha} \Rightarrow 2 = \frac{1+0.005T}{1+100\times 0.005}$$

$\Rightarrow T = 400°C$

NOTE We may use this expression as an approximation because the difference in the answers is appreciable. For accurate results one should use $R = R_0 e^{\alpha\Delta T}$

34. **(b)** From balanced wheat stone bridge $\frac{P}{Q} = \frac{R}{S}$

where $S = \frac{S_1 S_2}{S_1 + S_2}$

35. **(c)** The resistance of the electric bulb is

$$R = \frac{V^2}{P} = \frac{(220)^2}{100}$$

The power consumed when operated at 110 V is

$$P' = \frac{V^2}{R}$$

$$\Rightarrow P = \frac{(110)^2}{(220)^2/100} = \frac{100}{4} = 25\text{ W}$$

36. **(a)** Energy in capacitor $= \frac{1}{2}CV^2$

Work done by battery $= QV$

$= CV^2$

where C = Capacitance of capacitor

V = Potential difference,

e = emf of battery

$$\text{Required ratio} = \frac{\frac{1}{2}CV^2}{CV^2} = \frac{1}{2} \quad (\because V = e)$$

37. **(d)** Resistance of a metal conductor at temperature t°C is given by

$R_t = R_0(1 + \alpha t)$,

R_0 is the resistance of the wire at 0ºC and α is the temperature coefficient of resistance.

Resistance at 50°C, $R_{50} = R_0(1 + 50\alpha)$ ... (i)

Resistance at 100°C, $R_{100} = R_0(1 + 100\alpha)$ ... (ii)

From (i), $R_{50} - R_0 = 50\alpha R_0$ ... (iii)

From (ii), $R_{100} - R_0 = 100\alpha R_0$ ... (iv)

Dividing (iii) by (iv), we get

$$\frac{R_{50} - R_0}{R_{100} - R_0} = \frac{1}{2}$$

Here, $R_{50} = 5\Omega$ and $R_{100} = 6\Omega$

$$\therefore \frac{5 - R_0}{6 - R_0} = \frac{1}{2}$$

or, $6 - R_0 = 10 - 2R_0$ or, $R_0 = 4\Omega$.

38. **(b)** Given,

Balance point from one end, $\ell_1 = 20$ cm

From the condition for balance of metre bridge, we have

$$\frac{55}{R} = \frac{\ell_1}{100 - \ell_1}$$

$$\frac{55}{R} = \frac{20}{80}$$

$\Rightarrow R = 220\Omega$

39. **(a)** Let j be the current density.

Then $j \times 2\pi r^2 = I \Rightarrow j = \frac{I}{2\pi r^2}$

$$\therefore E = \rho j = \frac{\rho I}{2\pi r^2}$$

Now, $V_B - V_C$

$$= -\int_{a+b}^{a} \vec{E}\cdot\vec{dr} = -\int_{a+b}^{a} \frac{\rho I}{2\pi r^2}dr$$

$$= -\frac{\rho I}{2\pi}\left[-\frac{1}{r}\right]_{a+b}^{a} = \frac{\rho I}{2\pi a} - \frac{\rho I}{2\pi(a+b)}$$

On applying superposition as mentioned we get

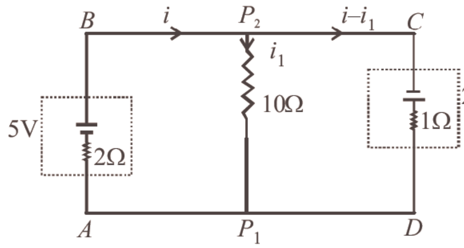
$$\Delta V_{BC} = 2 \times \Delta V'_{BC} = \frac{\rho I}{\pi a} - \frac{\rho I}{\pi(a+b)}$$

40. **(c)** As shown in Answer (a) $E = \frac{\rho I}{2\pi r^2}$

41. **(c)** Applying Kirchoff's second law in ABP_2P_1A, we get

$-2i + 5 - 10\,i_1 = 0$

$2i + 10i_1 = 5$ .....(i)



Again applying Kirchoff's second law in $P_2CDP_1P_2$ we get,

$10\,i_1 + 2 - i + i_1 = 0$

$2_i - 22i_1 = 4$ ....(ii)

From (i) and (ii)

$32i_1 = 1$

$\Rightarrow i_1 = \frac{1}{32}$ A from P_2 to P_1

42. (c) Initial energy of capacitor, $E_1 = \frac{q_1^2}{2C}$

Final energy of capacitor,

$$E_2 = \frac{1}{2}E_1 = \frac{q_1^2}{4C} = \frac{\left(\frac{q_1}{\sqrt{2}}\right)^2}{2C}$$

$\therefore t_1 =$ time for the charge to reduce to $\frac{1}{\sqrt{2}}$ of its initial value

and $t_2 =$ time for the charge to reduce to $\frac{1}{4}$ of its initial value

The charge as a function of time is

$q_2 = q_1 e^{-t/CR}$

$$\Rightarrow \ln\left(\frac{q_2}{q_1}\right) = -\frac{t}{CR}$$

$$\therefore \ln\left(\frac{1}{\sqrt{2}}\right) = \frac{-t_1}{CR} \quad ...(1)$$

$$\text{and } \ln\left(\frac{1}{4}\right) = \frac{-t_2}{CR} \quad ...(2)$$

By (1) and (2),

$$\frac{t_1}{t_2} = \frac{\ln\left(\frac{1}{\sqrt{2}}\right)}{\ln\left(\frac{1}{4}\right)} = \frac{1}{2}\frac{\ln\left(\frac{1}{2}\right)}{2\ln\left(\frac{1}{2}\right)} = \frac{1}{4}$$

43. (d) Let R_1 and R_2 be the resistances of two conductors, then

$R_1 = R_0[1 + \alpha_1 \Delta t]$

$R_2 = R_0[1 + \alpha_2 \Delta t]$

Here, R_0 is the resistance of conductor at 0°C

In Series, $R = R_1 + R_2$

$= R_0[2 + (\alpha_1 + \alpha_2)\Delta t]$

$$= 2R_0\left[1 + \left(\frac{\alpha_1 + \alpha_2}{2}\right)\Delta t\right]$$

$$\therefore \alpha_{eq} = \frac{\alpha_1 + \alpha_2}{2}$$

In Parallel, $\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$

$$= \frac{1}{R_0[1 + \alpha_1 \Delta t]} + \frac{1}{R_0[1 + \alpha_2 \Delta t]}$$

$$\Rightarrow \frac{1}{\frac{R_0}{2}(1 + \alpha_{eq}\Delta t)}$$

$$= \frac{1}{R_0(1 + \alpha_1 \Delta t)} + \frac{1}{R_0(1 + \alpha_2 \Delta t)}$$

$2(1 - \alpha_{eq}\Delta t) = (1 - \alpha_1 \Delta t)(1 - \alpha_2 \Delta t)$

$$\therefore \alpha_{eq} = \frac{\alpha_1 + \alpha_2}{2}$$

44. (a) Resistance of wire

$$R = \frac{\rho l}{A} = \frac{\rho l^2}{V} \quad (\because V = A\ell)$$

Hence, $R = \rho\frac{\ell^2}{V}$ = constant × ℓ^2

$\therefore$ Fractional change in resistance

$$\frac{\Delta R}{R} = 2\frac{\Delta l}{l}$$

$$100 \times \frac{\Delta R}{R} = 200 \times \left(\frac{d\ell}{\ell}\right)$$

$\because d\ell/\ell = 0.1\%$

$$\therefore \%\text{ change in } R = \left[200 \times \left\{\frac{0.1}{100}\right\}\right] = 0.2\%$$

$\therefore$ Resistance will increase by 0.2%.

45. (a)

Tolerance for one resistance,

R = 100 ± 5

$\Rightarrow$ 4R = 400 ± 20

Thus, tolerance of combination is also 5%.

46. **(c)** Potential gradient

$$\Rightarrow k = \frac{V}{\ell} = \frac{IR}{\ell} = \frac{I}{\ell}\left(\frac{\rho\ell}{A}\right) = \frac{I\rho}{A}$$

$$k = \frac{0.2\times 4\times 10^{-7}}{8\times 10^{-7}} = \frac{0.8}{8} = 0.1 \text{ V/m}$$

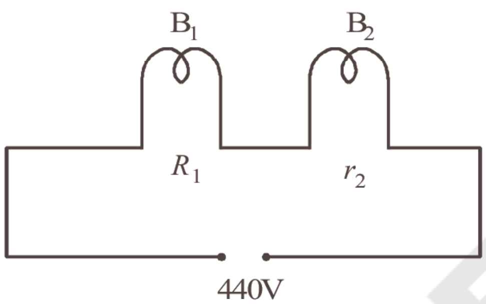
47. **(c)** Current capacity of 25 W bulb

$$I_1 = \frac{W_1}{V_1} = \frac{25}{220} \text{ Amp}$$

Current capacity of 100 W bulb

$$I_2 = \frac{W_2}{V_2} = \frac{100}{220} \text{ Amp}$$

The current flowing through the circuit



Resistance of 25 W bulb,

$$R_1 = \frac{V_1^2}{P_1} = \frac{(220)^2}{25};$$

Resistance of 100 W bulb

$$R_2 = \frac{V_2^2}{P} = \frac{(220)^2}{100}$$

$R_{eff} = R_1 + R_2$

Current flowing through circuit

$$I = \frac{440}{R_{eff}}$$

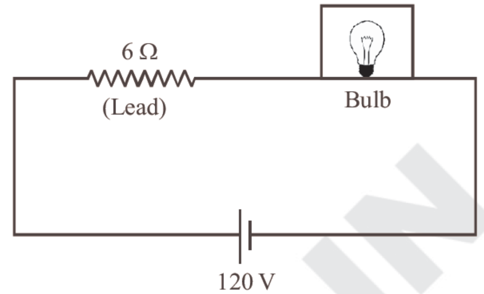
$$I = \frac{440}{\frac{(220)^2}{25} + \frac{(220)^2}{100}}$$

$$= \frac{440}{(220)^2\left[\frac{1}{25}+\frac{1}{100}\right]}; \quad I = \frac{40}{220} \text{ Amp}$$

$$\because I_1\left(=\frac{25}{220}A\right) < I\left(=\frac{40}{220}A\right) < I_2\left(=\frac{100}{200}A\right)$$

Thus the bulb rated 25 W–220 will fuse.

48. **(d)**



Power of bulb = 60 W (given)

$$\text{Resistance of bulb} = \frac{120\times 120}{60} = 240\Omega$$

$$\left[\because P = \frac{V^2}{R}\right]$$

Power of heater = 240W (given)

$$\text{Resistance of heater} = \frac{120\times 120}{240} = 60\Omega$$

Voltage across bulb before heater is switched on,

$$V_1 = \frac{240}{246}\times 120 = 117.73 \text{ volt}$$

Voltage across bulb after heater is switched on,

$$V_2 = \frac{48}{54}\times 120 = 106.66 \text{ volt}$$

Hence decrease in voltage

$V_1 - V_2 = 117.073 - 106.66 = 10.04$ Volt (approximately)

50. **(c)** Total power consumed by electrical appliances in the building, $P_{total} = 2500$W

Watt = Volt × ampere

$\Rightarrow$ $2500 = V\times I \Rightarrow 2500 = 220\,I$

$$\Rightarrow I = \frac{2500}{220} = 11.36 \approx 12\text{A}$$

(Minimum capacity of main fuse)

51. **(b)** $V = IR = (neAv_d)\rho\frac{\ell}{A}$

$$\therefore \rho = \frac{V}{v_d\, lne}$$

Here V = potential difference

l = length of wire

n = no. of electrons per unit volume of conductor.

e = no. of electrons

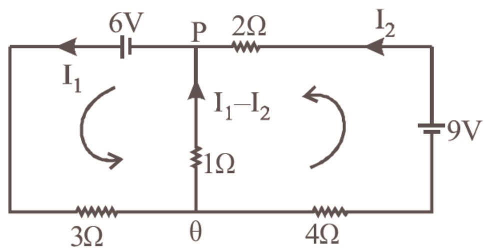
Placing the value of above parameters we get resistivity

$$\rho = \frac{5}{8\times10^{28}\times1.6\times10^{-19}\times2.5\times10^{-4}\times0.1}$$

$= 1.6 \times 10^{-5}\Omega m$

52. **(a)** From KVL

$-6 + 3I_1 + 1(I_i - I_2) = 0$



$6 = 3I_1 + I_1 - I_2$; $4I_1 - I_2 = 6$ ...(1)

$-9 + 2I_2 - (I_1 - I_2) + 3I_2 = 0$

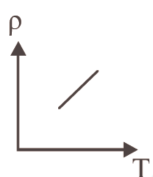
$-I_1 + 6I_2 = 9$ ...(2)

On solving (1) and (2)

$I_1 = 0.13A$

Direction Q to P, since $I_1 > I_2$.

53. **(a)**

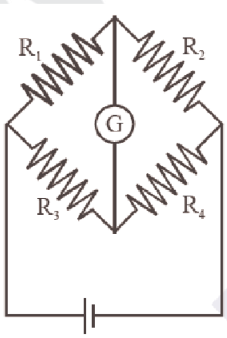


Metal (for limited range of temperature)

Semiconductor

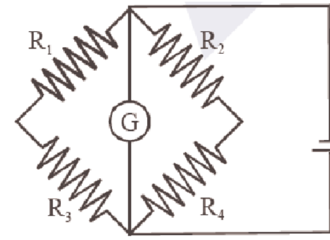
$\rho = \rho_o e^{\frac{-Eg}{K_B T}}$

54. **(d)** There is no change in null point, if the cell and the galvanometer are exchanged in a balanced wheatstone bridge.



On balancing condition $\frac{R_1}{R_3} = \frac{R_2}{R_4}$

After exchange

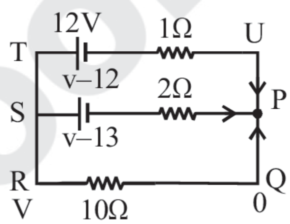


On balancing condition $\frac{R_1}{R_2} = \frac{R_3}{R}$

55. **(b)** The potential difference in each loop is zero.

$\therefore$ No current will flow or current in each resistance is Zero.

56. **(b)**



Using Kirchhoff's law at *P* we get

$$\frac{V-12}{1}+\frac{V-13}{2}+\frac{V-0}{10}=0$$

[Let potential at P, Q, U = 0 and at R = V

$$\Rightarrow \frac{V}{1}+\frac{V}{2}+\frac{V}{10}=\frac{12}{1}+\frac{13}{2}+\frac{0}{10}$$

$$\Rightarrow \frac{10+5+1}{10}V=\frac{24+13}{2}$$

$$\Rightarrow V\left(\frac{16}{10}\right)=\frac{37}{2}$$

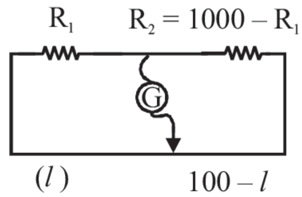
$$\Rightarrow V=\frac{37\times10}{16\times2}=\frac{370}{32}=11.56 \text{ volt}$$

57. **(b)** Using formula, internal resistance,

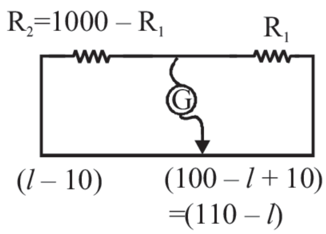
$$r=\left(\frac{l_1-l_2}{l_2}\right)s$$

$$=\left(\frac{52-40}{40}\right)\times5=1.5\Omega$$

58. (c) $R_1 + R_2 = 1000$
$\Rightarrow R_2 = 1000 - R_1$



On balancing condition
$R_1(100 - l) = (1000 - R_1)l$ ...(i)
On Interchanging resistance balance point shifts left by 10 cm



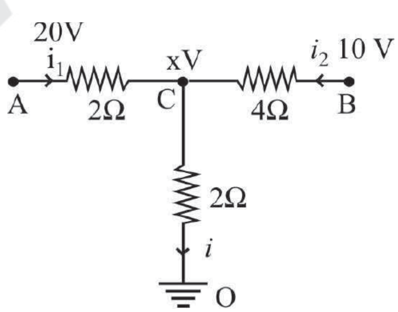
On balancing condition
$(1000 - R_1)(110 - l) = R_1(l - 10)$
or, $R_1(l - 10) = (1000 - R_1)(110 - l)$ ...(ii)

Dividing eqn (i) by (ii)

$$\frac{100-l}{l-10} = \frac{l}{110-l}$$

$\Rightarrow (100 - l)(110 - l) = l(l - 10)$
$\Rightarrow 11000 - 100l - 110l + l^2 = l^2 - 10l$
$\Rightarrow 11000 = 200l$
or, $l = 55$
Putting the value of 'l' in eqn (i)
$R_1(100 - 55) = (1000 - R_1)\,55$
$\Rightarrow R_1(45) = (1000 - R_1)\,55$
$\Rightarrow R_1(9) = (1000 - R_1)\,11$
$\Rightarrow 20R_1 = 11000$
$\therefore R_1 = 550\text{K}\Omega$

59. (b)



Let voltage at C = x volt

From kirchhoff's current law,

KCL : $i_1 + i_2 = i$

$$\frac{20-x}{2} + \frac{10-x}{4} = \frac{x-0}{2} \Rightarrow x = 10$$

$$\therefore i = \frac{V}{R} = \frac{x}{R} = \frac{10}{2}\ 5\text{A}$$

60. (b) Using colour code we have
$R = 27 \times 10^3\ \Omega \pm 10\%$
$= 27\ \text{k}\Omega \pm 10\%$

61. (c) Resistance, $R = \frac{\rho\ell}{A}$

$$R = \rho\frac{\ell}{A} \times \frac{\ell}{\ell} = \frac{\rho\ell^2}{V} \quad [\because \text{Volume (V)} = A\ell]$$

Since resistivity and volume remains constant therefore % change in resistance

$$\frac{\Delta R}{R} = \frac{2\Delta\ell}{\ell} = 2 \times (0.5) = 1\%$$

62. (a) Using, $I = neAv_d$

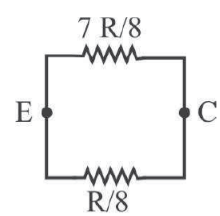
$\therefore$ Drift speed $v_d = \frac{I}{neA}$

$= 0.02 \times 10^{-3}\ \text{ms}^{-1}$

$= 0.02\ \text{mms}^{-1}$

63. (b) Here $R_{DA} = R_{AB} = R_{BC} = R/4$
and $R_{DE} = R_{EC} = R/8$
Now $R_{ED}, R_{DA}, R_{AB}, R_{BC}$ are in series.

$$\therefore R_s = \frac{R}{8} + \frac{R}{4} + \frac{R}{4} + \frac{R}{4} = \frac{R + 2R + 2R + 2R}{8} = \frac{7R}{8}$$



$$\therefore R_{eq} = \frac{\left(\frac{7R}{8}\right)\left(\frac{R}{8}\right)}{R} = \frac{7R}{64}$$

64. (c)

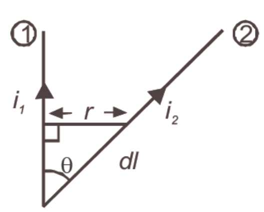
18 Moving Charges and Magnetism

1. If a current is passed through a spring then the spring will **[2002]**
(a) expand (b) compress
(c) remains same (d) none of these

2. If in a circular coil A of radius R, current I is flowing and in another coil B of radius $2R$ a current $2I$ is flowing, then the ratio of the magnetic fields B_A and B_B, produced by them will be **[2002]**
(a) 1 (b) 2
(c) 1/2 (d) 4

3. If an electron and a proton having same momenta enter perpendicular to a magnetic field, then **[2002]**
(a) curved path of electron and proton will be same (ignoring the sense of revolution)
(b) they will move undeflected
(c) curved path of electron is more curved than that of the proton
(d) path of proton is more curved.

4. Wires 1 and 2 carrying currents i_1 and i_2 respectively are inclined at an angle θ to each other. What is the force on a small element dl of wire 2 at a distance of r from wire 1 (as shown in figure) due to the magnetic field of wire 1? **[2002]**



(a) $\frac{\mu_0}{2\pi r} i_1 i_2\, dl \tan\theta$ (b) $\frac{\mu_0}{2\pi r} i_1 i_2\, dl \sin\theta$
(c) $\frac{\mu_0}{2\pi r} i_1 i_2\, dl \cos\theta$ (d) $\frac{\mu_0}{4\pi r} i_1 i_2\, dl \sin\theta$

5. The time period of a charged particle undergoing a circular motion in a uniform magnetic field is independent of its **[2002]**
(a) speed (b) mass
(c) charge (d) magnetic induction

6. A particle of mass M and charge Q moving with velocity $\vec{v}$ describe a circular path of radius R when subjected to a uniform transverse magnetic field of induction B. The work done by the field when the particle completes one full circle is **[2003]**
(a) $\left(\frac{Mv^2}{R}\right) 2\pi R$ (b) zero
(c) $BQ2\pi R$ (d) $BQv2\pi R$

7. An ammeter reads upto 1 ampere. Its internal resistance is 0.81ohm. To increase the range to 10 A the value of the required shunt is **[2003]**
(a) 0.03Ω (b) 0.3Ω
(c) 0.9Ω (d) 0.09Ω

8. A particle of charge -16×10^{-18} coulomb moving with velocity $10 ms^{-1}$ along the x-axis enters a region where a magnetic field of induction B is along the y-axis, and an electric field of magnitude 10^4 V/m is along the negative z-axis. If the charged particle continues moving along the x-axis, the magnitude of B is **[2003]**
(a) $10^3 Wb/m^2$ (b) $10^5 Wb/m^2$
(c) $10^{16} Wb/m^2$ (d) $10^{-3} Wb/m^2$

9. A current i ampere flows along an infinitely long straight thin walled tube, then the magnetic induction at any point inside the tube is **[2004]**
(a) $\frac{\mu_0}{4\pi}\cdot\frac{2i}{r}$ tesla (b) zero
(c) infinite (d) $\frac{2i}{r}$ tesla

10. A long wire carries a steady current. It is bent into a circle of one turn and the magnetic field at the centre of the coil is B. It is then bent into a circular loop of n turns. The magnetic field at the centre of the coil will be **[2004]**

(a) $2n\,B$ (b) $n^2\,B$
(c) nB (d) $2\,n^2\,B$

11. The magnetic field due to a current carrying circular loop of radius 3 cm at a point on the axis at a distance of 4 cm from the centre is 54 μT. What will be its value at the centre of loop? **[2004]**

(a) 125 μT (b) 150 μT
(c) 250 μT (d) 75 μT

12. Two long conductors, separated by a distance d carry current I_1 and I_2 in the same direction. They exert a force F on each other. Now the current in one of them is increased to two times and its direction is reversed. The distance is also increased to $3d$. The new value of the force between them is **[2004]**

(a) $-\frac{2F}{3}$ (b) $\frac{F}{3}$
(c) $-2\,F$ (d) $-\frac{F}{3}$

13. Two concentric coils each of radius equal to 2π cm are placed at right angles to each other. 3 ampere and 4 ampere are the currents flowing in each coil respectively. The magnetic induction in Weber/m^2 at the centre of the coils will be ($\mu_0 = 4\pi \times 10^{-7}$ Wb/A.m) **[2005]**

(a) 10^{-5} (b) 12×10^{-5}
(c) 7×10^{-5} (d) 5×10^{-5}

14. A charged particle of mass m and charge q travels on a circular path of radius r that is perpendicular to a magnetic field B. The time taken by the particle to complete one revolution is **[2005]**

(a) $\frac{2\pi q^2 B}{m}$ (b) $\frac{2\pi mq}{B}$
(c) $\frac{2\pi m}{qB}$ (d) $\frac{2\pi qB}{m}$

15. Two thin, long, parallel wires, separated by a distance 'd' carry a current of 'i' A in the same direction. They will **[2005]**

(a) repel each other with a force of $\mu_0 i^2/(2\pi d)$
(b) attract each other with a force of $\mu_0 i^2/(2\pi d)$
(c) repel each other with a force of $\mu_0 i^2/(2\pi d^2)$
(d) attract each other with a force of $\mu_0 i^2/(2\pi d^2)$

16. A uniform electric field and a uniform magnetic field are acting along the same direction in a certain region. If an electron is projected along the direction of the fields with a certain velocity then **[2005]**

(a) its velocity will increase
(b) Its velocity will decrease
(c) it will turn towards left of direction of motion
(d) it will turn towards right of direction of motion

17. A moving coil galvanometer has 150 equal divisions. Its current sensitivity is 10-divisions per milliampere and voltage sensitivity is 2 divisions per millivolt. In order that each division reads 1 volt, the resistance in ohms needed to be connected in series with the coil will be -**[2005]**

(a) 10^5 (b) 10^3
(c) 9995 (d) 99995

18. In a region, steady and uniform electric and magnetic fields are present. These two fields are parallel to each other. A charged particle is released from rest in this region. The path of the particle will be a **[2006]**

(a) helix (b) straight line
(c) ellipse (d) circle

19. A long solenoid has 200 turns per cm and carries a current i. The magnetic field at its centre is 6.28×10^{-2} Weber/m^2. Another long solenoid has 100 turns per cm and it carries a current $\frac{i}{3}$. The value of the magnetic field at its centre is **[2006]**

(a) 1.05×10^{-2} Weber/m^2
(b) 1.05×10^{-5} Weber/m^2
(c) 1.05×10^{-3} Weber/m^2
(d) 1.05×10^{-4} Weber/m^2

20. A long straight wire of radius a carries a steady current i. The current is uniformly distributed across its cross section. The ratio of the magnetic field at $a/2$ and $2a$ is **[2007]**

(a) 1/2 (b) 1/4
(c) 4 (d) 1

21. A current I flows along the length of an infinitely long, straight, thin walled pipe. Then **[2007]**

(a) the magnetic field at all points inside the pipe is the same, but not zero
(b) the magnetic field is zero only on the axis of the pipe
(c) the magnetic field is different at different points inside the pipe
(d) the magnetic field at any point inside the pipe is zero

22. A charged particle with charge q enters a region of constant, uniform and mutually orthogonal fields $\vec{E}$ and $\vec{B}$ with a velocity $\vec{v}$ perpendicular to both $\vec{E}$ and $\vec{B}$, and comes out without any change in magnitude or direction of $\vec{v}$. Then **[2007]**

(a) $\vec{v}=\vec{B}\times\vec{E}/E^2$ (b) $\vec{v}=\vec{E}\times\vec{B}/B^2$
(c) $\vec{v}=\vec{B}\times\vec{E}/B^2$ (d) $\vec{v}=\vec{E}\times\vec{B}/E^2$

23. A charged particle moves through a magnetic field perpendicular to its direction. Then **[2007]**

(a) kinetic energy changes but the momentum is constant
(b) the momentum changes but the kinetic energy is constant
(c) both momentum and kinetic energy of the particle are not constant
(d) both momentum and kinetic energy of the particle are constant

24. Two identical conducting wires AOB and COD are placed at right angles to each other. The wire AOB carries an electric current I_1 and COD carries a current I_2. The magnetic field on a point lying at a distance d from O, in a direction perpendicular to the plane of the wires AOB and COD, will be given by **[2007]**

(a) $\dfrac{\mu_0}{2\pi d}(I_1^2+I_2^2)$

(b) $\dfrac{\mu_0}{2\pi}\left(\dfrac{I_1+I_2}{d}\right)^{\frac{1}{2}}$

(c) $\dfrac{\mu_0}{2\pi d}\left(I_1^2+I_2^2\right)^{\frac{1}{2}}$

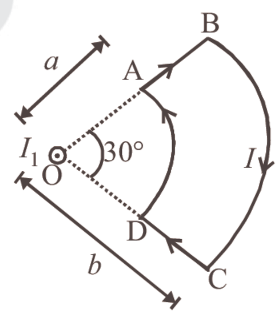
(d) $\dfrac{\mu_0}{2\pi d}\left(I_1+I_2\right)$

25. A horizontal overhead powerline is at height of 4m from the ground and carries a current of 100A from east to west. The magnetic field directly below it on the ground is ($\mu_0 = 4\pi \times 10^{-7}$ Tm A^{-1}) **[2008]**

(a) 2.5×10^{-7} T southward
(b) 5×10^{-6} T northward
(c) 5×10^{-6} T southward
(d) 2.5×10^{-7} T northward

Directions : *Question numbers 26 and 27 are based on the following paragraph.*

A current loop $ABCD$ is held fixed on the plane of the paper as shown in the figure. The arcs BC (radius $= b$) and DA (radius $= a$) of the loop are joined by two straight wires AB and CD. A steady current I is flowing in the loop. Angle made by AB and CD at the origin O is 30°. Another straight thin wire with steady current I_1 flowing out of the plane of the paper is kept at the origin. **[2009]**



26. The magnitude of the magnetic field (B) due to the loop $ABCD$ at the origin (O) is :

(a) $\dfrac{\mu_o I(b-a)}{24ab}$

(b) $\dfrac{\mu_o I}{4\pi}\left[\dfrac{b-a}{ab}\right]$

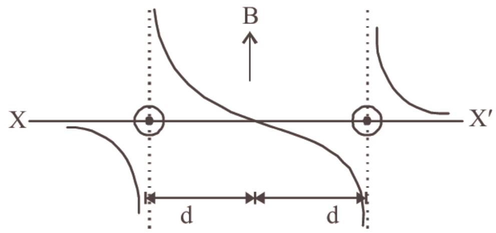
(c) $\dfrac{\mu_o I}{4\pi}[2(b-a)+\pi/3(a+b)]$

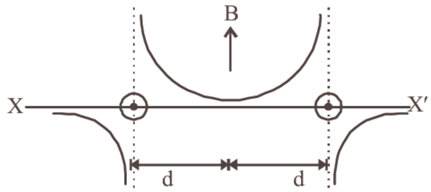
(d) zero

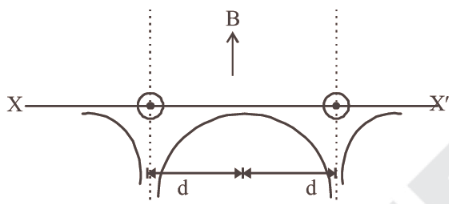
27. Due to the presence of the current I_1 at the origin:

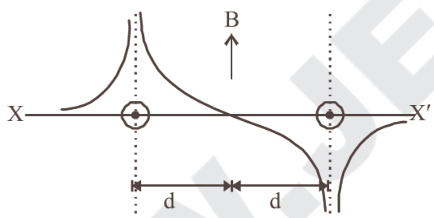
(a) The forces on AD and BC are zero.
(b) The magnitude of the net force on the loop is given by $\dfrac{I_1 I}{4\pi}\mu_o[2(b-a)+\pi/3(a+b]$.
(c) The magnitude of the net force on the loop is given by $\dfrac{\mu_o I I_1}{24ab}(b-a)$.
(d) The forces on AB and DC are zero.

28. Two long parallel wires are at a distance $2d$ apart. They carry steady equal currents flowing out of the plane of the paper as shown. The variation of the magnetic field B along the line XX′ is given by **[2010]**

(a) 

(b) 

(c) 

(d) 

29. A current I flows in an infinitely long wire with cross section in the form of a semi-circular ring of radius R. The magnitude of the magnetic induction along its axis is: **[2011]**

(a) $\frac{\mu_0 I}{2\pi^2 R}$ (b) $\frac{\mu_0 I}{2\pi R}$

(c) $\frac{\mu_0 I}{4\pi R}$ (d) $\frac{\mu_0 I}{\pi^2 R}$

30. An electric charge $+q$ moves with velocity $\vec{v} = 3\hat{i} + 4\hat{j} + \hat{k}$ in an electromagnetic field given by $\vec{E} = 3\hat{i} + \hat{j} + 2\hat{k}$ and $\vec{B} = \hat{i} + \hat{j} - 3\hat{k}$ The y-component of the force experienced by $+q$ is : **[2011 RS]**

(a) 11 q (b) 5 q

(c) 3 q (d) 2 q

31. A thin circular disc of radius R is uniformly charged with density $\sigma > 0$ per unit area. The disc rotates about its axis with a uniform angular speed ω. The magnetic moment of the disc is **[2011 RS]**

(a) $\pi R^4 \sigma\omega$ (b) $\frac{\pi R^4}{2}\sigma\omega$

(c) $\frac{\pi R^4}{4}\sigma\omega$ (d) $2\pi R^4 \sigma\omega$

32. Proton, deuteron and alpha particle of same kinetic energy are moving in circular trajectories in a constant magnetic field. The radii of proton, deuteron and alpha particle are respectively r_p, r_d and r_α. Which one of the following relation is correct? **[2012]**

(a) $r_\alpha = r_p = r_d$ (b) $r_\alpha = r_p < r_d$

(c) $r_\alpha > r_d > r_p$ (d) $r_\alpha = r_d > r_p$

33. This questions has Statement I and Statement II. Of the four choices given after the Statements, choose the one that best describes into two Statements.

Statement-I : Higher the range, greater is the resistance of ammeter.

Statement-II : To increase the range of ammeter, additional shunt needs to be used across it. **[2013]**

(a) Statement-I is true, Statement-II is true, Statement-II is the correct explanation of Statement-I.

(b) Statement-I is true, Statement-II is true, Statement-II is not the correct explanation of Statement-I.

(c) Statement-I is true, Statement-II is false.

(d) Statement-I is false, Statement-II is true.

34. A circular loop of radius 0.3 cm lies parallel to a much bigger circular loop of radius 20 cm. The centre of the small loop is on the axis of the bigger loop. The distance between their centres is 15 cm. If a current of 2.0 A flows through the smaller loop, then the flux linked with bigger loop is **[2013]**

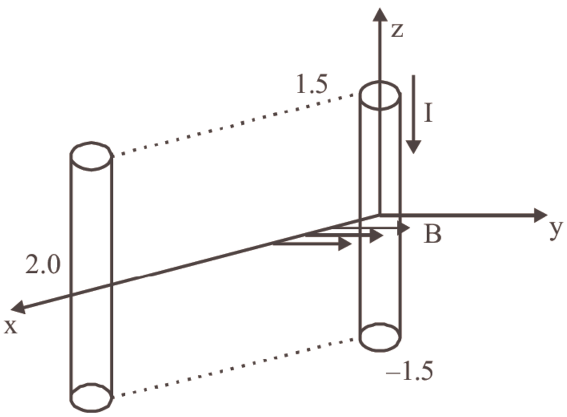
(a) 9.1×10^{-11} weber

(b) 6×10^{-11} weber

(c) 3.3×10^{-11} weber

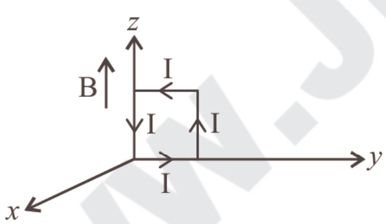
(d) 6.6×10^{-9} weber

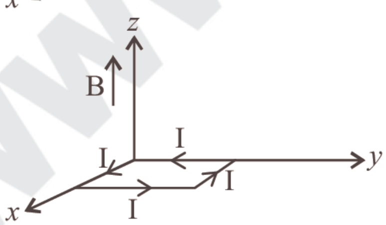
35. A conductor lies along the z-axis at $-1.5 \le z <$ 1.5 m and carries a fixed current of 10.0 A in $-\hat{a}_z$ direction (see figure). For a field $\vec{B} = 3.0 \times 10^{-4} e^{-0.2x}\, \hat{a}_y$ T, find the power required to move the conductor at constant speed to x = 2.0 m, y = 0 m in 5×10^{-3} s. Assume parallel motion along the x-axis. **[2014]**

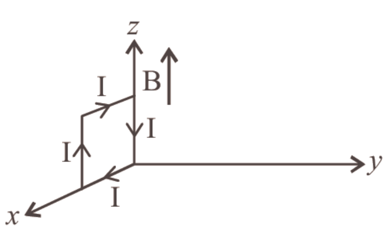


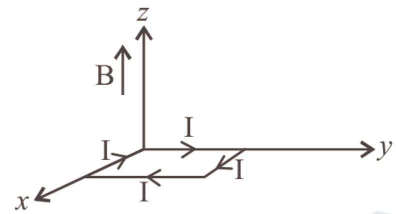
(a) 1.57 W (b) 2.97 W
(c) 14.85 W (d) 29.7 W

36. A rectangular loop of sides 10 cm and 5 cm carrying a current I of 12 A is placed in different orientations as shown in the figures below : **[2015]**

(A)


(B)


(C)


(D)


If there is a uniform magnetic field of 0.3 T in the positive z direction, in which orientations the loop would be in (i) stable equilibrium and (ii) unstable equilibrium?
(a) (B) and (D), respectively
(b) (B) and (C), respectively
(c) (A) and (B), respectively
(d) (A) and (C), respectively

37. Two coaxial solenoids of different radius carry current I in the same direction. $\vec{F}_1$ be the magnetic force on the inner solenoid due to the outer one and $\vec{F}_2$ be the magnetic force on the outer solenoid due to the inner one. Then : **[2015]**

(a) $\vec{F}_1$ is radially inwards and $\vec{F}_2 = 0$
(b) $\vec{F}_1$ is radially outwards and $\vec{F}_2 = 0$
(c) $\vec{F}_1 = \vec{F}_2 = 0$
(d) $\vec{F}_1$ is radially inwards and $\vec{F}_2$ is radially outwards

38. Two identical wires A and B, each of length 'l', carry the same current I. Wire A is bent into a circle of radius R and wire B is bent to form a square of side 'a'. If B_A and B_B are the values of magnetic field at the centres of the circle and square respectively, then the ratio $\frac{B_A}{B_B}$ is: **[2016]**

(a) $\frac{\pi^2}{16}$ (b) $\frac{\pi^2}{8\sqrt{2}}$
(c) $\frac{\pi^2}{8}$ (d) $\frac{\pi^2}{16\sqrt{2}}$

39. A galvanometer having a coil resistance of 100 Ω gives a full scale deflection, when a currect of 1 mA is passed through it. The value of the resistance, which can convert this galvanometer into ammeter giving a full scale deflection for a current of 10 A, is : **[2016]**
(a) 0.1 Ω (b) 3Ω
(c) 0.01Ω (d) 2Ω

40. When a current of 5 mA is passed through a galvanometer having a coil of resistance 15Ω, it shows full scale deflection. The value of the resistance to be put in series with the galvanometer to convert it into to voltmeter of range 0 – 10 V is **[2017]**

(a) $2.535 \times 10^3\ \Omega$ (b) $4.005 \times 10^3\ \Omega$

(c) $1.985 \times 10^3\ \Omega$ (d) $2.045 \times 10^3\ \Omega$

41. An electron, a proton and an alpha particle having the same kinetic energy are moving in circular orbits of radii r_e, r_p, r_α respectively in a uniform magnetic field B. The relation between r_e, r_p, r_α is : **[2018]**

(a) $r_e > r_p = r_\alpha$ (b) $r_e < r_p = r_\alpha$

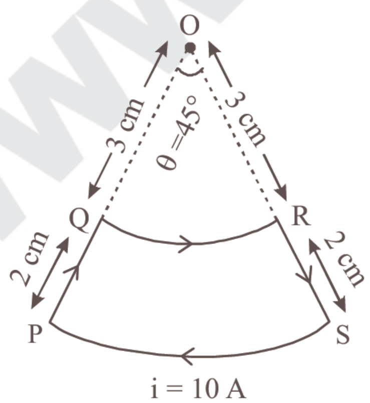
(c) $r_e < r_p < r_\alpha$ (d) $r_e < r_\alpha < r_p$

42. The dipole moment of a circular loop carrying a current I, is m and the magnetic field at the centre of the loop is B_1. When the dipole moment is doubled by keeping the current constant, the magnetic field at the centre of the loop is B_2. The ratio $\frac{B_1}{B_2}$ is: **[2018]**

(a) 2 (b) $\sqrt{3}$

(c) $\sqrt{2}$ (d) $\frac{1}{\sqrt{2}}$

43. A current loop, having two circular arcs joined by two radial lines is shown in the figure. It carries a current of 10 A. The magnetic field at point O will be close to: **[2019]**



(a) 1.0×10^{-7} T (b) 1.5×10^{-7} T

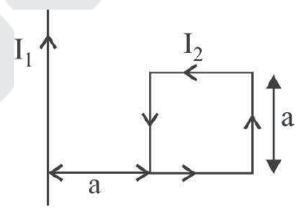
(c) 1.5×10^{-5} T (d) 1.0×10^{-5} T

44. A rectangular coil (Dimension 5 cm × 2.5 cm) with 100 turns, carrying a current of 3 A in the clock-wise direction, is kept centered at the origin and in the X-Z plane. A magnetic field of 1 T is applied along X-axis. If the coil is tilted through 45° about Z-axis, then the torque on the coil is: **[2019]**

(a) 0.38 Nm (b) 0.55 Nm

(c) 0.42 Nm (d) 0.27 Nm

45. A rigid square of loop of side '*a*' and carrying current I_2 is lying on a horizontal surface near a long current I_1 carrying wire in the same plane as shown in figure. The net force on the loop due to the wire will be: **[2019]**



(a) Repulsive and equal to $\frac{\mu_o I_1 I_2}{2\pi}$

(b) Attractive and equal to $\frac{\mu_o I_1 I_2}{3\pi}$

(c) Repulsive and equal to $\frac{\mu_o I_1 I_2}{4\pi}$

(d) Zero

46. A moving coil galvanometer has resistance 50 Ω and it indicates full deflection at 4 mA current. A voltmeter is made using this galvanometer and a 5 k Ω resistance. The maximum voltage, that can be measured using this voltmeter, will be close to: **[2019]**

(a) 40 V (b) 15 V

(c) 20 V (d) 10 V

Answer Key

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
(b)	(a)	(a)	(c)	(a)	(b)	(d)	(a)	(b)	(b)	(c)	(a)	(d)	(c)	(b)
16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
(b)	(c)	(b)	(a)	(d)	(d)	(b)	(b)	(c)	(c)	(a)	(d)	(a)	(d)	(a)
31	32	33	34	35	36	37	38	39	40	41	42	43	44	45
(c)	(b)	(d)	(a)	(b)	(a)	(c)	(b)	(c)	(c)	(b)	(c)	(d)	(d)	(c)
46														
(c)														

Solutions

1. **(b)** When current is passed through a spring then current flows parallel in the adjacent turns in the same direction. As a result the various turn attract each other and spring get compress.

2. **(a)** Magnetic field induction at the centre of current carrying circular coil of radius r is

$$B=\frac{\mu_0}{4\pi}\frac{I}{R}\times 2\pi$$

Here $B_A=\frac{\mu_0}{4\pi}\frac{I}{R}\times 2\pi$

and $B_B=\frac{\mu_0}{4\pi}\frac{2I}{2R}\times 2\pi$

$$\Rightarrow \frac{B_A}{B_B}=\frac{I/R}{2I/2R}=1$$

3. **(a)** When a moving charged particle is subjected to a perpendicular magnetic field, then it describes a circular path of radius.

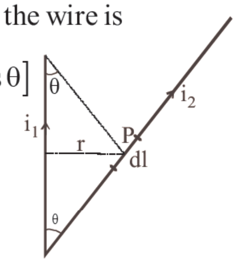
$$r=\frac{p}{qB}$$

where q = Charge of the particle
p = Momentum of the particle
B = Magnetic field

Here p, q and B are constant for electron and proton, therefore the radius will be same.

4. **(c)** Magnetic field due to current in wire 1 at point P distant r from the wire is

$$B=\frac{\mu_0}{4\pi}\frac{i_1}{r}[\cos\theta+\cos\theta]$$



$$B=\frac{\mu_0}{2\pi}\frac{i_1\cos\theta}{r}$$

This magnetic field is directed perpendicular to the plane of paper, inwards.

The force exerted due to this magnetic field on current element $i_2\,dl$ is

$$dF=i_2\,dl\,B\sin 90°$$

$$\therefore\ dF=i_2\,dlB$$

$$\Rightarrow\ dF=i_2\,dl\left(\frac{\mu_0}{4\pi}\frac{i_1\cos\theta}{r}\right)$$

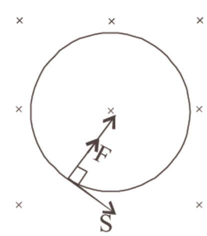
$$=\frac{\mu_0}{2\pi r}i_1 i_2\,dl\cos\theta$$

5. **(a)** The time period of a charged particle of charge q and mass m moving in a magnetic field (B) is $T=\frac{2\pi m}{qB}$

Clearly time period is independent of speed of the particle.

6. **(b)** The workdone, $dW=Fds\cos\theta$

The angle between force and displacement is 90°. Therefore work done is zero.



7. **(d)** $i_g\times G=(i-i_g)S$

$$\therefore\ S=\frac{i_g\times G}{i-i_g}=\frac{1\times 0.81}{10-1}=0.09\Omega$$

8. **(a)** Let

F_E = Force due to electric field

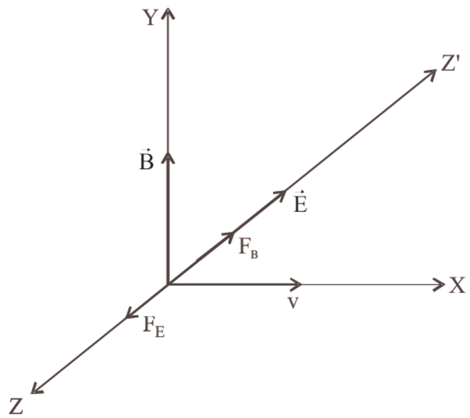
F_B = Force due to magnetic field

As particle is moving undeflected in the presence of both electric field as well as magnetic field.

Therefore $F_B = F_E$

$\Rightarrow qvB = qE$

$$\Rightarrow B = \frac{E}{v} = \frac{10^4}{10} = 10^3 \text{ weber/m}^2$$



9. (b) From Ampere's circuital law

$\int \vec{B} \cdot \vec{dl} = \mu_0 i$

$\Rightarrow B \times 2\pi r = \mu_0 i$

Here i is zero, for $r < R$, whereas R is the radius

$\therefore\ B = 0$

10. (b) Magentic field at the centre of a circular coil of radius R carrying current i is

$$B = \frac{\mu_0 i}{2R}$$

The circumference of the first loop $= 2\pi R$.

If it is bent into n circular coil of radius r'.

$n \times (2\pi r') = 2\pi R$

$\Rightarrow nr' = R$ ...(1)

New magnetic field, $B' = \frac{n \cdot \mu_0 i}{2r'}$ ...(2)

From (1) and (2),

$$B' = \frac{n\mu_0 i \cdot n}{2\pi R} = n^2 B$$

11. (c) The magnetic field at a point on the axis of a circular loop at a distance x from centre is,

$$B = \frac{\mu_0 i\ a^2}{2(x^2 + a^2)^{3/2}}$$

Magnetic field at the centre of loop is

$$B' = \frac{\mu_0 i}{2a}$$

$$\therefore\ B' = \frac{B \cdot (x^2 + a^2)^{3/2}}{a^3}$$

Put $x = 4$ & $a = 3$

$$\Rightarrow B' = \frac{54(5^3)}{3 \times 3 \times 3} = 250\ \mu T$$

12. (a) Force acting between two long conductor carrying current,

$$F = \frac{\mu_0}{4\pi} \frac{2I_1 I_2}{d} \times \ell \quad ...(i)$$

Where d = distance between the conductors

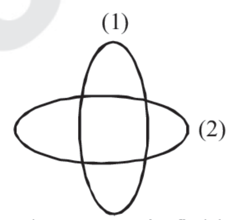
ℓ = length of conductor

In second case, $F' = -\frac{\mu_0}{4\pi} \frac{2(2I_1)I_2}{3d} \ell$ ...(ii)

From equation (i) and (ii), we have

$$\therefore\ \frac{F'}{F} = \frac{-2}{3}$$

13. (d)



The magnetic field due to circular coil (1) is

$$B_1 = \frac{\mu_0 i_1}{2r}$$

$$= \frac{\mu_0 i_1}{2(2\pi \times 10^{-2})}$$

$$= \frac{\mu_0 \times 3 \times 10^2}{4\pi}$$

Magnetic field due to coil (2)

Total magnetic field

$$B_2 = \frac{\mu_0 i_2}{2(2\pi \times 10^{-2})} = \frac{\mu_0 \times 4 \times 10^2}{4\pi}$$

Total magnetic field, $B = \sqrt{B_1^2 + B_2^2}$

$$= \frac{\mu_0}{4\pi} \cdot 5 \times 10^2$$

$\Rightarrow B = 10^{-7} \times 5 \times 10^2$

$\Rightarrow B = 5 \times 10^{-5}$ Wb/m^2

14. (c) Equating magnetic force to centripetal force,

$$\frac{mv^2}{r} = qvB \sin 90^\circ$$

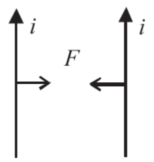
$\Rightarrow \frac{mv}{r} = Bq$

$\Rightarrow v = \frac{Bqr}{m}$

Time to complete one revolution,

$T = \frac{2\pi r}{v} = \frac{2\pi m}{qB}$

15. **(b)**



$$\frac{F}{\ell} = \frac{\mu_0 i_1 i_2}{2\pi d} = \frac{\mu_0 i^2}{2\pi d}$$

(attractive as current is in the same direction)

16. **(b)** Due to electric field, it experiences force and accelerates i.e. its velocity decreases.

17. **(c)** Resistance of Galvanometer,

$$G = \frac{\text{Current sensitivity}}{\text{Voltage sensitivity}} \Rightarrow G = \frac{10}{2} = 5\Omega$$

Here i_g = Full scale deflection current = $\frac{150}{10} = 15\text{ mA}$

V = voltage to be measured = 150 volts (such that each division reads 1 volt)

$$\Rightarrow R = \frac{150}{15 \times 10^{-3}} - 5 = 9995\Omega$$

18. **(b)** The charged particle will move along the lines of electric field (and magnetic field). Magnetic field will exert no force. The force by electric field will be along the lines of uniform electric field. Hence the particle will move in a straight line.

19. **(a)** Magnetic field due to long solenoid is given by $B = \mu_0 nI$

In first case $B_1 = \mu_0 n_1 I_1$

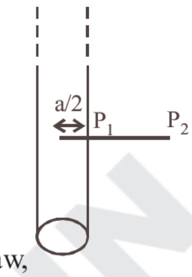
In second case, $B_2 = \mu_0 n_2 I_2$

$$\therefore \frac{B_2}{B_1} = \frac{\mu_0 n_2 i_2}{\mu_0 n_1 i_1}$$

$$\Rightarrow \frac{B_2}{6.28 \times 10^{-2}} = \frac{100 \times \frac{i}{3}}{200 \times i}$$

$$\Rightarrow B_2 = \frac{6.28 \times 10^{-2}}{6} = 1.05 \times 10^{-2}\ \text{Wb/m}^2$$

20. **(d)** Since uniform current is flowing through a straight wire, current enclosed in the amperean path formed at a distance $r_1\left(= \frac{a}{2}\right)$ is



$$i = \left(\frac{\pi r_1^2}{\pi a^2}\right) \times I,$$

where I is total current

Using Ampere circuital law,

$$\oint B \cdot \vec{dl} = \mu_0 i$$

$$\Rightarrow B_1 = \frac{\mu_0 \times \text{current enclosed}}{\text{Path}}$$

$$\Rightarrow B_1 = \frac{\mu_0 \times \left(\frac{\pi r_1^2}{\pi a^2}\right) \times I}{2\pi r_1} = \frac{\mu_0 \times I r_1}{2\pi a^2}$$

Now, magnetic field induction at point P_2,

$$B_2 = \frac{\mu_0}{2\pi} \cdot \frac{I}{(2a)} = \frac{\mu_0 I}{4\pi a}.$$

$$\therefore \frac{B_1}{B_2} = \frac{\mu_0 I r_1}{2\pi a^2} \times \frac{4\pi a}{\mu_0 I}$$

$$\Rightarrow \frac{B_1}{B_2} = \frac{2r_1}{a} = \frac{2 \times \frac{a}{2}}{a} = 1.$$

21. **(d)** There is no current inside the pipe. From Ampere's circuital law $\oint \vec{B} \cdot \vec{dl} = \mu_0 I$

$\because I = 0$

$\therefore B = 0$

22. **(b)** As velocity is not changing, charge particle must go undeflected, then

$qE = qvB$

$\Rightarrow v = \frac{E}{B}$

Also,

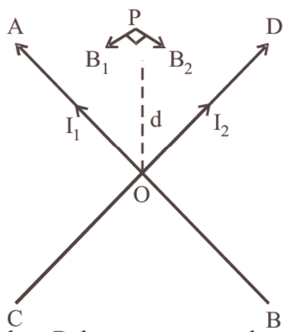
$$\left|\frac{\vec{E} \times \vec{B}}{B^2}\right| = \frac{E\ B \sin\theta}{B^2}$$

$$= \frac{E\ B\ \sin 90°}{B^2} = \frac{E}{B} = |\vec{v}| = v$$

23. **(b)** When a charged particle enters a magnetic field at a direction perpendicular to the direction of motion, the path of the motion is circular. In circular motion the direction of velocity changes at every point (the magnitude remains constant).

Therefore, the tangential momentum will change at every point. But kinetic energy will remain constant as it is given by $\frac{1}{2}mv^2$ and v^2 is the square of the magnitude of velocity which does not change.

24. (c) The direction of magnetic field induction due to current through AB and CD at P are indicated as B_1 and B_2. The magnetic fields at a point P, equidistant from AOB and COD will have directions perpendicular to each other, as they are placed normal to each other.



Magnetic field at P due to current through AB,

$$B_1 = \frac{\mu_0 I_1}{2\pi d}$$

Magnetic field at P due to current through CD,

$$B_2 = \frac{\mu_0 I_2}{2\pi d}$$

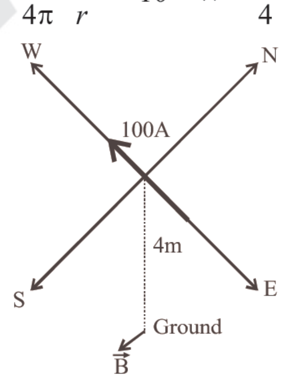
$\therefore$ Resultant field, $B = \sqrt{B_1^2 + B_2^2}$

$$\therefore \quad B = \sqrt{\left(\frac{\mu_0}{2\pi d}\right)^2 \left(I_1^2 + I_2^2\right)}$$

or, $B = \frac{\mu_0}{2\pi d}\left(I_1^2 + I_2^2\right)^{1/2}$

25. (c) The magnetic field is

$$B = \frac{\mu_0}{4\pi}\frac{2I}{r} = 10^{-7} \times \frac{2 \times 100}{4} = 5 \times 10^{-6}\ T$$



Current flows from east to west. Point is below the power line, using right hand thumb rule, the magnetic field is directed towards south.

26. (a) The magnetic field at O due to current in DA is

$B_1 = \frac{\mu_o}{4\pi}\frac{I}{a} \times \frac{\pi}{6}$ (directed vertically upwards)

The magnetic field at O due to current in BC is

$B_2 = \frac{\mu_o}{4\pi}\frac{I}{b} \times \frac{\pi}{6}$ (directed vertically downwards)

The magnetic field due to current AB and CD at O is zero.

Therefore the net magnetic field is

$B = B_1 - B_2$ (directed vertically upwards)

$$= \frac{\mu_o}{4\pi}\frac{I}{a}\frac{\pi}{6} - \frac{\mu_o}{4\pi}\frac{I}{b} \times \frac{\pi}{6}$$

$$= \frac{\mu_o I}{24}\left(\frac{1}{a} - \frac{1}{b}\right) = \frac{\mu_o I}{24ab}(b - a)$$

27. (d) $\vec{F} = I(\vec{\ell} \times \vec{B})$

The force on AD and BC due to current I_1 is zero. This is because the directions of current element $I\overrightarrow{d\ell}$ and magnetic field $\vec{B}$ are parallel.

28. (a) The magnetic field varies inversely with the distance for a long conductor. That is,

$$B \propto \frac{1}{d}$$

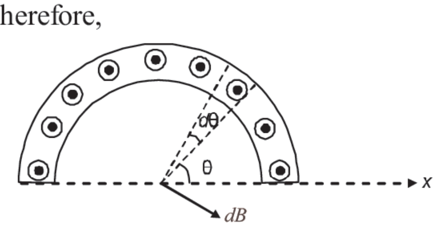
so, graph in option (a) is the correct one.

29. (d) Let R be the radius of semicircular ring. Let an elementary length dl is cut for finding magnetic field. So, $dl = Rd\theta$. Current in a small element, $dI = \frac{d\theta}{\pi}I$

Magnetic field due to the element

$$dB = \frac{\mu_0}{4\pi}\frac{2dI}{R} = \frac{\mu_0 I}{2\pi^2 R}$$

The component $dB\cos\theta$, of the field is cancelled by another opposite component. Therefore,



$$B_{net} = \int dB \sin\theta = \frac{\mu_0 I}{2\pi^2 R}\int_0^{\pi} \sin\theta d\theta = \frac{\mu_0 I}{\pi^2 R}$$

30. **(a)** The charge experiences both electric and magnetic force.

Electric force, $F_e = qE$

Magnetic force, $F_m = q(\vec{v}\times\vec{B})$

$\therefore$ Net force, $\vec{F} = q\left[\vec{E} + \vec{v}\times\vec{B}\right]$

$$= q\left[3\hat{i}+\hat{j}+2\hat{k}+\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 3 & 4 & 1\\ 1 & 1 & -3\end{vmatrix}\right]$$

$$= q\left[3\hat{i}+\hat{j}+2\hat{k}+\hat{i}(-12-1) -\hat{j}(-9-1)+k(3-4)\right]$$

$$= q\left[3\hat{i}+\hat{j}+2\hat{k}-13\hat{i}+10\hat{j}-\hat{k}\right]$$

$$= q\left[-10\hat{i}+11\hat{j}+\hat{k}\right]$$

$$F_y = 11q\hat{j}$$

Thus, the y component of the force.

31. **(c)** We know that geomagnetic ratio,

$$\frac{q}{2m} = \frac{\text{Magnetic dipole moment}}{\text{Angular momentum}}$$

Moment of inertia of disc $= \dfrac{mR^2}{2}$

Angular momentum,

$$L = \frac{mR^2}{2}\omega$$

Net charge on disc $= \sigma(\pi R^2)$

$\therefore$ Magnetic dipole moment (M)

$$M = \frac{q}{2m}\cdot\left(\frac{mR^2}{2}\right)\cdot\omega = \frac{\sigma(\pi R^2)}{2m}\left(\frac{mR^2}{2}\right)\omega$$

$$= \frac{1}{4}\sigma\cdot\pi R^4\omega.$$

32. **(b)** The centripetal force is provided by the magnetic force

$$\therefore \frac{mv^2}{R} = qvB \Rightarrow r = \frac{mv}{Bq} \qquad \therefore r \propto \frac{\sqrt{m}}{q}$$

$$\therefore r_p : r_d : r_\alpha = \frac{\sqrt{m_p}}{q_p} : \frac{\sqrt{m_d}}{q_d} : \frac{\sqrt{m_\alpha}}{q_\alpha}$$

$$= 1 : \sqrt{2} : 1$$

Thus we have, $r_\alpha = r_p < r_d$

33. **(d)** Statements I is false and Statement II is true

For ammeter, shunt resistance, $S = \dfrac{IgG}{I - Ig}$

Therefore for I to increase, S should decrease, So additional S can be connected across it.

34. **(a)** As we know, Magnetic flux, $\phi = B.A$

$$\frac{\mu_0(2)(20\times10^{-2})^2}{2[(0.2)^2+(0.15)^2]}\times\pi(0.3\times10^{-2})^2$$

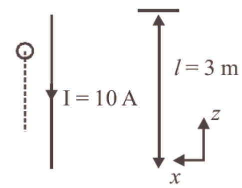
On solving

$= 9.216\times10^{-11} = 9.2\times10^{-11}$ weber

35. **(b)** Work done in moving the conductor is,

$$W = \int_0^2 F dx = \int_0^2 3.0\times10^{-4}e^{-0.2x}\times10\times3dx$$

$$= 9\times10^{-3}\int_0^2 e^{-0.2x}dx$$



$$= \frac{9\times10^{-3}}{0.2}[-e^{-0.2\times2}+1]$$

$$= \frac{9\times10^{-3}}{0.2}\times[1-e^{-0.4}]$$

$$= \frac{9\times10^{-3}\times(0.33)}{2} = \frac{2.97\times10^{-3}}{2}$$

Power required to move the conductor is,

$$P = \frac{W}{t}$$

$$P = \frac{2.97\times10^{-3}}{(0.2)\times5\times10^{-3}} = 2.97\,\text{W}$$

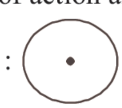
36. **(a)** For stable equilibrium $\vec{M} \parallel \vec{B}$

For unstable equilibrium $\vec{M} \parallel (-\vec{B})$

37. **(c)** $\vec{F}_1 = \vec{F}_2 = 0$

because of action and reaction pair

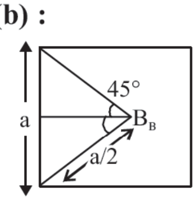
38. **(b)** Case (a) :



$B_A = \dfrac{\mu_0}{4\pi}\dfrac{I}{R}\times2\pi = \dfrac{\mu_0}{4\pi}\dfrac{I}{\ell/2\pi}\times2\pi$ ($\because 2\pi R = \ell$)

$$= \frac{\mu_0}{4\pi}\frac{I}{\ell}\times(2\pi)^2$$

Case (b) :



$$B_B = 4 \times \frac{\mu_0}{4\pi}\frac{I}{a/2}[\sin 45^\circ + \sin 45^\circ]$$

$$= 4\times\frac{\mu_0}{4\pi}\times\frac{I}{\ell/8}\times\frac{2}{\sqrt{2}} = \frac{\mu_0}{4\pi}\frac{I}{\ell}\times\frac{64}{\sqrt{2}} = \frac{\mu_0 I}{4\pi\ell}32\sqrt{2}$$

$[4a = l]$

$$\Rightarrow \frac{B_A}{B_B} = \frac{\pi^2}{8\sqrt{2}}$$

39. (c) $I_g G = (I - I_g)s$

$\therefore\ 10^{-3} \times 100 = (10 - 10^{-3}) \times S$

$\therefore\ S \approx 0.01\Omega$

40. (c) Given : Current through the galvanometer, $i_g = 5 \times 10^{-3}\ A$

Galvanometer resistance, $G = 15\Omega$

Let resistance R to be put in series with the galvanometer to convert it into a voltmeter.

$V = i_g(R + G)$

$10 = 5 \times 10^{-3}(R + 15)$

$\therefore\ R = 2000 - 15 = 1985 = 1.985 \times 10^3\ \Omega$

41. (b) As we know, radius of circular path in magnetic field

$$r = \frac{\sqrt{2Km}}{qB}$$

For electron, $r_e = \dfrac{\sqrt{2Km_e}}{eB}$ ....(i)

For proton, $r_p = \dfrac{\sqrt{2Km_p}}{eB}$ ....(ii)

For α particle,

$$r_\alpha = \frac{\sqrt{2Km_a}}{q_\alpha B} = \frac{\sqrt{2K4m_p}}{2eB} = \frac{\sqrt{2Km_p}}{eB} \quad ...(iii)$$

$\therefore\ r_e < r_p = r_\alpha \quad (\because m_e < m_p)$

42. (c) Magnetic field at the centre of loop,

$$B_1 = \frac{\mu_0 I}{2R}$$

Dipole moment of circular loop is m = IA

$m_1 = I.A = I.\pi R^2$ {R = Radius of the loop}

If moment is doubled (keeping current constant) R becomes $\sqrt{2}R$

$$m_2 = I.\pi\left(\sqrt{2}R\right)^2 = 2.I\pi R^2 = 2m_1$$

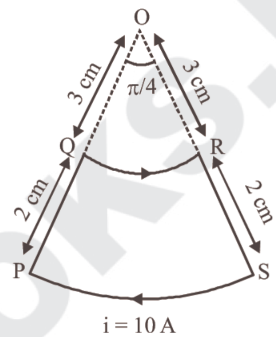
$$B_2 = \frac{\mu_0 I}{2\left(\sqrt{2}R\right)}$$

$$\therefore\ \frac{B_1}{B_2} = \frac{\frac{\mu_0 I}{2R}}{\frac{\mu_0 I}{2\left(\sqrt{2}R\right)}} = \sqrt{2}$$

43. (d) There will be no magnetic field at O due to wire PQ and RS

Magnetic field at 'O' due to arc QR $= \dfrac{\mu_0}{4\pi}\dfrac{(10)}{\left(3\times10^{-2}\right)}\times\dfrac{\pi}{4}$

(Perpendicular outwards)



Magnetic field at 'O' due to arc PS

$$= \frac{\mu_0}{4\pi}\times\frac{(10)}{\left(5\times10^{-2}\right)}\times\frac{\pi}{4} \text{ (Perpendicular inwards)}$$

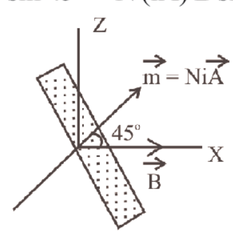
$\therefore$ Net magnetic field at 'O'

$$B = \frac{\mu_0}{4\pi}\times10\left[\frac{1}{\left(3\times10^{-2}\right)} - \frac{1}{\left(5\times10^{-2}\right)}\right]\times\frac{\pi}{4}$$

$$\Rightarrow B = \frac{\pi}{3}\times10^{-5}T \approx 1\times10^{-5}T$$

(Perpendicular outwards)

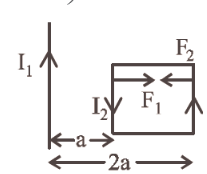
44. (d) $\tau = mB\sin 45^\circ = N(iA)B\sin 45^\circ$



$$= 100\times3(5\times2.5)\times10^{-4}\times1\times\frac{1}{\sqrt{2}} = 0.27\,\text{Nm}$$

45. (c) $F = F_1 - F_2 = \dfrac{\mu_0}{2\pi}\left(\dfrac{I_1 I_2}{a} - \dfrac{I_1 I_2}{2a}\right)\times a$

$= \dfrac{\mu_0 i_1 i_2}{4\pi}$ (Repulsive)



46. (c) $V = i_g(G + R) = 4\times10^{-3}$
$(50 + 5000) = 20V$

19 Magnetism and Matter

1. A thin rectangular magnet suspended freely has a period of oscillation equal to T. Now it is broken into two equal halves (each having half of the original length) and one piece is made to oscillate freely in the same field. If its period of oscillation is T', the ratio $\frac{T'}{T}$ is **[2003]**

(a) $\frac{1}{2\sqrt{2}}$ (b) $\frac{1}{2}$ (c) 2 (d) $\frac{1}{4}$

2. A magnetic needle lying parallel to a magnetic field requiers W units of work to turn it through 60^0. The torque needed to maintain the needle in this position will be **[2003]**

(a) $\sqrt{3}$W (b) W
(c) $\frac{\sqrt{3}}{2}$W (d) 2W

3. The magnetic lines of force inside a bar magnet **[2003]**

(a) are from north-pole to south-pole of the magnet
(b) do not exist
(c) depend upon the area of cross-section of the bar magnet
(d) are from south-pole to north-pole of the Magnet

4. Curie temperature is the temperature above which **[2003]**

(a) a ferromagnetic material becomes paramagnetic
(b) a paramagnetic material becomes diamagnetic
(c) a ferromagnetic material becomes diamagnetic
(d) a paramagnetic material becomes ferromagnetic

5. The length of a magnet is large compared to its width and breadth. The time period of its oscillation in a vibration magnetometer is 2s. The magnet is cut along its length into three equal parts and these parts are then placed on each other with their like poles together. The time period of this combination will be **[2004]**

(a) $2\sqrt{3}$ s (b) $\frac{2}{3}$s
(c) 2 s (d) $\frac{2}{\sqrt{3}}$s

6. The materials suitable for making electromagnets should have **[2004]**

(a) high retentivity and low coercivity
(b) low retentivity and low coercivity
(c) high retentivity and high coercivity
(d) low retentivity and high coercivity

7. A magnetic needle is kept in a non-uniform magnetic field. It experiences **[2005]**

(a) neither a force nor a torque
(b) a torque but not a force
(c) a force but not a torque
(d) a force and a torque

8. Needles N_1, N_2 and N_3 are made of a ferromagnetic, a paramagnetic and a diamagnetic substance respectively. A magnet when brought close to them will **[2006]**

(a) attract N_1 and N_2 strongly but repel N_3
(b) attract N_1 strongly, N_2 weakly and repel N_3 weakly
(c) attract N_1 strongly, but repel N_2 and N_3 weakly
(d) attract all three of them

9. Relative permittivity and permeability of a material ε_r and μ_r, respectively. Which of the following values of these quantities are allowed for a diamagnetic material? **[2008]**

(a) $\varepsilon_r = 0.5, \mu_r = 1.5$
(b) $\varepsilon_r = 1.5, \mu_r = 0.5$
(c) $\varepsilon_r = 0.5, \mu_r = 0.5$
(d) $\varepsilon_r = 1.5, \mu_r = 1.5$

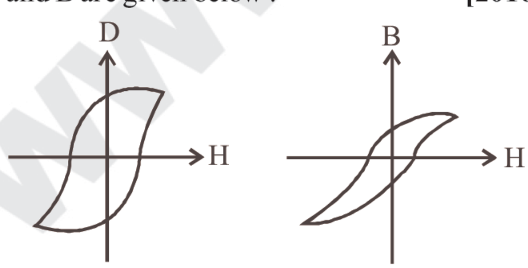
10. Two short bar magnets of length 1 cm each have magnetic moments 1.20 Am^2 and 1.00 Am^2 respectively. They are placed on a horizontal table parallel to each other with their N poles pointing towards the South. They have a common magnetic equator and are separated by a distance of 20.0 cm. The value of the resultant horizontal magnetic induction at the mid-point O of the line joining their centres is close to (Horizontal component of earth's magnetic induction is 3.6×10^{-5} Wb/m^2) **[2013]**

(a) 3.6×10^{-5} Wb/m^2
(b) 2.56×10^{-4} Wb/m^2
(c) 3.50×10^{-4} Wb/m^2
(d) 5.80×10^{-4} Wb/m^2

11. The coercivity of a small magnet where the ferromagnet gets demagnetized is 3×10^3 Am^{-1}. The current required to be passed in a solenoid of length 10 cm and number of turns 100, so that the magnet gets demagnetized when inside the solenoid, is: **[2014]**

(a) 30 mA (b) 60 mA
(c) 3 A (d) 6 A

12. Hysteresis loops for two magnetic materials A and B are given below : **[2016]**



(A) (B)

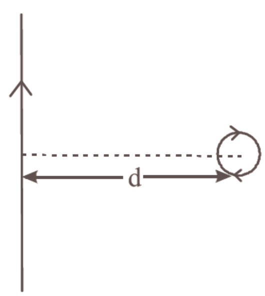
These materials are used to make magnets for elecric generators, transformer core and electromagnet core. Then it is proper to use :

(a) A for transformers and B for electric generators.
(b) B for electromagnets and transformers.
(c) A for electric generators and trasformers.
(d) A for electromagnets and B for electric generators

13. A bar magnet is demagnetized by inserthing it inside a solenoid of length 0.2 m, 100 turns, and carrying a current of 5.2 A. The coercivity of the bar magnet is: **[2019]**

(a) 285 A/m (b) 2600 A/m
(c) 520 A/m (d) 1200 A/m

14. An infinitely long current carrying wire and a small current carrying loop are in the plane of the paper as shown. The redius of the loop is a and distance of its centre from the wire is d ($d >> a$). If the loop applies a force F on the wire then: **[2019]**



(a) $F = 0$ (b) $F \propto \left(\frac{a}{d}\right)$
(c) $F \propto \left(\frac{a^2}{d^3}\right)$ (d) $F \propto \left(\frac{a}{d}\right)^2$

Answer Key

1	2	3	4	5	6	7	8	9	10	11	12	13	14	
(b)	(a)	(d)	(a)	(b)	(b)	(d)	(b)	(b)	(b)	(c)	(b)	(b)	(d)	

Solutions

1. **(b)** The time period of a rectangular magnet oscillating in earth's magnetic field is given by $T = 2\pi\sqrt{\frac{I}{MB_H}}$

where I = Moment of inertia of the rectangular magnet
M = Magnetic moment
B_H = Horizontal component of the earth's magnetic field
Initially, the time period of the magnet

$$T = 2\pi\sqrt{\frac{I}{MB_H}} \text{ where } I = \frac{1}{12}M\ell^2$$

Case 2
Magnet is cut into two identical pieces such that each piece has half the original length.

Then $T' = 2\pi\sqrt{\frac{I'}{M'B_H}}$

Moment of inertia of each part

$$I' = \frac{1}{12}\left(\frac{M}{2}\right)\left(\frac{\ell}{2}\right)^2 = \frac{I}{8}$$

and $M' = \frac{M}{2}$

$$\therefore \frac{T'}{T} = \sqrt{\frac{I'}{M} \times \frac{M}{I}}$$

$$= \sqrt{\frac{I/8}{M/2} \times \frac{M}{I}} = \sqrt{\frac{1}{4}} = \frac{1}{2}$$

2. **(a)** Workdone to turn a magnetic needle from angle θ_1 to θ_2 is given by
$W = MB(\cos\theta_1 - \cos\theta_2)$

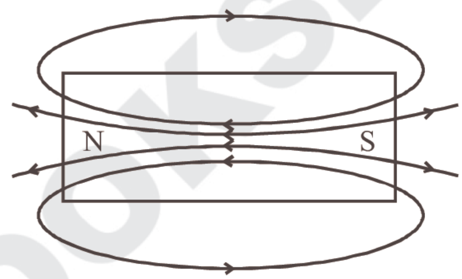
$\therefore W = MB(\cos 0° - \cos 60°)$

$$= MB\left(1 - \frac{1}{2}\right) = \frac{MB}{2}$$

$\therefore$ Torque, $\tau = MB \sin\theta = MB \sin 60°$

$$= \sqrt{3}\frac{MB}{2} = \sqrt{3}W$$

3. **(d)** The magnetif field lines of bar magnet form closed lines. As shown in the figure, the magnetic lines of force are directed from south to north inside a bar magnet. Outside the bar magnet magnetic field lines directed from north to south pole.



4. **(a)** The temperature above which a ferromagnetic substance becomes paramagnetic is called Curie's temperature.

5. **(b)** Initially, time period of magnet

$$T = 2\pi\sqrt{\frac{I}{MB}} = 25 \text{ where } I = \frac{1}{12}m\ell^2$$

When the magnet is cut into three pieces the pole strength will remain the same and Moment of inertia of each part,

$$(I') = \frac{1}{12}\left(\frac{m}{3}\right)\left(\frac{\ell}{3}\right) \times 3 = \frac{I}{9}$$

We have, Magnetic moment (M) = Pole strength (m) $\times \ell$
$\therefore$ New magnetic moment,

$$M' = m \times \left(\frac{\ell}{3}\right) \times 3 = m\ell = M$$

New time period, $T' = 2\pi\sqrt{\frac{I'}{M'B}}$

$$= 2\pi\sqrt{\frac{I}{9MB}}$$

$$\Rightarrow T' = \frac{T}{\sqrt{9}} = \frac{2}{3}s.$$

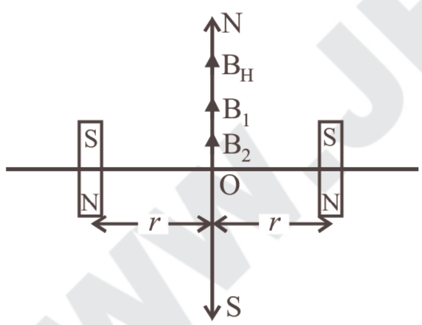
6. (b) Electromagnet should be amenable to magnetisation & demagnetization.
$\therefore$ Materials suitable for making electromagnets should have low retentivity and low coercivity should be low.

7. (d) A magnetic needle kept in non uniform magnetic field experience a force and torque due to unequal forces acting on poles.

8. (b) Ferromagnetic substance has magnetic domains whereas paramagnetic substances have magnetic dipoles which get attracted to a magnetic field. Ferromagnetic material magnetised strongly in the direction of magnetism field, Hence, N_1 will be attracted paramagnetic substance attract weekly in the direction of field. Hence, N_2 will weakly attracted. Diamagnetic substances do not have magnetic dipole but in the presence of external magnetic field due to their orbital motion of electrons these substances are repelled. Hence, N_3 will be repelled.

9. (b) For a diamagnetic material, the value of μ_r is slightly less than one. For any material, the value of $\in_r$ is always greater than 1.

10. (b) Given : $M_1 = 1.20\, Am^2$



$M_2 = 1.00\, Am^2$; $r = \frac{20}{2} cm = 0.1\text{m}$

$B_{net} = B_1 + B_2 + B_H$

$$B_{net} = \frac{\mu_0}{4\pi}\frac{(M_1 + M_2)}{r^3} + B_H$$

$$= \frac{10^{-7}(1.2+1)}{(0.1)^3} + 3.6\times10^{-5} = 2.56\times10^{-4}\ \text{wb/m}^2$$

11. (c) Magnetic field in solenoid $B = \mu_0 ni$

$$\Rightarrow \frac{B}{\mu_0} = ni$$

(Where n = number of turns per unit length)

$$\Rightarrow \frac{B}{\mu_0} = \frac{Ni}{L} \Rightarrow 3\times10^3 = \frac{100i}{10\times10^{-2}}$$

$\Rightarrow i = 3\text{A}$

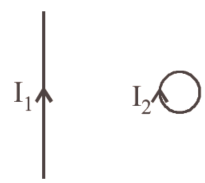
12. (b) Graph [A] is for material used for making permanent magnets (high coercivity)
Graph [B] is for making electromagnets and transformers.

13. (b) Corecivity, $H = \frac{B}{\mu_0}$ and

$$B = \mu_0 ni\left(n = \frac{N}{\ell}\right)$$

or, $\text{H} = \frac{N}{\ell}i = \frac{100}{0.2}\times 5.2 = 2600\ \text{A/m}$

14. (d) We know that $F = -\frac{dv}{dr}$ where r = distance of the loop from straight current carrying wire



Here

$$U = -\vec{m}\cdot\vec{B} = -I_2\pi a^2 \times \frac{\mu_0}{4\pi}\frac{I_1}{r}\times 2\times\cos 0$$

$$= -\frac{\mu_0 I_1 I_2 a^2}{2r}$$

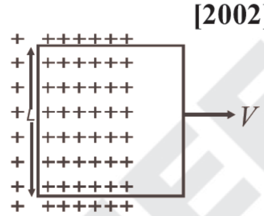
$$\therefore F = -\frac{d}{d(r)}\left[-\frac{\mu_0 I_1 I_2 a^2}{2r}\right] = -\frac{\mu_0 I_1 I_2 a^2}{r^2}$$

Here $r = d$

$$\therefore F \propto \frac{a^2}{d^2} \text{ (attractive)}$$

20 Electromagnetic Induction

1. A conducting square loop of side L and resistance R moves in its plane with a uniform velocity v perpendicular to one of its sides. A magnetic induction B constant in time and space, pointing perpendicular and into the plane at the loop exists everywhere with half the loop outside the field, as shown in figure. The induced emf is **[2002]**



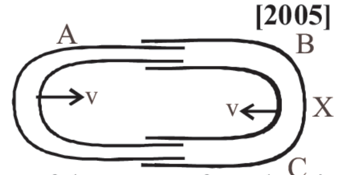
   (a) zero
   (b) RvB
   (c) vBL/R
   (d) vBL

2. Two coils are placed close to each other. The mutual inductance of the pair of coils depends upon **[2003]**
   (a) the rates at which currents are changing in the two coils
   (b) relative position and orientation of the two coils
   (c) the materials of the wires of the coils
   (d) the currents in the two coils

3. When the current changes from +2 A to –2A in 0.05 second, an e.m.f. of 8 V is induced in a coil. The coefficient of self -induction of the coil is **[2003]**
   (a) 0.2 H (b) 0.4 H
   (c) 0.8 H (d) 0.1 H

4. A metal conductor of length 1 m rotates vertically about one of its ends at angular velocity 5 radians per second. If the horizontal component of earth's magnetic field is 0.2×10^{-4}T, then the e.m.f. developed between the two ends of the conductor is **[2004]**
   (a) 5mV (b) 50 μV (c) 5 μV (d) 50mV

5. One conducting U tube can slide inside another as shown in figure, maintaining electrical contacts between the tubes. The magnetic field B is perpendicular to the plane of the figure . If each tube moves towards the other at a constant speed v, then the emf induced in the circuit in terms of B, l and v where l is the width of each tube, will be **[2005]**



   (a) $-Blv$
   (b) Blv
   (c) $2\ Blv$
   (d) zero

6. The self inductance of the motor of an electric fan is 10 H. In order to impart maximum power at 50 Hz, it should be connected to a capacitance of **[2005]**
   (a) 8 μF (b) 4 μF (c) 2 μF (d) 1 μF

7. In an AC generator, a coil with N turns, all of the same area A and total resistance R, rotates with frequency ω in a magnetic field B. The maximum value of emf generated in the coil is **[2006]**
   (a) N.A.B.R.ω (b) N.A.B
   (c) N.A.B.R. (d) N.A.B.ω

8. The flux linked with a coil at any instant 't' is given by $\phi = 10t^2 - 50t + 250$. The induced emf at $t = 3s$ is **[2006]**
   (a) –190 V (b) –10 V (c) 10 V (d) 190 V

9. Two coaxial solenoids are made by winding thin insulated wire over a pipe of cross-sectional area A = 10 cm^2 and length = 20 cm. If one of the solenoid has 300 turns and the other 400 turns, their mutual inductance is **[2008]**
   ($\mu_0 = 4\pi \times 10^{-7}$ Tm A^{-1})
   (a) $2.4\pi \times 10^{-5}$ H (b) $4.8\pi \times 10^{-4}$ H
   (c) $4.8\pi \times 10^{-5}$ H (d) $2.4\pi \times 10^{-4}$ H

10. A boat is moving due east in a region where the earth's magnetic field is 5.0×10^{-5} NA^{-1} m^{-1} due north and horizontal. The boat carries a vertical aerial 2 m long. If the speed of the boat is 1.50 ms^{-1}, the magnitude of the induced emf in the wire of aerial is: **[2011]**

(a) 0.75 mV (b) 0.50 mV
(c) 0.15 mV (d) 1 mV

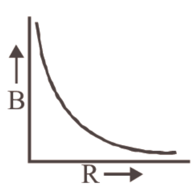
11. A horizontal straight wire 20 m long extending from east to west falling with a speed of 5.0 m/s, at right angles to the horizontal component of the earth's magnetic field 0.30×10^{-4} Wb/m^2. The instantaneous value of the e.m.f. induced in the wire will be **[2011 RS]**

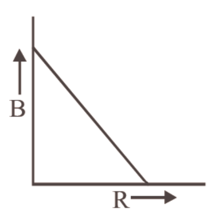
(a) 3 mV (b) 4.5 mV
(c) 1.5 mV (d) 6.0 mV

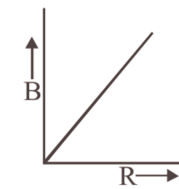
12. A coil is suspended in a uniform magnetic field, with the plane of the coil parallel to the magnetic lines of force. When a current is passed through the coil it starts oscillating; It is very difficult to stop. But if an aluminium plate is placed near to the coil, it stops. This is due to : **[2012]**

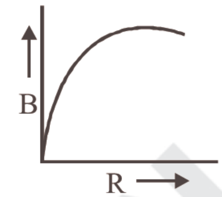
(a) developement of air current when the plate is placed
(b) induction of electrical charge on the plate
(c) shielding of magnetic lines of force as aluminium is a paramagnetic material.
(d) electromagnetic induction in the aluminium plate giving rise to electromagnetic damping.

13. A charge Q is uniformly distributed over the surface of non-conducting disc of radius R. The disc rotates about an axis perpendicular to its plane and passing through its centre with an angular velocity ω. As a result of this rotation a magnetic field of induction B is obtained at the centre of the disc. If we keep both the amount of charge placed on the disc and its angular velocity to be constant and vary the radius of the disc then the variation of the magnetic induction at the centre of the disc will be represented by the figure : **[2012]**

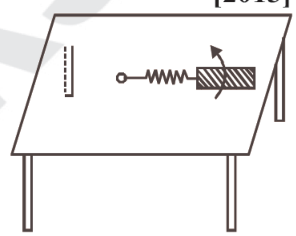
(a) 

(b) 

(c) 

(d) 

14. A metallic rod of length 'ℓ' is tied to a string of length 2ℓ and made to rotate with angular speed ω on a horizontal table with one end of the string fixed. If there is a vertical magnetic field 'B' in the region, the e.m.f. induced across the ends of the rod is **[2013]**

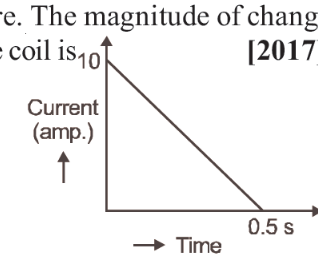


(a) $\frac{2B\omega\ell^2}{2}$

(b) $\frac{3B\omega\ell^2}{2}$

(c) $\frac{4B\omega\ell^2}{2}$ (d) $\frac{5B\omega\ell^2}{2}$

15. In a coil of resistance 100Ω, a current is induced by changing the magnetic flux through it as shown in the figure. The magnitude of change in flux through the coil is **[2017]**



(a) 250 Wb
(b) 275 Wb
(c) 200 Wb
(d) 225 Wb

16. A conducting circular loop made of a thin wire, has area 3.5×10^{-3}m^2 and resistance 10Ω. It is placed perpendicular to a time dependent magnetic field B(t) = (0.4T) sin (50πt). The the net charge flowing through the loop during t = 0 s and t = 10 ms is close to: **[2019]**

(a) 14 mC (b) 7 mC (c) 21 mC (d) 6 mC

17. The total number of turns and cross-section area in a solenoid is fixed. However, its length L is varied by adjusting the separation between windings. The inductance of solenoid will be proportional to: **[2019]**

(a) L (b) L^2 (c) 1/L^2 (d) 1/L

18. Consider a circular coil of wire carrying constant current I, forming a magnetic dipole. The magnetic flux through an infinite plane that contains the circular coil and excluding the circular coil area is given by ϕ_i. The

magnetic flux through the area of the circular coil area is given by ϕ_0. Which of the following option is correct? **[2020]**

(a) $\phi_i = \phi_0$ (b) $\phi_i > \phi_0$

(c) $\phi_i < \phi_0$ (d) $\phi_i = -\phi_0$

19. A long solenoid of radius R carries a time (t) - dependent current $I(t) = I_0 t(1-t)$. A ring of radius $2R$ is placed coaxially near its middle. During the time interval $0 \le t \le 1$, the induced current (I_R) and the induced $EMF(V_R)$ in the ring change as: **[2020]**

(a) Direction of I_R remains unchanged and V_R is maximum at $t = 0.5$

(b) At $t = 0.25$ direction of I_R reverses and V_R is maximum

(c) Direction of I_R remains unchanged and V_R is zero at $t = 0.25$

(d) At $t = 0.5$ direction of I_R reverses and V_R is zero

20. A loop ABCDEFA of straight edges has six corner points $A(0, 0, 0)$, $B\{5, 0, 0)$, $C(5, 5, 0)$, $D(0, 5, 0)$, $E(0, 5, 5)$ and $F(0, 0, 5)$. The magnetic field in this region is

$\vec{B} = (3\hat{i} + 4\hat{k})$T. The quantity of flux through the loop $ABCDEFA$ (in Wb) is ________. **[2020]**

Answer Key

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
(d)	(b)	(d)	(b)	(c)	(d)	(d)	(b)	(d)	(c)	(a)	(d)	(a)	(d)	(a)
16	17	18	19	20										
(None)	(d)	(d)	(b)	(175.00)										

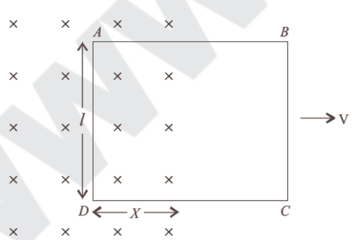
Solutions

1. **(d)** As the side BC is outside the field, no emf is induced across BC. Further, sides AB and CD are not cutting any flux. So, they will not centribute in flux.

Only side AD is cutting the flux 50 emf will be induced due to AD only.

The induced emf is

$$e = \frac{-d\phi}{dt} = -\frac{d(\vec{B}\cdot\vec{A})}{dt} = \frac{-d(BA\cos 0^\circ)}{dt}$$



$$\therefore e = -B\frac{dA}{dt} = -B\frac{d(\ell \times x)}{dt}$$

$$\therefore e = -B\ell\frac{dx}{dt} = -B\ell v$$

2. **(b)** Mutual inductance depends on the relative position and orientation of the two coils.

3. **(d)** Induced emf,

$$e = -\frac{\Delta\phi}{\Delta t} = \frac{-\Delta(LI)}{\Delta t} = -L\frac{\Delta I}{\Delta t}$$

$$\therefore |e| = L\frac{\Delta I}{\Delta t}$$

$$\Rightarrow 8 = L \times \frac{[2-(-2)]}{0.05}$$

$$\Rightarrow L = \frac{8 \times 0.05}{4} = 0.1\text{H}$$

4. **(b)** Given, length of conductor $\ell = 1$m,
Angular speed, $\omega = 5$ rad/s,
Magnetic field, $B = 0.2 \times 10^{-4}\,T$
EmF generated between two ends of conductor

$$\varepsilon = \frac{B\omega l^2}{2} = \frac{0.2 \times 10^{-4} \times 5 \times 1}{2} = 50\mu V$$

5. **(c)** Relative velocity of the tube of width l,
$= v - (-v)\ v = 2v$
$\therefore$ Induced emf. $= B.l\,(2v)$

6. **(d)** Maximum power (I^2R) is obtained when I is maximum and
For maximum current $X_L = X_C$, which yields

$$\therefore \omega L = \frac{1}{\omega C} \Rightarrow C = \frac{1}{\omega^2 L}$$

$$C = \frac{1}{(2\pi n)^2 L} = \frac{1}{4\pi^2 \times 50 \times 50 \times 10}$$

$$\therefore C = 0.1 \times 10^{-5}\,F = 1\mu F$$

7. (d) Flux = NBA cos ωt

EmF, e = rate of change of flux

$$e=-\frac{d\phi}{dt}=-\frac{d(N\vec{B}\cdot\vec{A})}{dt}$$

$$=-N\frac{d}{dt}(BA\cos\omega t)=NBA\omega\sin\omega t$$

Maximum emf occur when $\sin\omega t=1$

$\Rightarrow e_{max}=NBA\omega$

8. (b) Electric flux, $\phi=10t^2-50t+250$

Induced emf, $e=-\frac{d\phi}{dt}=-(20t-50)$

$e_{t=3}=-10\ V$

9. (d) Given, Area of cross-section of pipe, $A=10\ cm^2$

Length of pipe, $\ell=20$ cm

$$M=\frac{\mu_0N_1N_2A}{\ell}$$

$$=\frac{4\pi\times10^{-7}\times300\times400\times100\times10^{-4}}{0.2}$$

$$M=\frac{\mu_0N_1N_2A}{\ell}$$

$=2.4\pi\times10^{-4}$ H

10. (c) Given,

Length of aerial, ℓ = 2m Earth's magnetic field,

$B=5\times10^{-5}\ NA^{-1}m$

Induced emf $=vB_Hl=1.5\times5\times10^{-5}\times2$

$=15\times10^{-5}$

$=0.15$ mV

11. (a) Induced, emF, $\varepsilon=Bv\ell$

$=0.3\times10^{-4}\times5\times20$

$=3\times10^{-3}\ V=3$ mV.

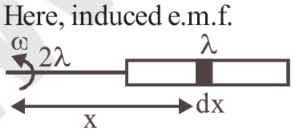
12. (d) Because of the Lenz's law of conservation of energy. Length of straignt wire, ℓ = 20m Earth's Magneti field,

$B=0.30\times10^{-4}\ Wb/m^2$.

13. (a) The magnetic field due to a disc is given as

$$B=\frac{\mu_0\omega Q}{2\pi R}\quad i.e.,\ B\propto\frac{1}{R}$$

14. (d) Here, induced e.m.f.



$$e=\int_{2\ell}^{3\ell}(\omega x)Bdx=B\omega\frac{[(3\ell)^2-(2\ell)^2]}{2}$$

$$=\frac{5B\ell^2\omega}{2}$$

15. (a) According to Faraday's law of electromagnetic induction, $\varepsilon=\frac{d\phi}{dt}$

Also, $\varepsilon=iR$

$\therefore\quad iR=\frac{d\phi}{dt}\quad\Rightarrow\quad\int d\phi=R\int idt$

Magnitude of change in flux ($d\phi$) = R × area under current vs time graph

or, $d\phi=100\times\frac{1}{2}\times\frac{1}{2}\times10=250$ Wb

16. (None)

Net charge

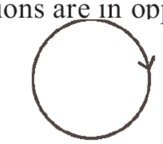
$$Q=\frac{\Delta\phi}{R}=\frac{1}{10}A(B_f-B_i)=\frac{1}{10}\times3.5\times10^{-3}\left(0.4\sin\frac{\pi}{2}-0\right)$$

$$=\frac{1}{10}(3.5\times10^{-3})(0.4-0)=1.4\times10^{-4}$$

No option matches, so it should be a bonus.

17. (d) Inductance $=\frac{\mu_0N^2A}{L}$

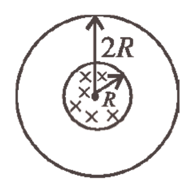
18. (d) As magnetic field lines form close loop, hence every magnetic field line creating magnetic flux through the inner region (ϕ_i) must be passing through the outer region. Since flux in two regions are in opposite region.



$\therefore\ \phi_i=-\phi_0$

19. (d) According to question, $I(t)=I_0t(1-t)$

$\therefore\ I=I_0t-I_0t^2$



$\phi=B\cdot A$

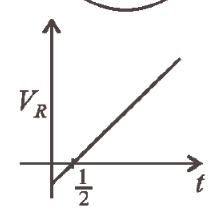
$\phi=(\mu_0nI)\times(\pi R^2)$

($\because\ B=\mu_0nI$ and $A=\pi R^2$)

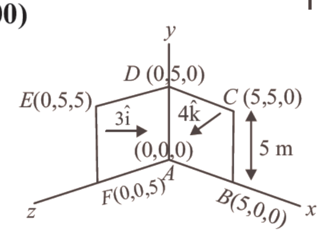
$V_R=\frac{-d\phi}{dt}$

$V_R=\mu_0n\pi R^2(I_0-2I_0t)$

$\Rightarrow V_R=0$ at $t=\frac{1}{2}s$



20. (175.00)



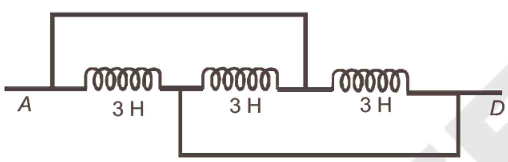
Flux through the loop $ABCDEFA$,

$\phi=\vec{B}\cdot\vec{A}=(3\hat{i}+4\hat{k})\cdot(25\hat{i}+25\hat{k})$

$\Rightarrow\phi=(3\times25)+(4\times25)=175$ weber

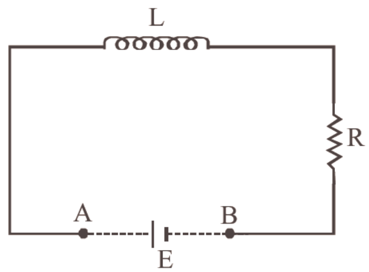
21 Alternating Current

1. The power factor of an AC circuit having resistance (R) and inductance (L) connected in series and an angular velocity ω is **[2002]**
   (a) $R/\omega L$ (b) $R/(R^2+\omega^2L^2)^{1/2}$
   (c) $\omega L/R$ (d) $R/(R^2-\omega^2L^2)^{1/2}$
2. The inductance between A and D is **[2002]**



   (a) 3.66 H (b) 9 H (c) 0.66 H (d) 1 H.
3. In a transformer, number of turns in the primary coil are 140 and that in the secondary coil are 280. If current in primary coil is 4 A, then that in the secondary coil is **[2002]**
   (a) 4 A (b) 2 A (c) 6 A (d) 10 A.
4. In an oscillating LC circuit the maximum charge on the capacitor is Q. The charge on the capacitor when the energy is stored equally between the electric and magnetic field is **[2003]**
   (a) $\frac{Q}{2}$ (b) $\frac{Q}{\sqrt{3}}$ (c) $\frac{Q}{\sqrt{2}}$ (d) Q
5. The core of any transformer is laminated so as to **[2003]**
   (a) reduce the energy loss due to eddy currents
   (b) make it light weight
   (c) make it robust and strong
   (d) increase the secondary voltage
6. Alternating current can not be measured by D.C. ammeter because **[2004]**
   (a) Average value of current for complete cycle is zero
   (b) A.C. Changes direction
   (c) A.C. can not pass through D.C. Ammeter
   (d) D.C. Ammeter will get damaged.
7. In an LCR series a.c. circuit, the voltage across each of the components, L, C and R is 50V. The voltage across the LC combination will be **[2004]**
   (a) 100 V (b) $50\sqrt{2}$ V
   (c) 50 V (d) 0 V (zero)
8. In a LCR circuit capacitance is changed from C to 2 C. For the resonant frequency to remain unchanged, the inductance should be changed from L to **[2004]**
   (a) $L/2$ (b) $2L$ (c) $4L$ (d) $L/4$
9. The phase difference between the alternating current and emf is $\frac{\pi}{2}$. Which of the following cannot be the constituent of the circuit? **[2005]**
   (a) R, L (b) C alone
   (c) L alone (d) L, C
10. A circuit has a resistance of 12 ohm and an impedance of 15 ohm. The power factor of the circuit will be **[2005]**
    (a) 0.4 (b) 0.8 (c) 0.125 (d) 1.25
11. A coil of inductance 300 mH and resistance 2Ω is connected to a source of voltage 2V. The current reaches half of its steady state value in **[2005]**
    (a) 0.1 s (b) 0.05 s
    (c) 0.3 s (d) 0.15 s
12. In a series resonant LCR circuit, the voltage across R is 100 volts and $R = 1$ kΩ with C = 2μF. The resonant frequency ω is 200 rad/s. At resonance the voltage across L is **[2006]**
    (a) 2.5×10^{-2} V (b) 40 V
    (c) 250 V (d) 4×10^{-3} V

13. An inductor ($L = 100$ mH), a resistor ($R = 100\,\Omega$) and a battery ($E = 100$ V) are initially connected in series as shown in the figure. After a long time the battery is disconnected after short circuiting the points A and B. The current in the circuit 1 ms after the short circuit is **[2006]**



(a) 1/eA (b) eA (c) 0.1 A (d) 1 A

14. In an a.c. circuit the voltage applied is $E = E_0 \sin \omega t$. The resulting current in the circuit is $I = I_0 \sin\left(\omega t - \frac{\pi}{2}\right)$. The power consumption in the circuit is given by **[2007]**

(a) $P = \sqrt{2} E_0 I_0$ (b) $P = \frac{E_0 I_0}{\sqrt{2}}$

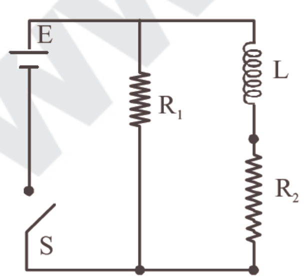
(c) P = zero (d) $P = \frac{E_0 I_0}{2}$

15. An ideal coil of 10H is connected in series with a resistance of 5Ω and a battery of 5V. 2second after the connection is made, the current flowing in ampere in the circuit is **[2007]**

(a) $(1 - e^{-1})$ (b) $(1 - e)$

(c) e (d) e^{-1}

16.

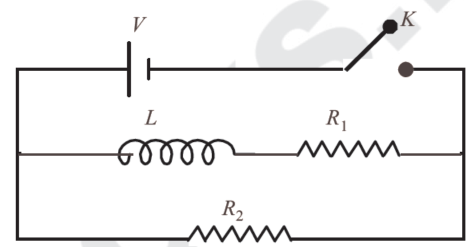


An inductor of inductance L = 400 mH and resistors of resistance $R_1 = 2\Omega$ and $R_2 = 2\Omega$ are connected to a battery of emf 12 V as shown in the figure. The internal resistance of the battery is negligible. The switch S is closed at $t = 0$. The potential drop across L as a function of time is **[2009]**

(a) $\frac{12}{t} e^{-3t}$ V (b) $6(1 - e^{-t/0.2})$V

(c) $12e^{-5t}$ V (d) $6e^{-5t}$ V

17. In the circuit shown below, the key K is closed at $t = 0$. The current through the battery is **[2010]**



(a) $\frac{VR_1R_2}{\sqrt{R_1^2 + R_2^2}}$ at $t = 0$ and $\frac{V}{R_2}$ at $t = \infty$

(b) $\frac{V}{R_2}$ at $t = 0$ and $\frac{V(R_1 + R_2)}{R_1R_2}$ at $t = \infty$

(c) $\frac{V}{R_2}$ at $t = 0$ and $\frac{VR_1R_2}{\sqrt{R_1^2 + R_2^2}}$ at $t = \infty$

(d) $\frac{V(R_1 + R_2)}{R_1R_2}$ at $t = 0$ and $\frac{V}{R_2}$ at $t = \infty$

18. In a series LCR circuit $R = 200\Omega$ and the voltage and the frequency of the main supply is 220V and 50 Hz respectively. On taking out the capacitance from the circuit the current lags behind the voltage by 30°. On taking out the inductor from the circuit the current leads the voltage by 30°. The power dissipated in the LCR circuit is **[2010]**

(a) 305 W (b) 210 W

(c) Zero W (d) 242 W

19. A fully charged capacitor C with initial charge q_0 is connected to a coil of self inductance L at $t = 0$. The time at which the energy is stored equally between the electric and the magnetic fields is: **[2011]**

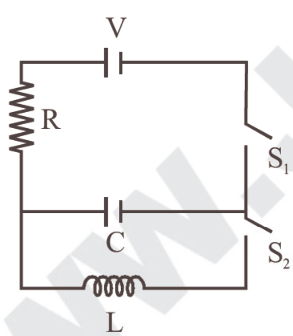
(a) $\frac{\pi}{4}\sqrt{LC}$ (b) $2\pi\sqrt{LC}$

(c) $\sqrt{LC}$ (d) $\pi\sqrt{LC}$

20. A resistor 'R' and 2μF capacitor in series is connected through a switch to 200 V direct supply. Across the capacitor is a neon bulb that lights up at 120 V. Calculate the value of R to make the bulb light up 5 s after the switch has been closed. ($\log_{10} 2.5 = 0.4$) **[2011]**
(a) $1.7 \times 10^5 \Omega$ (b) $2.7 \times 10^6 \Omega$
(c) $3.3 \times 10^7 \Omega$ (d) $1.3 \times 10^4 \Omega$

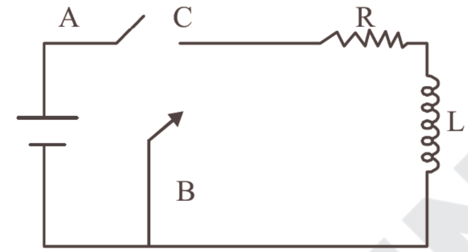
21. Combination of two identical capacitors, *a* resistor R and *a dc* voltage source of voltage 6V is used in an experiment on a (C-R) circuit. It is found that for a parallel combination of the capacitor the time in which the voltage of the fully charged combination reduces to half its original voltage is 10 second. For series combination the time for needed for reducing the voltage of the fully charged series combination by half is **[2011 RS]**
(a) 10 second (b) 5 second
(c) 2.5 second (d) 20 second

22. In an LCR circuit as shown below both switches are open initially. Now switch S_1 is closed, S_2 kept open. (q is charge on the capacitor and τ = RC is Capacitive time constant). Which of the following statement is correct? **[2013]**



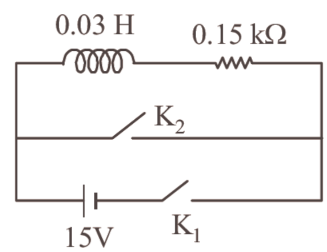
(a) Work done by the battery is half of the energy dissipated in the resistor
(b) At, $t = \tau$, $q = CV/2$
(c) At, $t = 2\tau$, $q = CV(1 - e^{-2})$
(d) At, $t = 2\tau$, $q = CV(1 - e^{-1})$

23. In the circuit shown here, the point 'C' is kept connected to point 'A' till the current flowing through the circuit becomes constant. Afterward, suddenly, point 'C' is disconnected from point 'A' and connected to point 'B' at time t = 0. Ratio of the voltage across resistance and the inductor at t = L/R will be equal to: **[2014]**



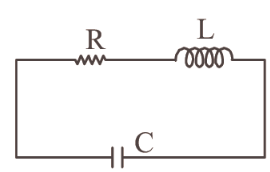
(a) $\frac{e}{1-e}$ (b) 1
(c) –1 (d) $\frac{1-e}{e}$

24. An inductor (L = 0.03 H) and a resistor (R = 0.15 kΩ) are connected in series to a battery of 15V EMF in a circuit shown below. The key K_1 has been kept closed for a long time. Then at $t = 0$, K_1 is opened and key K_2 is closed simultaneously. At $t = 1$ ms, the current in the circuit will be : ($e^5 \cong 150$) **[2015]**

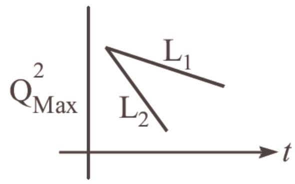


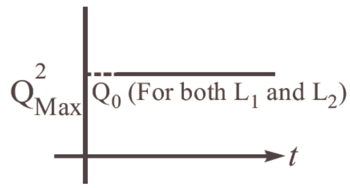
(a) 6.7 mA (b) 0.67 mA
(c) 100 mA (d) 67 mA

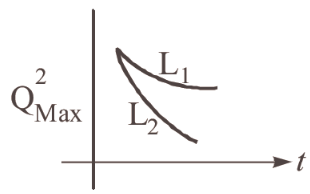
25. An LCR circuit is equivalent to a damped pendulum. In an LCR circuit the capacitor is charged to Q_0 and then connected to the L and R as shown below : **[2015]**

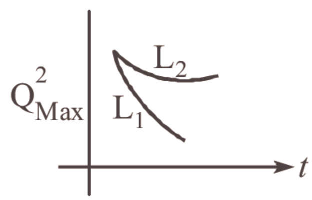


If a student plots graphs of the square of maximum charge $\left(Q^2_{Max}\right)$ on the capacitor with time(t) for two different values L_1 and L_2 ($L_1 > L_2$) of L then which of the following represents this graph correctly? (*plots are schematic and not drawn to scale*)

(a) 

(b) 

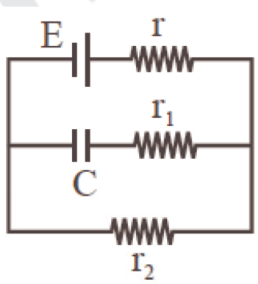
(c) 

(d) 

26. An arc lamp requires a direct current of 10 A at 80 V to function. If it is connected to a 220 V (rms), 50 Hz AC supply, the series inductor needed for it to work is close to : **[2016]**

(a) 0.044 H (b) 0.065 H
(c) 80 H (d) 0.08 H

27. In the given circuit diagram when the current reaches steady state in the circuit, the charge on the capacitor of capacitance C will be **[2017]**



(a) $CE\dfrac{r_2}{(r+r_2)}$ (b) $CE\dfrac{r_1}{(r_1+r)}$

(c) CE (d) $CE\dfrac{r_1}{(r_2+r)}$

28. In an a.c. circuit, the instantaneous e.m.f. and current are given by

$e = 100 \sin 30t$

$i = 20 \sin\left(30t - \dfrac{\pi}{4}\right)$

In one cycle of a.c., the average power consumed by the circuit and the wattless current are, respectively: **[2018]**

(a) 50W, 10A (b) $\dfrac{1000}{\sqrt{2}}$ W, 10A

(c) $\dfrac{50}{\sqrt{2}}$ W, 0 (d) 50W, 0

29. For an RLC circuit driven with voltage of amplitude v_m and frequency $\omega_0 = \dfrac{1}{\sqrt{LC}}$ the current exhibits resonance. The quality factor, Q is given by: **[2018]**

(a) $\dfrac{\omega_0 L}{R}$ (b) $\dfrac{\omega_0 R}{L}$

(c) $\dfrac{R}{(\omega_0 C)}$ (d) $\dfrac{CR}{\omega_0}$

30. A LCR circuit behaves like a damped harmonic oscillator. Comparing it with a physical spring-mass damped oscillator having damping constant 'b', the correct equivalence would be: **[2020]**

(a) $L \leftrightarrow m, C \leftrightarrow k, R \leftrightarrow b$

(b) $L \leftrightarrow \dfrac{1}{b}, C \leftrightarrow \dfrac{1}{m}, R \leftrightarrow \dfrac{1}{k}$

(c) $L \leftrightarrow k, C \leftrightarrow b, R \leftrightarrow m$

(d) $L \leftrightarrow m, C \leftrightarrow , R \leftrightarrow b$

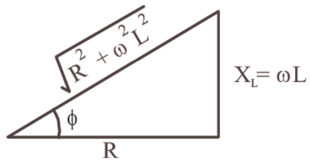
Answer Key

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
(b)	(d)	(b)	(c)	(a)	(a)	(d)	(a)	(a)	(b)	(a)	(c)	(a)	(c)	(a)
16	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**
(c)	(c)	(d)	(a)	(b)	(c)	(c)	(c)	(b)	(c)	(b)	(a)	(b)	(a)	(d)

Solutions

1. (b) Resistance of the inductor, $X_L = \omega L$

The impedance triangle for resistance (R) and inductor (L) connected in series is shown in the figure.



Net impedance of circuit $Z = \sqrt{X_L^2 + R^2}$

Power factor, $\cos\phi = \frac{R}{Z}$

$\Rightarrow \cos\phi = \frac{R}{\sqrt{R^2+\omega^2L^2}}$

2. (d) All three inductors are connected in parallel. The equivalent inductance L_p is given by

$$\frac{1}{L_p} = \frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3} = \frac{1}{3} + \frac{1}{3} + \frac{1}{3} = \frac{3}{3} = 1$$

$\therefore L_p = 1$

3. (b) Number of turns in primary

$N_p = 140$

Number of turns in secondary $N_s = 280$,

$I_p = 4A, I_s = ?$

Using transformation ratio for a transformer

$$\frac{I_s}{I_p} = \frac{N_p}{N_s}$$

$$\Rightarrow \frac{I_s}{4} = \frac{140}{280}$$

$\Rightarrow I_s = 2\,A$

4. (c) When the capacitor is completely charged, the total energy in the LC circuit is with the capacitor and that energy is given by

$$U_{max} = \frac{1}{2}\frac{Q^2}{C}$$

When half energy is with the capacitor in the form of electric field between the plates of the capacitor we get

$$\frac{U_{max}}{2} = \frac{1}{2}\frac{q'^2}{C}$$

Here q' is the charge on the plate of capacitor when energy is shared equally.

$$\therefore \frac{1}{2}\times\frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}\frac{q'^2}{C} \Rightarrow q' = \frac{Q}{\sqrt{2}}$$

5. (a) Laminated core provide less area of cross-section for the current to flow. Because of this, resistance of the core increases and current decreases there by decreasing the energy loss due to eddy current.

6. (a) D.C. ammeter measure average value of current. In AC current, average value of current in complete cycle is zero. Hence reading will be zero.

7. (d) In a series LCR circuit voltage across the inductor and capacitor are in opposite phase

$\therefore$ Net voltage difference across

$LC = 50 - 50 = 0$

8. (a) Resonant frequency, $F_r = \frac{1}{2\pi\sqrt{LC}}$

For resonant frequency to remain same

LC = constant

$\therefore LC = L'C'$

$\Rightarrow LC = L' \times 2C$

$\Rightarrow L' = \frac{L}{2}$

9. (a) Phase difference for R–L circuit lies

between $\left(0, \frac{\pi}{2}\right)$ but 0 or $\pi/2$

10. (b) Given, Resistance of circuit, $R = 12\,\Omega$

Imedance of circuit, $Z = 15\,\Omega$

Power factor $= \cos\phi = \frac{R}{Z} = \frac{12}{15} = \frac{4}{5} = 0.8$

11. (a) Current in inductor circuit is given by,

$$i = i_0\left(1 - e^{-\frac{Rt}{L}}\right)$$

$$\frac{i_0}{2} = i_0(1 - e^{-\frac{Rt}{L}}) \Rightarrow e^{-\frac{Rt}{L}} = \frac{1}{2}$$

Taking log on both the sides,

$$-\frac{Rt}{L} = \log 1 - \log 2$$

$$\Rightarrow t = \frac{L}{R}\log 2 = \frac{300\times10^{-3}}{2}\times 0.69$$

$\Rightarrow t = 0.1$ sec.

12. **(c)** At resonance $X_L = X_C$, $Z = R$

$\therefore$ Current, $I = \dfrac{V}{R} = \dfrac{100}{1000} = 0.1\ A$

At resonance,

$$X_L = X_C = \frac{1}{\omega C} = \frac{1}{200 \times 2 \times 10^{-6}} = 2500$$

Voltage across L is

$V_L = IX_L$

$= 0.1 \times 2500 = 250\,\text{V}$

13. **(a)** Initially, when steady state is achieved,

$i = \dfrac{E}{R}$

Let E is short circuited at $t = 0$. Then

At $t = 0$

Maximum current, $i_0 = \dfrac{E}{R} = \dfrac{100}{100} = 1A$

Let during decay of current at any time the current flowing is $-L\dfrac{di}{dt} - iR = 0$

$$\Rightarrow \frac{di}{i} = -\frac{R}{L}dt$$

$$\Rightarrow \int_{i_0}^{i} \frac{di}{i} = \int_0^t -\frac{R}{L}dt$$

$$\Rightarrow \log_e \frac{i}{i_0} = -\frac{R}{L}t$$

$$\Rightarrow i = i_0\, e^{-\frac{R}{L}t}$$

$$\Rightarrow i = \frac{E}{R}e^{-\frac{R}{L}t} = 1 \times e^{\frac{-100\times10^{-3}}{100\times10^{-3}}} = \frac{1}{e}$$

14. **(c)** We know that power consumed in a.c. circuit is given by,

$P = E_{\text{rms}}.I_{\text{rms}} \cos\phi$

Here, $E = E_0 \sin \omega t$

$I = I_0 \sin\left(\omega t - \dfrac{\pi}{2}\right)$

This means the phase difference, is $\phi = \dfrac{\pi}{2}$

$\because \cos\phi = \cos\dfrac{\pi}{2} = 2$

$\therefore\ P = E_{rms}.I_{rms}.\cos\dfrac{\pi}{2} = 0$

15. **(a)** Given, resistance of resistor, $R = 5\,\Omega$

Maximum current in the circuit

$I_0 = \dfrac{V}{R} = \dfrac{5}{5} = 1A$

When current is in growth in LR circuit

$$I = I_o\left(1 - e^{-\frac{R}{L}t}\right) = \frac{E}{R}\left(1 - e^{-\frac{R}{L}t}\right)$$

$$= \frac{5}{5}\left(1 - e^{-\frac{5}{10}\times 2}\right) = (1 - e^{-1})$$

16. **(c)** Growth in current in branch containing L and R_2 when switch is closed is given by

$$i = \frac{E}{R_2}[1 - e^{-R_2t/L}]$$

$$\Rightarrow \frac{di}{dt} = \frac{E}{R_2}\cdot\frac{R_2}{L}\cdot e^{-R_2t/L} = \frac{E}{L}e^{-\frac{R_2t}{L}}$$

Hence, potential drop across L

$$V_L = \frac{Ldi}{dt} = \left(\frac{E}{L}e^{-R_2t/L}\right)L$$

$$= Ee^{-R_2t/L} = 12e^{-\frac{2t}{400\times10^{-3}}} = 12\text{e}^{-5t}\text{V}$$

17. **(c)** At $t = 0$, no current will flow through L and R_1 as inductor will offer infinite resistance.

$\therefore$ Current through battery, $i = \dfrac{V}{R_2}$

At $t = \infty$, inductor behave as conducting wire

Effective resistance, $R_{eff} = \dfrac{R_1R_2}{R_1 + R_2}$

$\therefore$ Current through battery $= \dfrac{V}{R_{eff}}$

$= \dfrac{V(R_1 + R_2)}{R_1R_2}$

18. **(d)** When only the capacitance is removed phase difference between current and voltage is

$\tan\phi = \dfrac{X_L}{R}$

$\Rightarrow \tan\phi = \dfrac{\omega L}{R}$

$\Rightarrow \omega L = R\tan\phi = 200 \times \dfrac{1}{\sqrt{3}} = \dfrac{200}{\sqrt{3}}$

When only inductor is removed, phase difference between current and voltage is

$\therefore\ \tan\phi = \dfrac{1}{\omega CR}$

$\Rightarrow \dfrac{1}{\omega C} = R\tan\phi = 200 \times \dfrac{1}{\sqrt{3}} = \dfrac{200}{\sqrt{3}}$

Impedance of the circuit,

$$Z=\sqrt{R^2+\left(\frac{1}{\omega C}-\omega L\right)^2}$$

$$=\sqrt{(200)^2+\left(\frac{200}{\sqrt{3}}-\frac{200}{\sqrt{3}}\right)^2}=200\,\Omega$$

Power dissipated in the circuit

$$=V_{rms}I_{rms}\cos\phi$$

$$=V_{rms}\cdot\frac{V_{rms}}{Z}\cdot\frac{R}{Z}\left(\because \cos\phi=\frac{R}{Z}\right)$$

$$=\frac{V^2_{rms}R}{Z^2}=\frac{(220)^2\times 200}{(200)^2}$$

$$=\frac{220\times 220}{200}=242\text{ W}$$

19. (a) Energy stored in magnetic field $=\frac{1}{2}\text{Li}^2$

Energy stored in electric field $=\frac{1}{2}\frac{q^2}{C}$

Energy will be equal when

$$\therefore\ \frac{1}{2}Li^2=\frac{1}{2}\frac{q^2}{C}$$

$\tan\omega t=1$

$q=q_0\cos\omega t$

$$\Rightarrow \frac{1}{2}L(\omega q_0\sin\omega t)^2=\frac{(q_0\cos\omega t)^2}{2C}$$

$$\Rightarrow \omega=\frac{1}{\sqrt{LC}}$$

$$\Rightarrow \omega t=\frac{\pi}{4}$$

$$\Rightarrow t=\frac{\pi}{4}\sqrt{LC}$$

20. (b) We have, $V=V_0(1-e^{-t/RC})$

$\Rightarrow 120=200(1-e^{-t/RC})$

$$e^{-t/r}=\frac{200-120}{200}=\frac{80}{200}$$

$t=\log_e(2.5)$

$\Rightarrow t=RC\,\text{in}\,(2.5)$ $\quad[\because r=RC]$

$\Rightarrow R=2.71\times 10^6\,\Omega$

21. (c) Time constant for parallel combination $=2RC$

Time constant for series combination $=\frac{RC}{2}$

In first case :

$$V=V_0\left(1-e^{-\frac{t}{CR}}\right)\Rightarrow \frac{V_0}{2}=V_0-V_0e^{-\frac{t}{CR}}$$

$$V=V_0e^{-\frac{t_1}{2RC}}=\frac{V_0}{2}\qquad ...(1)$$

In second case :

In series grouping, equivalent capacitance $=\frac{C}{2}$

$$V=V_0e^{-\frac{t_2}{(RC/2)}}=\frac{V_0}{2}\qquad ....(2)$$

From (1) and (2)

$$\frac{t_1}{2RC}=\frac{t_2}{(RC/2)}$$

$$\Rightarrow t_2=\frac{t_1}{4}=\frac{10}{4}=2.5\text{ sec.}$$

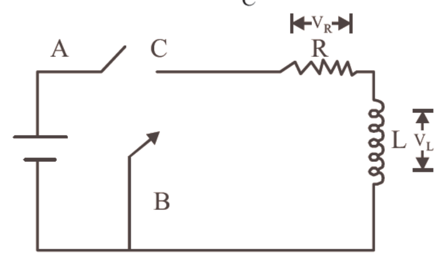
22. (c) Charge on he capacitor at any time t is given by $q=\text{CV}(1-e^{t/\tau})$

at $t=2\tau$

$q=\text{CV}(1-e^{-2})$

23. (c) Applying Kirchhoff's law of voltage in closed loop

$$-V_R-V_C=0\ \Rightarrow\ \frac{V_R}{V_C}=-1$$



24. (b) $I(0)=\frac{15\times 100}{0.15\times 10^3}=0.1A$

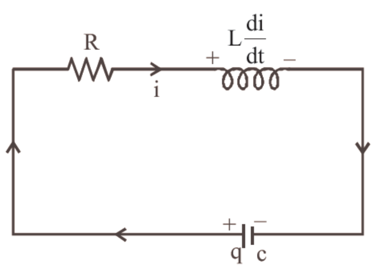
$I(\infty)=0$

$$I(t)=[\text{I}(0)-\text{I}(\infty)]\ e^{\frac{-t}{L/R}}+i(\infty)$$

$$I(t)=0.1\ e^{\frac{-t}{L/R}}=0.1\,e^{\frac{R}{L}}$$

$$I(t)=0.1\ e^{\frac{0.15\times 1000}{0.03}}=0.67mA$$

25. (c) From KVL at any time t



$$\frac{q}{c}-iR-L\frac{di}{dt}=0$$

$$i=-\frac{dq}{dt}\Rightarrow\frac{q}{c}+\frac{dq}{dt}R+\frac{Ld^2q}{dt^2}=0$$

$$\frac{d^2q}{dt^2}+\frac{R}{L}\frac{dq}{dt}+\frac{q}{Lc}=0$$

From damped harmonic oscillator, the amplitude is given by A $=A_o e-\frac{dt}{2m}$

Double differential equation

$$\frac{d^2x}{dt^2}+\frac{b}{m}\frac{dx}{dt}+\frac{k}{m}x=0$$

$$Q_{\max}=Q_o e^{-\frac{Rt}{2L}}\Rightarrow Q_{\max}^2=Q_o^2 e^{-\frac{Rt}{L}}$$

Hence damping will be faster for lesser self inductance.

26. (b) Here

$$i=\frac{e}{\sqrt{R^2+X_L^2}}=\frac{e}{\sqrt{R^2+\omega^2L^2}}=\frac{e}{\sqrt{R^2+4\pi^2v^2L^2}}$$

$$10=\frac{220}{\sqrt{64+4\pi^2(50)^2L}}$$

$$[\because R=\frac{V}{I}=\frac{80}{10}=8]$$

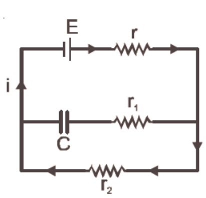
On solving we get

$$L=0.065\,H$$

27. (a) In steady state, flow of current through capacitor will be zero.

Current through the circuit,

$$i=\frac{E}{r+r_2}$$



Potential difference through capacitor

$$V_c=\frac{Q}{C}=E-ir=E-\left(\frac{E}{r+r_2}\right)r$$

$$\therefore\quad Q=CE\frac{r_2}{r+r_2}$$

28. (b) As we know, average power $P_{avg}=V_{rms}I_{rms}\cos\theta$

$$=\left(\frac{V_0}{\sqrt{2}}\right)\left(\frac{I_0}{\sqrt{2}}\right)\cos\theta=\left(\frac{100}{\sqrt{2}}\right)\left(\frac{20}{\sqrt{2}}\right)\cos 45°$$

$(\because\theta=45°)$

$$P_{avg}=\frac{1000}{\sqrt{2}}\text{ watt}$$

Wattless current $I=I_{rms}\sin\theta$

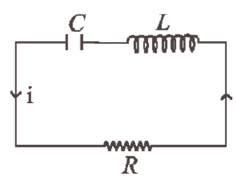
$$=\frac{I_0}{\sqrt{2}}\sin\theta=\frac{20}{\sqrt{2}}\sin 45°=10A$$

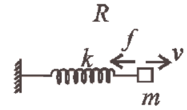
29. (a) Quality factor $Q=\frac{\omega_0}{2\Delta\omega}=\frac{\omega_0 L}{R}$

30. (d) In damped harmonic oscillation,

$$\frac{md^2x}{dt^2}=-kx-bv$$

$$\Rightarrow\frac{md^2x}{dt^2}+b\frac{dx}{dt}+kx=0\qquad\text{...(i)}$$





In LCR circuit, $\frac{-q}{C}-iR-\frac{Ldi}{dt}=0$

$$L\frac{d^2}{dt^2}+R\frac{dq}{dt}+\frac{q}{C}=0\qquad\text{...(ii)}$$

Comparing equations (i) & (ii)

$$L\leftrightarrow m,\ C\leftrightarrow\frac{1}{k},R\leftrightarrow b$$

22 Electromagnetic Waves

1. *Electromagnetic waves are transverse in nature* is evident by **[2002]**
   (a) polarization (b) interference
   (c) reflection (d) diffraction
2. An electromagnetic wave of frequency $v = 3.0$ MHz passes from vacuum into a dielectric medium with permittivity $\in = 4.0$. Then **[2004]**
   (a) wave length is halved and frequency remains unchanged
   (b) wave length is doubled and frequency becomes half
   (c) wave length is doubled and the frequency remains unchanged
   (d) wave length and frequency both remain unchanged.
3. An electromagnetic wave in vacuum has the electric and magnetic field $\vec{E}$ and $\vec{B}$, which are always perpendicular to each other. The direction of polarization is given by $\vec{X}$ and that of wave propagation by $\vec{k}$. Then **[2012]**
   (a) $\vec{X} \parallel \vec{B}$ and $\vec{k} \parallel \vec{B} \times \vec{E}$
   (b) $\vec{X} \parallel \vec{E}$ and $\vec{k} \parallel \vec{E} \times \vec{B}$
   (c) $\vec{X} \parallel \vec{B}$ and $\vec{k} \parallel \vec{E} \times \vec{B}$
   (d) $\vec{X} \parallel \vec{E}$ and $\vec{k} \parallel \vec{B} \times \vec{E}$
4. The magnetic field in a travelling electromagnetic wave has a peak value of 20 nT. The peak value of electric field strength is: **[2013]**
   (a) 3 V/m (b) 6 V/m
   (c) 9 V/m (d) 12 V/m
5. During the propagation of electromagnetic waves in a medium: **[2014]**
   (a) Electric energy density is double of the magnetic energy density.
   (b) Electric energy density is half of the magnetic energy density.
   (c) Electric energy density is equal to the magnetic energy density.
   (d) Both electric and magnetic energy densities are zero.
6. Match List - I (Electromagnetic wave type) with List - II (Its association/application) and select the correct option from the choices given below the lists: **[2014]**

	List 1		List 2
1.	Infrared waves	(i)	To treat muscular strain
2.	Radio waves	(ii)	For broadcasting
3.	X-rays	(iii)	To detect fracture of bones
4.	Ultraviolet rays	(iv)	Absorbed by the ozone layer of the atmosphere

	1	2	3	4
(a)	(iv)	(iii)	(ii)	(i)
(b)	(i)	(ii)	(iv)	(iii)
(c)	(iii)	(ii)	(i)	(iv)
(d)	(i)	(ii)	(iii)	(iv)

7. Arrange the following electromagnetic radiations per quantum in the order of increasing energy : **[2016]**

A : Blue light B : Yellow light
C : X-ray D : Radiowave.
(a) C, A, B, D (b) B, A, D, C
(c) D, B, A, C (d) A, B, D, C

8. An EM wave from air enters a medium. The electric fields are $\vec{E}_1 = E_{01}\hat{x}\cos\left[2\pi v\left(\frac{z}{c}-t\right)\right]$ in air and $\vec{E}_2 = E_{02}\hat{x}\cos\left[k(2z-ct)\right]$ in medium, where the wave number k and frequency v refer to their values in air. The medium is nonmagnetic. If $\in_{r_1}$ and $\in_{r_2}$ refer to relative permittivities of air and medium respectively, which of the following options is correct? **[2019]**

(a) $\frac{\in_{r_1}}{\in_{r_2}} = 4$ (b) $\frac{\in_{r_1}}{\in_{r_2}} = 2$

(c) $\frac{\in_{r_1}}{\in_{r_2}} = \frac{1}{4}$ (d) $\frac{\in_{r_1}}{\in_{r_2}} = \frac{1}{2}$

9. A plane electromagnetic wave of frequency 50 MHz travels in free space along the positive x-direction. At a particular point in space and time, $\vec{E} = 6.3\,\hat{j}$ V/m. The corresponding magnetic field $\vec{B}$, at that point will be: **[2019]**

(a) $18.9 \times 10^{-8}\,\hat{k}T$ (b) $2.1 \times 10^{-8}\,\hat{k}T$

(c) $6.3 \times 10^{-8}\,\hat{k}T$ (d) $18.9 \times 10^{8}\,\hat{k}T$

10. The magnetic field of a plane electromagnetic wave is given by:

$\vec{B} = B_0\hat{i}[\cos(kz-\omega t)] + B_1\hat{j}\cos(kz+\omega t)$

Where $B_0 = 3 \times 10^{-5}$ T and $B_1 = 2 \times 10^{-6}$ T.
The rms value of the force experienced by a stationary charge $Q = 10^{-4}$ C at $z = 0$ is closest to: **[2019]**

(a) 0.6 N (b) 0.1 N
(c) 0.9 N (d) 3×10^{-2} N

11. If the magnetic field in a plane electromagnetic wave is given by $\vec{B} = 3 \infty 10^{-8} \sin(1.6 \infty 10^3 x + 48 \infty 10^{10} t)\,\hat{j}$ T, then what will be expression for electric field? **[2020]**

(a) $\vec{E} = (60 \sin(1.6 \infty 10^3 x + 48 \infty 10^{10} t)$ v/m)

(b) $\vec{E} = (9 \sin(1.6 \infty 10^3 x + 48 \infty 10^{10} t)$v/m)

(c) $\vec{E} = (3 \infty 10^{-8} \sin(1.6 \infty 10^3 x + 48 \infty 10^{10} t)$ v/m)

(d) $\vec{E} = (3 \infty 10^{-8} \sin(1.6 \infty 10^3 x + 48 \infty 10^{10} t)$ v/m)

Answer Key

1	2	3	4	5	6	7	8	9	10	11
(a)	(a)	(b)	(b)	(c)	(d)	(c)	(c)	(b)	(a)	(b)

Solutions

1. **(a)** The phenomenon of polarisation is shown only by transverse waves. The vibration of electromagnetic wave are restricted through polarization in a direction perpendicular to wave propagation.

2. **(a)** Frequency remains unchanged during refraction

Velocity of EM wave in vacuum

$$V_{\text{vacuum}} = \frac{1}{\sqrt{\mu_0 \in_0}} = C$$

$$v_{\text{med}} = \frac{1}{\sqrt{\mu_0 \in_0 \times 4}} = \frac{c}{2}$$

$$\frac{\lambda_{\text{med}}}{\lambda_{\text{vacuum}}} = \frac{v_{\text{med}}}{v_{\text{vacuum}}} = \frac{c/2}{c} = \frac{1}{2}$$

$\therefore$ Wavelength is halved and frequency remains unchanged

3. **(b)** $\because$ The E.M. wave are transverse in nature *i.e.*,

$$= \frac{\vec{k} \times \vec{E}}{\mu} = \vec{H} \quad \ldots\text{(i)}$$

where $\vec{H} = \frac{\vec{B}}{\mu}$

and $\frac{\vec{k} \times \vec{H}}{\omega\varepsilon} = -\vec{E} \quad \ldots\text{(ii)}$

$\vec{k}$ is $\perp$ $\vec{H}$ and $\vec{k}$ is also $\perp$ to $\vec{E}$

The direction of wave propagation is parallel to $\vec{E} \times \vec{B}$.

The direction of polarization is parallel to electric field.

4. **(b)** From question,

$B_0 = 20\,nT = 20 \times 10^{-9} T$

($\because$ velocity of light in vacuum $C = 3 \times 10^8$ ms^{-1})

$\vec{E}_0 = \vec{B}_0 \times \vec{C}$

$|\vec{E}_0| = |\vec{B}| \cdot |\vec{C}| = 20 \times 10^{-9} \times 3 \times 10^8$

$= 6$ V/m.

5. **(c)** $E_0 = CB_0$ and $C = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$

Electric energy density $= \frac{1}{2}\varepsilon_0 E_0{}^2 = \mu_E$

Magnetic energy density $= \frac{1}{2}\frac{Bo^2}{\mu_0} = \mu_B$

Thus, $\mu_E = \mu_B$

Energy is equally divided between electric and magnetic field.

6. **(d)**

(1) Infrared rays are used to treat muscular strain because these are heat rays.

(2) Radio waves are used for broadcasting because these waves have very long wavelength ranging from few centimeters to few hundred kilometers.

(3) X-rays are used to detect fracture of bones because they have high penetrating power but they can't penetrate through denser medium like dones.

(4) Ultraviolet rays are absorbed by ozone of the atmosphere.

7. **(c)** $\xrightarrow{\text{E, Decreases}}$

γ-rays X-rays uv-rays Visible rays VIBGYOR IR rays Microwaves Radio waves

Radio wave < yellow light < blue light < X-rays

(Increasing order of energy)

8. **(c)** Velocity of EM wave is given by $v = \frac{1}{\sqrt{\mu \in}}$

Velocity in air $= \frac{\omega}{k} = C$

Velocity in medium = $\frac{C}{2}$

Here, $\mu_1 = \mu_2 = 1$ as medium is non-magnetic

$$\therefore \quad \frac{\frac{1}{\sqrt{\epsilon_{r_1}}}}{\frac{1}{\sqrt{\epsilon_{r_2}}}} = \frac{C}{\left(\frac{C}{2}\right)} = 2 \quad \Rightarrow \quad \frac{\epsilon_{r_1}}{\epsilon_{r_2}} = \frac{1}{4}$$

9. (b) As we know,

$$|B| = \frac{|E|}{C} = \frac{6.3}{3\times10^8} = 2.1\times10^{-8}T$$

As $\vec{V} \perp \vec{E} \perp \vec{B}$ therefore direction of $\vec{B}$ is in z-direction

$$\vec{B} = 2.1\times10^{-8}\hat{k}T$$

10. (a) $B_0 = \sqrt{B_0^2 + B_1^2} = \sqrt{30^2 + 2^2} \times 10^{-6}$

$\approx 30 \times 10^{-6}\,T$

$\therefore E_0 = cB = 3 \times 10^8 \times 30 \times 10^{-6}$

$= 9 \times 10^3$ V/m

$$E_{rms} = \frac{E_0}{\sqrt{2}} = \frac{9}{\sqrt{2}} \times 10^3 \text{ V/m}$$

Force on the charge,

$$F = E_{rms}Q = \frac{9}{\sqrt{2}} \times 10^3 \times 10^{-4} \simeq 0.64N$$

11. (b) Given,

$$\vec{B} = 3\times10^{-8}\sin(1.6\times10^3 x + 48\times10^{10} t)\hat{j}\,T$$

Using,

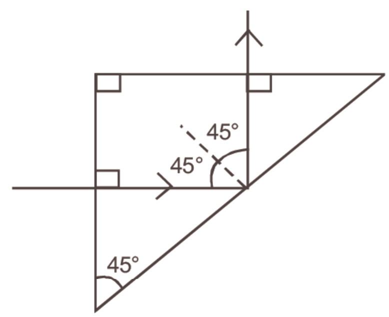
$E_0 = B_0 \times C = 3 \times 10^{-8} \times 3 \times 10^8 = 9$ V/m

Electric field,

$$\vec{E} = 9\sin(1.6\times10^3 x + 48\times10^{10} t)\hat{k} \text{ V/m}$$

23 Ray Optics and Optical Instruments

1. An astronomical telescope has a large aperture to **[2002]**
   (a) reduce spherical aberration
   (b) have high resolution
   (c) increase span of observation
   (d) have low dispersion
2. If two mirrors are kept at 60° to each other, then the number of images formed by them is **[2002]**
   (a) 5 (b) 6
   (c) 7 (d) 8
3. Which of the following is used in optical fibres? **[2002]**
   (a) total internal reflection
   (b) scattering
   (c) diffraction
   (d) refraction.
4. Consider telecommunication through optical fibres. Which of the following statements is **not** true? **[2003]**
   (a) Optical fibres can be of graded refractive index
   (b) Optical fibres are subject to electromagnetic interference from outside
   (c) Optical fibres have extremely low transmission loss
   (d) Optical fibres may have homogeneous core with a suitable cladding.
5. The image formed by an objective of a compound microscope is **[2003]**
   (a) virtual and diminished
   (b) real and diminished
   (c) real and enlarged
   (d) virtual and enlarged
6. To get three images of a single object, one should have two plane mirrors at an angle of **[2003]**
   (a) 60º (b) 90º
   (c) 120º (d) 30º
7. A light ray is incident perpendicularly to one face of a 90° prism and is totally internally reflected at the glass-air interface. If the angle of reflection is 45°, we conclude that the refractive index n **[2004]**



   (a) $n > \frac{1}{\sqrt{2}}$ (b) $n > \sqrt{2}$
   (c) $n < \frac{1}{\sqrt{2}}$ (d) $n < \sqrt{2}$
8. A plano convex lens of refractive index 1.5 and radius of curvature 30 cm, is silvered at the curved surface. Now this lens has been used to form the image of an object. At what distance from this lens an object be placed in order to have a real image of size of the object **[2004]**
   (a) 60 cm (b) 30 cm
   (c) 20 cm (d) 80 cm

9. A fish looking up through the water sees the outside world contained in a circular horizon. If the refractive index of water is $\frac{4}{3}$ and the fish is 12 cm below the surface, the radius of this circle in cm is **[2005]**

(a) $\frac{36}{\sqrt{7}}$ (b) $36\sqrt{7}$

(c) $4\sqrt{5}$ (d) $36\sqrt{5}$

10. A thin glass (refractive index 1.5) lens has optical power of $-5\,D$ in air. Its optical power in a liquid medium with refractive index 1.6 will be **[2005]**

(a) $-1D$ (b) $1\,D$

(c) $-25\,D$ (d) $25\,D$

11. The refractive index of a glass is 1.520 for red light and 1.525 for blue light. Let D_1 and D_2 be angles of minimum deviation for red and blue light respectively in a prism of this glass. Then, **[2006]**

(a) $D_1 < D_2$

(b) $D_1 = D_2$

(c) D_1 can be less than or greater than D_2 depending upon the angle of prism

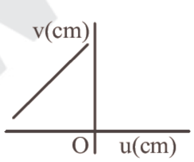
(d) $D_1 > D_2$

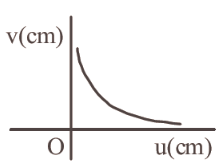
12. Two lenses of power $-15\ D$ and $+5\ D$ are in contact with each other. The focal length of the combination is **[2007]**

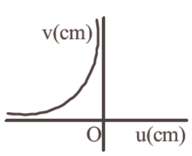
(a) $+10$ cm (b) -20 cm

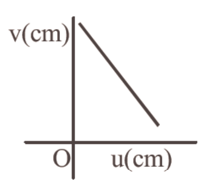
(c) -10 cm (d) $+20$ cm

13. A student measures the focal length of a convex lens by putting an object pin at a distance 'u' from the lens and measuring the distance 'v' of the image pin. The graph between 'u' and 'v' plotted by the student should look like **[2008]**

(a)


(b)


(c)


(d)


14. An experment is performed to find the refractive index of glass using a travelling microscope. In this experiment distances are measured by **[2008]**

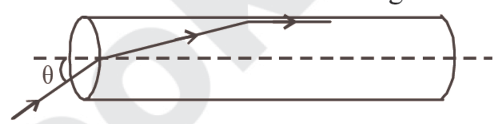
(a) a vernier scale provided on the microscope

(b) a standard laboratory scale

(c) a meter scale provided on the microscope

(d) a screw gauge provided on the microscope

15. A transparent solid cylindrical rod has a refractive index of $\frac{2}{\sqrt{3}}$. It is surrounded by air. A light ray is incident at the mid-point of one end of the rod as shown in the figure.



The incident angle θ for which the light ray grazes along the wall of the rod is : **[2009]**

(a) $\sin^{-1}\left(\sqrt{3}/2\right)$ (b) $\sin^{-1}\left(\frac{2}{\sqrt{3}}\right)$

(c) $\sin^{-1}\left(\frac{1}{\sqrt{3}}\right)$ (d) $\sin^{-1}(1/2)$

16. In an optics experiment, with the position of the object fixed, a student varies the position of a convex lens and for each position, the screen is adjusted to get a clear image of the object. A graph between the object distance u and the image distance v, from the lens, is plotted using the same scale for the two axes. A straight line passing through the origin and making an angle of 45° with the x-axis meets the experimental curve at P. The coordinates of P will be **[2009]**

(a) $\left(\frac{f}{2}, \frac{f}{2}\right)$ (b) (f, f)

(c) $(4f, 4f)$ (d) $(2f, 2f)$

17. Let the x-z plane be the boundary between two transparent media. Medium 1 in $z \geq 0$ has a refractive index of $\sqrt{2}$ and medium 2 with $z < 0$ has a refractive index of $\sqrt{3}$. A ray of light in medium 1 given by the vector $\vec{A} = 6\sqrt{3}\hat{i} + 8\sqrt{3}\hat{j} - 10\hat{k}$ is incident on the plane of separation. The angle of refraction in medium 2 is: **[2011]**

(a) 45° (b) 60°

(c) 75° (d) 30°

18. A car is fitted with a convex side-view mirror of focal length 20 cm. A second car 2.8 m behind the first car is overtaking the first car at a relative speed of 15 m/s. The speed of the image of the second car as seen in the mirror of the first one is : **[2011]**

(a) $\frac{1}{15}$ m/s (b) 10 m/s

(c) 15 m/s (d) $\frac{1}{10}$ m/s

19. A beaker contains water up to a height h_1 and kerosene of height h_2 above water so that the total height of (water + kerosene) is $(h_1 + h_2)$. Refractive index of water is μ_1 and that of kerosene is μ_2. The apparent shift in the position of the bottom of the beaker when viewed from above is **[2011 RS]**

(a) $\left(1+\frac{1}{\mu_1}\right)h_1-\left(1+\frac{1}{\mu_2}\right)h_2$

(b) $\left(1-\frac{1}{\mu_1}\right)h_1+\left(1-\frac{1}{\mu_2}\right)h_2$

(c) $\left(1+\frac{1}{\mu_1}\right)h_2-\left(1+\frac{1}{\mu_2}\right)h_1$

(d) $\left(1-\frac{1}{\mu_1}\right)h_2+\left(1-\frac{1}{\mu_2}\right)h_1$

20. When monochromatic red light is used instead of blue light in a convex lens, its focal length will **[2011 RS]**

(a) increase
(b) decrease
(c) remain same
(d) does not depend on colour of light

21. An object at 2.4 m in front of a lens forms a sharp image on a film 12 cm behind the lens. A glass plate 1 cm thick, of refractive index 1.50 is interposed between lens and film with its plane faces parallel to film. At what distance (from lens) should object shifted to be in sharp focus of film? **[2012]**

(a) 7.2 m (b) 2.4 m

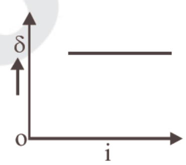
(c) 3.2 m (d) 5.6 m

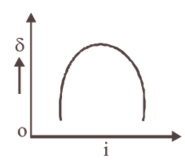
22. Diameter of a plano-convex lens is 6 cm and thickness at the centre is 3 mm. If speed of light in material of lens is 2×10^8 m/s, the focal length of the lens is **[2013]**

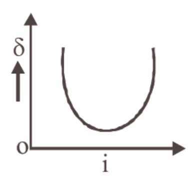
(a) 15 cm (b) 20 cm

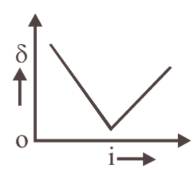
(c) 30 cm (d) 10 cm

23. The graph between angle of deviation (δ) and angle of incidence (i) for a triangular prism is represented by **[2013]**

(a) 

(b) 

(c) 

(d) 

24. A thin convex lens made from crown glass $\left(\mu=\frac{3}{2}\right)$ has focal length f. When it is measured in two different liquids having refractive indices $\frac{4}{3}$ and $\frac{5}{3}$, it has the focal lengths f_1 and f_2 respectively. The correct relation between the focal lengths is: **[2014]**

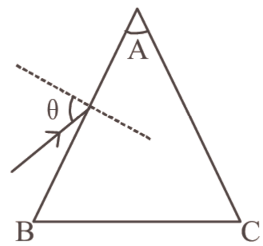
(a) $f_1 = f_2 < f$

(b) $f_1 > f$ and f_2 becomes negative

(c) $f_2 > f$ and f_1 becomes negative

(d) f_1 and f_2 both become negative

25. Monochromatic light is incident on a glass prism of angle A. If the refractive index of the material of the prism is μ, a ray, incident at an angle θ, on the face AB would get transmitted through the face AC of the prism provided : **[2015]**



(a) $\theta > \cos^{-1}\left[\mu \sin\left(A + \sin^{-1}\left(\frac{1}{\mu}\right)\right)\right]$

(b) $\theta < \cos^{-1}\left[\mu \sin\left(A + \sin^{-1}\left(\frac{1}{\mu}\right)\right)\right]$

(c) $\theta > \sin^{-1}\left[\mu \sin\left(A - \sin^{-1}\left(\frac{1}{\mu}\right)\right)\right]$

(d) $\theta < \sin^{-1}\left[\mu \sin\left(A - \sin^{-1}\left(\frac{1}{\mu}\right)\right)\right]$

26. An observer looks at a distant tree of height 10 m with a telescope of magnifying power of 20. To the observer the tree appears : **[2016]**

(a) 20 times taller (b) 20 times nearer
(c) 10 times taller (d) 10 times nearer

27. In an experiment for determination of refractive index of glass of a prism by $i - \delta$, plot it was found that a ray incident at angle 35°, suffers a deviation of 40° and that it emerges at angle 79°. In that case which of the following is closest to the maximum possible value of the refractive index? **[2016]**

(a) 1.7 (b) 1.8
(c) 1.5 (d) 1.6

28. A diverging lens with magnitude of focal length 25 cm is placed at a distance of 15 cm from a converging lens of magnitude of focal length 20 cm. A beam of parallel light falls on the diverging lens. The final image formed is : **[2017]**

(a) real and at a distance of 40 cm from the divergent lens
(b) real and at a distance of 6 cm from the convergent lens
(c) real and at a distance of 40 cm from convergent lens
(d) virtual and at a distance of 40 cm from convergent lens.

29. A convex lens is put 10 cm from a light source and it makes a sharp image on a screen, kept 10 cm from the lens. Now a glass block (refractive index 1.5) of 1.5 cm thickness is placed in contact with the light source. To get the sharp image again, the screem is shifted by a distance d. Then d is: **[2019]**

(a) 1.1 cm away from the lens
(b) 0
(c) 0.55 cm towards the lens
(d) 0.55 cm away from the lens

30. A concave mirror for face viewing has focal length of 0.4 m. The distance at which you hold the mirror from your face in order to see your image upright with a magnification of 5 is: **[2019]**

(a) 0.24m (b) 1.60m
(c) 0.32m (d) 0.16m

31. If we need a magnification of 375 from a compound microscope of tube length 150 mm and an objective of focal length 5 mm, the focal length of the eye-piece, should be close to: **[2020]**

(a) 22mm (b) 12mm
(c) 2mm (d) 33mm

Answer Key

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
(b)	(a)	(a)	(b)	(c)	(b)	(b)	(c)	(a)	(b)	(a)	(c)	(c)	(a)	(c)
16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
(d)	(a)	(a)	(b)	(a)	(d)	(c)	(c)	(b)	(c)	(b)	(c)	(c)	(d)	(c)
31														
(a)														

Solutions

1. **(b)** The resolving power of a telescope is

$$R.P = \frac{D}{1.22\lambda}$$

where D = diameter of the objective lens
λ = wavelength of light.

Clearly, $R.P \propto \frac{D}{\lambda}$

Resolving power of telescope resolution will be high if its objective is of large aperture.

2. **(a)** When two plane mirrors are inclined at each other at an angle θ then the number of the images (n) of a point object kept between the plane mirrors is

$$n = \frac{360°}{\theta} - 1,$$

(if $\frac{360°}{\theta}$ is even integer)

$\therefore$ Number of images formed $= \frac{360°}{60°} - 1 = 5$

3. **(a)** In an optical fibre, light is sent through the fibre without any loss by the phenomenon of total internal reflection. Total internal reflection of light waves confine the light rays inside the optical fiber.

4. **(b)** Optical fibres form a dielectric wave guide and are free from electromagnetic interference or radio frequency interference. There is extremely low transmission loss in optical fibre.

5. **(c)** A real, inverted and enlarged image of the object is formed by the objective lens of a compound microscope.

6. **(b)** The number of images formed is given by

$$n = \frac{360}{\theta} - 1$$

$$\Rightarrow \frac{360}{\theta} - 1 = 3$$

$$\Rightarrow \theta = \frac{360°}{4} = 90°$$

7. **(b)** For total internal reflection
Incident angle (i) > critical angle (i_c),

$\therefore \sin i > \sin i_c$

$\Rightarrow \sin 45° > \sin i_c$

$$\Rightarrow \sin i_c = \frac{1}{n}$$

$$\therefore \sin 45° > \frac{1}{n}$$

$$\Rightarrow \frac{1}{\sqrt{2}} > \frac{1}{n}$$

$\Rightarrow n > \sqrt{2}$

8. **(c)** The focal length(F) of the final mirror is

$$\frac{1}{F} = \frac{2}{f_\ell} + \frac{1}{f_m}$$

Using lens maker's formula

Here $\frac{1}{f_\ell} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

Here, $R_1 = \infty$
$R_2 = 30$ cm

$$= (1.5 - 1)\left[\frac{1}{\infty} - \frac{1}{-30}\right] = \frac{1}{60}$$

$$\therefore \frac{1}{F} = 2 \times \frac{1}{60} + \frac{1}{30/2} = \frac{1}{10}$$

$\therefore F = 10$ cm

Real image will be equal to the size of the object if the object distance

$u = 2F = 20$ cm

9. **(a)** From the figure it is clear that

$$\tan\theta_c = \frac{AB}{OA}$$

$\Rightarrow R = OA \tan\theta_c$

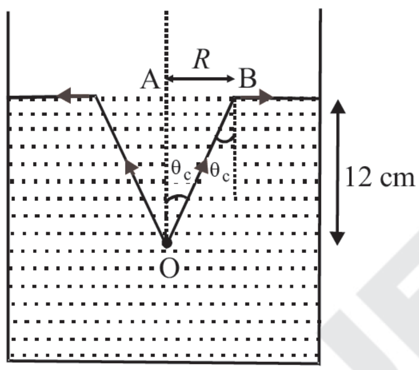
$$\Rightarrow R = \frac{OA\sin\theta_c}{\cos\theta_c}$$

$$\Rightarrow R = \frac{OA\sin\theta_c}{\sqrt{1-\sin^2\theta_c}}$$

$$\Rightarrow \tan\theta_c = \frac{R}{12} = \frac{\sin\theta_c}{\sqrt{1-\sin^2\theta_c}}$$

$$\because \sin\theta_c = \frac{1}{\mu} = \frac{3}{4}$$

$$\Rightarrow \tan\theta_c = \frac{3}{\sqrt{16-9}} = \frac{3}{\sqrt{7}} = \frac{R}{12}$$



$$\Rightarrow R = \frac{36}{\sqrt{7}} \text{ cm}$$

10. (b) According to lens maker's formula in air

$$\frac{1}{f_a} = \left({}_a\mu_g - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$\Rightarrow \frac{1}{f_a} = \left(\frac{1.5}{1} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \quad \text{.... (i)}$$

Using lens maker's formula in liquid medium,

$$\frac{1}{f_m} = \left(\frac{\mu_g}{\mu_m} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$\Rightarrow \frac{1}{f_m} = \left(\frac{1.5}{1.6} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \quad \text{.... (ii)}$$

Dividing (i) by (ii),

$$\frac{f_m}{f_a} = \left(\frac{1.5-1}{\frac{1.5}{1.6}-1}\right) = -8$$

$$P_a = -5 = \frac{1}{f_a}$$

$$\Rightarrow f_a = -\frac{1}{5}$$

$$\Rightarrow f_m = -8 \times f_a = -8 \times -\frac{1}{5} = \frac{8}{5}$$

$$P_m = \frac{\mu}{f_m} = \frac{1.6}{8} \times 5 = 1\text{D}$$

11. (a) When angle of prism is small,

Angle of deviation, $D = (\mu - 1)A$

Since $\lambda_b < \lambda_r$

$\Rightarrow \mu_r < \mu_b$

$\Rightarrow D_1 < D_2$

12. (c) When two thin lenses are in contact coaxially, power of combination is given by

$P = P_1 + P_2$

$= (-15 + 5)D$

$= -10D.$

Also, $P = \dfrac{1}{f}$

$$\Rightarrow f = \frac{1}{P} = \frac{1}{-10} \text{ metre}$$

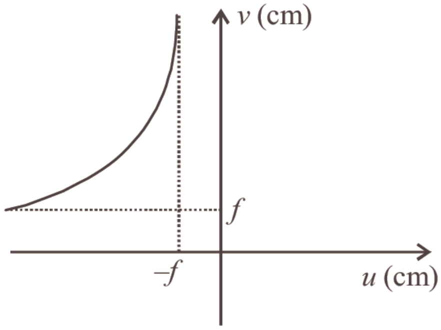
$$\therefore f = -\left(\frac{1}{10} \times 100\right) \text{cm} = -10 \text{ cm.}$$

13. (c) From the lens formula $\dfrac{1}{f} = \dfrac{1}{v} - \dfrac{1}{u}$

This graph suggest that when

$u = -f, v = +\infty$

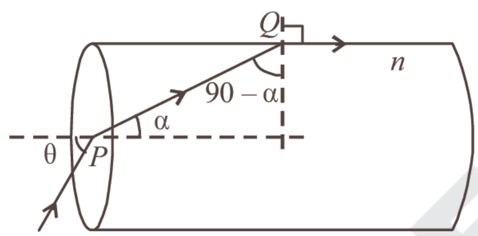
When u is at $-\infty$, $v = f$.



When the object is moved further away from the lens, v decreases but remains positive.

14. (a) To find the refractive index of glass using a travelling microscope, a vernier scale is provided on the microscope

15. (c)



Applying Snell's law for medium inside the cylinder and air at Q we get

$$n = \frac{\sin 90°}{\sin(90° - \alpha)} = \frac{1}{\cos\alpha}$$

$$\therefore \cos\alpha = \frac{1}{n}$$

$$\therefore \sin\alpha = \sqrt{1-\cos^2\alpha} = \sqrt{1-\frac{1}{n^2}} = \frac{\sqrt{n^2-1}}{n} \quad ...(i)$$

Applying Snell's Law for air and medium inside the cylinder at P we get

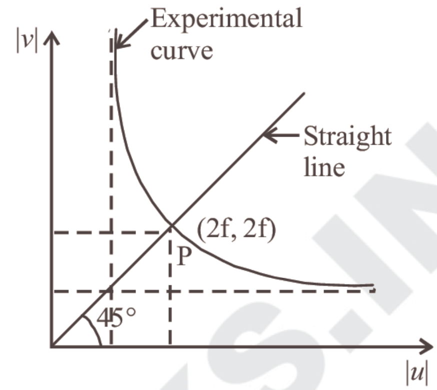
$$n = \frac{\sin\theta}{\sin\alpha}$$

$$\Rightarrow \sin\theta = n \times \sin\alpha = \sqrt{n^2-1}\,; \text{ [from (i)]}$$

$$\therefore \sin\theta = \sqrt{\left(\frac{2}{\sqrt{3}}\right)^2 - 1} = \sqrt{\frac{4}{3}-1} = \frac{1}{\sqrt{3}}$$

or $\quad \theta = \sin^{-1}\left(\frac{1}{\sqrt{3}}\right)$

16. (d)



For the graph to intersect y = x line. The value of $|v|$ and $|u|$ must be equal.

From lens formula

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

When $u = -2f, v = 2f$

Also $v = \dfrac{f}{1 + \dfrac{f}{u}}$

As $|u|$ increases, v decreases for $|u| > f$. The graph between $|v|$ and $|u|$ is shown in the figure. A straight line passing through the origin and making an angle of 45° with the x-axis meets the experimental curve at $P(2f, 2f)$.

17. (a) As refractive index for z > 0 and z ≤ 0 is different xy plane should be the boundry between two media.

Angle of incidence is given by

$$\cos(\pi - i) = \frac{\left(6\sqrt{3}\hat{i} + 8\sqrt{3}\hat{j} - 10\hat{k}\right).\hat{k}}{20}$$

$$-\cos i = -\frac{1}{2}$$

$$\Rightarrow \angle i = 60°$$

From Snell's law,

$$\frac{\sin i}{\sin r} = \frac{u_2}{u_1}$$

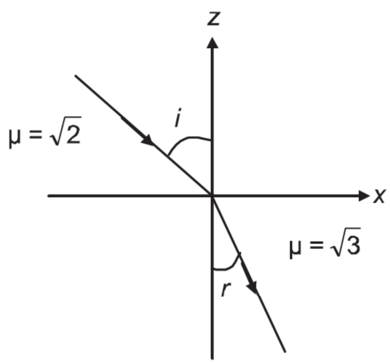
$$\Rightarrow \frac{\sin i}{\sin r} = \frac{\sqrt{3}}{\sqrt{2}}$$

$\Rightarrow \sqrt{2} \sin i = \sqrt{3} \sin r$

$\Rightarrow \sqrt{2} \sin 60° = \sqrt{3}$

$\Rightarrow \sqrt{2} \times \frac{\sqrt{3}}{2} = \sqrt{3} \sin r$

$\Rightarrow \angle r = 45°$



18. (a) From mirror formula

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$$

Differentiating the above equation, we get

$$\frac{dv}{dt} = -\frac{v^2}{u^2}\left(\frac{du}{dt}\right)$$

Also,

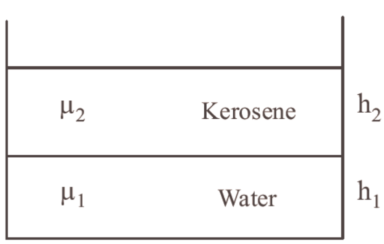
$$\frac{v}{u} = \frac{f}{u-f}$$

$$\Rightarrow \frac{dv}{dt} = -\left(\frac{f}{u-f}\right)^2 \frac{du}{dt}$$

$$\Rightarrow \frac{dv}{dt} = \left(\frac{0.2}{2.8-0.2}\right)^2 \times 15$$

$$\Rightarrow \frac{dv}{dt} = \frac{1}{15} \text{ m/s}$$

19. (b)



Apparent shift of the bottom due to water,

$$\Delta h_1 = h_1\left[1 - \frac{1}{\mu_1}\right]$$

Apparent shift of the bottom due to kerosene, Δh_2

$$= h_2\left[1 - \frac{1}{\mu_2}\right]$$

Thus, total apparent shift :

$= \Delta h_1 + \Delta h_2$

$$= h_1\left(1 - \frac{1}{\mu_1}\right) + h_2\left(1 - \frac{1}{\mu_2}\right)$$

20. (a) From the Cauchy

Formula, $\mu = A + \frac{B}{\lambda^2} + \frac{C}{\lambda^1}$

$$\therefore \mu \propto \frac{1}{\lambda}$$

As, $\lambda_{blue} < \lambda_{red}$

$\therefore \lambda_{blue} > \mu_{red}$

From lens maker's formula

and $\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

$$\Rightarrow \frac{1}{f_B} > \frac{1}{f_R} \quad \Rightarrow f_R > f_B.$$

21. (d) The focal length of the lens

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$$

$$= \frac{1}{12} + \frac{1}{240}$$

$$= \frac{20+1}{240} = \frac{21}{240}$$

$$f = \frac{240}{21} \text{cm}$$

When glass plate is interposed between lens and film, so shift produced will be

$$\text{Shift} = t\left(1 - \frac{1}{\mu}\right)$$

$$1\left(1 - \frac{1}{3/2}\right) = 1 \times \frac{1}{3}$$

Now image should be form at

$v' = 12 - \frac{1}{3} = \frac{35}{3}$ cm

Now the object distance u.
Using lens formula again

$$\frac{1}{f} = \frac{1}{v'} - \frac{1}{u}$$

$$\Rightarrow \frac{1}{u} = \frac{1}{v'} - \frac{1}{f}$$

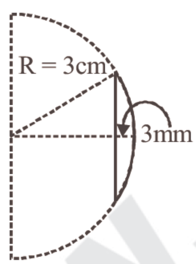
$$\Rightarrow \frac{1}{u} = \frac{3}{35} - \frac{21}{240} = \frac{1}{5}\left[\frac{3}{7} - \frac{21}{48}\right]$$

$$\Rightarrow \frac{1}{u} = \frac{1}{5}\left[\frac{48-49}{7 \times 16}\right]$$

$$\Rightarrow u = -7 \times 16 \times 5 = -560 \text{ cm} = -5.6 \text{ m}$$

22. **(c)** $\therefore\ n = \frac{\text{Velocity of light in vacuum}}{\text{Velocity of light in medium}}$

$\therefore\ n = \frac{3}{2}$



$3^2 + (R - 3mm)^2 = R^2$

$\Rightarrow 3^2 + R^2 - 2R(3mm) + (3mm)^2 = R^2$

$\Rightarrow R \approx 15$ cm

$$\frac{1}{f} = \left(\frac{3}{2} - 1\right)\left(\frac{1}{15}\right) \Rightarrow f = 30 \text{ cm}$$

23. **(c)** For the prism as the angle of incidence (i) increases, the angle of deviation (δ) first decreases goes to minimum value and then increases.

24. **(b)** By Lens maker's formula for convex lens

$$\frac{1}{f} = \left(\frac{\mu}{\mu_L} - 1\right)\left(\frac{2}{R}\right)$$

for, $\mu_{L_1} = \frac{4}{3}, f_1 = 4R$

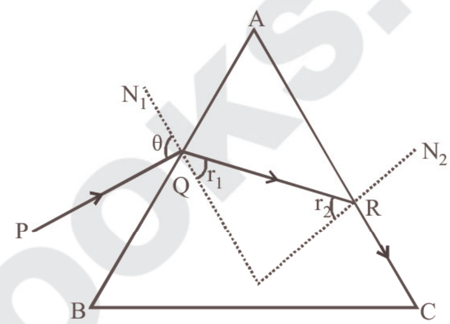
for $\mu_{L_2} = \frac{5}{3}, f_2 = -5R$

$\Rightarrow f_2 = (-)$ ve

25. **(c)** When $r_2 = C$, $\angle N_2RC = 90°$

Where C = critical angle

As $\sin C = \frac{1}{\mu} = \sin r_2$



Applying snell's law at 'R'

$\mu \sin r_2 = 1 \sin 90°$ ...(i)

Applying snell's law at 'Q'

$1 \times \sin\theta = \mu \sin r_1$ ...(ii)

But $r_1 = A - r_2$

So, $\sin\theta = \mu \sin(A - r_2)$

$\sin\theta = \mu \sin A \cos r_2 - \cos A$ ...(iii)

[using (i)]

From (1)

$$\cos r_2 = \sqrt{1 - \sin^2 r_2} = \sqrt{1 - \frac{1}{\mu^2}} \quad ...(iv)$$

By eq. (iii) and (iv)

$$\sin\theta = \mu \sin A \sqrt{1 - \frac{1}{\mu^2}} - \cos A$$

on further solving we can show for ray not to transmitted through face AC

$$\theta = \sin^{-1}\left[\mu \sin\left(A - \sin^{-1}\left(\frac{1}{\mu}\right)\right]\right.$$

So, for transmission through face AC

$$\theta > \sin^{-1}\left[\mu \sin\left(A - \sin^{-1}\left(\frac{1}{\mu}\right)\right]\right.$$

26. **(b)** A telescope magnifies by making the object appearing closer.

27. **(c)** We know that $i + e - A = \delta$

$35° + 79° - A = 40°$ $\therefore$ $A = 74°$

But $\mu = \dfrac{\sin\left(\dfrac{A+\delta_m}{2}\right)}{\sin A/2} = \dfrac{\sin\left(\dfrac{74+\delta_m}{2}\right)}{\sin\dfrac{74}{2}}$

$$= \frac{5}{3}\sin\left(37° + \frac{\delta_m}{2}\right)$$

μ_{max} can be $\frac{5}{3}$. That is μ_{max} is less than

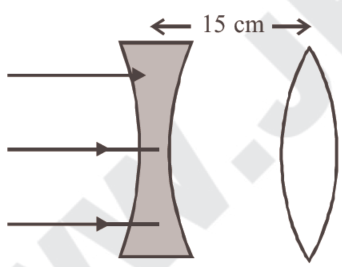
$\frac{5}{3} = 1.67$

But δ_m will be less than 40° so

$\mu < \frac{5}{3}\sin 57° < \frac{5}{3}\sin 60° \Rightarrow \mu = 1.5$

28. **(c)** As parallel beam incident on diverging lens will form image at focus.

$\therefore$ $v = -25\ cm$



$f = -25\ cm$ $f = 20\ cm$

The image formed by diverging lens is used as an object for converging lens,

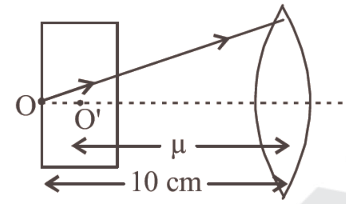
So for converging lens $u = -25 - 15 = -40\ cm$, $f = 20\ cm$

$\therefore$ Final image formed by converging lens

$$\frac{1}{V} - \frac{1}{-40} = \frac{1}{20}$$

or, $V = 40$ cm from converging lens real and inverted.

29. **(d)**



As the object and image distance is same, object is placed at 2f. Therefore 2f = 10 or f = 5 cm.

Shift due to slab, $d = t\left(1 - \frac{1}{\mu}\right)$

in the direction of incident ray

$\Rightarrow d = 1.5\left(1 - \frac{2}{3}\right) = 0.5$ cm

Now, $u = -9.5$ cm

Again using lens formula $\frac{1}{v} - \frac{1}{-9.5} = \frac{1}{5}$

$\Rightarrow v = 10.55$ cm

Thus, screen is shifted by a distance d $= 10.55 - 10 = 0.55$ cm away from the lens.

30. **(c)** $+5 = -\frac{v}{u} \Rightarrow v = -5u$

Using $\frac{1}{v} + \frac{1}{u} = \frac{1}{f} \Rightarrow \frac{1}{-5u} + \frac{1}{u} = \frac{-1}{0.4}$

$\therefore$ $u = -0.32$ m.

31. **(a)** According question, $M = 375$

$L = 150$ mm, $f_0 = 5$ mm and $f_e = ?$

Using, magnification, $M \simeq \frac{L}{f_0}\left(1 + \frac{D}{f_e}\right)$

$$\Rightarrow 375 = \frac{150}{5}\left(1 + \frac{250}{f_e}\right)$$

($\because D = 25$ cm $= 250$ mm)

$$\Rightarrow 12.5 = 1 + \frac{250}{f_e}$$

$$\Rightarrow f_e = \frac{250}{11.5} = 21.7 \approx 22\ mm$$

24 Wave Optics

1. To demonstrate the phenomenon of interference, we require two sources which emit radiation **[2003]**
(a) of nearly the same frequency
(b) of the same frequency
(c) of different wavelengths
(d) of the same frequency and having a definite phase relationship

2. The angle of incidence at which reflected light is totally polarized for reflection from air to glass (refractive index n), is **[2004]**
(a) $\tan^{-1}(1/n)$ (b) $\sin^{-1}(1/n)$
(c) $\sin^{-1}(n)$ (d) $\tan^{-1}(n)$

3. The maximum number of possible interference maxima for slit-separation equal to twice the wavelength in Young's double-slit experiment is **[2004]**
(a) three (b) five
(c) infinite (d) zero

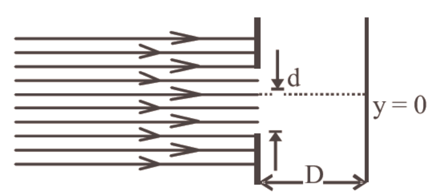
4. A Young's double slit experiment uses a monochromatic source. The shape of the interference fringes formed on a screen is **[2005]**
(a) circle (b) hyperbola
(c) parabola (d) straight line

5. If I_0 is the intensity of the principal maximum in the single slit diffraction pattern, then what will be its intensity when the slit width is doubled? **[2005]**
(a) $4I_0$ (b) $2I_0$
(c) $\frac{I_0}{2}$ (d) I_0

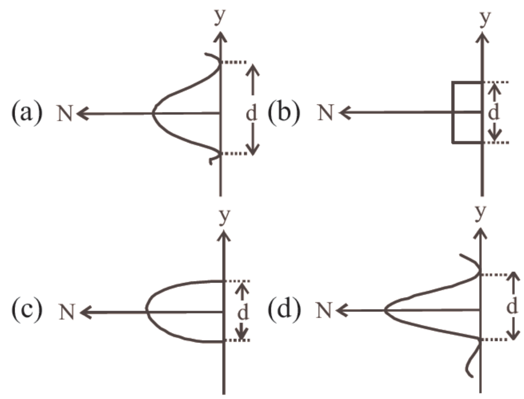
6. When an unpolarized light of intensity I_0 is incident on a polarizing sheet, the intensity of the light which does not get transmitted is **[2005]**
(a) $\frac{1}{4}I_0$ (b) $\frac{1}{2}I_0$
(c) I_0 (d) zero

7. In a Young's double slit experiment the intensity at a point where the path difference is $\frac{\lambda}{6}$ (λ being the wavelength of light used) is I. If I_0 denotes the maximum intensity, $\frac{I}{I_0}$ is equal to **[2007]**
(a) $\frac{3}{4}$ (b) $\frac{1}{\sqrt{2}}$
(c) $\frac{\sqrt{3}}{2}$ (d) $\frac{1}{2}$

8. In an experiment, electrons are made to pass through a narrow slit of width 'd' comparable to their wavelength. They are detected on a screen at a distance 'D' from the slit (see figure).



Which of the following graphs can be expected to represent the number of electrons 'N' detected as a function of the detector position 'y'($y = 0$ corresponds to the middle of the slit) **[2008]**



9. A mixture of light, consisting of wavelength 590 nm and an unknown wavelength, illuminates Young's double slit and gives rise to two overlapping interference patterns on the screen. The central maximum of both lights coincide. Further, it is observed that the third bright fringe of known light coincides with the 4th bright fringe of the unknown light. From this data, the wavelength of the unknown light is: **[2009]**

(a) 885.0 nm (b) 442.5 nm
(c) 776.8 nm (d) 393.4 nm

Directions : Questions number 10-12 are based on the following paragraph.

An initially parallel cylindrical beam travels in a medium of refractive index $\mu(I) = \mu_0 + \mu_2 I$, where μ_0 and μ_2 are positive constants and I is the intensity of the light beam. The intensity of the beam is decreasing with increasing radius

10. As the beam enters the medium, it will **[2010]**

(a) diverge
(b) converge
(c) diverge near the axis and converge near the periphery
(d) travel as a cylindrical beam

11. The initial shape of the wavefront of the beam is **[2010]**

(a) convex
(b) concave
(c) convex near the axis and concave near the periphery
(d) planar

12. The speed of light in the medium is **[2010]**

(a) minimum on the axis of the beam
(b) the same everywhere in the beam
(c) directly proportional to the intensity I
(d) maximum on the axis of the beam

13. This question has a paragraph followed by two statements, Statement – 1 and Statement – 2. Of the given four alternatives after the statements, choose the one that describes the statements.

A thin air film is formed by putting the convex surface of a plane-convex lens over a plane glass plate. With monochromatic light, this film gives an interference pattern due to light reflected from the top (convex) surface and the bottom (glass plate) surface of the film.

Statement – 1 : When light reflects from the air-glass plate interface, the reflected wave suffers a phase change of π.

Statement – 2 : The centre of the interference pattern is dark. **[2011]**

(a) Statement – 1 is true, Statement – 2 is true, Statement – 2 is the correct explanation of Statement – 1.
(b) Statement – 1 is true, Statement – 2 is true, Statement – 2 is not the correct explanation of Statement – 1.
(c) Statement – 1 is false, Statement – 2 is true.
(d) Statement – 1 is true, Statement – 2 is false.

14. At two points P and Q on screen in Young's double slit experiment, waves from slits S_1 and S_2 have a path difference of 0 and $\frac{\lambda}{4}$, respectively. The ratio of intensities at P and Q will be : **[2011 RS]**

(a) 2 : 1 (b) $\sqrt{2} : 1$
(c) 4 : 1 (d) 3 : 2

15. In a Young's double slit experiment, the two slits act as coherent sources of wave of equal amplitude A and wavelength λ. In another experiment with the same arrangement the two slits are made to act as incoherent sources of waves of same amplitude and wavelength. If the intensity at the middle point of the screen in the first case is I_1 and in the second case is I_2, then the ratio $\frac{I_1}{I_2}$ is **[2011 RS]**

(a) 2 (b) 1
(c) 0.5 (d) 4

16. **Statement - 1:** On viewing the clear blue portion of the sky through a Calcite Crystal, the intensity of transmitted light varies as the crystal is rotated.

Statement - 2: The light coming from the sky is polarized due to scattering of sun light by particles in the atmosphere. The scattering is largest for blue light. **[2011 RS]**

(a) Statement -1 is true, statement-2 is false.
(b) Statement-1 is true, statement-2 is true, statement-2 is the correct explanation of statement-1
(c) Statement-1 is true, statement-2 is true, statement-2 is not the correct explanation of statement-1
(d) Statement-1 is false, statement-2 is true.

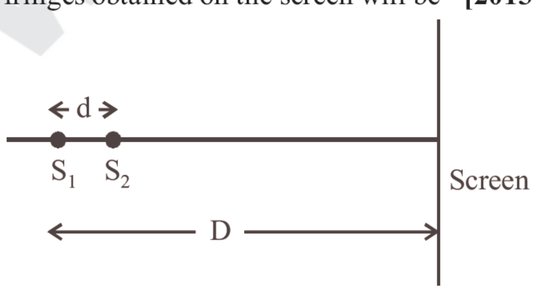
17. In Young's double slit experiment, one of the slit is wider than other, so that amplitude of the light from one slit is double of that other slit. If I_m be the maximum intensity, the resultant intensity I when they interfere at phase difference ϕ is given by : **[2012]**

(a) $\frac{I_m}{9}(4+5\cos\phi)$ (b) $\frac{I_m}{3}\left(1+2\cos^2\frac{\phi}{2}\right)$
(c) $\frac{I_m}{5}\left(1+4\cos^2\frac{\phi}{2}\right)$ (d) $\frac{I_m}{9}\left(1+8\cos^2\frac{\phi}{2}\right)$

18. A beam of unpolarised light of intensity I_0 is passed through a polaroid Aand then through another polaroid B which is oriented so that its principal plane makes an angle of 45° relative to that of A. The intensity of the emergent light is **[2013]**

(a) I_0 (b) $I_0/2$
(c) $I_0/4$ (d) $I_0/8$

19. Two coherent point sources S_1 and S_2 are separated by a small distance 'd' as shown. The fringes obtained on the screen will be **[2013]**



(a) points (b) straight lines
(c) semi-circles (d) concentric circles

20. Two beams, A and B, of plane polarized light with mutually perpendicular planes of polarization are seen through a polaroid. From the position when the beam A has maximum intensity (and beam B has zero intensity), a rotation of polaroid through 30° makes the two beams appear equally bright. If the initial intensities of the two beams are I_A and I_B respectively, then $\frac{I_A}{I_B}$ equals: **[2014]**

(a) 3 (b) $\frac{3}{2}$
(c) 1 (d) $\frac{1}{3}$

21. Assuming human pupil to have a radius of 0.25 cm and a comfortable viewing distance of 25 cm, the minimum separation between two objects that human eye can resolve at 500 nm wavelength is : **[2015]**

(a) 100 μm (b) 300 μm
(c) 1 μm (d) 30 μm

22. The box of a pin hole camera, of length L, has a hole of radius a. It is assumed that when the hole is illuminated by a parallel beam of light of wavelength λ the spread of the spot (obtained on the opposite wall of the camera) is the sum of its geometrical spread and the spread due to diffraction. The spot would then have its minimum size (say b_{min}) when : **[2016]**

(a) $a=\sqrt{\lambda L}$ and $b_{\min}=\sqrt{4\lambda L}$
(b) $a=\frac{\lambda^2}{L}$ and $b_{\min}=\sqrt{4\lambda L}$
(c) $a=\frac{\lambda^2}{L}$ and $b_{\min}=\left(\frac{2\lambda^2}{L}\right)$
(d) $a=\sqrt{\lambda l}$ and $b_{\min}=\left(\frac{2\lambda^2}{L}\right)$

23. In a Young's double slit experiment, slits are separated by 0.5 mm, and the screen is placed 150 cm away. A beam of light consisting of two wavelengths, 650 nm and 520 nm, is used to obtain interference fringes on the screen. The least distance from the common central maximum to the point where the bright fringes due to both the wavelengths coincide is : **[2017]**

(a) 9.75 mm (b) 15.6 mm
(c) 1.56 mm (d) 7.8 mm

24. Unpolarized light of intensity I passes through an ideal polarizer A. Another indentical polarizer B is placed behind A. The intensity of light beyond B is found to be $\frac{I}{2}$. Now another identical polarizer C is placed between A and B. The intensity beyond B is now found to be $\frac{I}{8}$. The angle between polarizer A and C is: **[2018]**

(a) 0° (b) 30°
(c) 45° (d) 60°

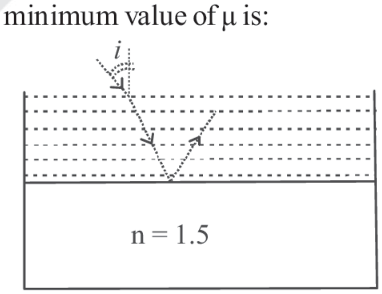
25. The angular width of the central maximum in a single slit diffraction pattern is 60°. The width of the slit is 1 μm. The slit is illuminated by monochromatic plane waves. If another slit of same width is made near it, Young's fringes can be observed on a screen placed at a distance 50 cm from the slits. If the observed fringe width is 1 cm, what is slit separation distance?
(i.e. distance between the centres of each slit.) **[2018]**

(a) 25 μm (b) 50 μm
(c) 75 μm (d) 100 μm

26. Two coherent sources produce waves of different intensities which interfere. After interference, the ratio of the maximum intensity to the minimum intensity is 16. The intensity of the waves are in the ratio: **[2019]**

(a) 16 : 9 (b) 25 : 9
(c) 4 : 1 (d) 5 : 3

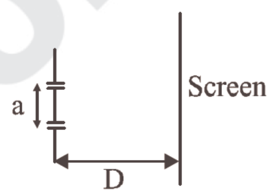
27. Consider a tank made of glass (refractive index 1.5) with a thick bottom. It is filled with a liquid of refractive index μ. A student finds that, irrespective of what the incident angle i (see figure) is for a beam of light entering the liquid, the light reflected from the liquid glass interface is never completely polarized. For this to happen, the minimum value of μ is: **[2019]**



(a) $\sqrt{\frac{5}{3}}$ (b) $\frac{3}{\sqrt{5}}$

(c) $\frac{5}{\sqrt{3}}$ (d) $\frac{4}{3}$

28. The figure shows a Young's double slit experimental setup. It is observed that when a thin transparent sheet of thickness t and refractive index μ is put in front of one of the slits, the central maximum gets shifted by a distance equal to n fringe widths. If the wavelength of light used is λ, t will be: **[2019]**



(a) $\frac{2nD\lambda}{a(\mu-1)}$ (b) $\frac{nD\lambda}{a(\mu-1)}$

(c) $\frac{D\lambda}{a(\mu-1)}$ (d) $\frac{2D\lambda}{a(\mu-1)}$

29. A polarizer - analyser set is adjusted such that the intensity of light coming out of the analyser is just 10% of the original intensity. Assuming that the polarizer - analyser set does not absorb any light, the angle by which the analyser need to be rotated further to reduce the output intensity to be zero, is: **[2020]**

(a) 71.6° (b) 18.4°
(c) 90° (d) 45°

30. Visible light of wavelength 6000×10^{-8} cm falls normally on a single slit and produces a diffraction pattern. It is found that the second diffraction minimum is at 60° from the central maximum. If the first minimum is produced at θ_1, then θ_1 is close to: **[2020]**

(a) 20° (b) 30°
(c) 25° (d) 45°

Answer Key

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
(d)	(d)	(b)	(d)	(a)	(b)	(a)	(d)	(b)	(b)	(d)	(a)	(b)	(a)	(a)
16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
(b)	(d)	(c)	(d)	(d)	(d)	(a)	(d)	(c)	(a)	(b)	(b)	(None)	(b)	(c)

Solutions

1. (d) To demonstrate the phenomenon of interference we require two sources of light which emit radiation of same frequency and having a definite phase relationship (a phase relationship that does not change with time)

2. (d) From the Brewster's law, angle of incidence for total polarization is given by $\tan\theta = n$

$\Rightarrow \theta = \tan^{-1} n$

Where n is the refractive index of the glass.

3. (b) For constructive interference path difference (As $\sin\theta \le 1$)

$d \sin\theta = n\lambda$

Given $d = 2\lambda$

$\therefore 2\lambda \sin\theta = n\lambda \Rightarrow \sin\theta = \frac{n}{2}$

$n = 0, 1, -1, 2, -2$ hence five maxima are possible.

4. (d) The light passing through the slits interfere and produce dark and bright band one screen. The shape of interference fringes formed on a screen in case of a monochromatic source is a straight line.

5. (a) $\mathrm{I} = \mathrm{I}_0\left(\frac{\sin\phi}{\phi}\right)^2$ and $\phi = \frac{\pi}{\lambda}(b\sin\theta)$

When the slit width is doubled, the amplitude of the wave at the centre of the screen is doubled, so the intensity at the centre is increased by a factor 4.

6. (b) From the law of Malus, $I = I_0\cos^2\theta$

When an unpolarised light is converted into plane polarised light by passing through polariod, its intensity become hafl.

$\therefore$ Intensity of polarized light $= \frac{I_0}{2}$

$\Rightarrow$ Intensity of untransmitted light

$$= I_0 - \frac{I_0}{2} = \frac{I_0}{2}$$

7. (a) For path difference of λ, the phase difference is 2π

For path difference of $\frac{\lambda}{6}$, the phase difference is

$$\frac{2\pi \times \lambda/6}{\lambda} = \frac{\pi}{3}$$

Resultant intensity

$$I = I_1 + I_2 + 2\sqrt{I_1}\sqrt{I_2}\cos\frac{\pi}{3}$$

$$\therefore\ I = I_1 + I_2 + \sqrt{I_1}\sqrt{I_2}$$

For two identical source, $I_1 = I_2 = I'$ (say) then $I = 3I'$

$$I_{max} = \left(\sqrt{I_1} + \sqrt{I_2}\right)^2$$

Maximum resultant intensity,

$$= \left(\sqrt{I'} + \sqrt{I'}\right)^2 = \left(2\sqrt{I'}\right)^2 = 4I'$$

$$\therefore\ \frac{I}{I_{max}} = \frac{3}{4}$$

ALTERNATE SOLUTION

The intensity of light at any point of the screen where the phase difference due to light coming from the two slits is ϕ is given by

$$\mathrm{I} = \mathrm{I_o}\cos^2\left(\frac{\phi}{2}\right)$$

Where I_0 is the maximum intensity.

NOTE This formula is applicable when $I_1 = I_2$.

Phase difference $\phi = \frac{2\pi}{\lambda} \times \frac{\lambda}{6} = \pi/3$

$$\therefore\ \frac{I}{I_0} = \cos^2\frac{\pi}{6} = \left(\frac{\sqrt{3}}{2}\right)^2 = \frac{3}{4}$$

8. **(d)** The electron beam will be diffracted and the maxima is obtained at $y = 0$.

Also, the diffraction pattern, should be wider than the slit width.

9. **(b)** Let λ be the wavelength of unknown light. Third bright fringe of known light coincides with the 4th bright fringe of the unknown light.

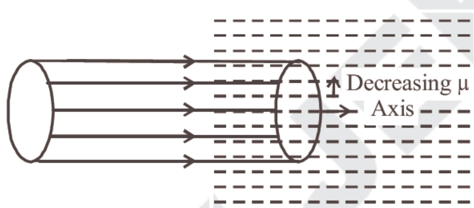
$$\therefore \frac{3\lambda_1 D}{d} = \frac{4\lambda D}{d}$$

$$\therefore \frac{3(590)D}{d} = \frac{4\lambda D}{d}$$

$$\Rightarrow \lambda = \frac{3}{4} \times 590 = 442.5 \text{ nm}$$

10. **(b)** When light beam is moving and as it enters the medium, the refractive index will decrease from the axis towards the periphery of the beam.

Therefore, the beam will conver less distance as one move from the axis to the periphery and hence the beam will converge.



11. **(d)** Initially the parallel beam is cylindrical. Therefore, the wavefront will be planar.

12. **(a)** The speed of light (v) in a medium of refractive index (μ) is given by

$\mu = \frac{c}{v}$, where c is the speed of light in vacuum

$$\therefore v = \frac{c}{\mu} = \frac{c}{\mu_0 + \mu_2(I)}$$

As I is decreasing with increasing radius, it is maximum on the axis of the beam. Therefore, v is minimum on the axis of the beam.

13. **(b)** A phase change of π rad appears when the ray reflects at the glass-air interface. As a result, there will be a destructive interference at the centre. So, the centre of the interference pattern is dark.

14. **(a)** Path difference at P $\Delta x_1 = 0$

$\therefore$ Phase difference at P will be

$$\Delta\phi_1 = \frac{2\pi}{\lambda}\Delta x_1$$

$$= \frac{2\pi}{\lambda} \times 0$$

$$= 0°$$

Resultant Intensity at P

$$I_1 = I_0 + I_0 + 2I_0 \cos 0° = 4I_0$$

Path difference at Q

$$\Delta x_2 = \frac{\lambda}{4}$$

$\therefore$ Phase difference at Q

$$\Delta = \frac{2\pi}{\lambda} \cdot \frac{\lambda}{4} = \left(\frac{\pi}{2}\right)$$

Resultant intensity at Q.

$$I_2 = I_0 + I_0 + 2I_0 \cos\frac{\pi}{2} = 2I_0$$

Thus, $\frac{I_1}{I_2} = \frac{4I_0}{2I_0} = \frac{2}{1}$

15. **(a)** For coherent sources, intensity at mid point

$I_1 \propto (a + a)^2$

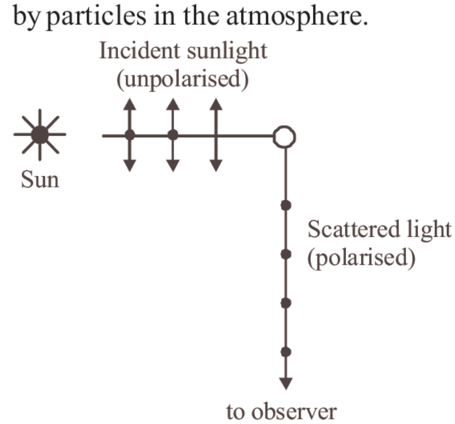
$\Rightarrow I_1 \propto (2a)^2$

For incoherent sources, intensity of mid point is

$I_2 \propto 2a^2$

$\therefore \frac{I_1}{I_2} = \frac{2}{1}$

16. **(b)** When viewed through a polaroid which is rotated then the light from a clear blue portion of the sky shows a rise and fall of intensity. The light coming from the sky is polarised due to scattering of sunlight by particles in the atmosphere.



17. **(d)** Let a, be the amplitude of light from first slit and a_2 be the amplitude of light from second slit.

$a_1 = a$, Then $a_2 = 2a$

Intensity $I \propto (\text{amplitude})^2$

$\therefore \quad I_1 = a_1^2 = a^2$

$I_2 = a_2^2 = 4a^2 = 4I$

$I_r = a_1^2 + a_2^2 + 2a_1 a_2 \cos\phi$

$= I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\phi$

$I_r = I_1 + 4I_1 + 2\sqrt{4I_1^2}\cos\phi$

$\Rightarrow \quad I_r = 5I_1 + 4I_1\cos\phi \qquad \ldots(1)$

Now, $I_{max} = (a_1 + a_2)^2 = (a + 2a)^2 = 9a^2$

$I_{max} = 9I_1 \Rightarrow I_1 = \dfrac{I_{max}}{9}$

Substituting in equation (1)

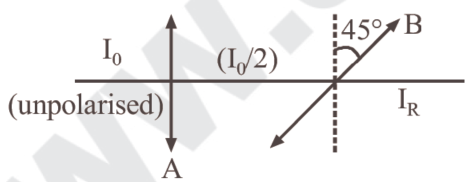
$$I_r = \frac{5I_{max}}{9} + \frac{4I_{max}}{9}\cos\phi$$

$$I_r = \frac{I_{max}}{9}[5 + 4\cos\phi]$$

$$I_r = \frac{I_{max}}{9}\left[5 + 8\cos^2\frac{\phi}{2} - 4\right]$$

$$I_r = \frac{I_{max}}{9}\left[1 + 8\cos^2\frac{\phi}{2}\right]$$

18. **(c)** Relation between intensities



$$I_r = \left(\frac{I_0}{2}\right)\cos^2(45°) = \frac{I_0}{2} \times \frac{1}{2} = \frac{I_0}{4}$$

19. **(d)** It will be concentric circles.

20. **(d)** According to malus law, intensity of emerging beam is given by,

$I = I_0 \cos^2\theta$

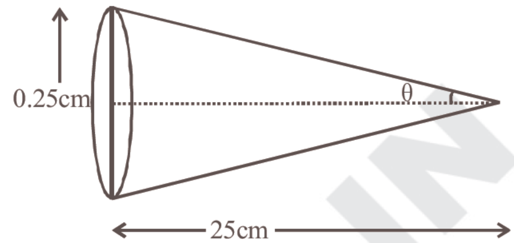
Now, $I_{A'} = I_A \cos^2 30°$

$I_{B'} = I_B \cos^2 60°$

As $I_{A'} = I_{B'}$

$$\Rightarrow \quad I_A \times \frac{3}{4} = I_B \times \frac{1}{4};\ \frac{I_A}{I_B} = \frac{1}{3}$$

21. **(d)** $\sin\theta = \dfrac{0.25}{25} = \dfrac{1}{100}$



Resolving power $= \dfrac{1.22\lambda}{2\mu\sin\theta} = 30\ \mu\text{m}$.

22. **(a)** Given geometrical spread $= a$

Diffraction spread $= \dfrac{\lambda}{a} \times L = \dfrac{\lambda L}{a}$

The sum $b = a + \dfrac{\lambda L}{a}$

For b to be minimum

$$\frac{db}{da} = 0 \qquad \frac{d}{da}\left(a + \frac{\lambda L}{a}\right) = 0$$

$a = \sqrt{\lambda L}$

$b\ \text{min} = \sqrt{\lambda L} + \sqrt{\lambda L} = 2\sqrt{\lambda L} = \sqrt{4\lambda L}$

23. **(d)** For common maxima, $n_1\lambda_1 = n_2\lambda_2$

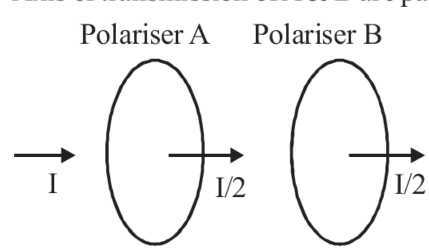
$$\Rightarrow \quad \frac{n_1}{n_2} = \frac{\lambda_2}{\lambda_1} = \frac{520 \times 10^{-9}}{650 \times 10^{-9}} = \frac{4}{5}$$

For λ_1

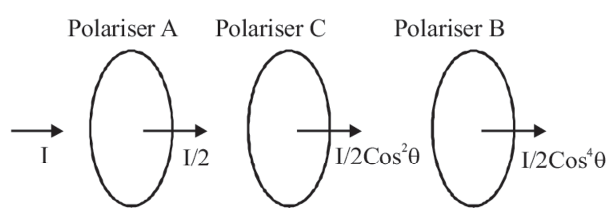
$$y = \frac{n_1\lambda_1 D}{d},\ \lambda_1 = 650\ \text{nm}$$

$$y = \frac{4 \times 650 \times 10^{-9} \times 1.5}{0.5 \times 10^{-3}} \quad \text{or,} \quad y = 7.8\ \text{mm}$$

24. **(c)** Axis of transmission of A & B are parallel.



After introducing polariser C between A and B,



$$\frac{I}{2}\cos^4\theta = \frac{I}{8} \Rightarrow \cos^4\theta = \frac{1}{4}$$

$$\Rightarrow \cos\theta = \frac{1}{\sqrt{2}} \text{ or, } \theta = 45°$$

25. (a) Angular width of central maxima $= \frac{2\lambda}{d}$

or, $\lambda = \frac{d}{2}$; Fringe width, $\beta = \frac{\lambda \times D}{d'}$

$$10^{-2} = \frac{d}{2} \times \frac{50\times10^{-2}}{d'} = \frac{10^{-6}\times50\times10^{-2}}{2\times d'}$$

Therefore, slit separation distance, d' = 25μm

26. (b) $\frac{I_{max}}{I_{min}} = \frac{\left(\sqrt{\frac{I_1}{I_2}}+1\right)^2}{\left(\sqrt{\frac{I_1}{I_2}}-1\right)^2} = 16\,(\text{given})$

$$\therefore \sqrt{\frac{I_1}{I_2}}+1 = 4\left(\sqrt{\frac{I_1}{I_2}}-1\right)$$

$$\therefore \sqrt{\frac{I_1}{I_2}} = \frac{5}{3} \therefore \frac{I_1}{I_2} = \frac{25}{9}$$

27. (b) For $i \approx 90°$ at air liquid interface we have by Snell's law

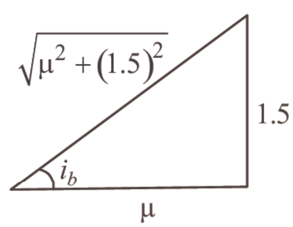
$$\mu = \frac{\sin 90°}{\sin r} \quad \therefore \sin r = \frac{1}{\mu}$$

According to Brewster's law, refractive index of liquid (μ) is equal to tangent of polarising angle

$$\because \tan i_b = \frac{1.5}{\mu}$$

$$\therefore \sin i_b = \frac{1.5}{\sqrt{\mu^2+1.5^2}}$$

Here $\sin r < \sin i_b$



$$\therefore \frac{1}{\mu} \le \frac{1.5}{\sqrt{\mu^2+(1.5)^2}}$$

or, $\sqrt{\mu^2+(1.5)^2} \le 1.5\times\mu$

$\Rightarrow \mu^2 + (1.5)^2 \le (\mu\times1.5)^2$

i.e., minimum

value of μ should be $\frac{3}{\sqrt{5}}$

28. (None) Shift $= n\beta$ (given)

$$\therefore D\frac{(\mu-1)t}{a} = \frac{n\lambda D}{a} \left[\because \text{Shift} = \frac{D(\mu-1)t}{a}\right]$$

or $t = \frac{n\lambda}{(\mu-1)}$

29. (b) According to question, the intensity of light coming out of the analyser is just 10% of the original intensity (I_0)

Using, $I = I_0\cos^2\theta$

$$\Rightarrow \frac{I_0}{10} = I_0\cos^2\theta \Rightarrow \frac{1}{10} = \cos^2\theta$$

$$\Rightarrow \cos\theta = \frac{1}{\sqrt{10}} = 0.316 \Rightarrow \theta \approx 71.6°$$

Therefore, the angle by which the analyser need to be rotated further to reduced the output intensity to be zero

$\phi = 90° - \theta = 90° - 71.6° = 18.4°$

30. (c) Given, $\lambda = 6000 \times 10^{-8}$ cm

Second diffraction minimum at 60° *i.e.*, $\theta_2 = 60°$

Using, $d\sin\theta = n\lambda$

$d\sin\theta_2 = 2\lambda$ (for 2nd minima)

$\Rightarrow \quad d\sin 60° = 2\lambda$

$$\Rightarrow d\times\left(\frac{\sqrt{3}}{2}\right) = 2\lambda \qquad ...(i)$$

$$\Rightarrow \frac{\lambda}{d} = \frac{\sqrt{3}}{4}$$

For first minima,

$d\sin\theta_1 = \lambda$

$$\Rightarrow \sin\theta_1 = \frac{\lambda}{d} = \frac{\sqrt{3}}{4} = 0.43 \Rightarrow \theta_1 < 30°$$

Hence closest option, $\theta_1 \approx 25°$

25 Dual Nature of Radiation and Matter

1. Sodium and copper have work functions 2.3 eV and 4.5 eV respectively. Then the ratio of the wavelengths is nearest to **[2002]**
   (a) 1 : 2 (b) 4 : 1
   (c) 2 : 1 (d) 1 : 4
2. Formation of covalent bonds in compounds exhibits **[2002]**
   (a) wave nature of electron
   (b) particle nature of electron
   (c) both wave and particle nature of electron
   (d) none of these
3. Two identical photocathodes receive light of frequencies f_1 and f_2. If the velocites of the photo electrons (of mass m) coming out are respectively v_1 and v_2, then **[2003]**
   (a) $v_1^2 - v_2^2 = \frac{2h}{m}(f_1 - f_2)$
   (b) $v_1 + v_2 = \left[\frac{2h}{m}(f_1 + f_2)\right]^{1/2}$
   (c) $v_1^2 + v_2^2 = \frac{2h}{m}(f_1 + f_2)$
   (d) $v_1 - v_2 = \left[\frac{2h}{m}(f_1 - f_2)\right]^{1/2}$
4. A radiation of energy E falls normally on a perfectly reflecting surface. The momentum transferred to the surface is **[2004]**
   (a) Ec (b) $2E/c$
   (c) E/c (d) E/c^2
5. According to Einstein's photoelectric equation, the plot of the kinetic energy of the emitted photo electrons from a metal vs the frequency, of the incident radiation gives a straight line whose slope **[2004]**
   (a) depends both on the intensity of the radiation and the metal used
   (b) depends on the intensity of the radiation
   (c) depends on the nature of the metal used
   (d) is the same for the all metals and independent of the intensity of the radiation
6. The work function of a substance is 4.0 eV. The longest wavelength of light that can cause photoelectron emission from this substance is approximately **[2004]**
   (a) 310 nm (b) 400 nm
   (c) 540 nm (d) 220 nm
7. A photocell is illuminated by a small bright source placed 1 m away. When the same source of light is placed $\frac{1}{2}$ m away, the number of electrons emitted by photocathode would **[2005]**
   (a) increase by a factor of 4
   (b) decrease by a factor of 4
   (c) increase by a factor of 2
   (d) decrease by a factor of 2

8. If the kinetic energy of a free electron doubles, it's deBroglie wavelength changes by the factor **[2005]**

(a) 2 (b) $\frac{1}{2}$

(c) $\sqrt{2}$ (d) $\frac{1}{\sqrt{2}}$

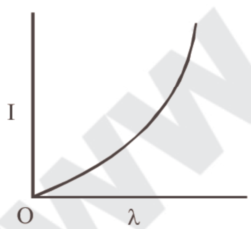
9. The threshold frequency for a metallic surface corresponds to an energy of 6.2 eV and the stopping potential for a radiation incident on this surface is 5 V. The incident radiation lies in **[2006]**

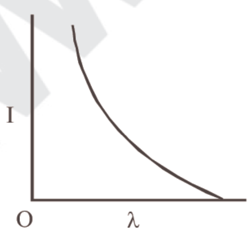
(a) ultra-violet region
(b) infra-red region
(c) visible region
(d) *X*-ray region

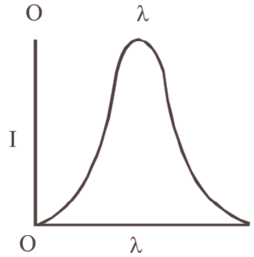
10. The time taken by a photoelectron to come out after the photon strikes is approximately **[2006]**

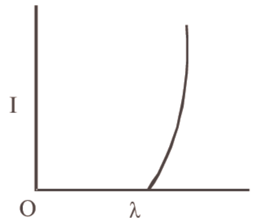
(a) 10^{-4} s (b) 10^{-10} s
(c) 10^{-16} s (d) 10^{-1} s

11. The anode voltage of a photocell is kept fixed. The wavelength λ of the light falling on the cathode is gradually changed. The plate current I of the photocell varies as follows **[2006]**

(a) 

(b) 

(c) 

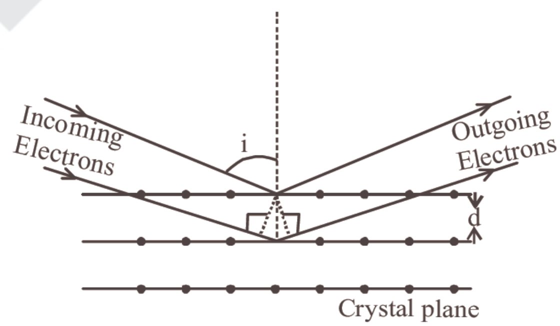
(d) 

12. Photon of frequency ν has a momentum associated with it. If c is the velocity of light, the momentum is **[2007]**

(a) hν / c (b) ν /c
(c) h ν c (d) hν / c^2

Directions: Question No. 13 and 14 are based on the following paragraph.

Wave property of electrons implies that they will show diffraction effects. Davisson and Germer demonstrated this by diffracting electrons from crystals. The law governing the diffraction from a crystal is obtained by requiring that electron waves reflected from the planes of atoms in a crystal interfere constructively (see figure).



13. Electrons accelerated by potential *V* are diffracted from a crystal. If d = 1Å and i = 30°, *V* should be about **[2008]**

($h = 6.6 \times 10^{-34}$ Js, $m_e = 9.1 \times 10^{-31}$ kg, $e = 1.6 \times 10^{-19}$ C)

(a) 2000 V (b) 50 V
(c) 500 V (d) 1000 V

14. If a strong diffraction peak is observed when electrons are incident at an angle '*i*' from the normal to the crystal planes with distance '*d*' between them (see figure), de Broglie wavelength λ_{dB} of electrons can be calculated by the relationship (*n* is an integer) **[2008]**

(a) $d \sin i = n\lambda_{dB}$ (b) $2d \cos i = n\lambda_{dB}$
(c) $2d \sin i = n\lambda_{dB}$ (d) $d \cos i = n\lambda_{dB}$

15. The surface of a metal is illuminted with the light of 400 nm. The kinetic energy of the ejected photoelectrons was found to be 1.68 eV. The work function of the metal is :**[2009]**
(hc = 1240 eV.nm)

(a) 1.41 eV (b) 1.51 eV
(c) 1.68 eV (d) 3.09 eV

Question (16–18) has Statement – 1 and Statement – 2. Of the four choices given after the statements, choose the one that best describes these two statements.

(a) Statement -1 is true, Statement -2 is true ; Statement -2 is the correct explanation of Statement -1.
(b) Statement -1 is true, Statement -2 is true; Statement -2 is **not** the correct explanation of Statement -1
(c) Statement -1 is false, Statement -2 is true.
(d) Statement -1 is true, Statement -2 is false.

16. **Statement -1 :** When ultraviolet light is incident on a photocell, its stopping potential is V_0 and the maximum kinetic energy of the photoelectrons is K_{max}. When the ultraviolet light is replaced by X-rays, both V_0 and K_{max} increase.

Statement -2 : Photoelectrons are emitted with speeds ranging from zero to a maximum value because of the range of frequencies present in the incident light. **[2010]**

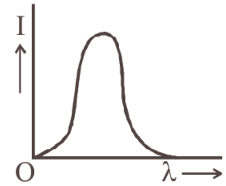
17. **Statement – 1:** A metallic surface is irradiated by a monochromatic light of frequency $v > v_0$ (the threshold frequency). The maximum kinetic energy and the stopping potential are $K_{\max}$ and V_0 respectively. If the frequency incident on the surface is doubled, both the $K_{\max}$ and V_0 are also doubled.

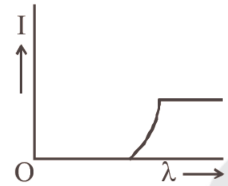
Statement – 2 : The maximum kinetic energy and the stopping potential of photoelectrons emitted from a surface are linearly dependent on the frequency of incident light. **[2011]**

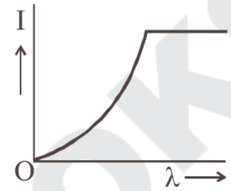
18. **Statement 1:** Davisson-Germer experiment established the wave nature of electrons.

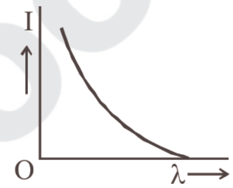
Statement 2 : If electrons have wave nature, they can interfere and show diffraction. **[2012]**

19. The anode voltage of a photocell is kept fixed. The wavelength λ of the light falling on the cathode is gradually changed. The plate current I of the photocell varies as follows : **[2013]**

(a)


(b)


(c)


(d)


20. The radiation corresponding to $3 \rightarrow 2$ transition of hydrogen atom falls on a metal surface to produce photoelectrons. These electrons are made to enter a magnetic field of 3×10^{-4} T. If the radius of the largest circular path followed by these electrons is 10.0 mm, the work function of the metal is close to: **[2014]**

(a) 1.8 eV (b) 1.1 eV
(c) 0.8 eV (d) 1.6 eV

21. Match **List - I** (Fundamental Experiment) with **List - II** (its conclusion) and select the correct option from the choices given below the list: **[2015]**

List-I	List-II
A. Franck-Hertz Experiment	(i) Particle nature of light
B. Photo-electric experiment	(ii) Discrete energy levels of atom
C. Davison-Germer experiment	(iii) Wave nature of electron
	(iv) Structure of atom

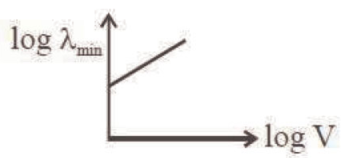
(a) (A)-(ii); (B)-(i); (C)-(iii)
(b) (A)-(iv); (B)-(iii); (C)-(ii)
(c) (A)-(i); (B)-(iv); (C)-(iii)
(d) (A)-(ii); (B)-(iv); (C)-(iii)

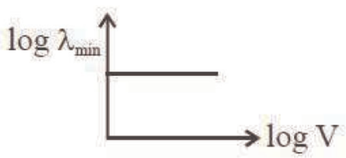
22. Radiation of wavelength λ, is incident on a photocell. The fastest emitted electron has speed v. If the wavelength is changed to $\frac{3\lambda}{4}$, the speed of the fastest emitted electron will be: **[2016]**

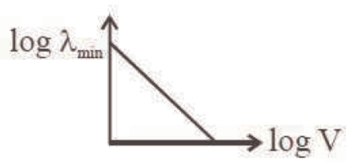
(a) $v\left(\frac{4}{3}\right)^{\frac{1}{2}}$ (b) $v\left(\frac{3}{4}\right)^{\frac{1}{2}}$

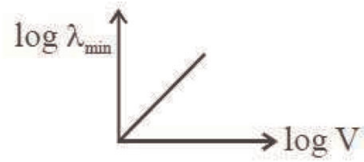
(c) $> v\left(\frac{4}{3}\right)^{\frac{1}{2}}$ (d) $< v\left(\frac{4}{3}\right)^{\frac{1}{2}}$

23. An electron beam is accelerated by a potential difference V to hit a metallic target to produce X-rays. It produces continuous as well as characteristic X-rays. If λ_{min} is the smallest possible wavelength of X-ray in the spectrum, the variation of log λ_{min} with log V is correctly represented in : **[2017]**

(a) 

(b) 

(c) 

(d) 

24. Surface of certain metal is first illuminated with light of wavelength $\lambda_1 = 350$ nm and then, by light of wavelength $\lambda_2 = 540$ nm. It is found that the maximum speed of the photo electrons in the two cases differ by a factor of (2) The work function of the metal (in eV) is close to: **[2019]**

(Energy of photon $= \frac{1240}{\lambda(\text{in nm})} eV$)

(a) 1.8 (b) 2.5
(c) 5.6 (d) 1.4

25. The electric field of light wave is given as
$\vec{E} = 10^3 \cos \left(\frac{2\pi x}{5\times10^{-7}} - 2\pi\times6\times10^{14} t\right)\hat{x}\frac{N}{C}$

This light falls on a metal plate of work function 2eV. The stopping potential of the photo-electrons is: **[2019]**

Given, E (in eV) $= \frac{12375}{\lambda(\text{in Å})}$

(a) 2.0 V (b) 0.72 V
(c) 0.48 V (d) 2.48 V

26. A beam of electromagnetic radiation of intensity $6.4 \infty 10^{-5}$ W/cm^2 is comprised of wavelength, $\lambda = 310$ nm. It falls normally on a metal (work function $\varphi = 2$eV) of surface area of 1 cm^2. If one in 10^3 photons ejects an electron, total number of electrons ejected in 1 s is 10^x. ($hc = 1240$ eVnm, 1 eV $= 1.6 \infty 10^{-19}$ J), then x is .... **[2020]**

Answer Key														
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
(c)	(a)	(a)	(b)	(d)	(a)	(a)	(d)	(a)	(b)	(b)	(a)	(b)	(b)	(a)
16	17	18	19	20	21	22	23	24	25	26				
(d)	(c)	(a)	(d)	(b)	(a)	(c)	(c)	(a)	(c)	(11.00)				

Solutions

1. **(c)** We know that work function,

$$E = h\upsilon = \frac{hC}{\lambda}$$

where

h = Planck's constant

C = velocity of light

λ = wavelength of light

$$\therefore \frac{E_{Na}}{E_{Cu}} = \frac{\lambda_{Cu}}{\lambda_{Na}}$$

$$\Rightarrow \frac{\lambda_{Na}}{\lambda_{Cu}} = \frac{E_{Cu}}{E_{Na}} = \frac{4.5}{2.3} \approx \frac{2}{1}$$

2. **(a)** Covalent bonds are formed by sharing of electrons with different compounds. Formation of covalent bond is best explained by molecular orbital theory.

3. **(a)** Let work function be W and v_1 and v_2 be the velocity of electrons for frequencies f_1 and f_2.

Using Einstein's photo electric equation for one photodiode, we get

$$hf_1 - W = \frac{1}{2}mv_1^2 \qquad \text{....(i)}$$

Using Einstein's photo electric equation for another photodiode we get,

$$hf_2 - W = \frac{1}{2}mv_2^2 \qquad \text{....(ii)}$$

Subtracting (ii) from (i) we get

$$(hf_1 - W) - (hf_2 - W) = \frac{1}{2}mv_1^2 - \frac{1}{2}mv_2^2$$

$$\therefore h(f_1 - f_2) = \frac{m}{2}(v_1^2 - v_2^2)$$

$$\therefore v_1^2 - v_2^2 = \frac{2h}{m}(f_1 - f_2)$$

4. **(b)** Momentum of photon of energy E is $= \frac{E}{c}$

When a photon hits a perfectly reflecting surface, it reflects black in opposite direction with same energy and momentum.

$\therefore$ Change in momentum $= \frac{E}{C} - \left(\frac{-E}{C}\right) = \frac{2E}{C}$

This is equal to momentum transferred to the surface.

5. **(d)** From the Einstein photoelectric equation $K.E. = h\nu - \phi$

Here, ϕ = work function of metal

h = Plank's constant

slope of graph of $K.E.$ & ν is h (Plank's constant) which is same for all metals.

6. **(a)** Work function of metal (ϕ) is given by

$$\phi = \frac{hc}{\lambda}$$

$$\Rightarrow \lambda = \frac{hc}{\phi}$$

$$\Rightarrow \lambda = \frac{6.63 \times 10^{-34} \times 3 \times 10^8}{4 \times 1.6 \times 10^{-19}} = 310 \text{ nm}$$

7. **(a)** We know that

Intensity, $I \propto \frac{1}{r^2}$;

$$\frac{I_1}{I_2} = \left(\frac{r_1/2}{r_1}\right)^2 = \frac{1}{4}$$

$I_2 \to 4$ times I_1

When intensity becomes 4 times, no. of photoelectrons emitted would increase by 4 times, since number of electrons emitted

per second is directly proportional to intensity.

8. (d) de-Broglie wavelength,

$$\lambda = \frac{h}{p} = \frac{h}{mv} \quad .... \text{(i)}$$

but $K.E = \frac{1}{2}mv^2$

$$\Rightarrow \quad K.E = \frac{(mv)^2}{2m}$$

$$\Rightarrow \quad mv = \sqrt{2m\,K.E}$$

$$\lambda = \frac{h}{\sqrt{2m\,K.E}}$$

$$\therefore \ \lambda \propto \frac{1}{\sqrt{K.E}}$$

So, if $K.E.$ is doubled, wavelength becomes $\frac{\lambda}{\sqrt{2}}$

9. (a) Work function, $\phi = 6.2\,\text{eV} = 6.2 \times 1.6 \times 10^{-19}$ J

Stopping potential, $V = 5$ volt

From the Einstein's photoelectric equation

$$\frac{hc}{\lambda} - \phi = \text{eV}_0$$

$$\Rightarrow \lambda = \frac{hc}{\phi + eV_0}$$

$$= \frac{6.6 \times 10^{-34} \times 3 \times 10^8}{1.6 \times 10^{-19}(6.2+5)} \approx 10^{-7}\ \text{m}$$

This range lies in ultra violet range.

10. (b) The photoelectric emission is an instantaneous process without any apparent time lag. It is known that emission starts in the time of the order of 10^{-9} second. So, the approximate time taken by a photoelectron to come out after the photon strikes is 10^{-10} second.

11. (b) As the wavelength λ is increased, there will be a value of wavelength above which photoelectric current becomes zero.

12. (a) Energy of a photon of frequency ν is given by $E = h\nu$. ...(i)

Also, $E = mc^2$ ...(ii)

On equating (i) and (ii) we have

$mc^2 = h\nu$

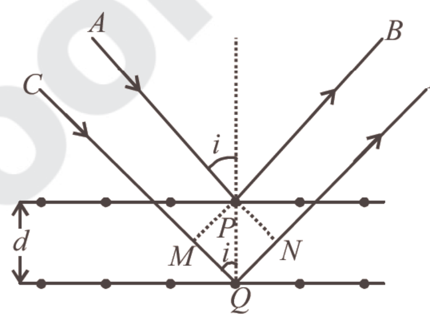
$$\Rightarrow \quad mc = \frac{h\nu}{c}$$

$$\Rightarrow \quad p = \frac{h\nu}{c}$$

13. (b) The path difference between the rays APB and CQD is

$\Delta x = \text{MQ} + \text{QN} = d\cos i + d\cos i$

$\Delta x = 2d\cos i$



For constructive interference the path difference is integral multiple of wavelength

$\therefore \ n\lambda = 2d\cos i$

From de-broglie concept

Wavelength,

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2\text{mK.E}}} = \frac{h}{\sqrt{2\text{meV}}}$$

$$\therefore \ \frac{nh}{\sqrt{2\text{meV}}} = 2d\cos i$$

Squaring both side

$$\frac{n^2h^2}{2meV} = 4d^2\cos^2 i$$

For first order interference $n = 1$

$$\therefore \ V = \frac{h^2}{8med^2\cos^2 i}$$

$$= \frac{(6.6\times10^{-34})^2}{8\times9.1\times10^{-31}\times1.6\times10^{-19}\times(10^{-10})^2\times\cos^2 30}$$

$= 50\,\text{V}$

14. **(b)** For constructive interference,

$2d \cos i = n\lambda_{dB}$

15. **(a)** Wavelength of incident light, $\lambda = 400$ nm

$hc = 1240$ eV.nm

$K.E = 1.68$ eV

Using Einstein's photoelectric equation

$$\frac{hc}{\lambda} - W = K.E$$

$$\Rightarrow W = \frac{hc}{\lambda} - K.E$$

$$\Rightarrow W = \frac{1240}{400} - 1.68$$

$= 3.1 - 1.68$

$= 1.41$ eV

16. **(d)** We know that

$eV_0 = K_{max} = h\nu - \phi$

where, ϕ is the work function.

X-rays have higher frequency (v) than ultraviolet rays. Therefore as v increases $K.E$ and V_0 both increases.

The kinetic energy ranges from zero to maximum because of loss of energy due to subsequent collisions before getting ejected.

17. **(c)** By Einstein photoelectric equation,

$K_{max} = eV_0 = hv - hv_0$

When v is doubled, K_{max} and V_0 become more than double.

18. **(a)** Davisson Germer experiment showed that electron beams can undergo diffraction when passed through atomic crystal. This established wave nature of electron as waves can exhibit interference and diffraction.

19. **(d)** As λ is increased, there will be a value of λ above which photoelectrons will be cease to come out so photocurrent will become zero. Hence (d) is correct answer.

20. **(b)** Radius of circular path followed by electron is given by,

$$r = \frac{m\upsilon}{qB} = \frac{\sqrt{2meV}}{eB} = \frac{1}{B}\sqrt{\frac{2m}{e}V}$$

$$\Rightarrow V = \frac{B^2 r^2 e}{2m} = 0.8V$$

For transition between 3 to 2.

$$E = 13.6\left(\frac{1}{4} - \frac{1}{9}\right) = \frac{13.6 \times 5}{36} = 1.88eV$$

Work function = 1.88 eV – 0.8 eV = 1.08 eV ≈ 1.1eV

21. **(a)** Frank-Hertz experiment - Discrete energy levels of atom, Photoelectric effect - Particle nature of light.

Davison - Germer experiment - wave nature of electron.

22. **(c)** $h\frac{c}{\lambda} - h\nu_0 = \frac{1}{2}\text{mv}^2$

$$\therefore \frac{4}{3}\frac{hc}{\lambda} - h\nu_0 = \frac{1}{2}mv'^2$$

$$\therefore \frac{v'^2}{v^2} = \frac{\frac{4}{3}\nu - \nu_0}{\nu - \nu_0} \quad \therefore v' = \text{v}\sqrt{\frac{\frac{4}{3}\nu - \nu_0}{\nu - \nu_0}}$$

$$\therefore v' > v\sqrt{\frac{4}{3}}$$

23. **(c)** In X-ray tube, $\lambda_{\min} = \frac{hc}{eV}$

$$\text{In } \lambda_{\min} = In\left(\frac{hc}{e}\right) - InV$$

Clearly, slope of log λ_{min} versus log V graph is negative hence option (c) correctly depicts.

24. **(a)** From Einstein's photoelectric equation,

$$\frac{hc}{\lambda_1} - \phi = \frac{1}{2}m(2v)^2 \quad ...(i)$$

$$\text{and } \frac{hc}{\lambda_2} - \phi = \frac{1}{2}mv^2 \quad ....(ii)$$

From eqn. (i) & (ii)

$$\Rightarrow \frac{\frac{hc}{\lambda_1} - \phi}{\frac{hc}{\lambda_2} - \phi} = 4 \Rightarrow \frac{hc}{\lambda_1} - \phi = \frac{4hc}{\lambda_2} - 4\phi$$

$$\Rightarrow \frac{4hc}{\lambda_2} - \frac{hc}{\lambda_1} = 3\phi \quad \Rightarrow \phi = \frac{1}{3}hc\left(\frac{4}{\lambda_2} - \frac{1}{\lambda_1}\right)$$

$$= \frac{1}{3} \times 1240\left(\frac{4 \times 350 - 540}{350 \times 540}\right)$$
$$= 1.8 \text{ eV}$$

25. (c) Here $\omega = 2\pi \times 6 \times 10^{14}$

$\Rightarrow f = 6 \times 10^{14}$ Hz

Wavelength

$$\lambda = \frac{c}{f} = \frac{3 \times 10^8}{6 \times 10^{14}} = 0.5 \times 10^{-6} m = 5000\text{Å}$$

Given $E = \frac{12375}{5000} = 2.48 eV$

Using E = W + eV_s

$\Rightarrow 2.48 = 2 + eV_s$

or $V_s = 0.48$ V

26. (11.00) Energy of proton

$$E = \frac{hc}{\lambda} = \frac{1240}{310} = 4eV > 2eV \; [= \phi]$$

(so emission of photoelectron will take place)
$= 4 \times 1.6 \times 10^{-19} = 6.4 \times 10^{-19}$ joule

$$N = \frac{6.4 \times 10^{-5} \times 1}{4 \times 6.4 \times 10^{-19}} = 10^{14}$$

No. of photoelectrons emitted per second

$= \frac{10^{14}}{10^3} = 10^{11}$ ($\because$ 1 in 10^3 photons ejects an electron)

$\therefore$ Value of $X = 11.00$

26 Atoms

1. If 13.6 eV energy is required to ionize the hydrogen atom, then the energy required to remove an electron from $n = 2$ is **[2002]**
(a) 10.2 eV (b) 0 eV
(c) 3.4 eV (d) 6.8 eV

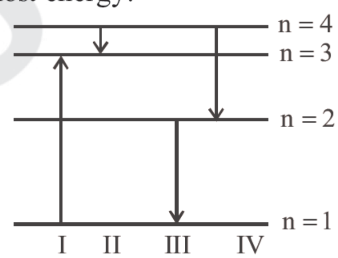
2. Which of the following atoms has the lowest ionization potential? **[2003]**
(a) ${}^{14}_{7}N$ (b) ${}^{133}_{55}Cs$
(c) ${}^{40}_{18}Ar$ (d) ${}^{16}_{8}O$

3. The wavelengths involved in the spectrum of deuterium $\left({}^{2}_{1}D\right)$ are slightly different from that of hydrogen spectrum, because **[2003]**
(a) the size of the two nuclei are different
(b) the nuclear forces are different in the two cases
(c) the masses of the two nuclei are different
(d) the attraction between the electron and the nucleus is differernt in the two cases

4. If the binding energy of the electron in a hydrogen atom is 13.6eV, the energy required to remove the electron from the first excited state of Li^{++} is **[2003]**
(a) 30.6 eV (b) 13.6 eV
(c) 3.4 eV (d) 122.4 eV

5. The manifestation of band structure in solids is due to **[2004]**
(a) Bohr's correspondence principle
(b) Pauli's exclusion principle
(c) Heisenberg's uncertainty principle
(d) Boltzmann's law

6. The diagram shows the energy levels for an electron in a certain atom. Which transition shown represents the emission of a photon with the most energy? **[2005]**



(a) IV (b) III
(c) II (d) I

7. Which of the following transitions in hydrogen atoms emit photons of highest frequency? **[2007]**
(a) $n = 1$ to $n = 2$ (b) $n = 2$ to $n = 6$
(c) $n = 6$ to $n = 2$ (d) $n = 2$ to $n = 1$

8. Suppose an electron is attracted towards the origin by a force $\frac{k}{r}$ where 'k' is a constant and 'r' is the distance of the electron from the origin. By applying Bohr model to this system, the radius of the n^{th} orbital of the electron is found to be 'r_n' and the kinetic energy of the electron to be 'T_n'. Then which of the following is true? **[2008]**
(a) $T_n \propto \frac{1}{n^2}, r_n \propto n^2$
(b) T_n independent of n, $r_n \propto n$
(c) $T_n \propto \frac{1}{n}, r_n \propto n$
(d) $T_n \propto \frac{1}{n}, r_n \propto n^2$

9. The transition from the state $n = 4$ to $n = 3$ in a hydrogen like atom results in ultraviolet radiation. Infrared radiation will be obtained in the transition from : **[2009]**

(a) $3 \to 2$ (b) $4 \to 2$

(c) $5 \to 4$ (d) $2 \to 1$

10. Energy required for the electron excitation in Li^{++} from the first to the third Bohr orbit is : **[2011]**

(a) 36.3 eV (b) 108.8 eV

(c) 122.4 eV (d) 12.1 eV

11. Hydrogen atom is excited from ground state to another state with principal quantum number equal to 4. Then the number of spectral lines in the emission spectra will be : **[2012]**

(a) 2 (b) 3

(c) 5 (d) 6

12. In a hydrogen like atom electron make transition from an energy level with quantum number n to another with quantum number (n – 1). If n>>1, the frequency of radiation emitted is proportional to : **[2013]**

(a) $\frac{1}{n}$ (b) $\frac{1}{n^2}$

(c) $\frac{1}{n^{3/2}}$ (d) $\frac{1}{n^3}$

13. Hydrogen $({}_1H^1)$, Deuterium $({}_1H^2)$, singly ionised Helium $({}_2He^4)^+$, and doubly ionised lithium $({}_3Li^6)^{++}$ all have one electron around the nucleus. Consider an electron transition from n = 2 to n = 1. If the wavelengths of emitted radiation are λ_1, λ_2, λ_3 and λ_4 respectively then approximately which one of the following is correct? **[2014]**

(a) $4\lambda_1 = 2\lambda_2 = 2\lambda_3 = \lambda_4$

(b) $\lambda_1 = 2\lambda_2 = 2\lambda_3 = \lambda_4$

(c) $\lambda_1 = \lambda_2 = 4\lambda_3 = 9\lambda_4$

(d) $\lambda_1 = 2\lambda_2 = 3\lambda_3 = 4\lambda_4$

14. As an electron makes a transition from an excited state to the ground state of a hydrogen - like atom/ion : **[2015]**

(a) kinetic energy decreases, potential energy increases but total energy remains same

(b) kinetic energy and total energy decrease but potential energy increases

(c) its kinetic energy increases but potential energy and total energy decrease

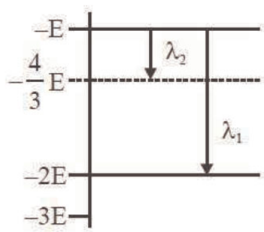
(d) kinetic energy, potential energy and total energy decrease

15. A particle A of mass m and initial velocity v collides with a particle B of mass $\frac{m}{2}$ which is at rest. The collision is head on, and elastic. The ratio of the de-Broglie wavelengths λ_A to λ_B after the collision is **[2017]**

(a) $\frac{\lambda_A}{\lambda_B} = \frac{2}{3}$ (b) $\frac{\lambda_A}{\lambda_B} = \frac{1}{2}$

(c) $\frac{\lambda_A}{\lambda_B} = \frac{1}{3}$ (d) $\frac{\lambda_A}{\lambda_B} = 2$

16. Some energy levels of a molecule are shown in the figure. The ratio of the wavelengths $r = \lambda_1/\lambda_2$, is given by **[2017]**



(a) $r = \frac{3}{4}$ (b) $r = \frac{1}{3}$

(c) $r = \frac{4}{3}$ (d) $r = \frac{2}{3}$

17. An electron from various excited states of hydrogen atom emit radiation to come to the ground state. Let λ_n, λ_g be the de Broglie wavelength of the electron in the n^{th} state and the ground state respectively. Let Λ_n be the wavelength of the emitted photon in the transition from the n^{th} state to the ground state. For large n, (A, B are constants) **[2018]**

(a) $\Lambda_n \approx A + \frac{B}{\lambda_n^2}$ (b) $\Lambda_n \approx A + B\lambda_n$

(c) $\Lambda_n^2 \approx A + B\lambda_n^2$ (d) $\Lambda_n^2 \approx \lambda$

18. If the series limit frequency of the Lyman series is v_1, then the series limit frequency of the P-fund series is : **[2018]**

(a) 25 ν_L (b) 16 ν_L
(c) $\nu_L/16$ (d) $\nu_L/25$

19. Taking the wavelength of first Balmer line in hydrogen spectrum ($n = 3$ to $n = 2$) as 660 nm, the wavelength of the 2nd Balmer line ($n = 4$ to $n = 2$) will be; **[2019]**

(a) 889.2 nm (b) 488.9 nm
(c) 642.7 nm (d) 388.9 nm

20. The time period of revolution of electron in its ground state orbit in a hydrogen atom is $1.6 \infty 10^{-16}$ s. The frequency of revolution of the electron in its first excited state (in s^{-1}) is: **[2020]**

(a) $1.6 \infty 10^{14}$ (b) $7.8 \infty 10^{14}$
(c) $6.2 \infty 10^{15}$ (d) $5.6 \infty 10^{12}$

Answer Key

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
(d)	(d)	(b)	(d)	(a)	(b)	(a)	(d)	(b)	(b)	(d)	(a)	(b)	(a)	(a)
16	17	18	19	20										
(b)	(a)	(d)	(b)	(b)										

Solutions

1. **(c)** The energy required to remove the electron from the n^{th} orbit of hydrogen is given by

$$E_n = \frac{13.6}{n^2} \text{ eV /atom}$$

For $n = 2$, $E_n = \frac{13.6}{4} = 3.4\,eV$

Therefore the energy required to remove electron from $n = 2$ is + 3.4 eV.

2. **(b)** Ionisation potential is the minimum energy required to eject electron from the outer most orbital of an atom. As $_{55}Cs^{133}$ has larger size among the given four atoms. So electron present in the outermost orbit will be away from nucleus and the electrostatic force experienced by electrons due to nucleus will be minimum.

3. **(c)** The wavelength of spectrum is given by

$$\frac{1}{\lambda} = Rz^2\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

where $R = \dfrac{1.097 \times 10^7}{1 + \dfrac{m}{M}}$

where m = mass of electron

M = mass of nucleus.

Thus, wavelength involved in the spectrum of hudrogen like atom depends upon masses of nucleus. The mass number of hydrogen and deuterium is 1 and 2 respectively, so spectrum of deuterium will be different from hydrogen.

4. **(a)** Energy for hudrogen like atom is

$$E_n = -\frac{13.6}{n^2} Z^2 \text{eV/atom}$$

where Z = atomic number

n = orbital quantum number

For lithium ion $Z = 3$; $n = 2$ (for first excited state)

$$\therefore\ E_n = -\frac{13.6}{2^2} \times 3^2 = -30.6 \text{ eV}$$

5. **(b)** The electronic configuration of number of subshells existing in a shell and number of electrons entering each subshell can be found by Pauli's exclusion principle. Hence, on the basis of Pauli's exclusion principle, the manifestation of band structure in solids can be explained.

6. **(b)** Eenrgy of radiation that corresponds to energy difference between two energy levels n_1 and n_2 is given as

$$E = Rhc\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$$

$$\therefore\ E \,\alpha \left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

E will be maximum for the transition for which $\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$ is maximum. Here n_2 is the higher energy level.

Clearly, $\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$ is maximum for the third transition, *i.e.*, $2 \rightarrow 1$. I transition is showing the absorption of energy.

7. **(d)** We have to find the frequency of emitted photons. For emission of photons electron should makes a transition from higher energy level to lower energy level. so, option (a) and (b) are incorrect.

Frequency of emitted photon is given by

$$h\nu = -13.6\left(\frac{1}{n_2^2} - \frac{1}{n_1^2}\right)$$

For transition from $n = 6$ to $n = 2$,

$$\nu_1 = \frac{-13.6}{h}\left(\frac{1}{6^2} - \frac{1}{2^2}\right) = \frac{2}{9} \times \left(\frac{13.6}{h}\right)$$

For transition from $n = 2$ to $n = 1$,

$$\nu_2 = \frac{-13.6}{h}\left(\frac{1}{2^2} - \frac{1}{1^2}\right) = \frac{3}{4} \times \left(\frac{13.6}{h}\right).$$

$$\therefore\ \nu_1 < \nu_2$$

8. **(b)** Given,

Centripetal force $= \frac{k}{r}$

Then

$$\frac{k}{r} = \frac{mv^2}{r}$$

$\Rightarrow \quad k = mv^2$

$\Rightarrow \quad Tn = \frac{1}{2}mv^2 = \frac{1}{2}k$

Tn is independent of n

Also,

Angular momentum, $L = \frac{nh}{2\pi}$

$\Rightarrow \quad mvr_n = \frac{nh}{2\pi} \quad (\because L = mvr)$

$\Rightarrow \quad r_n = \frac{nh}{2\pi\sqrt{km}} \quad \left(\begin{array}{l}\because m^2v^2 = km \\ \text{or } mv = \sqrt{km}\end{array}\right)$

Clearly, $r_n \propto n$

9. **(c)** It is given that transition from the state n = 4 to n = 3 in a hydrogen like atom result in ultraviolet radiation. For infrared radiation $\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$ should be less. The only option is $5 \rightarrow 4$.

10. **(b)** Energy of excitation (ΔE) is

$$\Delta E = 13.6\,z^2\left(\frac{1}{n_1} - \frac{1}{n_2}\right) eV$$

$$\Rightarrow \Delta E = 13.6\,(3)^2\left(\frac{1}{1^2} - \frac{1}{3^2}\right) = 108.8\,eV$$

11. **(d)** For ground state, the principal quantum no. (n) = 1. Principal quantum number 4 belongs to 3rd excited state.

The possible number of the spectral lines from a state n to ground state is

$$= \frac{n(n-1)}{2} = \frac{4(4-1)}{2} = 6$$

12. **(d)** $\Delta E = hv$

$$\nu = \frac{\Delta E}{h} = k\left[\frac{1}{(n-1)^2} - \frac{1}{n^2}\right] = \frac{k(2n-1)}{n^2(n-1)^2}$$

$$\approx \frac{2k}{n^3} \quad \text{or} \quad \nu \propto \frac{1}{n^3}$$

13. **(c)** Wave number $\frac{1}{\lambda} = RZ^2\left[\frac{1}{n_1^2} - \frac{1}{n^2}\right]$

$$\Rightarrow \lambda \propto \frac{1}{Z^2}$$

$\therefore \lambda Z^2$ = constant

By question $n = 1$ and $n_1 = 2$

Then, $\lambda_1 = \lambda_2 = 4\lambda_3 = 9\lambda_4$

14. **(c)** Kinetic energy of electron is

$$\text{K.E.} \propto \left(\frac{Z}{N}\right)^2$$

When the electron makes transition from excited state to ground state, then n increases and kinetic energy increases.

Total energy = – KE

$\therefore$ Total energy also decreases.

Potential energy is lowest for ground state.

15. **(d)** From question, $m_A = m$; $m_B = \frac{m}{2}$

$u_A = u \quad u_B = 0$

Let after collision velocity of $A = v_1$ and velocity of $B = v_2$

Applying law of conservation of momentum,

$$mu = mv_1 + \left(\frac{m}{2}\right)v_2$$

or, $2u = 2v_1 + v_2$ ....(*i*)

By law of collision

$$e = \frac{v_2 - v_1}{u - 0}$$

or, $u = v_2 - v_1$ ....(*ii*)

[$\because$ collision is elastic, $e = 1$]

using eqns (*i*) and (*ii*)

$v_1 = \frac{u}{3}$ and $v_2 = \frac{4}{3}u$

de-Broglie wavelength $\lambda = \frac{h}{p}$

$$\therefore \quad \frac{\lambda_A}{\lambda_B} = \frac{P_B}{P_A} = \frac{\frac{m}{2} \times \frac{4}{3}u}{m \times \frac{u}{3}} = 2$$

16. (b) From energy level diagram, using $\Delta E = \frac{hc}{\lambda}$

For wavelength λ_1

$$\Delta E = -E - (-2E) = \frac{hc}{\lambda_1}$$

$$\therefore \quad \lambda_1 = \frac{hc}{E}$$

For wavelength λ_2

$$\Delta E = -E - \left(-\frac{4E}{3}\right) = \frac{hc}{\lambda_2}$$

$$\therefore \quad \lambda_2 = \frac{hc}{\left(\frac{E}{3}\right)} \quad \therefore r = \frac{\lambda_1}{\lambda_2} = \frac{1}{3}$$

17. (a) Wavelength of emitted photon from n^{th} state to the ground state,

$$\frac{1}{\Lambda_n} = RZ^2\left(\frac{1}{1^2} - \frac{1}{n^2}\right)$$

$$\Lambda_n = \frac{1}{RZ^2}\left(1 - \frac{1}{n^2}\right)^{-1}$$

Since n is very large, using binomial theorem

$$\Lambda_n = \frac{1}{RZ^2}\left(1 + \frac{1}{n^2}\right)$$

$$\Lambda_n = \frac{1}{RZ^2} + \frac{1}{RZ^2}\left(\frac{1}{n^2}\right)$$

As we know,

$$\lambda_n = \frac{2\pi r}{n} = 2\pi\left(\frac{n^2h^2}{4\pi^2 mZe^2}\right)\frac{1}{n} \propto n$$

$$\Lambda_n \approx A + \frac{B}{\lambda_n^2}$$

18. (d) $h\nu_L = E_\infty - E_1$ ...(i)

$h\nu_f = E_\infty - E_5$ ...(ii)

$$E \propto \frac{z^2}{n^2} \Rightarrow \frac{E_5}{E_1} = \left(\frac{1}{5}\right)^2 = \frac{1}{25}$$

$$\text{Eqn (i) / (ii)} \Rightarrow \frac{h\nu_L}{h\nu_f} = \frac{E_1}{E_5}$$

$$\Rightarrow \frac{\nu_L}{\nu_f} = \frac{25}{1} \Rightarrow \nu_f = \frac{\nu_L}{25}$$

19. (b) $\frac{1}{\lambda_1} = R\left(\frac{1}{2^2} - \frac{1}{3^2}\right) = \frac{5R}{36}$

$$\frac{1}{\lambda_2} = R\left(\frac{1}{2^2} - \frac{1}{4^2}\right) = \frac{3R}{16}$$

$$\therefore \quad \frac{\lambda_2}{\lambda_1} = \frac{80}{108}$$

$$\lambda_2 = \frac{80}{108}\lambda_1 = \frac{80}{108} \times 660 = 488.9\text{nm}.$$

20. (b) For first excited state $n' = 3$

Time period $T \propto \frac{n^3}{z^2}$

$$\Rightarrow \frac{T_2}{T_1} = \frac{n'^3}{n^3}$$

$$\therefore T_2 = 8T_1 = 8 \times 1.6 \times 10^{-16} s$$

$$\therefore \text{Frequency, } v = \frac{1}{T_2} = \frac{1}{8 \times 1.6 \times 10^{-16}}$$

$$\approx 7.8 \times 10^{14}\, Hz$$

27 Nuclei

1. At a specific instant emission of radioactive compound is deflected in a magnetic field. The compound can emit
(i) electrons (ii) protons
(iii) He^{2+} (iv) neutrons
The emission at instant can be **[2002]**
(a) i, ii, iii (b) i, ii, iii, iv
(c) iv (d) ii, iii

2. If N_0 is the original mass of the substance of half-life period $t_{1/2} = 5$ years, then the amount of substance left after 15 years is **[2002]**
(a) $N_0/8$ (b) $N_0/16$
(c) $N_0/2$ (d) $N_0/4$

3. When a U^{238} nucleus originally at rest, decays by emitting an alpha particle having a speed 'u', the recoil speed of the residual nucleus is **[2003]**
(a) $\frac{4u}{238}$ (b) $-\frac{4u}{234}$
(c) $\frac{4u}{234}$ (d) $-\frac{4u}{238}$

4. A radioactive sample at any instant has its disintegration rate 5000 disintegrations per minute. After 5 minutes, the rate is 1250 disintegrations per minute. Then, the decay constant (per minute) is **[2003]**
(a) 0.4 ln 2 (b) 0.2 ln 2
(c) 0.1 ln 2 (d) 0.8 ln 2

5. A nucleus with $Z = 92$ emits the following in a sequence:
$\alpha, \beta^-, \beta^- \alpha, \alpha, \alpha, \alpha, \alpha, \beta^-, \beta^-, \alpha, \beta^+, \beta^+, \alpha$
Then Z of the resulting nucleus is **[2003]**
(a) 76 (b) 78 (c) 82 (d) 74

6. Which of the following **cannot** be emitted by radioactive substances during their decay? **[2003]**
(a) Protons (b) Neutrinoes
(c) Helium nuclei (d) Electrons

7. In the nuclear fusion reaction
$$^2_1H + ^3_1H \rightarrow ^4_2He + n$$
given that the repulsive potential energy between the two nuclei is $\sim 7.7 \times 10^{-14}$ J, the temperature at which the gases must be heated to initiate the reaction is nearly
[Boltzmann's Constant $k = 1.38 \times 10^{-23}$ J/K] **[2003]**
(a) 10^7 K (b) 10^5 K
(c) 10^3 K (d) 10^9 K

8. A nucleus disintegrated into two nuclear parts which have their velocities in the ratio of 2 : 1. The ratio of their nuclear sizes will be **[2004]**
(a) $3^{1/2} : 1$ (b) $1 : 2^{1/3}$
(c) $2^{1/3} : 1$ (d) $1 : 3^{1/2}$

9. The binding energy per nucleon of deuteron $\left(^2_1H\right)$ and helium nucleus $\left(^4_2He\right)$ is 1.1 MeV and 7 MeV respectively. If two deuteron nuclei react to form a single helium nucleus, then the energy released is **[2004]**
(a) 23.6 MeV (b) 26.9 MeV
(c) 13.9 MeV (d) 19.2 MeV

10. If radius of the $^{27}_{13}Al$ nucleus is estimated to be 3.6 fermi then the radius of $^{125}_{52}Te$ nucleus be nearly **[2005]**
(a) 8 fermi (b) 6 fermi
(c) 5 fermi (d) 4 fermi

11. Starting with a sample of pure ^{66}Cu, $\frac{7}{8}$ of it decays into Zn in 15 minutes. The corresponding half life is **[2005]**

(a) 15 minutes (b) 10 minutes

(c) $7\frac{1}{2}$ minutes (d) 5 minutes

12. The intensity of gamma radiation from a given source is I. On passing through 36 mm of lead, it is reduced to $\frac{I}{8}$. The thickness of lead which will reduce the intensity to $\frac{I}{2}$ will be **[2005]**

(a) 9mm (b) 6mm

(c) 12mm (d) 18mm

13. A nuclear transformation is denoted by $X(n, \alpha)\ {}^{7}_{3}Li$. Which of the following is the nucleus of element X? **[2005]**

(a) ${}^{10}_{5}B$ (b) ${}^{12}C_6$

(c) ${}^{11}_{4}Be$ (d) ${}^{9}_{5}B$

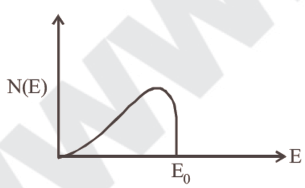
14. When ${}_3Li^7$ nuclei are bombarded by protons, and the resultant nuclei are ${}_4Be^8$, the emitted particles will be **[2006]**

(a) alpha particles (b) beta particles

(c) gamma photons (d) neutrons

15. The energy spectrum of β-particles [number N(E) as a function of β-energy E] emitted from a radioactive source is **[2006]**

(a)



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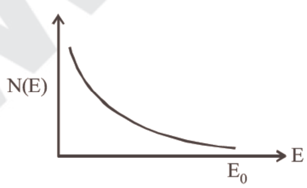

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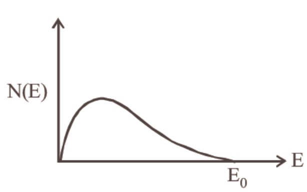
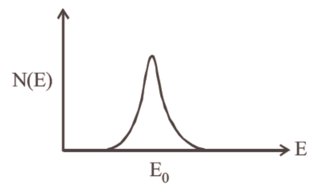


(d)



16. If the binding energy per nucleon in ${}^{7}_{3}Li$ and ${}^{4}_{2}He$ nuclei are 5.60 MeV and 7.06 MeV respectively, then in the reaction

$$p + {}^{7}_{3}Li \longrightarrow 2\ {}^{4}_{2}He$$

energy of proton must be **[2006]**

(a) 28.24 MeV (b) 17.28 MeV

(c) 1.46 MeV (d) 39.2 MeV

17. The 'rad' is the correct unit used to report the measurement of **[2006]**

(a) the ability of a beam of gamma ray photons to produce ions in a target

(b) the energy delivered by radiation to a target

(c) the biological effect of radiation

(d) the rate of decay of a radioactive source

18. If M_O is the mass of an oxygen isotope ${}_8O^{17}$, M_P and M_N are the masses of a proton and a neutron respectively, the nuclear binding energy of the isotope is **[2007]**

(a) $(M_O - 17M_N)c^2$

(b) $(M_O - 8M_P)c^2$

(c) $(M_O - 8M_P - 9M_N)c^2$

(d) M_Oc^2

19. In gamma ray emission from a nucleus **[2007]**

(a) only the proton number changes

(b) both the neutron number and the proton number change

(c) there is no change in the proton number and the neutron number

(d) only the neutron number changes

20. The half-life period of a radio-active element X is same as the mean life time of another radio-active element Y. Initially they have the same number of atoms. Then **[2007]**

(a) X and Y decay at same rate always

(b) X will decay faster than Y

(c) Y will decay faster than X

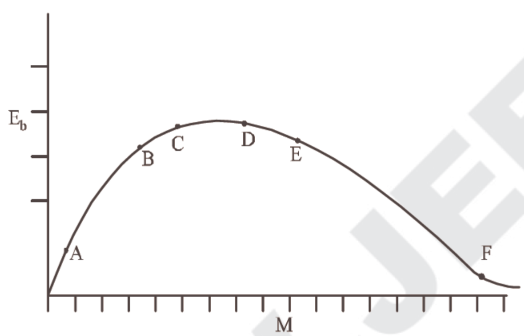
(d) X and Y have same decay rate initially

21. This question contains Statement-1 and statement-2. Of the four choices given after the statements, choose the one that best describes the two statements. **[2008]**

Statement-1:
Energy is released when heavy nuclei undergo fission or light nuclei undergo fusion and
Statement-2 :
For heavy nuclei, binding energy per nucleon increases with increasing Z while for light nuclei it decreases with increasing Z.

(a) Statement-1 is false, Statement-2 is true
(b) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1
(c) Statement-1 is true, Statement-2 is true; Statement-2 is not a correct explanation for Statement-1
(d) Statement-1 is true, Statement-2 is false

22.



The above is a plot of binding energy per nucleon E_b, against the nuclear mass M; A, B, C, D, E, F correspond to different nuclei. Consider four reactions : **[2009]**

(i) $A+B \rightarrow C+\varepsilon$
(ii) $C \rightarrow A+B+\varepsilon$
(iii) $D+E \rightarrow F+\varepsilon$ and
(iv) $F \rightarrow D+E+\varepsilon$,

where ε is the energy released? In which reactions is ε positive?

(a) (i) and (iii) (b) (ii) and (iv)
(c) (ii) and (iii) (d) (i) and (iv)

DIRECTIONS: Questions number 23-24 are based on the following paragraph.

A nucleus of mass $M + \Delta$m is at rest and decays into two daughter nuclei of equal mass $\frac{M}{2}$ each. Speed of light is c.

23. The binding energy per nucleon for the parent nucleus is E_1 and that for the daughter nuclei is E_2. Then **[2010]**

(a) $E_2 = 2E_1$ (b) $E_1 > E_2$
(c) $E_2 > E_1$ (d) $E_1 = 2E_2$

24. The speed of daughter nuclei is **[2010]**

(a) $c\frac{\Delta m}{M+\Delta m}$ (b) $c\sqrt{\frac{2\Delta m}{M}}$

(c) $c\sqrt{\frac{\Delta m}{M}}$ (d) $c\sqrt{\frac{\Delta m}{M+\Delta m}}$

25. A radioactive nucleus (initial mass number A and atomic number Z emits 3 α-particles and 2 positrons. The ratio of number of neutrons to that of protons in the final nucleus will be **[2010]**

(a) $\frac{A-Z-8}{Z-4}$ (b) $\frac{A-Z-4}{Z-8}$

(c) $\frac{A-Z-12}{Z-4}$ (d) $\frac{A-Z-4}{Z-2}$

26. The half life of a radioactive substance is 20 minutes. The approximate time interval $(t_2 - t_1)$ between the time t_2 when $\frac{2}{3}$ of it had decayed and time t_1 when $\frac{1}{3}$ of it had decayed is : **[2011]**

(a) 14 min (b) 20 min
(c) 28 min (d) 7 min

27. After absorbing a slowly moving neutron of mass m_N (momentum ≈ 0) a nucleus of mass M breaks into two nuclei of masses m_1 and $5m_1$ $(6m_1 = M + m_N)$ respectively. If the de Broglie wavelength of the nucleus with mass m_1 is λ, the de Broglie wavelength of the nucleus will be **[2011]**

(a) 5λ (b) $\lambda/5$ (c) λ (d) 25λ

28. **Statement - 1 :** A nucleus having energy E_1 decays by β^- emission to daughter nucleus having energy E_2, but the β^- rays are emitted with a continuous energy spectrum having end point energy $E_1 - E_2$.
Statement - 2 : To conserve energy and momentum in β^- decay at least three particles must take part in the transformation. **[2011 RS]**
(a) Statement-1 is correct but statement-2 is not correct.
(b) Statement-1 and statement-2 both are correct and statement-2 is the correct explanation of statement-1.
(c) Statement-1 is correct, statement-2 is correct and statement-2 is not the correct explanation of statement-1
(d) Statement-1 is incorrect, statement-2 is correct.

29. Assume that a neutron breaks into a proton and an electron. The energy released during this process is : (mass of neutron = 1.6725×10^{-27} kg, mass of proton = 1.6725×10^{-27} kg, mass of electron = 9×10^{-31} kg). **[2012]**
(a) 0.51 MeV (b) 7.10 MeV
(c) 6.30 MeV (d) 5.4 MeV

30. Half-lives of two radioactive elements A and B are 20 minutes and 40 minutes, respectively. Initially, the samples have equal number of nuclei. After 80 minutes, the ratio of decayed number of A and B nuclei will be : **[2016]**
(a) 1 : 4 (b) 5 : 4
(c) 1 : 16 (d) 4 : 1

31. A radioactive nucleus A with a half life T, decays into a nucleus B. At $t = 0$, there is no nucleus B. At sometime t, the ratio of the number of B to that of A is 0.3. Then, t is given by **[2017]**
(a) $t = T \log(1.3)$ (b) $t = \frac{T}{\log(1.3)}$
(c) $t = T\frac{\log 2}{\log 1.3}$ (d) $t = T\frac{\log 1.3}{\log 2}$

32. A sample of radioactive material A, that has an activity of 10 mCi (1 Ci = 3.7×10^{10} decays/s), has twice the number of nuclei as another sample of a different radioactive material B which has an acitvity of 20 mCi. The correct choices for half-lives of A and B would then be respectively: **[2019]**
(a) 5 days and 10 days
(b) 10 days and 40 days
(c) 20 days and 5 days
(d) 20 days and 10 days

Answer Key

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
(a)	(a)	(c)	(a)	(b)	(a)	(d)	(b)	(a)	(b)	(d)	(c)	(a)	(c)	(c)
16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
(b)	(c)	(c)	(c)	(c)	(d)	(d)	(c)	(b)	(b)	(b)	(c)	(b)	(a)	(b)
31	32													
(d)	(c)													

Solutions

1. **(a)** Charged particles are deflected in magnetic field. Electrons, protons and He^{2+} all are charged species. Hence, correct option is (a).

2. **(a)** After every half-life, the mass of the substance reduces to half its initial value.

$$N_0 \xrightarrow{5\text{ years}} \frac{N_0}{2} \xrightarrow{5\text{ years}} \frac{N_0/2}{2}$$

$$= \frac{N_0}{4} \xrightarrow{5\text{years}} \frac{N_0/4}{2} = \frac{N_0}{8}$$

ALTERNATE SOLUTION

Let N_0 is the initial amount of substance and N is the amount left after decay.

Number of half lives $n = \frac{t}{t_{1/2}} = \frac{15}{5} = 3$

We know that

$$N = N_0\left(\frac{1}{2}\right)^n = N_0\left(\frac{1}{2}\right)^3 = \frac{N_0}{8}$$

3. **(c)** Mass of α particle, $m_\alpha = 4\,u$
Mass of nucleus after fission, $m_n = 234u$
From conservation of linear momentum we have
$238 \times 0 = 4\,u + 234\,v$

$$\therefore\ v = -\frac{4}{234}u$$

$$\therefore\ \text{Speed} = |\vec{v}| = \frac{4}{234}u$$

4. **(a)** Initial activity, $A_o = 5000$ disintegration per minute
Activity after 5 min, $A = 1250$ disintegration per minute
$A = A_o e^{-\lambda t}$

$$\Rightarrow e^{-\lambda t} = \frac{A_o}{A}$$

$$\Rightarrow \lambda = \frac{1}{t}\log_e\frac{A_o}{A} = \frac{1}{5}\log_e\frac{5000}{1250}$$

$$= \frac{2}{5}\log_e 2 = 0.4\log_e 2$$

5. **(b)** The number of α-particles released = 8
Decrease in atomic number = $8 \times 2 = 16$
The number of β^--particles released = 4
Increase in atomic number = $4 \times 1 = 4$
Also the number of β^+ particles released is 2, which should decrease the atomic number by 2.
Therefore the final atomic number of resulting nucleus
$= Z - 16 + 4 - 2 = Z - 14$
$= 92 - 14 = 78$

6. **(a)** The radioactive substances emit α-particles (Helium nucleus), β-particles (electrons) and neutrinoes. Protons cannot be emitted by radioactive substances during their decay.

7. **(d)** The average kinetic energy per molecule at temperature T is

$$= \frac{3}{2}kT$$

Where k = Boltzmann's constant
This kinetic energy should be able to provide the repulsive potential energy

$$\therefore\ \frac{3}{2}kT = 7.7 \times 10^{-14}$$

$$\Rightarrow T = \frac{2 \times 7.7 \times 10^{-14}}{3 \times 1.38 \times 10^{-23}} = 3.7 \times 10^9\,K$$

8. **(b)** Given :

$$\frac{v_1}{v_2} = \frac{2}{1}$$

From conservation of momentum

$$m_1 v_1 = m_2 v_2$$

$$\Rightarrow \left(\frac{m_1}{m_2}\right) = \left(\frac{v_2}{v_1}\right) = \frac{1}{2}$$

We know that mass of nucleus, $m \propto A$
Nuclear size $R \propto A^{1/3} \propto m^{1/3}$

$$\frac{R_1}{R_2} = \left(\frac{m_1}{m_2}\right)^{1/3} \Rightarrow \frac{R_1^3}{R_2^3} = \frac{1}{2}$$

$$\Rightarrow \left(\frac{R_1}{R_2}\right) = \left(\frac{1}{2}\right)^{1/3}$$

9. **(a)** The chemical reaction of process is

$$2\,{}_1^2\text{H} \rightarrow {}_2^4\text{He}$$

Binding energy of two deuterons,
$4 \times 1.1 = 4.4$ MeV
Binding energy of helium nucleus = $4 \times 7 =$ 28 MeV
Energy released $= 28 - 4.4 = 23.6$ MeV

10. **(b)** Radius of a nucleus,

$$R = R_0(A)^{1/3}$$

Here, R_0 is a constant
A = atomic mass number

$$\therefore \frac{R_1}{R_2} = \left(\frac{A_1}{A_2}\right)^{1/3} = \left(\frac{27}{125}\right)^{1/3} = \frac{3}{5}$$

$$\Rightarrow R_2 = \frac{5}{3} \times 3.6 = 6 \text{ fermi}$$

11. (d) It is given that

$\frac{7}{8}$ of Cu decays in 15 minutes.

$\therefore$ Cu left undecayed is

$$N = 1 - \frac{7}{8} = \frac{1}{8} = \left(\frac{1}{2}\right)^3$$

$\therefore$ No. of half lifes = 3

$$n = \frac{t}{T}$$

$$\Rightarrow 3 = \frac{15}{T}$$

$$\Rightarrow T = \text{half life period} = \frac{15}{3} = 5 \text{ minutes}$$

ALTERNATE SOLUTION

$$N = N_0 (1 - e^{-\lambda t})$$

$$\Rightarrow \frac{N_0 - N}{N_0} = e^{-\lambda t} \qquad \therefore \frac{1}{8} = e^{-\lambda t}$$

$$3 \ \ell n\, 2 = \lambda t \text{ or } \lambda = \frac{3 \times 0.693}{15} = 0.1386$$

Half-lifeperiod,

$$t_{1/2} = \frac{0.693}{\lambda} = \frac{0.693}{0.1386} = 5 \text{ minutes}$$

12. (c) Let intensity of gamma radiation from source be I_0.

Intensity $I = I_0 . e^{-\mu d}$,

Where d is the thickness of lead.

Applying logarithm on both sides,

$$-\mu d = \log\left(\frac{I}{I_0}\right)$$

For $d = 36$ mm, intensity $= \frac{I}{8}$

$$-\mu \times 36 = \log\left(\frac{I/8}{I}\right) \text{ ....(i)}$$

For intensity $I/2$, thickness $= d$

$$-\mu \times d = \log\left(\frac{I/2}{I}\right) \text{ ....}(ii)$$

Dividing (i) by (ii),

$$\frac{36}{d} = \frac{\log\left(\frac{1}{8}\right)}{\log\left(\frac{1}{2}\right)} = \frac{3\log\left(\frac{1}{2}\right)}{\log\left(\frac{1}{2}\right)} = 3 \text{ or } d = \frac{36}{3}$$

$= 12$ mm

13. (a) $_ZX^A + {}_0n^1 \longrightarrow {}_3Li^7 + {}_2He^4$

Using conservation of mass number

$A + 1 = 4 + 7$

$\Rightarrow A = 10$

Using conservation of charge number

$Z + 0 = 2 + 3$

$\Rightarrow Z = 5$

It is boron $_5B^{10}$

14. (c) ${}^7_3\text{Li} + {}^1_1\text{p} \longrightarrow {}^8_4\text{Be} + {}^0_0\gamma$

We see that both proton number and mass number are equal in both sides, so emitted particle should be massless gamma photons.

15. (c) The range of energy of β-particles is from zero to some maximum value.

16. (b) Given,

Binding energy per nucleon of ${}^7_3\text{Li} = 5.60$ MeV

Binding energy per nucleon of ${}^4_2\text{He} = 7.06$ MeV

Let E be the energy of proton, then

$$E + 7 \times 5.6 = 2 \times [4 \times 7.06]$$

$$\Rightarrow E = 56.48 - 39.2 = 17.28 \text{MeV}$$

17. (c) The risk posed to a human being by any radiation exposure depends partly upon the absorbed dose, the amount of energy absorbed per gram of tissue. Absorbed dose is expressed in rad. Thus, it is used to report biological effect of radiation.

18. (c) Number of protons in oxygen isotope, $Z = 8$

Number of neutrons = 17 – 8 = 9

Binding energy

$= [ZM_P + (A - Z)M_N - M]c^2$

$= [8M_P + (17 - 8)M_N - M]c^2$

$= [8M_P + 9M_N - M]c^2$

$= [8M_P + 9M_N - M_o]c^2$

19. **(c)** There is no change in the proton number and the neutron number as the γ-emission takes place as a result of excitation or de-excitation of nuclei. γ-rays have no charge or mass.

20. **(c)** Let λ_X and λ_Y be the decay constant of X and Y.

Half life of X, = average life of Y

$$T_{1/2} = T_{av}$$

$$\Rightarrow \frac{0.693}{\lambda_X} = \frac{1}{\lambda_Y}$$

$$\Rightarrow \lambda_X = (0.693).\lambda_Y$$

$$\therefore \lambda_X < \lambda_Y.$$

Now, the rate of decay is given by

$$-\left(\frac{dN}{dt}\right)_x = \lambda_X N_0$$

$$-\left(\frac{dN}{dt}\right)_y = \lambda_y N_0$$

As the rate of decay is directly proportional to decay constant, Y will decay faster than X.

21. **(d)** We know that energy is released when heavy nuclei undergo fission or light nuclei undergo fusion. Therefore statement (1) is correct.

The second statement is false because for heavy nuclei the binding energy per nucleon decreases with increasing Z and for light nuclei, B.E/nucleon increases with increasing Z.

22. **(d)** For $A + B \rightarrow C + \varepsilon$, ε is positive. This is because binding energy for C is greater than the binding energy for A and B.

Again for $F \rightarrow D + E + \varepsilon$, ε is positive. This is because binding energy for D and E is greater than binding energy for F.

23. **(c)** In nuclear fission, the binding energy per nucleon of daughter nuclei is always greater than the parent nucleus.

24. **(b)** Mass defect, $\Delta M = \left[(M+\Delta m) - \left(\frac{M}{2}+\frac{M}{2}\right)\right]$

$$= [M + \Delta m - M] = \Delta m$$

Energy released, $Q = \Delta M c^2 = \Delta m c^2$ ...(i)

From the law of conservation of momentum

$$(M+\Delta m)\times 0 = \frac{M}{2}v_1 - \frac{M}{2}\times v_2$$

$$\Rightarrow v_1 = v_2$$

Now, $Q = \frac{1}{2}\left(\frac{M}{2}\right)v_1^2 + \frac{1}{2}\left(\frac{M}{2}\right)v_2^2 - \frac{1}{2}(M+\Delta m)\times(0)^2$

$$= \frac{M}{2}v_1^2 \; (\because v_1 = v_2) \quad \text{...(ii)}$$

From equation (i) and (ii), we get

$$\left(\frac{M}{2}\right)v_1^2 = \Delta mc^2$$

$$\Rightarrow v_1^2 = \frac{2\Delta mc^2}{M}$$

$$\Rightarrow V_1 = c\sqrt{\frac{2\Delta m}{M}}$$

25. **(b)** When a radioactive nucleus emits 1 α-particle, the mass number decreases by 4 units and atomic number decreases by 2 units. When a radioactive nucleus emits 1 positron the atomic number decreases by 1 unit but mass number remains constant.

$\therefore$ Mass number of final nucleus = $A - 12$

Atomic number of final nucleus = $Z - 8$

$\therefore$ Number of neutrons, $N_n = (A-12)-(Z-8)$
$= A - Z - 4$

Number of protons, $N_p = Z - 8$

$$\therefore \text{Required ratio} = \frac{N_n}{N_p} = \frac{A-Z-4}{Z-8}$$

26. **(b)** Number of undecayed atom after time t_2;

$$\frac{N_0}{3} = N_0 e^{-\lambda t_2} \quad \text{...(i)}$$

Number of undecayed atom after time t_1;

$$\frac{2N_0}{3} = N_0 e^{-\lambda t_1} \quad \text{...(ii)}$$

Dividing (ii) by (i), we get

$$2 = e^{\lambda(t_2 - t_1)}$$

$$\Rightarrow \text{In } 2 = \lambda(t_2 - t_1)$$

$$\Rightarrow t_2 - t_1 = \text{In } 2/\lambda$$

27. **(c)** Initial momentum of system, $p_i = 0$

Let p_1 and p_2 be the momentum of broken nuclei of masses m_1 and $5m_1$ respectively.

$p_f = p_1 + p_2$

From the conservation of momentum

$p_i = p_f$

$0 = p_1 + p_2$

$p_1 = -p_2$

From de Broglie relation, wavelength

$\lambda_1 = \dfrac{h}{p_1}$

$\lambda_2 = \dfrac{h}{p_2}$

$|\lambda_1| = |\lambda_2|$

$\lambda_1 = \lambda_2 = \lambda.$

28. **(b)** **Statement-1:** A nucleus having energy E_1 decays by β- emission to daughter nucleus having energy E_2 then β- rays are emitted with continuous energy spectrum with energy $E_1 - E_2$.

Statement-2: For energy conservation and momentum conservation at least three particles, daughter nucleus, β particle and antineutrino are required.

29. **(a)** ${}_0^1\text{n} \longrightarrow {}_1^1\text{H} + {}_{-1}\text{e}^0 + \overline{\nu} + Q$

The mass defect during the process

$\Delta m = m_n - m_H - m_e = 1.6725 \times 10^{-27} - (1.6725 \times 10^{-27} + 9 \times 10^{-31}\text{kg})$

$= -9 \times 10^{-31}$ kg

The energy released during the process

$E = \Delta mc^2$

$E = 9 \times 10^{-31} \times 9 \times 10^{16} = 81 \times 10^{-15}$ Joules

$E = \dfrac{81 \times 10^{-15}}{1.6 \times 10^{-19}} = 0.511\text{MeV}$

30. **(b)** For $A_{t_{1/2}} = 20$ min, $t = 80$ min, number of half lifes $n = 4$

$\therefore$ Nuclei remaining $= \dfrac{N_0}{2^4}$. Therefore nuclei decayed

$= N_0 - \dfrac{N_0}{2^4}$

For $B_{t_{1/2}} = 40$ min., $t = 80$ min, number of half lifes $n = 2$

$\therefore$ Nuclei remaining $= \dfrac{N_0}{2^2}$. Therefore nuclei decayed

$= N_0 - \dfrac{N_0}{2^2}$

$\therefore$ Required ratio $= \dfrac{N_0 - \dfrac{N_0}{2^4}}{N_0 - \dfrac{N_0}{2^2}} = \dfrac{1-\dfrac{1}{16}}{1-\dfrac{1}{4}}$

$= \dfrac{15}{16} \times \dfrac{4}{3} = \dfrac{5}{4}$

31. **(d)** Let initially there are total N_0 number of nuclei

At time t

$\dfrac{N_B}{N_A} = 0.3(\text{given})$

$\Rightarrow N_B = 0.3N_A$

$N_0 = N_A + N_B = N_A + 0.3N_A$

$\therefore N_A = \dfrac{N_0}{1.3}$

As we know $N_t = N_0 e^{-\lambda t}$

or, $\dfrac{N_0}{1.3} = N_0 e^{-\lambda t}$

$\dfrac{1}{1.3} = e^{-\lambda t}$

$\Rightarrow \; ln(1.3) = \lambda t$

or, $t = \dfrac{ln(1.3)}{\lambda}$

$\Rightarrow \; t = \dfrac{ln(1.3)}{\dfrac{ln(2)}{T}} = \dfrac{ln(1.3)}{ln(2)} T$

32. **(c)** Activity $A = \lambda N$

For material, A

$10 = (2N_0)\lambda_A$

For material, B $\quad 20 = N_0\lambda_B$

$\Rightarrow \; \lambda_B = 4\lambda_B$

$\therefore \; T_{1/2A} = 4T_{1/2B} \left[\because T_{1/2} = \dfrac{0.693}{\lambda}\right]$

i.e., 20 days half-lives for A and 5 days $\left(T_{1/2}\right)_B$ for material B.

28 Semiconductor Electronics : Materials, Devices and Simple Circuits

1. At absolute zero, Si acts as **[2002]**
   (a) non-metal (b) metal
   (c) insulator (d) none of these
2. By increasing the temperature, the specific resistance of a conductor and a semiconductor
   (a) increases for both **[2002]**
   (b) decreases for both
   (c) increases, decreases
   (d) decreases, increases
3. The energy band gap is maximum in **[2002]**
   (a) metals (b) superconductors
   (c) insulators (d) semiconductors.
4. The part of a transistor which is most heavily doped to produce large number of majority carriers is **[2002]**
   (a) emitter
   (b) base
   (c) collector
   (d) can be any of the above three.
5. A strip of copper and another of germanium are cooled from room temperature to 80K. The resistance of **[2003]**
   (a) each of these decreases
   (b) copper strip increases and that of germanium decreases
   (c) copper strip decreases and that of germanium increases
   (d) each of these increases
6. The difference in the variation of resistance with temeperature in a metal and a semiconductor arises essentially due to the difference in the
   (a) crystal sturcture **[2003]**
   (b) variation of the number of charge carriers with temperature
   (c) type of bonding
   (d) variation of scattering mechanism with temperature
7. In the middle of the depletion layer of a reverse-biased *p-n* junction, the **[2003]**
   (a) electric field is zero
   (b) potential is maximum
   (c) electric field is maximum
   (d) potential is zero
8. When npn transistor is used as an amplifier **[2004]**
   (a) electrons move from collector to base
   (b) holes move from emitter to base
   (c) electrons move from base to collector
   (d) holes move from base to emitter
9. For a transistor amplifier in common emitter configuration for load impedance of 1kΩ ($h_{fe} = 50$ and $h_{oe} = 25$) the current gain is **[2004]**
   (a) -24.8 (b) -15.7
   (c) -5.2 (d) -48.78
10. A piece of copper and another of germanium are cooled from room temperature to 77K, the resistance of **[2004]**
    (a) copper increases and germanium decreases
    (b) each of them decreases
    (c) each of them increases
    (d) copper decreases and germanium increases
11. When p-n junction diode is forward biased then **[2004]**
    (a) both the depletion region and barrier height are reduced
    (b) the depletion region is widened and barrier height is reduced
    (c) the depletion region is reduced and barrier height is increased
    (d) Both the depletion region and barrier height are increased

12. The electrical conductivity of a semiconductor increases when electromagnetic radiation of wavelength shorter than 2480 nm is incident on it. The band gap in (eV) for the semiconductor is **[2005]**

(a) 2.5 eV (b) 1.1 eV
(c) 0.7 eV (d) 0.5 eV

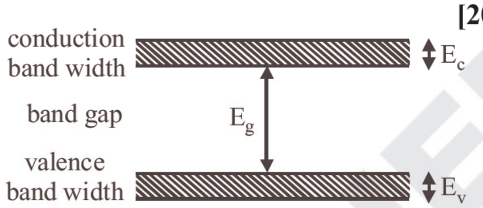
13. In a common base amplifier, the phase difference between the input signal voltage and output voltage is **[2005]**

(a) π (b) $\frac{\pi}{4}$
(c) $\frac{\pi}{2}$ (d) 0

14. In a full wave rectifier circuit operating from 50 Hz mains frequency, the fundamental frequency in the ripple would be **[2005]**

(a) 25 Hz (b) 50 Hz
(c) 70.7 Hz (d) 100 Hz

15. If the lattice constant of this semiconductor is decreased, then which of the following is correct? **[2006]**



(a) All E_c, E_g, E_v increase
(b) E_c and E_v increase, but E_g decreases
(c) E_c and E_v decrease, but E_g increases
(d) All E_c, E_g, E_v decrease

16. In a common base mode of a transistor, the collector current is 5.488 mA for an emitter current of 5.60 mA. The value of the base current amplification factor (β) will be **[2006]**

(a) 49 (b) 50
(c) 51 (d) 48

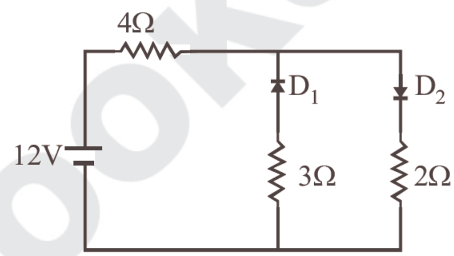
17. A solid which is not transparent to visible light and whose conductivity increases with temperature is formed by **[2006]**

(a) Ionic bonding
(b) Covalent bonding
(c) Vander Waals bonding
(d) Metallic bonding

18. If the ratio of the concentration of electrons to that of holes in a semiconductor is $\frac{7}{5}$ and the ratio of currents is $\frac{7}{4}$, then what is the ratio of their drift velocities? **[2006]**

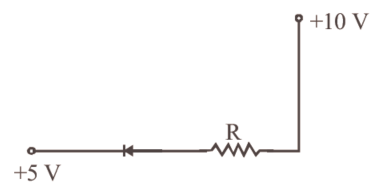
(a) $\frac{5}{8}$ (b) $\frac{4}{5}$
(c) $\frac{5}{4}$ (d) $\frac{4}{7}$

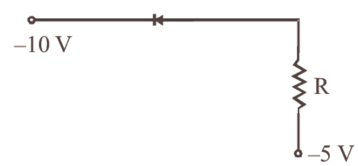
19. The circuit has two oppositively connected ideal diodes in parallel. What is the current flowing in the circuit? **[2006]**

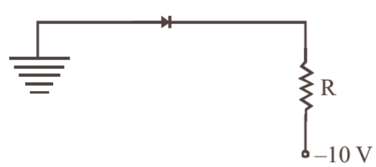


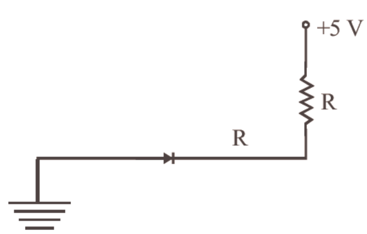
(a) 1.71 A (b) 2.00 A
(c) 2.31 A (d) 1.33 A

20. In the following, which one of the diodes reverse biased? **[2006]**

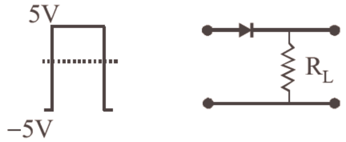
(a)


(b)


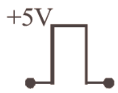
(c)


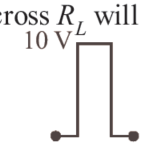
(d)


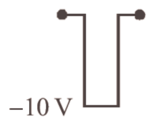
21. If in a *p-n* junction diode, a square input signal of 10 V is applied as shown **[2007]**



Then the output signal across R_L will be

(a) 

(b) 

(c) 

(d) 

22. Carbon, silicon and germanium have four valence electrons each. At room temperature which one of the following statements is most appropriate ? **[2007]**

(a) The number of free electrons for conduction is significant only in Si and Ge but small in C.

(b) The number of free conduction electrons is significant in C but small in Si and Ge.

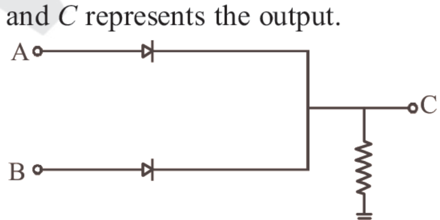
(c) The number of free conduction electrons is negligibly small in all the three.

(d) The number of free electrons for conduction is significant in all the three.

23. A working transistor with its three legs marked *P*, *Q* and *R* is tested using a multimeter. No conduction is found between *P* and *Q*. By connecting the common (negative) terminal of the multimeter to *R* and the other (positive) terminal to *P* or *Q*, some resistance is seen on the multimeter. Which of the following is true for the transistor? **[2008]**

(a) It is an npn transistor with *R* as base
(b) It is a pnp transistor with *R* as base
(c) It is a pnp transistor with *R* as emitter
(d) It is an npn transistor with *R* as collector

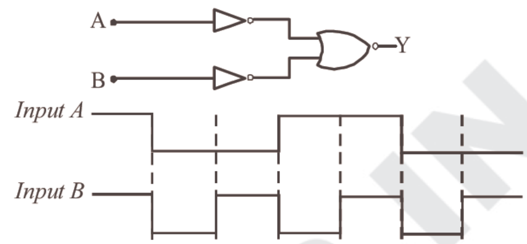
24. In the circuit below, *A* and *B* represent two inputs and *C* represents the output. **[2008]**



The circuit represents

(a) NOR gate (b) AND gate
(c) NAND gate (d) OR gate

25. The logic circuit shown below has the input waveforms '*A*' and '*B*' as shown. Pick out the correct output waveform. **[2009]**



Output is

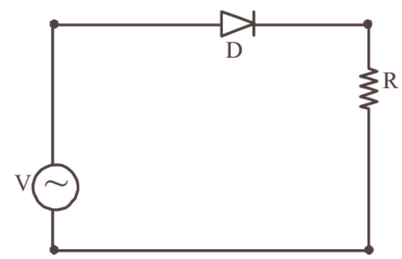
(a) 

(b) 

(c) 

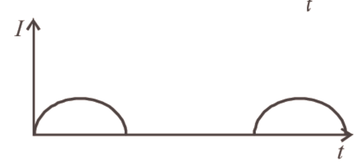
(d) 

26. A *p-n* junction (*D*) shown in the figure can act as a rectifier. An alternating current source (*V*) is connected in the circuit. **[2009]**

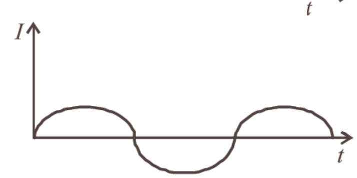


The current (*I*) in the resistor (*R*) can be shown by :

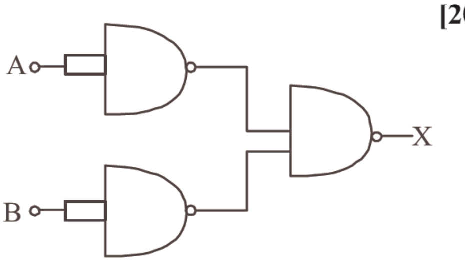
(a) 

(b) 

(c) 

(d) 

27. The combination of gates shown below yields **[2010]**

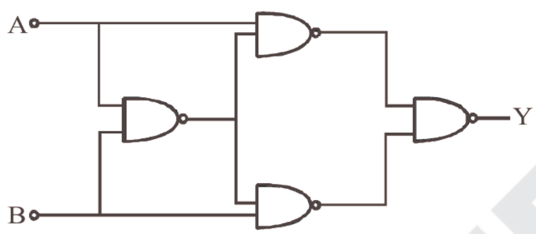


(a) OR gate (b) NOT gate
(c) XOR gate (d) NAND gate

28. The output of an OR gate is connected to both the inputs of a NAND gate. The combination will serve as a: **[2011 RS]**

(a) NOT gate (b) NOR gate
(c) AND gate (d) OR gate

29. Truth table for system of four NAND gates as shown in figure is : **[2012]**



(a)

A	B	Y
0	0	0
0	1	1
1	0	1
1	1	0

(b)

A	B	Y
0	0	0
0	1	0
1	0	1
1	1	1

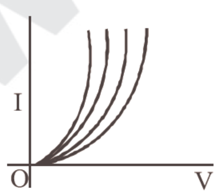
(c)

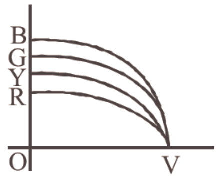
A	B	Y
0	0	1
0	1	1
1	0	0
1	1	0

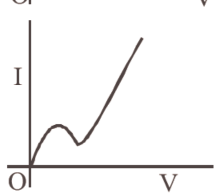
(d)

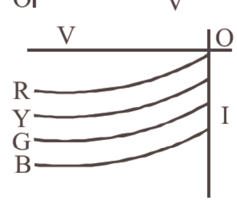
A	B	Y
0	0	1
0	1	0
1	0	1
1	1	1

30. The I-V characteristic of an LED is **[2013]**

(a) 

(b) 

(c) 

(d) 

31. The forward biased diode connection is: **[2014]**

(a) 

(b) 

(c) 

(d) 

32. A red LED emits light at 0.1 watt uniformly around it. The amplitude of the electric field of the light at a distance of 1 m from the diode is : **[2015]**

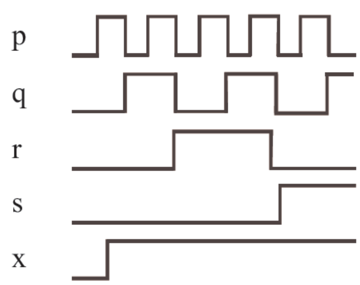
(a) 5.48 V/m (b) 7.75 V/m
(c) 1.73 V/m (d) 2.45 V/m

33. For a common emitter configuration, if α and β have their usual meanings, the incorrect relationship between α and β is : **[2016]**

(a) $a = \frac{b}{1+b}$ (b) $a = \frac{b^2}{1+b^2}$

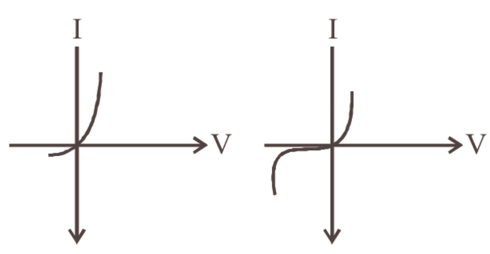
(c) $\frac{1}{a} = \frac{1}{b} + 1$ (d) $a = \frac{b}{1-b}$

34. If p, q, r, s are inputs to a gate and x is its output, then, as per the following time graph, the gate is : **[2016]**

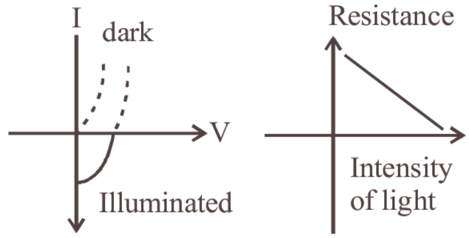


(a) OR (b) NAND
(c) NOT (d) AND

35. Identify the semiconductor devices whose characteristics are given below, in the order (p), (q), (r), (s) : **[2016]**



(p) (q)



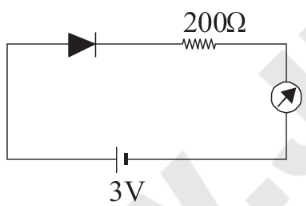
(r) (s)

(a) Solar cell, Light dependent resistance, Zener diode, simple diode
(b) Zener diode, Solar cell, simple diode, Light dependent resistance
(c) Simple diode, Zener diode, Solar cell, Light dependent resistance
(d) Zener diode, Simple diode, Light dependent resistance, Solar cell

36. In a common emitter amplifier circuit using an n-p-n transistor, the phase difference between the input and the output voltages will be : **[2017]**

(a) 135° (b) 180°
(c) 45° (d) 90°

37. The reading of the ammeter for a silicon diode in the given circuit is : **[2018]**



(a) 0 (b) 15 mA
(c) 11.5 mA (d) 13.5 mA

38. Mobility of electrons in a semiconductor is defined as the ratio of their drift velocity to the applied electric field. If, for an n-type semiconductor, the density of electrons is 10^{19} m^{-3} and their mobility is $1.6 m^2/(V.s)$ then the resistivity of the semiconductor (since it is an n-type semiconductor contribution of holes is ignored) is close to: **[2019]**

(a) $2\,\Omega m$ (b) $4\,\Omega m$
(c) $0.4\,\Omega m$ (d) $0.2\,\Omega m$

39. An NPN transistor is used in common emitter configuration as an amplifier with 1 kΩ load resistance. Signal voltage of 10 mV is applied across the base-emitter. This produces a 3 mA change in the collector current and 15 μA change in the base current of the amplifier. The input resistance and voltage gain are: **[2019]**

(a) 0.33 kΩ, 1.5 (b) 0.67 kΩ, 300
(c) 0.67 kΩ, 200 (d) 0.33 kΩ, 300

40. A LCR circuit behaves like a damped harmonic oscillator. Comparing it with a physical spring-mass damped oscillator having damping constant 'b', the correct equivalence would be: **[2020]**

(a) $L \leftrightarrow m, C \leftrightarrow k, R \leftrightarrow b$

(b) $L \leftrightarrow \frac{1}{b}, \ C \leftrightarrow \frac{1}{m}, \ R \leftrightarrow \frac{1}{k}$

(c) $L \leftrightarrow k, C \leftrightarrow b, R \leftrightarrow m$

(d) $L \leftrightarrow m, C \leftrightarrow \frac{1}{k}, \ R \leftrightarrow b$

Answer Key

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
(c)	(c)	(c)	(a)	(c)	(b)	(a)	(d)	(d)	(d)	(a)	(d)	(d)	(d)	(c)
16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
(a)	(b)	(c)	(b)	(d)	(a)	(a)	(b)	(d)	(d)	(b)	(a)	(b)	(a)	(a)
31	32	33	34	35	36	37	38	39	40					
(a)	(d)	(b,d)	(a)	(c)	(b)	(c)	(c)	(b)	(d)					

Solutions

1. **(c)** Pure silicon, at OK, will contain all the electrons in bounded state. The conduction band will be empty. So there will be no free electrons (in conduction band) and holes (in valence band). Therefore no electrons from valence band are able to shift to conduction band due to thermal agitation. Pure silicon will act as insulator.

2. **(c)** Specific resistance (resistivity) is given by

$$\rho = \frac{m}{ne^2\tau}$$

where n = no. of free electrons per unit volume and τ = average relaxation time

For a conductor with rise in temperature n increases. Increase in temperature results increase in number of collision between free electrons due to which relaxation time T decreases. But the decrease in τ is more dominant than increase in n resulting an increase in the value of ρ.

For a semiconductor with rise in temperature, n increases and τ decreases. But the increase in n is more dominant than decrease in τ resulting in a decrease in the value of ρ.

ALTERNATE SOLUTION

$$\rho_2 = \rho_1(1 + \alpha\Delta T)$$

For conductor α is positive

$\therefore \rho_2 > \rho_1$ for ΔT positive i.e., increase in temperature.

For semi conductor α is negative

$\therefore \rho_2 < \rho_1$ for ΔT positive.

3. **(c)** In insulators, valence band is completely filled while conduction band is empty. The energy band gap is maximum in insulators.

4. **(a)** Emitter main function is to supply the majority charge carrriers towards the collector. Therefore emitter is most heavily doped.

5. **(c)** Copper is a conductor and in conductor resistance decreases with decrease in temperature. Germanium is a semicon ductor. In semi-conductor resistance increases with decrease in temperature.

6. **(b)** When the temperature increases, certain bounded electrons become free which tend to promote conductivity. Simultaneously number of collisions between electrons and positive kernels increases which decrease the relaxation time.

7. **(a)** In reverse biasing the width of depletion region increases, and current flowing through diode is zero. Thus, electric field is zero at middle of depletion region.

8. **(c)** In npn transistor, electrons moves from emitter to base.

9. **(d)** In common emitter configuration for transistor amplifier current gain

$$A_i = \frac{-h_{fe}}{1 + b_{oe}R_L}$$

Where h_{fe} and h_{oe} are hybrid parameters.

$$\therefore A_i = \frac{-50}{1 + 25\times10^{-6}\times1\times10^3}$$

$$= -48.78$$

10. **(d)** Copper is a conductor, so its resistance decreases on decreasing temperature whereas germanium is semiconductor therefore on decreasing temperature resistance increases.

11. **(a)** In forward biasing, the p type is connected to positive terminal and n type is connected with negative terminal. So holes from p region and electron from n region are pushed towards the Junction which reduces the width of depletion layer. Also, distance between diffused holes and electrons decrease, which decrease electric field hence barrier potential.

12. **(d)** Band gap = energy of photon of wavelength 2480 nm. So,

$$\text{Band gap, } E_g = \frac{hc}{\lambda}$$

$$= \left(\frac{6.63\times10^{-34}\times3\times10^8}{2480\times10^{-9}}\right)\times\frac{1}{1.6\times10^{-19}}eV$$

$$= 0.5\text{ eV}$$

13. **(d)** In common base amplifier circuit, input and output voltage are in the same phase. So, the phase difference between input voltage signal and output voltage signal is zero.

14. (d) Input frequency, $f = 50$ Hz

$\Rightarrow$ Time period $T = \dfrac{1}{50}$ S

For full wave rectifier, $T_1 = \dfrac{T}{2} = \dfrac{1}{100}$

$\Rightarrow \; f_1 = \dfrac{1}{T_1} = 100$ Hz.

15. (c) A crystal structure is made up of a unit cell arranged in a particular way; which is periodically repeated in three dimensions on a lattice. The spacing between unit cells in various directions is called its lattice constants. As lattice constants increases the band-gap (E_g), also increases which means more energy would be required by electrons to reach the conduction band from the valence band. Automatically E_c and E_v decreases.

16. (a) Collector current, $I_C = 5.488$ mA,
Emitter current $I_e = 5.6$ mA

$$\alpha = \frac{I_c}{I_e} = \frac{5.488}{5.6},$$

$$\beta = \frac{\alpha}{1-\alpha} = 49$$

17. (b) Van der Waal's bonding is attributed to the attractive forces between molecules of a liquid. The conductivity of semiconductors (covalent bonding) and insulators (ionic bonding) increases with increase in temperature.
Solid which is formed by covalent bond is not transparent to visible light and its conductivity increase with temperature.

18. (c) Relation between drift velocity and current is $I = nAeV_d$

$$\frac{I_e}{I_h} = \frac{n_e eAv_e}{n_h eAv_h}$$

$$\Rightarrow \frac{7}{4} = \frac{7}{5} \times \frac{v_e}{v_h}$$

$$\Rightarrow \frac{v_e}{v_h} = \frac{5}{4}$$

19. (b) D_2 is forward biased.
D_1 is reversed biased. So, it will act like an open circuit.
So effective resistance of the circuit

$R = 4 + 2 = 6\Omega \qquad \therefore i = \dfrac{E}{R} = \dfrac{12}{6} = 2$ A

20. (d) In option (d) n-side is connected to high potential than P Junction.
So, it is reverse baised.

21. (a) The current will flow through R_L when the diode is forward biased.

22. (a) Si and Ge are semiconductors but C is an insulator. In Si and Ge at room temperature, the energy band gap is low due to which electrons in the covalent bonds gains kinetic energy and break the bond and move to conduction band. As a result, hole is created in valence band. So, the number of free electrons is significant in Si and Ge.

23. (b) It is a p-n-p transistor with R as base.

24. (d)

The truth table for the above circuit is :

A	B	C
1	1	1
1	0	1
0	1	1
0	0	0

when either A or B conducts, the gate conducts. It means $C = A + B$ which is for OR gate

25. (d) The final boolean expression

$Y = \overline{(\overline{A}+\overline{B})} = \overline{\overline{A}}.\overline{\overline{B}} = A \times B$.

Thus, it is an AND gate for which truth table is

A	B	Y
0	0	0
0	1	0
1	0	0
1	1	1

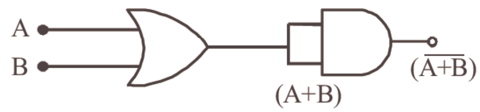
26. (b) The given circuit will work as half wave rectifier as it conducts during the positive half cycle of input AC.
Forward biased in one half cycle and reverse biased in the other half cycle].

27. (a) The final boolean expression of these gates is,

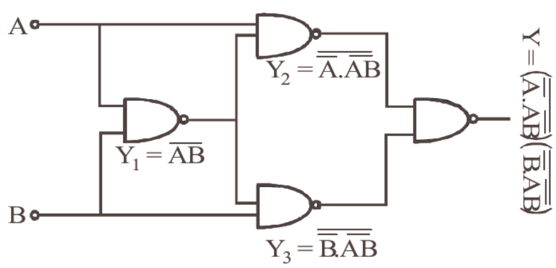
$X = \overline{\left(\overline{A}.\overline{B}\right)} = \overline{\overline{A}} + \overline{\overline{B}} = A + B \Rightarrow$ OR gate

It means OR gate is formed.

28. (b) When both inputs of NAND gate are jointed to form a single input, it behaves as NOT gate
OR + NOT = NOR.
$\overline{\left(A+B\right)}$ = NOR gate



29. **(a)**



By expanding this Boolen expression

$$Y = A.\overline{B} + B.\overline{A}$$

Thus the truth table for this expression should be (a).

30. **(a)** For same value of current higher value of voltage is required for higher frequency hence (a) is correct answer.

31. **(a)**

For forward bias, p-side must be at higher potential than n-side. $\Delta V = (+)Ve$

32. **(d)** Using $U_{av} = \frac{1}{2}\varepsilon_0 E_0{}^2$

But $U_{av} = \frac{P}{4\pi r^2 \times c}$

$\therefore \quad \frac{P}{4\pi r^2} = \frac{1}{2}\varepsilon_0 E_0{}^2 \times c$

$E_0^2 = \frac{2P}{4pr^2 e_0 c} = \frac{2 \times 0.1 \times 9 \times 10^9}{1 \times 3 \times 10^8}$

$\therefore \quad E_0 = \sqrt{6} = 2.45 \text{V/m}$

33. **(b, d)** We know that $\alpha = \frac{I_c}{I_e}$ and $\beta = \frac{I_c}{I_b}$

Also $I_e = I_b + I_c$

$$\therefore \ \alpha = \frac{Ic}{I_b + I_c} = \frac{\frac{I_c}{I_b}}{1 + \frac{I_c}{I_b}} = \frac{\beta}{1+\beta}$$

Option (b) and (d) are therefore incorrect.

34. **(a)** In case of an 'OR' gate the input is zero when all inputs are zero. If any one input is ' 1', then the output is '1'.

35. **(c)** Graph (p) is for a simple diode.
Graph (q) is showing the V Break down used for zener diode.
Graph (r) is for solar cell which shows cut-off voltage and open circuit current.
Graph (s) shows the variation of resistance h and hence current with intensity of light.

36. **(b)** In common emitter configuration for n-p-n transistor input and output signals are 180° out of phase *i.e.,* phase difference between output and input voltage is 180°.

37. **(c)** Clearly from fig. given in question, Silicon diode is in forward bias.
$\therefore$ Potential barrier across diode

$\Delta V = 0.7$ volts

Current,

$$I = \frac{V - \Delta V}{R} = \frac{3 - 0.7}{200} = \frac{2.3}{200} = 11.5 \text{mA}$$

38. **(c)** $\rho = \frac{1}{\sigma} = \frac{1}{n_e e \mu_e} \left[\begin{array}{l} \because \ \sigma = e(n_e\mu_e + n_h\mu_h) \\ \text{Here } n_h\mu_h \text{ is neglected} \end{array} \right]$

$$= \frac{1}{10^{19} \times 1.6 \times 10^{-19} \times 1.6}$$

or $\rho = 0.4\,\Omega\text{m}$

39. **(b)** Given $\Delta V_i = 10 \times 10^{-3} V$
$\Delta I_c = 3 \times 10^{-3}$A
$\Delta I_b = 15 \times 10^{-6}$A

$$R_i = \frac{\Delta V_i}{\Delta I_b} = \frac{10 \times 10^{-3}}{15 \times 10^{-6}} = 0.67 k\Omega$$

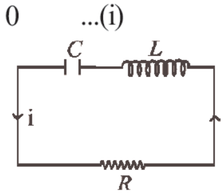
$\therefore$ Voltage gain $= \frac{\Delta I_c}{\Delta I_b} \times \frac{R_0}{R_i}$

$$= \left(\frac{3 \times 10^{-3}}{15 \times 10^{-6}}\right) \times \frac{1000}{670} = 200 \times \frac{1000}{670} \approx 300$$

40. **(d)** In damped harmonic oscillation,

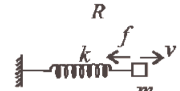
$$\frac{md^2x}{dt^2} = -kx - bv$$

$$\Rightarrow \frac{md^2x}{dt^2} + b\frac{dx}{dt} + kx = 0 \quad \text{...(i)}$$



In LCR circuit,

$$\frac{-q}{C} - iR - \frac{Ldi}{dt} = 0$$



$$L\frac{d^2}{dt^2} + R\frac{dq}{dt} + \frac{q}{C} = 0 \quad \text{...(ii)}$$

Comparing equations (i) & (ii)

$$L \leftrightarrow m, C \leftrightarrow \frac{1}{k}, R \leftrightarrow b$$

29 Communication Systems

1. This question has Statement – 1 and Statement – 2. Of the four choices given after the statements, choose the one that best describes the two statements. **[2011]**

Statement – 1 : Sky wave signals are used for long distance radio communication. These signals are in general, less stable than ground wave signals.

Statement – 2 : The state of ionosphere varies from hour to hour, day to day and season to season.

(a) Statement–1 is true, Statement–2 is true, Statement–2 is the correct explanation of Statement–1.
(b) Statement–1 is true, Statement–2 is true, Statement–2 is not the correct explanation of Statement – 1.
(c) Statement – 1 is false, Statement – 2 is true.
(d) Statement – 1 is true, Statement – 2 is false.

2. Which of the following four alternatives is not correct ? We need modulation : **[2011 RS]**

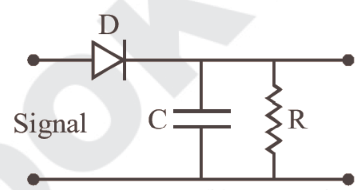
(a) to reduce the time lag between transmission and reception of the information signal
(b) to reduce the size of antenna
(c) to reduce the fractional band width, that is the ratio of the signal band width to the centre frequency
(d) to increase the selectivity

3. A radar has a power of 1kW and is operating at a frequency of 10 GHz. It is located on a mountain top of height 500 m. The maximum distance upto which it can detect object located on the surface of the earth

(Radius of earth = 6.4×10^6m) is : **[2012]**

(a) 80 km (b) 16 km (c) 40 km (d) 64 km

4. A diode detector is used to detect an amplitudemodulated wave of 60% modulation by using a condenser of capacity 250 picofarad in parallel with a load resistance 100 kilo ohm. Find the maximum modulated frequency which could be detected by it. **[2013]**



(a) 10.62 MHz (b) 10.62 kHz
(c) 5.31 MHz (d) 5.31 kHz

5. A signal of 5 kHz frequency is amplitude modulated on a carrier wave of frequency 2 MHz. The frequencies of the resultant signal is/are : **[2015]**

(a) 2005 kHz, 2000 kHz and 1995 kHz
(b) 2000 kHz and 1995 kHz
(c) 2 MHz only
(d) 2005 kHz and 1995 kHz

6. Choose the correct statement :

(a) In frequency modulation the amplitude of the high frequency carrier wave is made to vary in proportion to the amplitude of the audio signal.
(b) In frequency modulation the amplitude of the high frequency carrier wave is made to vary in proportion to the frequency of the audio signal.
(c) In amplitude modulation the amplitude of the high frequency carrier wave is made to vary in proportion to the amplitude of the audio signal.
(d) In amplitude modulation the frequency of the high frequency carrier wave is made to vary in proportion to the amplitude of the audio signal. **[2016]**

7. In amplitude modulation, sinusoidal carrier frequency used is denoted by ω_c and the signal frequency is denoted by ω_m. The bandwidth $(\Delta\omega_m)$ of the signal is such that $\Delta\omega_m < \omega_c$. Which of the following frequencies is not contained in the modulated wave ? **[2017]**

(a) $\omega_m + \omega_c$ (b) $\omega_c - \omega_m$
(c) ω_m (d) ω_c

8. A telephonic communication service is working at carrier frequency of 10 GHz. Only10% of it is utilized for transmission. How many telephonic channels can be transmitted simultaneously if each channel requires a bandwidth of 5 kHz? **[2018]**

(a) 2×10^3 (b) 2×10^4

(c) 2×10^5 (d) 2×10^6

9. A signal Acoswt is transmitted using v0 sin w modulated (AM) signal is: **[2019]**

(a) $v_0 \sin\omega_0 \;\; t + \frac{A}{2}\sin$

$\left(\omega_0 - \omega\right)t + \frac{A}{2}\,(\omega_0 + \omega)\,t$

(b) $v_0 \sin[\omega_0(1 + 0.01\, A\sin\omega t)t]$

(c) $v_0 \sin\omega_0 t + A\cos\omega t$

(d) $(v_0 + A)$ coswt $\sin\omega_0 t$

Answer Key

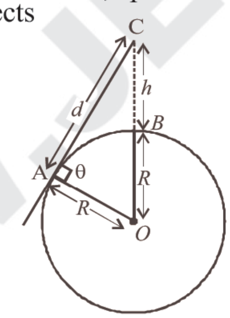
1	2	3	4	5	6	7	8	9						
(b)	(a)	(a)	(b)	(a)	(c)	(c)	(c)	(a)						

Solutions

1. **(b)** For long distance communication, sky wave signals are used.
Also, the state of ionosphere varies every time. So, both statements are correct.

2. **(a)** Low frequencies cannot be transmitted to long distances. Therefore, they are super imposed on a high frequency carrier signal by a process known as modulation.
Speed of electro-magnetic waves will not change due to modulation. So there will be time lag between transmission and reception of the information signal.

3. **(a)** Let d is the maximum distance, upto which it can detect the objects



From ΔAOC

$$OC^2 = AC^2 + AO^2$$

$$(h+R)^2 = d^2 + R^2$$

$$\Rightarrow \quad d^2 = (h+R)^2 - R^2$$

$$d = \sqrt{(h+R)^2 - R^2}\;;\; d = \sqrt{h^2 + 2hR}$$

$$d = \sqrt{500^2 + 2 \times 6.4 \times 10^6} = 80\,\text{km}$$

4. **(b)** **Given :** Resistance R = 100 kilo ohm
$= 100 \times 10^3\,\Omega$
Capacitance C = 250 picofarad
$= 250 \times 10^{-12}$F
$\tau = RC = 100 \times 10^3 \times 250 \times 10^{-12}$ sec
$= 2.5 \times 10^7 \times 10^{-12}$ sec
$= 2.5 \times 10^{-5}$ sec
The higher frequency whcih can be detected with tolerable distortion is

$$f = \frac{1}{2\pi m_a RC} = \frac{1}{2\pi \times 0.6 \times 2.5 \times 10^{-5}}\,Hz$$

$$= \frac{100 \times 10^4}{25 \times 1.2\pi}\,Hz = \frac{4}{1.2\pi} \times 10^4\,Hz$$

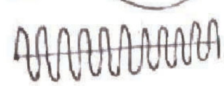
$= 10.61$ KHz

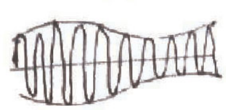
This condition is obtained by applying the condition that rate of decay of capacitor voltage must be equal or less than the rate of decay modulated singnal voltage for proper detection of mdoulated signal.

5. **(a)** Amplitude modulated wave consists of three frequencies are $\omega_c + \omega_m, \omega, \omega_c - \omega_m$
i.e. 2005 kHz, 2000kHz, 1995 kHz

6. **(c)** In amplitude modulation, the amplitude of the high frequency carrier wave made to vary in proportional to the amplitude of audio signal.


Audio signal


Carrier wave


Amplitude modulated wave

7. **(c)** Modulated carrier wave contains frequency ω_c and $\omega_c \pm \omega_m$

8. **(c)** If n = no. of channels
10% of 10 GHz = n × 5 KHz or,
$\Rightarrow$ n = 2×10^5

9. **(a)** The equation of amplitude modulated wave
$m = (v_0 + A\cos\omega t)\sin\omega t$
$= v_0 \sin\omega_0 t + A\cos\omega t \sin\omega_0 t$

$$= v_0 \sin\omega_0 t + \frac{A}{2}\,[\sin(\omega_0 - \omega)t + \sin(\omega_0 + \omega)t]$$

1 Some Basic Concepts of Chemistry

1. In a compound C, H and N atoms are present in 9 : 1 : 3.5 by weight. Molecular weight of compound is 108. Molecular formula of compound is **[2002]**
(a) $C_2H_6N_2$ (b) C_3H_4N
(c) $C_6H_8N_2$ (d) $C_9H_{12}N_3$

2. With increase of temperature, which of these changes? **[2002]**
(a) molality
(b) weight fraction of solute
(c) molarity
(d) mole fraction.

3. Number of atoms in 558.5 gram Fe (at. wt. of Fe = 55.85 g mol^{-1}) is **[2002]**
(a) twice that in 60 g carbon
(b) 6.023×10^{22}
(c) half that in 8 g He
(d) $558.5 \times 6.023 \times 10^{23}$

4. What volume of hydrogen gas, at 273 K and 1 atm. pressure will be consumed in obtaining 21.6 g of elemental boron (atomic mass = 10.8) from the reduction of boron trichloride by hydrogen? **[2003]**
(a) 67.2 L (b) 44.8 L
(c) 22.4 L (d) 89.6 L

5. 25mL of a solution of barium hydroxide on titration with a 0.1 molar solution of hydrochloric acid gave a titre value of 35mL. The molarity of barium hydroxide solution was **[2003]**
(a) 0.14 (b) 0.28
(c) 0.35 (d) 0.07

6. 6.02×10^{20} molecules of urea are present in 100 mL of its solution. The concentration of urea solution is **[2004]**
(a) 0.02 M (b) 0.01 M
(c) 0.001 M (d) 0.1 M
(Avogadro constant, $N_A = 6.02 \times 10^{23}$ mol^{-1})

7. To neutralise completely 20 mL of 0.1 M aqueous solution of phosphorous acid (H_3PO_3), the value of 0.1 M aqueous KOH solution required is **[2004]**
(a) 40 mL (b) 20 mL
(c) 10 mL (d) 60 mL

8. Two solutions of a substance (non electrolyte) are mixed in the following manner: 480 mL of 1.5 M first solution + 520 mL of 1.2 M second solution. What is the molarity of the final mixture? **[2005]**
(a) 2.70 M (b) 1.344 M
(c) 1.50 M (d) 1.20 M

9. If we consider that 1/6, in place of 1/12, mass of carbon atom as the relative atomic mass unit, the mass of one mole of the substance will **[2005]**
(a) be a function of the molecular mass of the substance
(b) remain unchanged
(c) increase two fold
(d) decrease twice

10. How many moles of magnesium phosphate, $Mg_3(PO_4)_2$ will contain 0.25 mole of oxygen atoms? **[2006]**
(a) 1.25×10^{-2} (b) 2.5×10^{-2}
(c) 0.02 (d) 3.125×10^{-2}

11. Density of a 2.05M solution of acetic acid in water is 1.02 g/mL. The molality of the solution is **[2006]**

(a) 2.28 mol kg^{-1}
(b) 0.44 mol kg^{-1}
(c) 1.14 mol kg^{-1}
(d) 3.28 mol kg^{-1}

12. The density (in g mL^{-1}) of a 3.60 M sulphuric acid solution that is 29% H_2SO_4 (molar mass = 98 g mol^{-1}) by mass will be **[2007]**

(a) 1.45 (b) 1.64
(c) 1.88 (d) 1.22

13. In the reaction, **[2007]**

$$2Al(s) + 6HCl(aq) \longrightarrow 2Al^{3+}(aq) + 6Cl^{-}(aq) + 3H_2(g)$$

(a) 11.2 L H_2(g) at STP is produced for every mole of HCl(aq) consumed
(b) 6 L HCl(aq) is consumed for every 3 L of H_2(g) produced
(c) 33.6 L H_2(g) is produced regardless of temperature and pressure for every mole of Al that reacts
(d) 67.2 H_2(g) at STP is produced for every mole of Al that reacts.

14. The molality of a urea solution in which 0.0100 g of urea, $[(NH_2)_2CO]$ is added to 0.3000 dm^3 of water at STP is : **[2011RS]**

(a) 5.55×10^{-4} m (b) 33.3 m
(c) 3.33×10^{-2} m (d) 0.555 m

15. A gaseous hydrocarbon gives upon combustion 0.72 g of water and 3.08 g. of CO_2. The empirical formula of the hydrocarbon is : **[2013]**

(a) C_2H_4 (b) C_3H_4
(c) C_6H_5 (d) C_7H_8

16. 3 g of activated charcoal was added to 50 mL of acetic acid solution (0.06N) in a flask. After an hour it was filtered and the strength of the filtrate was found to be 0.042 N. The amount of acetic acid adsorbed (per gram of charcoal) is : **[2015]**

(a) 42 mg (b) 54 mg
(c) 18 mg (d) 36 mg

17. The molecular formula of a commercial resin used for exchanging ions in water softening is $C_8H_7SO_3^{-}\,Na^{+}$ (Mol. wt. 206). What would be the maximum uptake of Ca^{2+} ions by the resin when expressed in mole per gram resin? **[2015]**

(a) $\frac{2}{309}$ (b) $\frac{1}{412}$
(c) $\frac{1}{103}$ (d) $\frac{1}{206}$

18. At 300 K and 1 atm, 15 mL of a gaseous hydrocarbon requires 375 mL air containing 20% O_2 by volume for complete combustion. After combustion the gases occupy 330 mL. Assuming that the water formed is in liquid form and the volumes were measured at the same temperature and pressure, the formula of the hydrocarbon is: **[2016]**

(a) C_4H_8 (b) C_4H_{10}
(c) C_4H_6 (d) C_3H_8

19. The most abundant elements by mass in the body of a healthy human adult are :

Oxygen (61.4%) ; Carbon (22.9%), Hydrogen (10.0%) ; and Nitrogen (2.6%). The weight which a 75 kg person would gain if all ^1H atoms are replaced by ^2H atoms is **[2017]**

(a) 15 kg (b) 37.5 kg
(c) 7.5 kg (d) 10 kg

20. 1 gram of a carbonate (M_2CO_3) on treatment with excess HCl produces 0.01186 mole of CO_2. The molar mass of M_2CO_3 in g mol^{-1} is : **[2017]**

(a) 1186 (b) 84.3
(c) 118.6 (d) 11.86

21. The ratio of mass percent of C and H of an organic compound ($C_XH_YO_Z$) is 6 : 1. If one molecule of the above compound ($C_XH_YO_Z$) contains half as much oxygen as required to burn one molecule of compound C_XH_Y completely to CO_2 and H_2O. The empirical formula of compound $C_XH_YO_Z$ is : **[2018]**

(a) $C_3H_6O_3$ (b) C_2H_4O
(c) $C_3H_4O_2$ (d) $C_2H_4O_3$

22. For a reaction,

$N_2(g) + 3H_2(g) \rightarrow 2\,NH_3(g)$; identify dihydrogen ($H_2$) as a limiting reagent in the following reaction mixtures. **[2019]**

(a) 56 g of N_2 + 10 g of H_2

(b) 35 g of N_2 + 8 g of H_2

(c) 28 g of N_2 + 6 g of H_2

(d) 14 g of N_2 + 4 g of H_2

23. Amongst the following statements, that which was not proposed by Dalton was: **[2020]**

(a) Chemical reactions involve reorganization of atoms. These are neither created nor destroyed in a chemical reaction.

(b) All the atoms of a given element have identical properties including identical mass. Atoms of different elements differ in mass.

(c) When gases combine or reproduced in a chemical reaction they do so in a simple ratio by volume provided all gases are at the same T & P.

(d) Matter consists of indivisible atoms.

Answer Key

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
(c)	(c)	(a)	(a)	(d)	(b)	(a)	(b)	(d)	(d)	(a)	(d)	(a)	(a)	(d)	(c)
17	**18**	**19**	**20**	**21**	**22**	**23**									
(b)	(n)	(c)	(b)	(d)	(a)	(c)									

Solutions

1. (c)

Element	%	Relative no. of atoms	Simplest ratio of atoms
C	9	$\frac{9}{12} = \frac{3}{4}$	3
H	1	$\frac{1}{1} = 1$	4
N	3.5	$\frac{3.5}{14} = \frac{1}{4}$	1

Empirical formula $= C_3H_4N$

$(C_3H_4N)_n = 108$

$(12 \times 3 + 4 \times 1 + 14)_n = 108$

$(54)_n = 108$

$$n = \frac{108}{54} = 2$$

$\therefore$ Molecular formula $= C_6H_8N_2$

2. (c) Among all the given options, molarity changes with temperature because the term molarity involves volume which increases on increasing temperature.

3. (a) $\text{Fe (No. of moles)} = \frac{558.5}{55.85} = 10\text{ mol}$

C (No. of moles) in 60 g of C = 60/12 = 5 mol.

4. (a) $2BCl_3 + 3H_2 \rightarrow 2B + 6HCl$

or $BCl_3 + \frac{3}{2}H_2 \rightarrow B + 3HCl$

Now, since 10.8 g boron requires hydrogen

$= \frac{3}{2} \times 22.4\text{L at S.T.P}$

Hence 21.6 boron requires hydrogen

$= \frac{3}{2} \times \frac{22.4}{10.8} \times 21.6 = 67.2\text{L at S.T.P.}$

5. (d) $Ba(OH)_2$ HCl

$N_1V_1 = N_2V_2$

$N_1 \times 25 = 0.1 \times 35$

$N_1 = 0.14$

Since, $Ba(OH)_2$ is diacid base

Hence $N = M \times 2$ or $M = \frac{N}{2}$

$M = 0.07\,M$

6. (b) Moles of urea present in 100 mL of sol.

$$= \frac{6.02\times10^{20}}{6.02\times10^{23}} \text{ mol}$$

$$\therefore M = \frac{6.02\times10^{20}\times1000}{6.02\times10^{23}\times100} = 0.01M$$

[$\because$ M = Moles of solute present in 1L of solution]

7. (a) H_3PO_3 KOH

$N_1V_1 = N_2V_2$

(Note : H_3PO_3 is dibasic, $\therefore M = 2N$)

$20 \times 0.2 = 0.1 \times V_2$ (Thus. 0.1 M = 0.2 N)

$\therefore V_2 = 40$ mL

8. (b) From the molarity equation

$$M_1V_1 + M_2V_2 = MV$$

Let M be the molarity of final mixture,

$$M = \frac{M_1V_1 + M_2V_2}{V} \text{ where } V = V_1 + V_2$$

$$M = \frac{480\times1.5 + 520\times1.2}{480+520} = 1.344\text{ M}$$

9. (d) Relative atomic mass =

$$\frac{\text{Mass of one atom of the element}}{1/12^{\text{th}}\text{ part of the mass of one atom of carbon} - 12}$$

or $\dfrac{\text{Mass of one atom of the element}}{\text{Mass of one atom of the C} - 12}\times12$

Now if we use $\frac{1}{6}$ in place of $\frac{1}{12}$ the formula becomes

Relative atomic mass

$$= \frac{\text{Mass of one atom of element}}{\text{Mass of one atom of carbon}}\times6$$

$\therefore$ Relative atomic mass decrease twice.

10. (d) 1 Mole of $Mg_3(PO_4)_2$ contains 8 moles of oxygen atoms

$\therefore$ 8 mole of oxygen atoms $\equiv$ 1 mole of $Mg_3(PO_4)_2$

0.25 mole of oxygen atom $\equiv \frac{1}{8}\times0.25$ mole of $Mg_3(PO_4)_2$

$= 3.125\times10^{-2}$ mole of $Mg_3(PO_4)_2$

11. (a) Apply the formula $d = M\left(\frac{1}{m}+\frac{M_2}{1000}\right)$

$$\therefore 1.02 = 2.05\left(\frac{1}{m}+\frac{60}{1000}\right)$$

On solving we get, m = 2.288 mol/kg

12. (d) Since molarity of solution is 3.60 M. It means 3.6 moles of H_2SO_4 is present in its 1 litre solution.

Mass of 3.6 moles of H_2SO_4

= Moles × Molecular mass

= 3.6 × 98 g = 352.8 g

$\therefore$ 1000 mL solution has 352.8 g of H_2SO_4

Given that 29 g of H_2SO_4 is present in

= 100 g of solution

$\therefore$ 352.8 g of H_2SO_4 is present in

$= \frac{100}{29}\times352.8$ g of solution

= 1216 g of solution

$$\text{Density} = \frac{\text{Mass}}{\text{Volume}} = \frac{1216}{1000}$$

= 1.216 g/mL = 1.22 g/mL

13. (a) $2Al(s) + 6HCl(aq) \rightarrow 2Al^{3+}(aq) + 6Cl^-(aq) + 3H_2(g)$

$\because$ 6 moles of HCl produces = 3 moles of H_2

= 3 × 22.4 L of H_2 at S.T.P

$\therefore$ 1 mole of HCl produces

$= \frac{3\times22.4}{6}$ L of H_2 at S.T.P

= 11.2 L of H_2 at STP

14. (a) Molality = Moles of solute / Mass of solvent in kg

$$\text{Molality} = \frac{0.01/60}{0.3} = \frac{0.01}{60\times0.3};$$

$= 5.55\times10^{-4}$ m

15. (d) $\because$ 18 g, H_2O contains = 2 g H

$\therefore$ 0.72 g H_2O contains

$= \frac{2}{18}\times0.72\text{ g} = 0.08\text{ g H}$

$\because$ 44 g CO_2 contains = 12 g C

$\therefore$ 3.08 g CO_2 contains

$= \frac{12}{44} \times 3.08 = 0.84$ g C

$\therefore C : H = \frac{0.84}{12} : \frac{0.08}{1}$

$= 0.07 : 0.08 = 7 : 8$

$\therefore$ Empirical formula $= C_7H_8$

16. (c) Let the weight of acetic acid initially be w_1 in 50 mL of 0.060 N solution.

$N = \frac{w_1 \times 1000}{M.wt. \times 50}$ (Normality = 0.06 N)

$0.06 = \frac{w_1 \times 1000}{60 \times 50}$

$\Rightarrow w_1 = \frac{0.06 \times 60 \times 50}{1000} = 0.18$ g = 180 mg.

After an hour, the strength of acetic acid = 0.042 N

so, let the weight of acetic acid be w_2

$N = \frac{w_2 \times 1000}{60 \times 50}$

$0.042 = \frac{w_2 \times 1000}{3000}$

$\Rightarrow w_2 = 0.126$ g = 126 mg

So amount of acetic acid adsorbed per 3g

= 180 – 126 mg = 54 mg

$\therefore$ amount of acetic acid absorbed per g = 54/3 = 18 mg

17. (b) 2 mole of water softner require 1 mole of Ca^{2+} ion

So, 1 mole of water softner require $\frac{1}{2}$ mole of Ca^{2+} ion

Thus, $\frac{1}{2 \times 206} = \frac{1}{412}$ mol/g will be maximum uptake.

18. (N) $C_xH_y(g) + \left(\frac{4x+y}{4}\right)O_2(g) \longrightarrow xCO_2(g) + \frac{y}{2}H_2O(l)$

Volume of O_2 used $= \frac{20}{100} \times 375 = 75$ mL

Volume of air = 375 – 75 = 300 mL

Total volume of gases after combustion

= vol. of CO_2 + vol. of air = 330 mL

Volume of CO_2 = 330 – 300 = 30 mL

15 mL C_xH_y gives = 30 mL CO_2

1 mL C_xH_y gives $= \frac{30}{15} = 2$ mL CO_2

At constant T and P; Volume $\propto$ mole

$\therefore$ 1 mol C_xH_y = 2 mol CO_2

$x = 2$

$\left(\frac{4x+y}{4}\right) = \frac{75}{15}$

$4x + y = 20$

$y = 20 - 4 \times 2 = 12$

Hence, formula of the hydrocarbon is C_2H_{12}.

19. (c) Percentage (by mass) of elements given in the body of a healthy human adult is :-

Oxygen = 61.4%, Carbon = 22.9%,

Hydrogen = 10.0% and Nitrogen = 2.6%

$\because$ Total weight of person = 75 kg

$\therefore$ Mass due to ^1H is $= 75 \times \frac{10}{100} = 7.5$ kg

If ^1H atoms are replaced by ^2H atoms.

Mass gain by person would be = 7.5 kg

20. (b) Given chemical eq^n

$\underset{1\,g}{M_2CO_3} + 2HCl \rightarrow 2MCl + H_2O + \underset{0.01186\,mol}{CO_2}$

From the above chemical eq^n.

$nM_2CO_3 = nCO_2$

$\frac{1}{\text{Molar mass of } M_2CO_3} = 0.01186$

$\therefore$ Molar mass of $M_2CO_3 = \frac{1}{0.01186}$

Molar mass = 84.3 g/mol

21. **(d)**

Element	Relative mass	Relative mole	Simplest whole number ratio
C	6	$\frac{6}{12}=0.5$	1
H	1	$\frac{1}{1}=1$	2

So, $x=1, y=2$

Equation for combustion of C_xH_y

$$C_xH_y+\left(x+\frac{y}{4}\right)O_2 \longrightarrow xCO_2+\frac{y}{2}H_2O$$

Oxygen atoms required $=2\left(x+\frac{y}{4}\right)$

As mentioned,

$$2\left(x+\frac{y}{4}\right)=2z;\ \left(x+\frac{y}{4}\right)=z$$

Now putting the values of x and y

$\Rightarrow\ \left(1+\frac{2}{4}\right)=z\ \ \Rightarrow\ z=1.5$

$\therefore$ Molecule $(C_xH_yO_z)$ can be written as

$C_1H_2O_{3/2}$

$\Rightarrow\ C_2H_4O_3$

22. **(a)** According to the stoichiometry of balanced equation 28 g N_2 react with 6 g H_2

$$\underset{\substack{1\text{ mole}\\28\text{ g}}}{N_2}+\underset{\substack{3\text{ mole}\\6\text{ g}}}{3H_2}\longrightarrow 2NH_3$$

$\therefore$ For 56 g of N_2, 12 g of H_2 is required.

23. **(c)** Except (c) all postulates was given by the Dalton.

2 Structure of Atom

1. In a hydrogen atom, if energy of an electron in ground state is 13.6. eV, then that in the 2nd excited state is **[2002]**
   (a) 1.51 eV (b) 3.4 eV
   (c) 6.04 eV (d) 13.6 eV.

2. Uncertainty in position of a minute particle of mass 25 g in space is 10^{-5} m. What is the uncertainty in its velocity (in ms^{-1})?
   ($h = 6.6 \times 10^{-34}$ Js) **[2002]**
   (a) 2.1×10^{-34} (b) 0.5×10^{-34}
   (c) 2.1×10^{-28} (d) 0.5×10^{-23}.

3. The number of *d*-electrons retained in Fe^{2+} (At. no. of Fe = 26) ion is **[2003]**
   (a) 4 (b) 5 (c) 6 (d) 3

4. The orbital angular momentum for an electron revolving in an orbit is given by $\sqrt{l(l+1)}.\frac{h}{2\pi}$. This momentum for an *s*-electron will be given by **[2003]**
   (a) zero (b) $\frac{h}{2\pi}$
   (c) $\sqrt{2}.\frac{h}{2\pi}$ (d) $+\frac{1}{2}.\frac{h}{2\pi}$

5. Which one of the following groupings represents a collection of isoelectronic species ?(At. nos. : Cs : 55, Br : 35) **[2003]**
   (a) N^{3-}, F^-, Na^+ (b) Be, Al^{3+}, Cl^-
   (c) Ca^{2+}, Cs^+, Br (d) Na^+, Ca^{2+}, Mg^{2+}

6. In Bohr series of lines of hydrogen spectrum, the third line from the red end corresponds to which one of the following inter-orbit jumps of the electron for Bohr orbits in an atom of hydrogen **[2003]**
   (a) $5 \to 2$ (b) $4 \to 1$
   (c) $2 \to 5$ (d) $3 \to 2$

7. The de Broglie wavelength of a tennis ball of mass 60 g moving with a velocity of 10 metres per second is approximately **[2003]**
   (a) 10^{-31} metres (b) 10^{-16} metres
   (c) 10^{-25} metres (d) 10^{-33} metres
   Planck's constant, $h = 6.63 \times 10^{-34}$ Js

8. Which of the following sets of quantum numbers is correct for an electron in 4*f* orbital ? **[2004]**
   (a) $n = 4, l = 3, m = +1, s = +½$
   (b) $n = 4, l = 4, m = -4, s = -½$
   (c) $n = 4, l = 3, m = +4, s = +½$
   (d) $n = 3, l = 2, m = -2, s = +½$

9. Consider the ground state of Cr atom (X = 24). The number of electrons with the azimuthal quantum numbers, $l = 1$ and 2 are, respectively **[2004]**
   (a) 16 and 4 (b) 12 and 5
   (c) 12 and 4 (d) 16 and 5

10. The wavelength of the radiation emitted, when in a hydrogen atom electron falls from infinity to stationary state 1, would be (Rydberg constant $= 1.097 \times 10^7 m^{-1}$) **[2004]**
    (a) 406 nm (b) 192 nm
    (c) 91 nm (d) 9.1×10^{-8} nm

11. Which one of the following sets of ions represents the collection of isoelectronic species? **[2004]**
    (a) $K^+, Cl^-, Mg^{2+}, Sc^{3+}$
    (b) $Na^+, Ca^{2+}, Sc^{3+}, F^-$
    (c) $K^+, Ca^{2+}, Sc^{3+}, Cl^-$
    (d) $Na^+, Mg^{2+}, Al^{3+}, Cl^-$
    (Atomic nos. : F = 9, Cl = 17, Na = 11, Mg = 12, Al = 13, K = 19, Ca = 20, Sc = 21)

12. In a multi-electron atom, which of the following orbitals described by the three quantum members will have the same energy in the absence of magnetic and electric fields? **[2005]**

(A) $n=1, l=0, m=0$ (B) $n=2, l=0, m=0$
(C) $n=2, l=1, m=1$ (D) $n=3, l=2, m=1$
(E) $n=3, l=2, m=0$

(a) (D) and (E) (b) (C) and (D)
(c) (B) and (C) (d) (A) and (B)

13. Of the following sets which one does NOT contain isoelectronic species? **[2005]**

(a) BO_3^{3-}, CO_3^{2-}, NO_3^-
(b) SO_3^{2-}, CO_3^{2-}, NO_3^-
(c) CN^-, N_2, C_2^{2-}
(d) PO_4^{3-}, SO_4^{2-}, ClO_4^-

14. According to Bohr's theory, the angular momentum of an electron in 5th orbit is **[2006]**

(a) $10\,h/\pi$ (b) $2.5\,h/\pi$
(c) $25\,h/\pi$ (d) $1.0\,h/\pi$

15. Uncertainty in the position of an electron (mass $= 9.1 \times 10^{-31}$ kg) moving with a velocity 300 ms^{-1}, accurate upto 0.001% will be **[2006]**

(a) 1.92×10^{-2} m (b) 3.84×10^{-2} m
(c) 19.2×10^{-2} m (d) 5.76×10^{-2} m

($h = 6.63 \times 10^{-34}$ Js)

16. Which one of the following sets of ions represents a collection of isoelectronic species? **[2006]**

(a) N^{3-}, O^{2-}, F^-, S^{2-}
(b) Li^+, Na^+, Mg^{2+}, Ca^{2+}
(c) K^+, Cl^-, Ca^{2+}, Sc^{3+}
(d) Ba^{2+}, Sr^{2+}, K^+, Ca^{2+}

17. Which of the following sets of quantum numbers represents the highest energy of an atom? **[2007]**

(a) $n=3, l=0, m=0, s=+1/2$
(b) $n=3, l=1, m=1, s=+1/2$
(c) $n=3, l=2, m=1, s=+1/2$
(d) $n=4, l=0, m=0, s=+1/2$.

18. Which one of the following constitutes a group of the isoelectronic species? **[2008]**

(a) C_2^{2-}, O_2^-, CO, NO
(b) $NO^+, C_2^{2-}, CN^-, N_2$
(c) $CN^-, N_2, O_2^{2-}, C_2^{2-}$
(d) N_2, O_2^-, NO^+, CO

19. The ionization enthalpy of hydrogen atom is 1.312×10^6 J mol^{-1}. The energy required to excite the electron in the atom from $n=1$ to $n=2$ is **[2008]**

(a) 8.51×10^5 J mol^{-1} (b) 6.56×10^5 J mol^{-1}
(c) 7.56×10^5 J mol^{-1} (d) 9.84×10^5 J mol^{-1}

20. Calculate the wavelength (in nanometer) associated with a proton moving at 1.0×10^3 ms^{-1}. (Mass of proton $= 1.67 \times 10^{-27}$ kg and $h = 6.63 \times 10^{-34}$ Js) **[2009]**

(a) 0.40 nm (b) 2.5 nm
(c) 14.0 nm (d) 0.32 nm

21. In an atom, an electron is moving with a speed of 600 m/s with an accuracy of 0.005%. Certainity with which the position of the electron can be located is ($h = 6.6 \times 10^{-34}$ kg m^{2}s^{-1}, mass of electron, $e_m = 9.1 \times 10^{-31}$ kg) : **[2009]**

(a) 5.10×10^{-3} m (b) 1.92×10^{-3} m
(c) 3.84×10^{-3} m (d) 1.52×10^{-4} m

22. The energy required to break one mole of Cl – Cl bonds in Cl_2 is 242 kJ mol^{-1}. The longest wavelength of light capable of breaking a single Cl – Cl bond is ($c = 3 \times 10^8$ ms^{-1} and $N_A = 6.02 \times 10^{23}$ mol^{-1}). **[2010]**

(a) 594 nm (b) 640 nm
(c) 700 nm (d) 494 nm

23. Ionisation energy of He^+ is 19.6×10^{-18} J atom^{-1}. The energy of the first stationary state ($n=1$) of Li^{2+} is **[2010]**

(a) 4.41×10^{-16} J atom^{-1}
(b) -4.41×10^{-17} J atom^{-1}
(c) -2.2×10^{-15} J atom^{-1}
(d) 8.82×10^{-17} J atom^{-1}

24. The frequency of light emitted for the transition $n=4$ to $n=2$ of the He^+ is equal to the transition in H atom corresponding to which of the following? **[2011RS]**

(a) $n=2$ to $n=1$ (b) $n=3$ to $n=2$
(c) $n=4$ to $n=3$ (d) $n=3$ to $n=1$

25. The electrons identified by quantum numbers n and l : **[2012]**

(*A*) $n=4, l=1$ (*B*) $n=4, l=0$

(*C*) $n=3, l=2$ (*D*) $n=3, l=1$

can be placed in order of increasing energy as :

(a) $(C)<(D)<(B)<(A)$

(b) $(D)<(B)<(C)<(A)$

(c) $(B)<(D)<(A)<(C)$

(d) $(A)<(C)<(B)<(D)$

26. The increasing order of the ionic radii of the given isoelectronic species is : **[2012]**

(a) $Cl^-, Ca^{2+}, K^+, S^{2-}$

(b) $S^{2-}, Cl^-, Ca^{2+}, K^+$

(c) $Ca^{2+}, K^+, Cl^-, S^{2-}$

(d) $K^+, S^{2-}, Ca^{2+}, Cl^-$

27. Energy of an electron is given by $E = -2.178 \times 10^{-18} J\left(\frac{Z^2}{n^2}\right)$. Wavelength of light required to excite an electron in an hydrogen atom from level $n=1$ to $n=2$ will be : **[2013]**

($h = 6.62 \times 10^{-34}$ Js and $c = 3.0 \times 10^8$ ms^{-1})

(a) 1.214×10^{-7} m (b) 2.816×10^{-7} m

(c) 6.500×10^{-7} m (d) 8.500×10^{-7} m

28. The correct set of four quantum numbers for the valence electrons of rubidium atom ($Z=37$) is: **[2014]**

(a) $5, 0, 0, +\frac{1}{2}$ (b) $5, 1, 0, +\frac{1}{2}$

(c) $5, 1, 1, +\frac{1}{2}$ (d) $5, 0, 1, +\frac{1}{2}$

29. Which of the following is the energy of a possible excited state of hydrogen? **[2015]**

(a) –3.4 eV (b) +6.8 eV

(c) +13.6 eV (d) –6.8 eV

30. A stream of electrons from a heated filaments was passed two charged plates kept at a potential difference V esu. If e and m are charge and mass of an electron, respectively, then the value of h/λ (where λ is wavelength associated with electron wave) is given by: **[2016]**

(a) $\sqrt{meV}$ (b) $\sqrt{2meV}$

(c) meV (d) $2meV$

31. The radius of the second Bohr orbit for hydrogen atom is : **[2017]**

(Plank's const. $h = 6.6262 \times 10^{-34}$ Js ; mass of electron $= 9.1091 \times 10^{-31}$ kg ; charge of electron $e = 1.60210 \times 10^{-19}$ C ; permittivity of vaccum $\epsilon_0 = 8.854185 \times 10^{-12}$ kg^{-1} m^{-3} A^2)

(a) 1.65Å (b) 4.76Å

(c) 0.529Å (d) 2.12Å

32. The group having isoelectronic species is : **[2017]**

(a) $O^{2-}, F^-, Na^+, Mg^{2+}$

(b) O^-, F^-, Na, Mg^+

(c) O^{2-}, F^-, Na, Mg^{2+}

(d) O^-, F^-, Na^+, Mg^{2+}

33. For emission line of atomic hydrogen from $n_i = 8$ to $n_f = n$, the plot of wave number ($\bar{\nu}$) against $\left(\frac{1}{n^2}\right)$ will be (The Rydberg constant, R_H is in wave number unit) **[2019]**

(a) Linear with intercept $-R_H$

(b) Non linear

(c) Linear with slope R_H

(d) Linear with slope $-R_H$

34. For any given series of spectral lines of atomic hydrogen, let $\Delta\bar{\nu} = \bar{\nu}_{max} - \bar{\nu}_{min}$ be the difference in maximum and minimum frequencies in cm^{-1}. The ratio $\Delta\bar{\nu}_{Lyman}/\Delta\bar{\nu}_{Balmer}$ is : **[2019]**

(a) 4 : 1 (b) 9 : 4

(c) 5 : 4 (d) 27 : 5

35. The number of orbitals associated with quantum numbers $n=5, m_s=+\frac{1}{2}$ is: **[2020]**

(a) 11 (b) 25

(c) 50 (d) 15

Answer Key														
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
(a)	(c)	(c)	(a)	(a)	(a)	(d)	(a)	(b)	(c)	(c)	(a)	(b)	(b)	(a)
16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
(c)	(c)	(b)	(d)	(a)	(b)	(d)	(b)	(a)	(b)	(c)	(a)	(a)	(a)	(b)
31	32	33	34	35										
(d)	(a)	(d)	(b)	(b)										

Solutions

1. **(a)** 2nd excited state will be the 3rd energy level.

$$E_n = \frac{13.6}{n^2}\text{eV}$$

or $E_3 = \frac{13.6}{9}\text{eV} = 1.51\text{eV}$.

2. **(c)** $\Delta x.\Delta p = \frac{h}{4\pi}$; or $\Delta x.m.\Delta v = \frac{h}{4\pi}$

$$\therefore \Delta v = \frac{6.62\times10^{-34}}{4\times3.14\times0.025\times10^{-5}}$$

$$= 2.1\times10^{-28}\,\text{ms}^{-1}$$

3. **(c)** $Fe^{++}(26-2=24) = 1s^2 2s^2 2p^6 3s^2 3p^6 4s^0 3d^6$
hence no. of d electrons retained is 6.
[Two 4s electron are removed]

4. **(a)** For s-electron, $l = 0$
$\therefore$ Orbital angular momentum

$$= \sqrt{0(0+1)}\frac{h}{2\pi} = 0$$

5. **(a)** N^{3-}, F^- and Na^+ contain 10 electrons each.

6. **(a)** The lines falling in the visible region comprise Balmer series. Hence the third line from red would be $n_1 = 2$, $n_2 = 5$ i.e. $5 \to 2$.

7. **(d)** $\lambda = \frac{h}{mv} = \frac{6.63\times10^{-34}}{60\times10^{-3}\times10}$

$$= 1.105\times10^{-33} \approx 10^{-33}\,\text{m}$$

8. **(a)** The possible quantum numbers for 4f electron are

$n = 4$, $l = 3$, $m = -3, -2 -1, 0, 1, 2, 3$ and

$s = \pm\frac{1}{2}$

Of various possiblities only option (a) is possible.

9. **(b)** Electronic configuration of Cr atom
$(Z = 24) = 1s^2 2s^2 2p^6 3s^2 3p^6 3d^5 4s^1$
when $l = 1$, p-subshell,
Numbers of electrons = 12
when $l = 2$, d-subshell,
Numbers of electrons = 5

10. **(c)** $\frac{1}{\lambda} = R\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$

$$\frac{1}{\lambda} = 1.097\times10^7\left(\frac{1}{1} - \frac{1}{\infty}\right) = 1.097\times10^7$$

$\lambda = 91.15\times10^{-9}\,\text{m} \approx 91\,\text{nm}$

11. **(c)** ${}_{19}K^+$, ${}_{20}Ca^{2+}$, ${}_{21}Sc^{3+}$, ${}_{17}Cl^-$
each contains 18 electrons.

12. **(a)** The energy of an orbital is given by $(n + l)$ rule. $(n + l)$ value for option (E) and (D) is $(3 + 2) = 5$ hence they will have same energy, since their n values are also same.

13. **(b)** Calculating number of electrons

a. $$\left.\begin{array}{l} BO_3^{3-} \longrightarrow 5+8\times3+3=32 \\ CO_3^{2-} \longrightarrow 6+8\times3+2=32 \\ NO_3^{-} \longrightarrow 7+8\times3+1=32 \end{array}\right\}\text{iso-electronic species}$$

b. $\left.\begin{array}{l} SO_3^{2-} \longrightarrow 16+8\times3+2=42 \\ CO_3^{2-} \longrightarrow 32 \\ NO_3^{-} \longrightarrow 32 \end{array}\right\}$ not iso-electronic species

c. $\left.\begin{array}{l} CN^{-} \longrightarrow 6+7+1=14 \\ N_2 \longrightarrow 7\times2=14 \\ C_2^{2-} \longrightarrow 6\times2+2=14=14 \end{array}\right\}$ iso-electronic species

d. $\left.\begin{array}{l} PO_4^{3-} \longrightarrow 15+8\times4+3=50 \\ SO_4^{2-} \longrightarrow 16+8+2=50 \\ ClO_4^{-} \longrightarrow 17+8\times4+1=50 \end{array}\right\}$ iso-electronic species

Hence the species in option (b) are not iso-electronic.

14. (b) Angular momentum of an electron in n^{th} orbital is given by,

$$mvr = \frac{nh}{2\pi}$$

For $n = 5$, we have

Angular momentum of electron

$$= \frac{5h}{2\pi} = \frac{2.5h}{\pi}$$

15. (a) Given $m = 9.1 \times 10^{-31}$kg

$h = 6.6 \times 10^{-34}$Js

$$\Delta v = \frac{300 \times .001}{100} = 0.003\ ms^{-1}$$

From Heisenberg's uncertainity principle

$$\Delta x = \frac{6.62\times10^{-34}}{4\times3.14\times0.003\times9.1\times10^{-31}}$$

$= 1.92 \times 10^{-2}$ m

16. (c) (a) $N^{3-} = 7 + 3 = 10e^-$,

$O^{2-} \longrightarrow 8 + 2 = 10e^-$

$F^- = 9 + 1 = 10e^-$,

$S^{2-} \longrightarrow 16 + 2 = 18e^-$

(not iso electronic)

(b) $Li^+ = 3 - 1 = 2e^-$, $Na^+ = 11 - 1 = 10e^-$,

$Mg^{2+} = 12 - 2 = 10e^-$

$Ca^{2+} = 20 - 2 = 18e^-$ (not isoelectronic)

(c) $K^+ = 19 - 1 = 18e^-$, $Cl^- = 17 + 1 = 18e^-$,

$Ca^{2+} = 20 - 2 = 18e^-$, $Sc^{3+} = 21 - 3 = 18e^-$

(isoelectronic)

(d) $Ba^{2+} = 56 - 2 = 54e^-$, $Sr^{2+} = 38 - 2 = 36e^-$

$K^+ = 19 - 1 = 18e^-$, $Ca^{2+} = 20 - 2 = 18e^-$

(not isoelectronic)

17. (c) (a) $n = 3, l = 0$ means 3*s*-orbital and $n + l = 3$

(b) $n = 3, l = 1$ means 3*p*-orbital $n + l = 4$

(c) $n = 3, l = 2$ means 3*d*-orbital $n + l = 5$

(d) $n = 4, l = 0$ means 4*s*-orbital $n + l = 4$

Increasing order of energy among these orbitals is

$3s < 3p < 4s < 3d$

$\therefore$ 3*d* has highest energy.

18. (b) Species having same number of electrons are **isoelectronic.** Calculating the number of electrons in each species given here, we get.

CN^- $(6 + 7 + 1 = 14)$; N_2 $(7 + 7 = 14)$;

O_2^{2-} $(8 + 8 + 2 = 18)$; C_2^{2-} $(6 + 6 + 2 = 14)$;

O_2^- $(8 + 8 + 1 = 17)$; NO^+ $(7 + 8 - 1 = 14)$

CO $(6 + 8 = 14)$; NO $(7 + 8 = 15)$

From the above calculation we find that all the species listed in choice (b) have 14 electrons each so it is the correct answer.

19. (d) (ΔE), The energy required to excite an electron in an atom of hydrogen from $n = 1$ to $n = 2$ is ΔE (difference in energy E_2 and E_1)

Values of E_2 and E_1 are,

$$E_2 = \frac{-1.312\times10^6\times(1)^2}{(2)^2}$$

$= -3.28 \times 10^5$ J mol^{-1}

ΔE is given by the relation,

$E_1 = -1.312 \times 10^6$ J mol^{-1}

$\therefore \Delta E = E_2 - E_1 = [-3.28 \times 10^5] - [-1.312 \times 10^6]$ J mol^{-1}

$= (-3.28 \times 10^5 + 1.312 \times 10^6)$ J mol^{-1}

$= 9.84 \times 10^5$ J mol^{-1}

20. **(a)** $\lambda = \frac{h}{mv} = \frac{6.63 \times 10^{-34}}{1.67 \times 10^{-27} \times 1 \times 10^{3}}$

$= 3.97 \times 10^{-10}\, m = 0.397\, nm$

21. **(b)** According to Heisenberg uncertainty principle.

$\Delta x.m\Delta v = \frac{h}{4\pi}$; $\Delta x = \frac{h}{4\pi m \Delta v}$

Here $\Delta v = \frac{600 \times 0.005}{100} = 0.03$

So, $\Delta x = \frac{6.6 \times 10^{-34}}{4 \times 3.14 \times 9.1 \times 10^{-31} \times 0.03}$

$= 1.92 \times 10^{-3}\, m$

22. **(d)** Energy required to break single Cl – Cl bond

$= \frac{242 \times 10^{3}}{6.023 \times 10^{23}} = \frac{hc}{\lambda}$

$= \frac{6.626 \times 10^{-34} \times 3 \times 10^{8}}{\lambda}$

$\therefore \lambda = \frac{6.626 \times 10^{-34} \times 3 \times 10^{8} \times 6.023 \times 10^{23}}{242 \times 10^{3}}$

$= 0.4947 \times 10^{-6}\, m = 494.7\, nm$

23. **(b)** $I.E = \frac{Z^2}{n^2} \times 13.6\, eV$ ...(i)

or $\frac{I_1}{I_2} = \frac{Z_1^{\,2}}{n_1^{\,2}} \times \frac{n_2^{\,2}}{Z_2^{2}}$ ...(ii)

Given $I_1 = -19.6 \times 10^{-18}$, $Z_1 = 2$,

$n_1 = 1$, $Z_2 = 3$ and $n_2 = 1$

Substituting these values in equation (ii).

$-\frac{19.6 \times 10^{-18}}{I_2} = \frac{4}{1} \times \frac{1}{9}$

or $I_2 = -19.6 \times 10^{-18} \times \frac{9}{4}$

$= -4.41 \times 10^{-17}$ J/atom

24. **(a)** For He^{+}

$v = RZ^2\left(\frac{1}{2^2} - \frac{1}{4^2}\right)$ Hz

For H,

$v = R\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$ Hz

For same frequency,

$Z^2\left(\frac{1}{2^2} - \frac{1}{4^2}\right) = \left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$

Since, $Z = 2$

$\therefore \quad \frac{1}{n_1^2} - \frac{1}{n_2^2} = \frac{1}{1^2} - \frac{1}{2^2}$

$\therefore \quad n_1 = 1$ & $n_2 = 2$

25. **(b)** (*A*) $4p$ (*B*) $4s$

(*C*) $3d$ (*D*) $3p$

Accroding to Bohr Bury's $(n + l)$ rule, increasing order of energy $(D) < (B) < (C) < (A)$.

NOTE If the two orbitals have same value of $(n + l)$ then the orbital with lower value of n will be filled first.

26. **(c)** Among isoelectronic species ionic radii increases as the negatives charge increases.

Order of ionic radii $Ca^{2+} < K^{+} < Cl^{-} < S^{2-}$

The number of electrons remains the same but nuclear charge increases with increase in the atomic number causing decrease in size.

27. **(a)** $\Delta E = 2.178 \times 10^{-18}\left(\frac{1}{1^2} - \frac{1}{2^2}\right) = \frac{hc}{\lambda}$

$\Rightarrow 2.178 \times 10^{-18} \times \frac{3}{4} = \frac{hc}{\lambda}$

$= \frac{6.62 \times 10^{-34} \times 3 \times 10^{8}}{\lambda}$

$\lambda = \frac{6.62 \times 10^{-34} \times 3 \times 10^{8} \times 4}{2.178 \times 10^{-18} \times 3}$

$= 1.214 \times 10^{-7} m$

28. **(a)** The electronic configuration of Rubidium (Rb = 37) is

$1s^2 2s^2 2p^6 3s^2 3p^6 3d^{10} 4s^2 4p^6 5s^1$

Since last electron enters in $5s$ orbital

Hence $n = 5$, $l = 0$, $m = 0$, $s = \pm\frac{1}{2}$

29. **(a)** Total energy $= -\frac{13.6}{n^2} Z^2 \text{eV}$
where $n = 2, 3, 4$ ....
Putting $n = 2$

$$E_T = -\frac{13.6}{4} = -3.4 \text{ eV}$$

30. **(b)** As electron of charge 'e' is passed through 'V' volt, kinetic energy of electron will be eV
Wavelength of electron wave (λ)

$$= \frac{h}{\sqrt{2m.\text{K.E}}}$$

$$\lambda = \frac{h}{\sqrt{2m\text{eV}}} \quad \therefore \frac{h}{\lambda} = \sqrt{2m\text{eV}}$$

31. **(d)** Radius of n^{th} Bohr orbit in H-atom
$= 0.53\, n^2 Å$
Radius of II Bohr orbit $= 0.53 \times (2)^2$
$= 2.12$ Å

32. **(a)** Isoelectronic species have same no. of electrons.

ions	O^{2-}	F^-	Na^+	Mg^{2+}
	$8+2$	$9+1$	$11-1$	$12-2$
No. of e^- =	10	10	10	10
therefore	O^{2-},	F^-,	Na^+,	Mg^{2+}

are isoelectronic

33. **(d)** As we know,

$$\bar{v} = -R_H\left(\frac{1}{n_2^2} - \frac{1}{n_1^2}\right) Z^2 \text{ (where, } Z = 1)$$

After putting the values, we get

$$\bar{v} = -R_H\left(\frac{1}{n^2} - \frac{1}{8^2}\right)$$

$$\Rightarrow \bar{v} = \frac{R_H}{64} - \frac{R_H}{n^2}$$

Comparing to $y = mx + c$, we get

$$x = \frac{1}{n^2} \text{ and } m = -R_H \text{ (slope)}$$

34. **(b)** $\bar{\nu} \propto \Delta E$
For H-atom

$$\bar{\nu} = R\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$$

For Lyman series,

$$\bar{\nu}(\max) = 13.6\left(1 - \frac{1}{\infty}\right)$$

$$\bar{\nu}(\min) = 13.6\left(1 - \frac{1}{4}\right)$$

$$\therefore \bar{\nu}_{max} - \bar{\nu}_{min} = 13.6\left(\frac{1}{4}\right)$$

For Balmer series,

$$\bar{\nu}(\max) = 13.6\left(\frac{1}{4} - \frac{1}{\infty}\right)$$

$$\bar{\nu}(\min) = 13.6\left(\frac{1}{4} - \frac{1}{9}\right)$$

$$\therefore \bar{\nu}_{max} - \bar{\nu}_{min} = 13.6\left(\frac{1}{9}\right)$$

$$\frac{\Delta\bar{\nu}_{Lyman}}{\Delta\bar{\nu}_{Balmer}} = \frac{9}{4}$$

35. **(b)** The possible number of orbitals in a shell in term of 'n' is n^2
$\therefore n = 5$; $n^2 = 25$

3 Classification of Elements and Periodicity in Properties

1. According to the Periodic Law of elements, the variation in properties of elements is related to their [2003]
(a) nuclear masses
(b) atomic numbers
(c) nuclear neutron-proton number ratios
(d) atomic masses

2. Which one of the following is an amphoteric oxide ? [2003]
(a) Na_2O (b) SO_2 (c) B_2O_3 (d) ZnO

3. Which one of the following ions has the highest value of ionic radius ? [2004]
(a) O^{2-} (b) B^{3+} (c) Li^{+} (d) F^{-}

4. Among Al_2O_3, SiO_2, P_2O_3 and SO_2 the correct order of acid strength is [2004]
(a) $Al_2O_3 < SiO_2 < SO_2 < P_2O_3$
(b) $SiO_2 < SO_2 < Al_2O_3 < P_2O_3$
(c) $SO_2 < P_2O_3 < SiO_2 < Al_2O_3$
(d) $Al_2O_3 < SiO_2 < P_2O_3 < SO_2$

5. The formation of the oxide ion $O^{2-}(g)$ requires first an exothermic and then an endothermic step as shown below [2004]
$O(g) + e^{-} = O^{-}(g)\ \Delta H^0 = -142\ \text{kJmol}^{-1}$
$O^{-}(g) + e^{-} = O^{2-}(g)\ \Delta H^0 = 844\ \text{kJmol}^{-1}$
This is because
(a) O^{-} ion will tend to resist the addition of another electron
(b) Oxygen has high electron affinity
(c) Oxygen is more elecronegative
(d) O^{-} ion has comparatively larger size than oxygen atom

6. Which of the following oxides is amphoteric in character? [2005]
(a) SnO_2 (b) SiO_2 (c) CO_2 (d) CaO

7. In which of the following arrangements, the order is NOT according to the property indicated against it? [2005]
(a) $Li < Na < K < Rb$:
Increasing metallic radius
(b) $I < Br < F < Cl$:
Increasing electron gain enthalpy (with negative sign)
(c) $B < C < N < O$
Increasing first ionization enthalpy
(d) $Al^{3+} < Mg^{2+} < Na^{+} < F^{-}$
Increasing ionic size

8. Following statements regarding the periodic trends of chemical reactivity of alkali metals and halogens are given. Which of these statements gives the correct picture? [2006]
(a) Chemical reactivity increases with increase in atomic number down the group in both the alkali metals and halogens
(b) In alkali metals the reactivity increases but in the halogens it decreases with increase in atomic number down the group
(c) The reactivity decreases in the alkali metals but increases in the halogens with increase in atomic number down the group
(d) In both, alkali metals and halogens-chemical reactivity decreases with increase in atomic number down the group

9. In which of the following arrangements, the sequence is *not* strictly according to the property written against it? [2008]
(a) $HF < HCl < HBr, HI$: increasing acid strength
(b) $NH_3 < PH_3 < AsH_3 < SbH_3$: increasing basic strength
(c) $B < C < O < N$: increasing first ionization enthalpy
(d) $CO_2 < SiO_2 < SnO_2 < PbO_2$: increasing oxidising power

10. The correct sequence which shows decreasing order of the ionic radii of the elements is **[2010]**
(a) $Al^{3+} > Mg^{2+} > Na^{+} > F^{-} > O^{2-}$
(b) $Na^{+} > Mg^{2+} > Al^{3+} > O^{2-} > F^{-}$
(c) $Na^{+} > F^{-} > Mg^{2+} > O^{2-} > Al^{3+}$
(d) $O^{2-} > F^{-} > Na^{+} > Mg^{2+} > Al^{3+}$

11. The correct order of electron gain enthalpy with negative sign of F, Cl, Br and I, having atomic numbers 9, 17, 35 and 53 respectively, is : **[2011RS]**
(a) F > Cl > Br > I (b) Cl > F > Br > I
(c) Br > Cl > I > F (d) I > Br > Cl > F

12. Which of the following represents the correct order of increasing first ionization enthalpy for Ca, Ba, S, Se and Ar ? **[2013]**
(a) Ca < S < Ba < Se < Ar
(b) S < Se < Ca < Ba < Ar
(c) Ba < Ca < Se < S < Ar
(d) Ca < Ba < S < Se < Ar

13. The ionic radii (in Å) of N^{3-}, O^{2-} and F^{-} are respectively : **[2015]**
(a) 1.71, 1.40 and 1.36
(b) 1.71, 1.36 and 1.40
(c) 1.36, 1.40 and 1.71
(d) 1.36, 1.71 and 1.40

14. Which of the following atoms has the highest first ionization energy? **[2016]**
(a) K (b) Sc
(c) Rb (d) Na

15. In general, the properties that decrease and increase down a group in the periodic table, respectively, are: **[2019]**
(a) atomic radius and electronegativity.
(b) electron gain enthalpy and electronegativity.
(c) electronegativity and atomic radius.
(d) electronegativity and electron gain enthalpy.

16. The element having greatest difference between its first and second ionization energies, is: **[2019]**
(a) Ca (b) Sc
(c) Ba (d) K

17. The electron gain enthalpy (in kJ/mol) of fluorine, chlorine, bromine and iodine, respectively, are: **[2020]**
(a) –296, –325, –333 and –349
(b) –349, –333, –325 and –296
(c) –333, –349, –325 and –296
(d) –333, –325, –349 and –296

Answer Key

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
(b)	(d)	(a)	(d)	(a)	(a)	(c)	(b)	(b)	(d)	(b)	(c)	(a)	(b)	(c)
16	17													
(d)	(c)													

Solutions

1. **(b)** According to modern periodic law, the properties of the elements are repeated after certain regular intervals when these elements are arranged in order of their increasing atomic numbers.

2. **(d)** Na_2O (basic), SO_2 and B_2O_3 (acidic) and ZnO is amphoteric.

3. **(a)** O^{2-} and F^{-} are isoelectronic ($10e^{-}$). Hence have same number of electron but different number of proton, F^{-} has 9 proton whereas O^{2-} has 8 proton. Therefore F^{-} has greater nuclear charge compared to O^{2-}. So O^{2-} will have greater ionic radius, i.e.

$$O^{2-} > F^{-}$$

Further Li^{+} and B^{3+} are also isoelectronic ($2e^{-}$) therefore, no. of proton in Li is 3 and in B is 5, So,

$$Li^{+} > B^{3+}$$

Hence, the correct order of atomic size is

$$O^{2-} > F^{-} > Li^{+} > B^{3+}$$

Therefore, O^{2-} has the highest value of ionic radius.

4. **(d)** As the size decreases the basic nature of oxides changes to acidic nature i.e., acidic nature increases.

$$\underset{\text{Acidic}}{SO_2 > P_2O_3} > \underset{\text{Weak acidic}}{SiO_2} > \underset{\text{Amphoteric}}{Al_2O_3}$$

SO_2 and P_2O_3 are acidic as their corresponding acids H_2SO_3 and H_3PO_3 are strong acids.

5. **(a)** O^- ion exerts a force of repulsion on the incoming electron. The energy is required to overcome it.

6. **(a)** CaO is basic as it forms strong base $Ca(OH)_2$ on reaction with water.

$$CaO + H_2O \longrightarrow Ca(OH)_2$$

CO_2 is acidic as it dissolves in water forming unstable carbonic acid.

$$H_2O + CO_2 \longrightarrow H_2CO_3$$

Silica (SiO_2) is insoluble in water and acts as a very weak acid.

SnO_2 is amphoteric as it reacts with both acid and base.

$$SnO_2 + 2H_2SO_4 \longrightarrow Sn(SO_4)_2 + 2H_2O$$

$$SnO_2 + 2KOH \longrightarrow K_2SnO_3 + H_2O$$

7. **(c)** In a period the value of ionisation potential increases from left to right with breaks where the atoms have somewhat stable configuration. In the given list N has half-filled stable orbitals. Hence N has highest ionisation energy. Thus the correct order is

$$B < C < O < N$$

and not as given in option (c)

8. **(b)** The alkali metals are highly reactive because their first ionisation potential is very low and hence they have great tendency to give up electron to form unipositive ion.

NOTE On moving down in group I from Li to Cs, ionisation enthalpy decreases hence the reactivity increases. The halogens are most reactive elements due to their low bond dissociation energy, high electron affinity and high enthalpy of hydration of halide ion. However, their reactivity decreases with increase in atomic number.

9. **(b)** In hydrides of 15th group elements, basic character decreases on descending the group i.e.

$NH_3 > PH_3 > AsH_3 > SbH_3$.

10. **(d)** All the given species contains 10 e^- each i.e. isoelectronic. For isoelectronic species anion having high negative charge is largest in size and the cation having high positive charge is smallest.

11. **(b)** As we move down in a group, electron gain enthalpy becomes less negative because the size of the atom increases and the distance of added electron from the nucleus increases. Negative electron gain enthalpy of F is less than Cl. This is due to the fact that when an electron is added to F, the added electron goes to the smaller $n = 2$ energy level and experiences significant repulsion from the other electrons present in this level. In Cl, the electron goes to the larger $n = 3$ energy level and consequently occupies a larger region of space leading to much less electron–electron repulsion.

So the correct order is

$$Cl > F > Br > I.$$

12. **(c)** On moving down a group size increases, hence ionisation enthalpy decreases. Hence Se < S and Ba < Ca. Further, Ar being an inert gas has maximum IE.

13. **(a)** For isoelectronic species, size of anion increases as negative charge increases. Thus the correct order is (a).

14. **(b)** Alkali metals have the lowest ionization energy in each period, on the other hand Sc is a *d* - block element.

Transition metals have smaller atomic radii and higher nuclear charge leading to high ionisation energy.

15. **(c)** Generally, electronegativity decreases down the group as the size increases. This can also be formulated as:

$$\text{Electronegativity} \propto \frac{1}{\text{size}}$$

16. **(d)** Alkali metals have high difference in the first and second ionisation energy as they achieve stable noble gas configuration after first ionisation.

17. **(c)** Chlorine has highest electron gain enthalpy (most negative) among the given elements, the electron gain enthalpy decreases down the group i.e., moves to least negative.

4 Chemical Bonding and Molecular Structure

1. In which of the following species the interatomic bond angle is 109° 28′? **[2002]**

(a) NH_3, BF_4^- (b) NH_4^+, BF_3

(c) NH_3, BF_4 (d) NH_2^-, BF_3

2. Which of the following are arranged in an increasing order of their bond strength? **[2002]**

(a) $O_2^- < O_2 < O_2^+ < O_2^{2-}$

(b) $O_2^{2-} < O_2^- < O_2 < O_2^+$

(c) $O_2^- < O_2^{2-} < O_2 < O_2^+$

(d) $O_2^+ < O_2 < O_2^- < O_2^{2-}$

3. Hybridisation of the underline atom changes in:

(a) $\underline{Al}H_3$ changes to AlH_4^- **[2002]**

(b) $H_2\underline{O}$ changes to H_3O^+

(c) $\underline{N}H_3$ changes to NH_4^+

(d) in all cases

4. An ether is more volatile than an alcohol having the same molecular formula. This is due to **[2003]**

(a) alcohols having resonance structures

(b) inter-molecular hydrogen bonding in ethers

(c) inter-molecular hydrogen bonding in alcohols

(d) dipolar character of ethers

5. Which one of the following pairs of molecules will have permanent dipole moments for both members? **[2003]**

(a) NO_2 and CO_2 (b) NO_2 and O_3

(c) SiF_4 and CO_2 (d) SiF_4 and NO_2

6. Which one of the following compounds has the smallest bond angle in its molecule? **[2003]**

(a) OH_2 (b) SH_2 (c) NH_3 (d) SO_2

7. The pair of species having identical shapes for molecules of both species is **[2003]**

(a) XeF_2, CO_2 (b) BF_3, PCl_3

(c) PF_5, IF_5 (d) CF_4, SF_4

8. The correct order of bond angles (smallest first) in H_2S, NH_3, BF_3 and SiH_4 is **[2004]**

(a) $H_2S < NH_3 < SiH_4 < BF_3$

(b) $NH_3 < H_2S < SiH_4 < BF_3$

(c) $H_2S < SiH_4 < NH_3 < BF_3$

(d) $H_2S < NH_3 < BF_3 < SiH_4$

9. The bond order in NO is 2.5 while that in NO^+ is 3. Which of the following statements is true for these two species? **[2004]**

(a) Bond length in NO^+ is equal to that in NO

(b) Bond length in NO is greater than in NO^+

(c) Bond length in NO^+ is greater than in NO

(d) Bond length is unpredictable

10. The states of hybridization of boron and oxygen atoms in boric acid (H_3BO_3) are respectively **[2004]**

(a) sp^3 and sp^2 (b) sp^2 and sp^3

(c) sp^2 and sp^2 (d) sp^3 and sp^3

11. Which one of the following has the regular tetrahedral structure? **[2004]**

(a) BF_4^- (b) SF_4

(c) XeF_4 (d) $[Ni(CN)_4]^{2-}$

(Atomic nos. : B = 5, S = 16, Ni = 28, Xe = 54)

12. The maximum number of 90º angles between bond pair-bond pair of electrons is observed in [2004]
(a) dsp^2 hybridization
(b) sp^3d hybridization
(c) dsp^3 hybridization
(d) sp^3d^2 hybridization

13. Lattice energy of an ionic compound depends upon [2005]
(a) Charge on the ion and size of the ion
(b) Packing of ions only
(c) Size of the ion only
(d) Charge on the ion only

14. Which of the following molecules/ions does not contain unpaired electrons? [2006]
(a) N_2^+ (b) O_2 (c) O_2^{2-} (d) B_2

15. In which of the following molecules/ions are all the bonds **not** equal? [2006]
(a) XeF_4 (b) BF_4^- (c) SF_4 (d) SiF_4

16. The decreasing values of bond angles from NH_3 (106º) to SbH_3 (101º) down group-15 of the periodic table is due to [2006]
(a) decreasing *lp*-*bp* repulsion
(b) decreasing electronegativity
(c) increasing *bp*-*bp* repulsion
(d) increasing *p*-orbital character in sp^3

17. Which of the following species exhibits the diamagnetic behaviour? [2007]
(a) NO (b) O_2^{2-} (c) O_2^+ (d) O_2

18. The charge/size ratio of a cation determines its polarizing power. Which one of the following sequences represents the increasing order of the polarizing power of the cationic species, K^+, Ca^{2+}, Mg^{2+}, Be^{2+}? [2007]
(a) $Ca^{2+} < Mg^{2+} < Be^+ < K^+$
(b) $Mg^{2+} < Be^{2+} < K^+ < Ca^{2+}$
(c) $Be^{2+} < K^+ < Ca^{2+} < Mg^{2+}$
(d) $K^+ < Ca^{2+} < Mg^{2+} < Be^{2+}$

19. In which of the following ionization processes, the bond order has increased and the magnetic behaviour has changed? [2007]
(a) $N \rightarrow N_2^+$ (b) $C_2 \rightarrow C_2^+$
(c) $NO \rightarrow NO^+$ (d) $O_2 \rightarrow O_2^+$

20. Which of the following hydrogen bonds is strongest? [2007]
(a) O – H - - - F (b) O – H - - - H
(c) F – H - - - F (d) O – H - - - O

21. Which one of the following pairs of species have the same bond order? [2008]
(a) CN^- and NO^+ (b) CN^- and CN^+
(c) O_2^- and CN^- (d) NO^+ and CN^+

22. The bond dissociation energy of B – F in BF_3 is 646 kJ mol^{-1}, whereas that of C – F in CF_4 is 515 kJ mol^{-1}. The correct reason for higher B – F bond dissociation energy as compared to that of C – F bond is [2008]
(a) stronger σ bond between B and F in BF_3 as compared to that between C and F in CF_4.
(b) significant $p\pi - p\pi$ interaction between B and F in BF_3 whereas there is no possibility of such interaction between C and F in CF_4.
(c) lower degree of $p\pi - p\pi$ interaction between B and F in BF_3 than that between C and F in CF_4.
(d) smaller size of B– atom as compared to that of C– atom.

23. Using MO theory, predict which of the following species has the shortest bond length? [2008]
(a) O_2^+ (b) O_2^- (c) O_2^{2-} (d) O_2^{2+}

24. The number of types of bonds between two carbon atoms in calcium carbide is : [2011RS]
(a) One sigma, one pi (b) Two sigma, one pi
(c) Two sigma, two pi (d) One sigma, two pi

25. *ortho*-Nitrophenol is less soluble in water than *p*- and *m*- nitrophenols because : [2012]
(a) *o*-Nitrophenol is more volatile steam than those of *m*- and *p*-isomers.
(b) *o*-Nitrophenol shows intramolecular H-bonding
(c) *o*-Nitrophenol shows intermolecular H-bonding
(d) Melting point of *o*-nitrophenol is lower than those of *m*- and *p*-isomers.

26. In which of the following pairs, the two species are not isostructural ? [2012]

(a) CO_3^{2-} and NO_3^- (b) PCl_4^+ and $SiCl_4$

(c) PF_5 and BrF_5 (d) AlF_6^{3-} and SF_6

27. Which one of the following molecules is expected to exhibit diamagnetic behaviour ? [2013]

(a) C_2 (b) N_2 (c) O_2 (d) S_2

28. Which of the following is the wrong statement [2013]

(a) ONCl and ONO^- are not isoelectronic.

(b) O_3 molecule is bent

(c) Ozone is violet-black in solid state

(d) Ozone is diamagnetic gas.

29. In which of the following pairs of molecules/ ions, both the species are not likely to exist ? [2013]

(a) H_2^+, He_2^{2-} (b) H_2^-, He_2^{2-}

(c) H_2^{2+}, He_2 (d) H_2^-, He_2^{2+}

30. Stability of the species Li_2, Li_2^- and Li_2^+ increases in the order of : [2013]

(a) $Li_2 < Li_2^+ < Li_2^-$ (b) $Li_2^- < Li_2^+ < Li_2$

(c) $Li_2 < Li_2^- < Li_2^+$ (d) $Li_2^- < Li_2 < Li_2^+$

31. Which one of the following properties is **not** shown by NO? [2014]

(a) It is diamagnetic in gaseous state

(b) It is neutral oxide

(c) It combines with oxygen to form nitrogen dioxide

(d) Its bond order is 2.5

32. The species in which the N atom is in a state of *sp* hybridization is : [2016]

(a) NO_3^- (b) NO_2 (c) NO_2^+ (d) NO_2^-

33. Which of the following species is not paramagnetic ? [2017]

(a) NO (b) CO (c) O_2 (d) B_2

34. According to molecular orbital theory, which of the following will not be a viable molecule? [2018]

(a) He_2^{2+} (b) He_2^+ (c) H_2^- (d) H_2^{2-}

35. Which of the following compounds contain(s) no covalent bond(s)? [2018]

KCl, PH_3, O_2, B_2H_6, H_2SO_4

(a) KCl, B_2H_6, PH_3 (b) KCl, H_2SO_4

(c) KCl (d) KCl, B_2H_6

36. Total number of lone pair of electrons in I_3^- ion is : [2018]

(a) 3 (b) 6 (c) 9 (d) 12

37. According to molecular orbital theory, which of the following is true with respect to Li_2^+ and Li_2^-? [2019]

(a) Li_2^+ is unstable and Li_2^- is stable

(b) Li_2^+ is stable and Li_2^- is unstable

(c) Both are stable

(d) Both are unstable

38. Among the following, the molecule expected to be stabilized by anion formation is: C_2, O_2, NO, F_2 [2019]

(a) C_2 (b) F_2 (c) NO (d) O_2

39. The dipole moments of CCl_4, $CHCl_3$ and CH_4 are in the order: [2020]

(a) $CHCl_3 < CH_4 = CCl_4$

(b) $CCl_4 < CH_4 < CHCl_3$

(c) $CH_4 < CCl_4 < CHCl_3$

(d) $CH_4 = CCl_4 < CHCl_3$

40. The theory that can completely/properly explain the nature of bonding in $[Ni(CO)_4]$ is: [2020]

(a) Werner's theory

(b) Molecular orbital theory

(c) Crystal field theory

(d) Valence bond theory

41. The relative strength of interionic/ intermolecular forces in decreasing order is: [2020]

(a) dipole-dipole > ion-dipole > ion-ion

(b) ion-dipole > ion-ion > dipole-dipole

(c) ion-dipole > dipole-dipole > ion-ion

(d) ion-ion > ion-dipole > dipole-dipole

Answer Key														
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
(a)	(b)	(a)	(c)	(b)	(b)	(a)	(a)	(b)	(b)	(a)	(d)	(a)	(c)	(c)
16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
(b)	(b)	(d)	(c)	(c)	(a)	(b)	(d)	(d)	(b)	(c)	(a, b)	(N)	(c)	(b)
31	32	33	34	35	36	37	38	39	40	41				
(a)	(c)	(b)	(d)	(c)	(c)	(c)	(a)	(d)	(b)	(d)				

Solutions

1. **(a)** In NH_3 and BF_4^-, the hybridisation is sp^3 and the bond angle is almost 109º 28'.

2. **(b)** $O_2^+(15) = KK\,\sigma 2s^2\,\sigma^* 2s^2\,\sigma 2p_z^2$
$\pi 2p_x^2 = \pi 2p_y^2\,\pi^* 2p_x^1 = \pi 2p_y^0$

Bond order $= \frac{1}{2}(8–3) = \frac{5}{2} = 2.5$

$O_2(16) = KK\,\sigma 2s^2\,\sigma^* 2s^2\,\sigma 2p_z^2$
$\pi 2p_x^2 = \pi 2p_y^2\,\pi^* 2p_x^1 = \pi^* 2p_y^1$

Bond order $= \frac{1}{2}(8-4) = 2$

$O_2^-(17) = KK\,\sigma 2s^2\,\sigma^* 2s^2\,\sigma 2p_z^2$
$\pi 2p_x^2 = \pi 2p_y^2\,\pi^* 2p_x^2 = \pi^* 2p_y^1$

Bond order $= \frac{1}{2}(8-5) = 1.5$

$O_2^{2-}(18) = KK\,\sigma 2s^2\,\sigma^* 2s^2\,\sigma 2p_z^2$
$\pi 2p_x^2 = \pi 2p_y^2\,\pi^* 2p_x^2 = \pi^* 2p_y^2$

Bond order $= \frac{1}{2}(8–6) = 1$

NOTE As we know that as the bond order decreases, stability also decreases and hence the bond strength also decreases. Therefore the correct order of their increasing bond strength is
$O_2^{2-} < O_2^- < O_2 < O_2^+$

3. **(a)** Hybridisation $= \frac{1}{2}\left[\begin{pmatrix}\text{No. of electrons}\\ \text{in valence}\\ \text{shell of atom}\end{pmatrix} + \begin{pmatrix}\text{No.of monovalent}\\ \text{atoms around it}\end{pmatrix} - \begin{pmatrix}\text{Charge on}\\ \text{cation}\end{pmatrix} + \begin{pmatrix}\text{Charge on}\\ \text{anion}\end{pmatrix}\right]$

(a) For AlH_3,

Hybridisation of Al atom $= \frac{1}{2}[3+3-0+0]$
$= 3 = sp^2$

For AlH_4^-,

Hybridisation of Al atom $= \frac{1}{2}[3+4-0+1]$
$= 4 = sp^3$

(b) For H_2O,
Hybridisation of O atom

$= \frac{1}{2}[6+2-0+0] = 4 = sp^3$

For H_3O^+, Hybridisation of O atom

$= \frac{1}{2}[6+3-1+0] = 4 = sp^3$

(c) For NH_3,
Hybridisation of N atom

$= \frac{1}{2}[5+3-0+0] = 4 = sp^3$

For NH_4^+, Hybridisation of N atom

$= \frac{1}{2}[5+4-1+0] = 4 = sp^3$

Thus hybridisation changes only in option (a).

4. **(c)** In ether, there is no H-bonding while alcohols have intermolecular H-bonding.

5. **(b)** Both NO_2 and O_3 have angular shape and hence will have net dipole moment.

6. **(b)** In H_2S, due to low electronegativity of sulphur the *l.p. - l.p.* repulsion is more than *b.p. - b.p.* repulsion and hence the bond angle is minimum.

	SO_2	H_2O	H_2S	NH_3
Bond angle	119.5°	104.5°	92.5°	106.5°

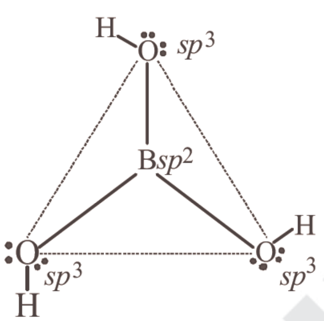
7. **(a)** Both XeF_2 and CO_2 have a linear structure.
F — Xe — F   O = C = O

8. **(a)** The order of bond angles

$$\underset{120^\circ}{BF_3} > \underset{109^\circ 28'}{SiH_4} > \underset{107^\circ}{NH_3} > \underset{92.5^\circ}{H_2S}$$

9. **(b)** Now since bond order of NO^+ (3) is higher than that of NO (2.5). Thus bond length of NO^+ will be shorter.

10. **(b)**

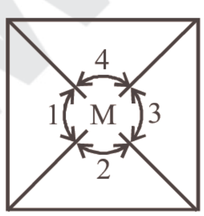


11. **(a)** XeF_4 (sp^3d^2, square planar),

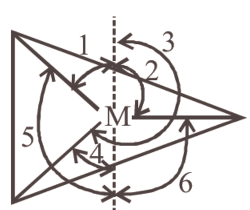
$[Ni(CN)_4]^{2-}$ (dsp^2, square planar),

BF_4^- (sp^3, tetrahedral), SF_4 (sp^3d, see saw shaped)

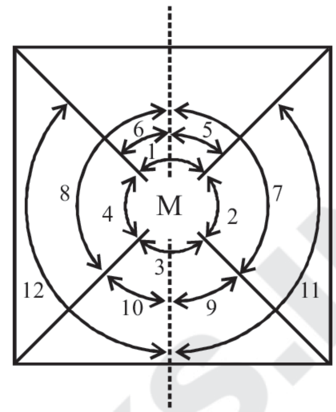
12. **(d)**



dsp^2 hybridisation
Number of 90° angle between bonds = 4



sp^3d or dsp^3 hybridisation
Number of 90° angle between bonds = 6



sp^3d^2 Hybridisation

Number of 90° angle between bonds = 12

13. **(a)** The value of lattice energy depends on the charge present on the two ions and the distance between them.

14. **(c)** The distribution of electrons in MOs is as follows :

N_2^+(electrons 13)

$$KK\sigma 2s^2\sigma^*2s^2\sigma 2p_z^2\pi 2p_x^2 = \pi 2p_y^1$$

O_2 (electron 16) = KK $\sigma 2s^2\ \sigma^* 2s^2\ \sigma 2p_x^{\ 2}$

$$\pi 2p_x^2 = \pi 2p_y^{\ 2}\ \pi^* 2p_x^1 = \pi^* 2p_y^1$$

O_2^{2-} (electron 18) = KK $\sigma 2s^2\ \sigma^* 2s^2\ \sigma 2p_x^{\ 2}$

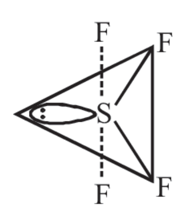
$$\pi 2p_x^2 = \pi 2p_y^2\ \pi^* 2p_x^2 = \pi^* 2p_y^{\ 2}$$

B_2 (electrons 10)

$$KK\ \sigma 2s^2\ \sigma^* 2s^2\ \pi 2p_x^1 = \pi 2p_y^1$$

Only O_2^{2-} does not contain any unpaired electron.

15. **(c)** In SF_4 the hybridisation is sp^3d and the shape of molecule is



It contains two different bonds i.e., axial and equatorial.

16. **(b)** The bond angle decreases on moving down the group due to decrease in bond pair-bond pair repulsion.

NH_3	PH_3	AsH_3	SbH_3	BiH_3
107°	94°	92°	91°	90°

NOTE This can also be explained due to decrease in electronegativity from N to Bi.

17. **(b)** Diamagnetic species have no unpaired electrons whereas paramagnetic species has one or more unpaired electrons.
For electronic configuration of O_2^+, O_2 and O_2^{2-}, consult Q. 2.
O_2 and O_2^+ have 2 and 1 unpaired electron respectively, while O_2^{2-} has no unpaired electron

$NO \rightarrow KK\sigma 2s^2\sigma^* 2s^2 \sigma 2p_z^2$
$\pi 2p_x^2 = \pi 2p_y^2\ \pi^* 2p_x^1 = \pi^* 2p_y^0$
(unpaired electron)

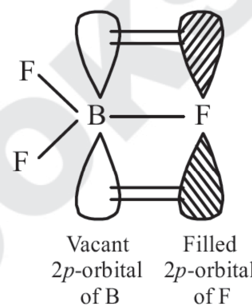
18. **(d)** Smaller the size and higher the charge, more will be the polarising power of the cation. Since the order of the size of cation is $K^+ > Ca^{2+} > Mg^{2+} > Be^{2+}$, so the correct order of polarising power is
$K^+ < Ca^{2+} < Mg^{2+} < Be^{2+}$

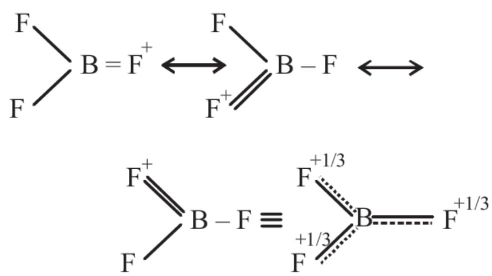
19. **(c)** (a) N_2 : bond order 3, diamagnetic
N_2^+ : bond order 2.5, paramagnetic
(b) C_2 : bond order 2, diamagnetic
C_2^+ : bond order 1.5, paramagnetic
(c) NO : bond order 2.5, paramagnetic
NO^+ : bond order 3, diamagnetic
(d) O_2 : bond order 2, paramagnetic
O_2^+ : bond order 2.5, paramagnetic

20. **(c)** **NOTE** Greater the difference between electronegativity of bonded atoms, stronger will be bond. Since F is most electronegative, hence F – H ...... F is the strongest bond.

21. **(a)** For any species to have same bond order we can expect them to have same number of electrons. Calculating the number of electrons in various species.
$O_2^-\ (8+8+1=17)$; $CN^-\ (6+7+1=14)$
$NO^+(7+8-1=14)$; $CN^+\ (6+7-1=12)$
We find CN^- and NO^+ both have 14 electrons, so they have same bond order.

22. **(b)** **NOTE** The delocalised $p\pi - p\pi$ bonding between filled p-orbital of F and vacant p-orbital of B leads to shortening of B – F bond length which results in higher bond dissociation energy of the B – F bond.





23. **(d)** Bond order =

$$\frac{\text{No. of bonding electrons} - \text{No. of antibonding electrons}}{2}$$

Bond order in $O_2^+ = \frac{10-5}{2} = 2.5$

Bond order in $O_2^- = \frac{10-7}{2} = 1.5$

Bond order in $O_2^{2-} = \frac{10-8}{2} = 1$

Bond order in $O_2^{2+} = \frac{10-4}{2} = 3$

Since, bond order $\propto \frac{1}{\text{Bond length}}$

$\therefore$ Bond length is shortest in O_2^{2+}.

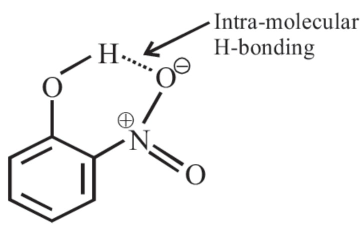
24. **(d)** Calcium carbide exists as Ca^{2+} and C_2^{2-}. According to the molecular orbital model, C_2^{2-} should have following molecular orbital configuration :

$KK\, \sigma 2s^2\, \sigma^* 2s^2$

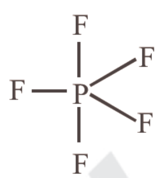
$\pi 2p_x^2 = \pi\, 2p_y^2\;\; \sigma 2p_z^2$

Thus M.O. configuration suggests that it contains one σ & two π bonds.

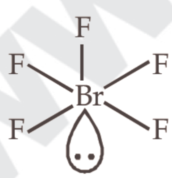
25. **(b)** Compounds involved in chelation become non-polar. Consequently such compounds are soluble in non-polar solvents like ether, benzene etc. and are only sparingly soluble in water, whereas meta and para isomers are more soluble in water & less soluble in non-polar solvents.



26. **(c)** PF_5 is trigonal bipyramidal



BrF_5 is square pyramidal (distorted)



27. **(a, b)** The molecular orbital structures of C_2 and N_2 are

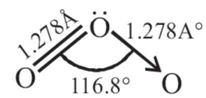
$N_2 = KK\,\sigma 2s^2\, \sigma^* 2s^2 \pi 2p_x^2 = \pi 2p_y^2\, \sigma 2p_z^2$

$C_2 = KK\,\sigma 2s^2\, \sigma^* 2s^2 \pi 2py_x^2 = \pi 2p_y^2$

Both N_2 and C_2 have paired electrons, hence they are diamagnetic.

28. (N) All options are correct,

(a) $\left.\begin{array}{l} ONCl = 8+7+17 = 32e^- \\ ONO^- = 8+7+8+1 = 24e^- \end{array}\right\}$ not isoelectronic

(b) 

The central atom is sp^2 hybridised with one lone pair.

(c) It is a pale blue gas. At – 249.7°, it forms violet black crystals.

(d) It is diamagnetic in nature due to absence of unpaired electrons.

29. **(c)** $H_2^{2+} = \sigma 1s^0\, \sigma^* 1s^0$

Bond order for $H_2^{2+} = \frac{1}{2}(0-0) = 0$

$He_2 = \sigma 1s^2 \sigma^* 1s^2$

Bond order for $He_2 = \frac{1}{2}(2-2) = 0$

So both H_2^{2+} and He_2 do not exist.

30. **(b)** $Li_2 = \sigma 1s^2\, \sigma^* 1s^2\, \sigma 2s^2$

$\therefore$ Bond order $= \frac{1}{2}(4 \quad 2) \quad 1$

$Li_2^+ = \sigma 1s^2\, \sigma^* 1s^2\, \sigma 2s^1$

B.O. $\frac{1}{2}(3 \quad 2) \quad 0.5$

$Li_2^- = \sigma 1s^2\, \sigma^* 1s^2 \sigma 2s^2 \sigma^* 2s^1$

B.O. $= \frac{1}{2}(4 \quad 3) \quad 0.5$

The bond order of Li_2^+ and Li_2^- is same but Li_2^+ is more stable than Li_2^- because Li_2^+ is smaller in size and has 2 electrons in antibonding orbitals whereas Li_2^- has 3 electrons in antibonding orbitals. Hence Li_2^+ is more stable than Li_2^-.

31. **(a)** Nitric oxide is paramagnetic in the gaseous state because of the presence of one unpaired electron in its outermost shell.

The electronic configuration of NO is

$\sigma KK \sigma^2 2s\, \sigma^* 2s^2 \sigma 2p_z^2\; \pi 2p_x^2$

$= \pi 2\, p_y^2 \pi^* 2p_x^1 = \pi^* 2p_y^0$

32. **(c)** Hybridisation (H) = [No. of valence electrons of central atom + No. of monovalent atoms attached to it + (–ve charge if any) – (+ve charge if any)]

$NO_2^+ =$ i.e. *sp* hybridisation

$NO_2^- =$ i.e. sp^2 hybridisation

$NO_3^- =$ i.e. sp^2 hybridisation

The Lewis structure of NO_2 shows a bent molecular geometry with trigonal planar electron pair geometry hence the hybridization will be sp^2.

33. (b)

(a) NO $\rightarrow$ One unpaired electron is present in π^* molecular orbit, hence paramagnetic.

(b) CO (14) $\rightarrow$

$$KK\sigma 2s^2 \pi 2p_x^2 = \pi 2p_y^2 \sigma 2p_z^2 \sigma^* 2s^2$$

No unpaired electron, hence diamagnetic.

(c) O_2 (16) $\rightarrow$

$$KK\sigma 2s^2 \sigma^* 2s^2 \sigma 2p_z^2 \pi 2p_x^2 = \pi 2p_y^2 \pi^* 2p_x^1 = \pi^* 2p_y^1$$

Two unpaired electrons, hence paramagnetic.

(d) $B_2 (10) \rightarrow KK\, \sigma 2s^2 \sigma^* 2s^2 \pi 2p_x^1 = \pi 2p_y^1$

B_2 contains two unpaired electrons, hence paramagnetic

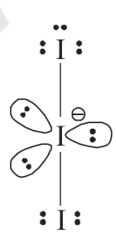
34. (d)

Species	No. of e⁻s	Elec. conf.	Bond order
He_2^+	$(4-1=3)$	$\sigma_{1s^2}\sigma^*_{1s^1}$	$\frac{2-1}{2}=0.5$
H_2^-	$(2+1=3)$	$\sigma_{1s^2}\sigma^*_{1s^1}$	$\frac{2-1}{2}=0.5$
H_2^{2-}	$(2+2=4)$	$\sigma_{1s^2}\sigma^*_{1s^2}$	$\frac{2-2}{2}=0$
He_2^{2+}	$(4-2=2)$	σ_{1s^2}	$\frac{2-0}{2}=1$

Molecule having zero bond order will not be a viable molecule.

35. (c) KCl is an ionic compound while others (PH_3, O_2, B_2H_6, and H_2SO_4) are covalent compounds.

36. (c)



$\therefore$ Total number of lone pair of electrons is 9.

37. (c) Electronic configuratios of Li_2^+ and Li_2^-:

$Li_2^+ : \sigma 1s^2 \sigma^* 1s^2 \sigma 2s^1$

$Li_2^- : \sigma 1s^2 \sigma^* 1s^2 \sigma 2s^2 \sigma^* 2s^1$

Now,

$$\text{Bond order of } Li_2^+ = \frac{1}{2}(3-2) = \frac{1}{2}$$

$$\text{Bond order of } Li_2^- = \frac{1}{2}(4-3) = \frac{1}{2}$$

Here, both Li_2^+ and Li_2^- have positive bond order, thus both are stable.

38. (a) Configuration of C_2

$$= \sigma 1s^2 \sigma^* 1s^2\, \sigma 2s^2 \sigma^* 2s^2\, \pi 2p_x^2 = \pi 2p_y^2$$

Configuration of C_2^-

$$= \sigma 1s^2\, \sigma^* 1s^2\, \sigma 2s^2\, \sigma^* 2s^2\, \pi 2p_x^2 = \pi 2p_y^2\;\; \sigma 2p_z^1$$

Bond order

$$= \frac{\text{No. of bonding } e^- - \text{No. of antibonding } e^-}{2}$$

C_2 has *s-p* mixing and the HOMO is $\pi 2p_x = \pi 2p_y$ and LUMO is $\sigma 2p_z$. So, the extra electron will occupy bonding molecular orbital and this will lead to an increase in bond order.

C_2^- has more bond order than C_2.

39. (d) $\mu_{CCl_4} = \mu_{CH_4} = 0$ due to symmetrical structure but $\mu_{CHCl_3} \neq 0$. So dipole moment order is :

$CHCl_3 > CH_4 = CCl_4$

40. (b) The covalant character of the bonding (M – C σ and M – C π bonding) which exists between the metal and the carbon atom of the CO can only be explained by the molecular orbital theory.

41. (d) Among given intermolecular forces, ionic interactions are stronger as compared to van der Waal interaction

Thus, correct order is ion-ion > ion-dipole > dipole-dipole.

5

States of Matter

1. For an ideal gas, number of moles per litre in terms of its pressure P, gas constant R and temperature T is **[2002]**
   (a) PT/R (b) PRT
   (c) P/RT (d) RT/P.
2. Value of gas constant R is **[2002]**
   (a) 0.082 litre atm
   (b) 0.987 cal mol^{-1} K^{-1}
   (c) 8.3 J mol^{-1} K^{-1}
   (d) 83 erg mol^{-1} K^{-1}.
3. Kinetic theory of gases proves **[2002]**
   (a) only Boyle's law
   (b) only Charles' law
   (c) only Avogadro's law
   (d) all of these.
4. According to the kinetic theory of gases, in an ideal gas, between two successive collisions a gas molecule travels **[2003]**
   (a) in a wavy path
   (b) in a straight line path
   (c) with an accelerated velocity
   (d) in a circular path
5. As the temperature is raised from 20 ºC to 40 ºC, the average kinetic energy of neon atoms changes by which factor ? **[2004]**
   (a) $313/293$ (b) $\sqrt{(313/293)}$
   (c) $1/2$ (d) 2
6. In van der Waals equation of state of the gas law, the constant 'b' is a measure of **[2004]**
   (a) volume occupied by the molecules
   (b) intermolecular attractions
   (c) intermolecular repulsions
   (d) intermolecular collisions per unit volume
7. Which one of the following statements is NOT true about the effect of an increase in temperature on the distribution of molecular speeds in a gas? **[2005]**
   (a) The area under the distribution curve remains the same as under the lower temperature
   (b) The distribution becomes broader
   (c) The fraction of the molecules with the most probable speed increases
   (d) The most probable speed increases
8. If 10^{-4} dm^3 of water is introduced into a 1.0 dm^3 flask at 300 K, how many moles of water are in the vapour phase when equilibrium is established ? **[2010]**
   (Given : Vapour pressure of H_2O at 300 K is 3170 Pa; $R = 8.314$ J K^{-1} mol^{-1})
   (a) 5.56×10^{-3} mol (b) 1.53×10^{-2} mol
   (c) 4.46×10^{-2} mol (d) 1.27×10^{-3} mol
9. When r, P and M represent rate of diffusion, pressure and molecular mass, respectively, then the ratio of the rates of diffusion (r_A/r_B) of two gases A and B, is given as **[2011RS]**
   (a) $(P_A/P_B)(M_B/M_A)^{1/2}$
   (b) $(P_A/P_B)^{1/2}(M_B/M_A)$
   (c) $(P_A/P_B)(M_A/M_B)^{1/2}$
   (d) $(P_A/P_B)^{1/2}(M_A/M_B)$

10. The molecular velocity of any gas is : **[2011RS]**
(a) inversely proportional to absolute temperature.
(b) directly proportional to square of temperature.
(c) directly proportional to square root of temperature.
(d) inversely proportional to the square root of temperature.

11. The compressibility factor for a real gas at high pressure is : **[2012]**
(a) $1+\frac{RT}{Pb}$ (b) 1
(c) $1+\frac{Pb}{RT}$ (d) $1-\frac{Pb}{RT}$

12. For gaseous state, if most probable speed is denoted by C*, average speed by $\overline{C}$ and mean square speed by C, then for a large number of molecules the ratios of these speeds are: **[2013]**
(a) $C^*:\overline{C}:C=1.225:1.128:1$
(b) $C^*:\overline{C}:C=1.128:1.225:1$
(c) $C^*:\overline{C}:C=1:1.128:1.225$
(d) $C^*:\overline{C}:C=1:1.225:1.128$

13. If Z is a compressibility factor, van der Waals equation at low pressure can be written as: **[2014]**
(a) $Z=1+\frac{RT}{Pb}$ (b) $Z=1-\frac{a}{VRT}$
(c) $Z=1-\frac{Pb}{RT}$ (d) $Z=1+\frac{Pb}{RT}$

14. The ratio of masses of oxygen and nitrogen in a particular gaseous mixture is 1 : 4. The ratio of number of their molecules is: **[2014]**
(a) 1 : 4 (b) 7 : 32 (c) 1 : 8 (d) 3 : 16

15. The intermolecular interaction that is dependent on the inverse cube of distance between the molecules is : **[2015]**
(a) London force
(b) hydrogen bond
(c) ion - ion interaction
(d) ion - dipole interaction

16. Two closed bulbs of equal volume (V) containing an ideal gas initially at pressure p_i and temperature T_1 are connected through a narrow tube of negligible volume. The temperature of one of the bulbs is then raised to T_2. The final pressure p_f is : **[2016]**
(a) $2p_i\left(\frac{T_2}{T_1+T_2}\right)$ (b) $2p_i\left(\frac{T_1T_2}{T_1+T_2}\right)$
(c) $p_i\left(\frac{T_1T_2}{T_1+T_2}\right)$ (d) $2p_i\left(\frac{T_1}{T_1+T_2}\right)$

17. 0.5 moles of gas A and x moles of gas B exert a pressure of 200 Pa in a container of volume 10 m^3 at 1000 K. Given R is the gas constant in $JK^{-1}\ mol^{-1}$, x is: **[2019]**
(a) $\frac{2R}{4+R}$ (b) $\frac{2R}{4-R}$
(c) $\frac{4+R}{2R}$ (d) $\frac{4-R}{2R}$

18. Consider the van der Waals constants, a and b, for the following gases, **[2019]**

Gas	Ar	Ne	Kr	Xe
a/ (atm $dm^6\ mol^{-2}$)	1.3	0.2	5.1	4.1
b/ ($10^{-2}\ dm^3\ mol^{-1}$)	3.2	1.7	1.0	5.0

Which gas is expected to have the highest critical temperature?
(a) Kr (b) Ne (c) Xe (d) Ar

Answer Key

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
(c)	(c)	(d)	(b)	(a)	(a)	(c)	(d)	(a)	(c)	(c)	(c)	(b)	(b)	(b)
16	17	18												
(a)	(d)	(a)												

Solutions

1. (c) PV = nRT
(number of moles = n/V)
$\therefore$ n/V = P/RT.

2. (c) Value of gas constant
$(R) = 0.08205 L\ atm\ K^{-1} mol^{-1}$
$= 8.314 \times 10^7\ ergs\ K^{-1} mol^{-1}$
$= 8.314 JK^{-1} mol^{-1} = 1.987 cal\ K^{-1} mol^{-1}$
[For conversion, these terms can be used
1.0 L atm = 101.3J
1 J = 10^7 ergs
1 J = 0.239 cal
or 1 cal = 4.184 J]

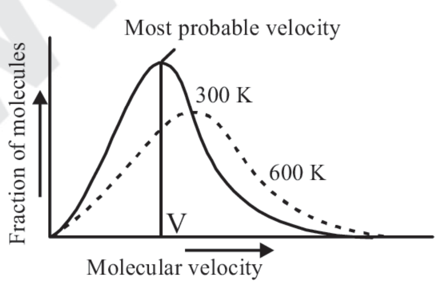
3. (d) Kinetic theory of gases proves all the given gas laws.

4. (b) According to kinetic theory of gases, gas molecules are in a state of constant rapid motion in all possible directions, colloiding in a random manner with one another and with the walls of the container and between two successive collisions, molecules travel in a straight line path but show haphazard motion due to collisions.

5. (a) $\dfrac{\text{K.E of neon at } 40\,°C}{\text{K.E of neon at } 20\,°C} = \dfrac{\frac{3}{2}K \times 313}{\frac{3}{2}K \times 293} = \dfrac{313}{293}$

6. (a) In van der Waals equation, 'b' is for volume correction.

7. (c) Distribution of molecular velocities at two different temperatures is shown below.



NOTE At higher temperature more molecules have higher velocities and less molecules have lower velocities.

As evident from fig., it is clear that with the increase in temperature the most probable velocity increases but the fraction of such molecules decreases.

8. (d) From the ideal gas equation :
$PV = nRT$
The volume occupied by H_2O molecule in vapour phase is $(1 - 10^{-4})dm^3 = 0.99\ dm^3$
i.e., $1\,dm^3 \approx 1\ L$
Given, $P = 3170\,Pa \approx 3170 \times 10^{-5}$ atm
$R = 8.314\ JK^{-1}\ mol^{-1}$
or $\dfrac{8.314\ JK^{-1} mol^{-1} \times 1.02\ atm}{101.3\ J}$
$R = 0.0821\ L\,atm\,K^{-1} mol^{-1}$
Thus, $T = 300\,K$
$n = \dfrac{PV}{RT} = \dfrac{3170 \times 10^{-5} atm \times 0.99}{0.0821\ Latm\ K^{-1} mol^{-1} \times 300\,K}$
$= 1.27 \times 10^{-3}$ mol

9. (a) $r \propto \dfrac{P}{\sqrt{M}}$
$\dfrac{r_A}{r_B} = \dfrac{P_A}{P_B}\sqrt{\dfrac{M_B}{M_A}}$

10. (c) The different type of molecular velocities possessed by gas molecules are
(i) Most probable velocity $(v_{mp}) = \sqrt{\dfrac{2RT}{M}}$
(ii) Average velocity $(\bar{v}) = \sqrt{\dfrac{8RT}{\pi M}}$
(iii) Root mean square velocity
$(v_{rms}) = \sqrt{\dfrac{3RT}{M}}$
In all the above cases
Velocity $\propto \sqrt{T}$

11. (c) $\left(P + \dfrac{a}{V^2}\right)(V - b) = RT$
At high pressure $\dfrac{a}{V^2}$ can be neglected

$\therefore \quad PV - Pb = RT$ or $PV = RT + Pb$

$$\frac{PV}{RT} = 1 + \frac{Pb}{RT}$$

$Z = 1 + \frac{Pb}{RT}$; $Z > 1$ at high pressure

12. (c) Most probable speed $(C^*) = \sqrt{\frac{2RT}{M}}$

Average Speed $(\overline{C}) = \sqrt{\frac{8RT}{\pi M}}$

Root mean square velocity $(C) = \sqrt{\frac{3RT}{M}}$

$$C^* : \overline{C} : C = \sqrt{\frac{2RT}{M}} : \sqrt{\frac{8RT}{\pi M}} : \sqrt{\frac{3RT}{M}}$$

$$= 1 : \sqrt{\frac{4}{\pi}} : \sqrt{\frac{3}{2}} = 1 : 1.128 : 1.225$$

13. (b) Compressibility factor $(Z) = \frac{PV}{RT}$

(For one mole of real gas)

van der Waals equation

$$(P + \frac{a}{V^2})(V - b) = RT$$

At low pressure, volume is very large and hence correction term b can be neglected in comparison to very large volume of V.

i.e. $V - b \approx V$

$$\left(P + \frac{a}{V^2}\right) V = RT$$

$$PV + \frac{a}{V} = RT$$

$$PV = RT - \frac{a}{V}$$

$$\frac{PV}{RT} = 1 - \frac{a}{VRT}$$

Hence, $Z = 1 - \frac{a}{VRT}$

14. (b) Number of moles of $O_2 = \frac{m}{32}$

Number of moles of $N_2 = \frac{4m}{28} = \frac{m}{7}$

$\therefore$ Ratio $= \frac{m}{32} : \frac{m}{7} = 7 : 32$

15. (b) Hydrogen bond is a type of strong electrostatic dipole-dipole interaction and dependent on the inverse cube of distance between the molecules.

16. (a) For a given mass of an ideal gas, the volume and amount (moles) of the gas are directly proportional if the temperature and pressure are constant. i.e $V \propto n$

Hence in the given case.

Initial moles and final moles are equal

$(n_T)_i = (n_T)_f$

$$\frac{P_i V}{RT_i} + \frac{P_i V}{RT_i} = \frac{P_f V}{RT_1} + \frac{P_f V}{RT_2}$$

$$2\frac{P_i}{T_i} = \frac{P_f}{T_1} + \frac{P_f}{T_2} \quad ; \quad P_f = 2P_i\left(\frac{T_2}{T_1 + T_2}\right)$$

17. (d) Ideal gas equation: $PV = nRT$

After putting the values we get,

$200 \times 10 = (0.5 + x) \times R \times 1000$

(total no. of moles are $0.5 + x$)

$$x = \frac{4 - R}{2R}$$

18. (a) Critical temperature $= \frac{8a}{27Rb}$

Value of $\frac{a}{b}$ is highest for Kr. Therefore, Kr has greatest value of critical temperature.

6 Thermodynamics

1. If an endothermic reaction is non-spontaneous at freezing point of water and becomes feasible at its boiling point, then **[2002]**
   (a) ΔH is –ve, ΔS is +ve
   (b) ΔH and ΔS both are +ve
   (c) ΔH and ΔS both are –ve
   (d) ΔH is +ve, ΔS is –ve

2. A heat engine abosrbs heat Q_1 at temperature T_1 and heat Q_2 at temperature T_2. Work done by the engine is $J(Q_1 + Q_2)$. This data **[2002]**
   (a) violates 1st law of thermodynamics
   (b) violates 1st law of themodynamics if Q_1 is –ve
   (c) violates 1st law of thermodynamics of Q_2 is –ve
   (d) does not violate 1st law of themodynamics.

3. For the reactions, **[2002]**
   $2C + O_2 \rightarrow 2\,CO_2$; $\Delta H = -0393$ J
   $2Zn + O_2 \rightarrow 2ZnO$; $\Delta H = -412$ J
   (a) carbon can oxidise Zn
   (b) oxidation of carbon is not feasible
   (c) oxidation of Zn is not feasible
   (d) Zn can oxidise carbon.

4. The heat required to raise the temperature of body by 1 K is called **[2002]**
   (a) specific heat (b) thermal capacity
   (c) water equivalent (d) none of these.

5. The internal energy change when a system goes from state A to B is 40 kJ/mol. If the system goes from A to B by a reversible path and returns to state A by an irreversible path what would be the net change in internal energy ? **[2003]**
   (a) > 40 kJ (b) < 40 kJ
   (c) Zero (d) 40 kJ

6. If at 298 K the bond energies of C — H, C — C, C = C and H — H bonds are respectively 414, 347, 615 and 435 kJ mol^{-1}, the value of enthalpy change for the reaction
   $H_2C = CH_2(g) + H_2(g) \rightarrow H_3C - CH_3(g)$ at 298 K will be **[2003]**
   (a) -250 kJ (b) $+125$ kJ
   (c) -125 kJ (d) $+250$ kJ

7. In an irreversible process taking place at constant T and P and in which only pressure-volume work is being done, the change in Gibbs free energy (dG) and change in entropy (dS), satisfy the criteria **[2003]**
   (a) $(dS)_{V,E} > 0, (dG)_{T,P} < 0$
   (b) $(dS)_{V,E} = 0, (dG)_{T,P} = 0$
   (c) $(dS)_{V,E} = 0, (dG)_{T,P} > 0$
   (d) $(dS)_{V,E} < 0, (dG)_{T,P} < 0$

8. The correct relationship between free energy change in a reaction and the corresponding equilibrium constant K_c is **[2003]**
   (a) $-\Delta G = RT \ln K_c$ (b) $\Delta G^o = RT \ln K_c$
   (c) $-\Delta G^o = RT \ln K_c$ (d) $\Delta G = RT \ln K_c$

9. The enthalpy change for a reaction does **not** depend upon **[2003]**
   (a) use of different reactants for the same product.
   (b) the nature of intermediate reaction steps.
   (c) the differences in initial or final temperatures of involved substances.
   (d) the physical states of reactants and products.

10. An ideal gas expands in volume from 1×10^{-3} to 1×10^{-2} m^3 at 300 K against a constant pressure of 1×10^5 Nm^{-2}. The work done is **[2004]**

(a) 270 kJ (b) – 900 kJ
(c) – 900 J (d) 900 kJ

11. The enthalpies of combustion of carbon and carbon monoxide are – 393.5 and – 283 kJ mol^{-1} respectively. The enthalpy of formation of carbon monoxide per mole is **[2004]**

(a) – 676.5 kJ (b) 676.5 kJ
(c) 110.5 kJ (d) – 110.5 kJ

12. Consider the reaction : $N_2 + 3H_2 \rightarrow 2NH_3$ carried out at constant temperature and pressure. If ΔH and ΔU are the enthalpy and internal energy changes for the reaction, which of the following expressions is true ? **[2005]**

(a) $\Delta H > \Delta U$ (b) $\Delta H < \Delta U$
(c) $\Delta H = \Delta U$ (d) $\Delta H = 0$

13. If the bond dissociation energies of XY, X_2 and Y_2 (all diatomic molecules) are in the ratio of 1 : 1 : 0.5 and ΔH_f for the formation of XY is – 200 kJ mol^{-1}. The bond dissociation energy of X_2 will be **[2005]**

(a) 400 kJ mol^{-1} (b) 300 kJ mol^{-1}
(c) 200 kJ mol^{-1} (d) 100 kJ mol^{-1}

14. An ideal gas is allowed to expand both reversibly and irreversibly in an isolated system. If T_i is the initial temperature and T_f is the final temperature, which of the following statements is correct? **[2006]**

(a) $(T_f)_{rev} = (T_f)_{irrev}$
(b) $T_f = T_i$ for both reversible and irreversible processes
(c) $(T_f)_{irrev} > (T_f)_{rev}$
(d) $T_f > T_i$ for reversible process but $T_f = T_i$ for irreversible process

15. The standard enthalpy of formation ($\Delta_f H^o$) at 298 K for methane, CH_4 (g) is –74.8 kJ mol^{-1}. The additional information required to determine the average energy for C – H bond formation would be **[2006]**

(a) the first four ionization energies of carbon and electron gain enthalpy of hydrogen
(b) the dissociation energy of hydrogen molecule, H_2
(c) the dissociation energy of H_2 and enthalpy of sublimation of carbon
(d) latent heat of vapourization of methane

16. The enthalpy changes for the following processes are listed below : **[2006]**

$Cl_2(g) = 2Cl(g)$, 242.3 kJ mol^{-1}
$I_2(g) = 2I(g)$, 151.0 kJ mol^{-1}
$ICl(g) = I(g) + Cl(g)$, 211.3 kJ mol^{-1}
$I_2(s) = I_2(g)$, 62.76 kJ mol^{-1}

Given that the standard states for iodine and chlorine are $I_2(s)$ and $Cl_2(g)$, the standard enthalpy of formation for ICl(g) is : **[2006]**

(a) +16.8 kJ mol^{-1} (b) +244.8 kJ mol^{-1}
(c) –14.6 kJ mol^{-1} (d) –16.8 kJ mol^{-1}

17. $(\Delta H - \Delta U)$ for the formation of carbon monoxide (CO) from its elements at 298 K is **[2006]**
(R = 8.314 J K^{-1} mol^{-1})

(a) –2477.57 J mol^{-1} (b) 2477.57 J mol^{-1}
(c) –1238.78 J mol^{-1} (d) 1238.78 J mol^{-1}

18. In conversion of lime-stone to lime,
$CaCO_3(s) \rightarrow CaO(s) + CO_2(g)$ the values of $\Delta H°$ and $\Delta S°$ are + 179.1 kJ mol^{-1} and 160.2 J/K respectively at 298 K and 1 bar. Assuming that $\Delta H°$ and $\Delta S°$ do not change with temperature, temperature above which conversion of limestone to lime will be spontaneous is **[2007]**

(a) 1118 K (b) 1008 K
(c) 1200 K (d) 845 K.

19. Assuming that water vapour is an ideal gas, the internal energy change (ΔU) when 1 mol of water is vapourised at 1 bar pressure and 100 °C, (given : molar enthalpy of vapourisation of water at 1 bar and 373 K = 41 kJ mol^{-1} and R = 8.3 J mol^{-1} K^{-1}) will be **[2007]**

(a) 41.00 kJ mol^{-1} (b) 4.100 kJ mol^{-1}
(c) 3.7904 kJ mol^{-1} (d) 37.904 kJ mol^{-1}

20. Identify the correct statement regarding a spontaneous process: **[2007]**

(a) Lowering of energy in the process is the only criterion for spontaneity.

(b) For a spontaneous process in an isolated system, the change in entropy is positive.

(c) Endothermic processes are never spontaneous.

(d) Exothermic processes are always spontaneous.

21. Oxidising power of chlorine in aqueous solution can be determined by the parameters indicated below:

$$\frac{1}{2}Cl_2(g) \xrightarrow{\frac{1}{2}\Delta_{diss}H^o} Cl(g) \xrightarrow{\Delta_{eg}H^o} Cl^-(g) \xrightarrow{\Delta_{Hyd}H^o} Cl^-(aq)$$

The energy involved in the conversion of

$\frac{1}{2}Cl_2(g)$ to $Cl^-(aq)$ **[2008]**

(using the data,

$\Delta_{diss}H^{\circ}_{Cl_2} = 240\ kJ\ mol^{-1}$,

$\Delta_{eg}H^{\circ}_{Cl} = -349\ kJ\ mol^{-1}$,

$\Delta_{hyd}H^{\circ}_{Cl^-} = -381\ kJ\ mol^{-1}$), will be

(a) $+152\ kJ\ mol^{-1}$ (b) $-610\ kJ\ mol^{-1}$

(c) $-850\ kJ\ mol^{-1}$ (d) $+120\ kJ\ mol^{-1}$

22. Standard entropy of X_2, Y_2 and XY_3 are 60, 40 and 50 $J\ K^{-1}mol^{-1}$, respectively. For the reaction,

$\frac{1}{2}X_2 + \frac{3}{2}Y_2 \rightarrow XY_3, \Delta H = -30kJ$, to be at equilibrium, the temperature will be **[2008]**

(a) 1250 K (b) 500 K

(c) 750 K (d) 1000 K

23. On the basis of the following thermochemical data : $(\Delta_f G^\circ H^+_{(aq)} = 0)$ **[2009]**

$$H_2O(l) \rightarrow H^+(aq) + OH^-(aq); \Delta H = 57.32kJ$$

$$H_2(g) + \frac{1}{2}O_2(g) \longrightarrow H_2O(l);\ \Delta H = -286.20kJ$$

The value of enthalpy of formation of OH^- ion at 25 °C is:

(a) –228.88 kJ (b) +228.88 kJ

(c) –343.52 kJ (d) –22.88 kJ

24. The standard enthalpy of formation of NH_3 is $-46.0\ kJ\ mol^{-1}$. If the enthalpy of formation of H_2 from its atoms is $-436\ kJ\ mol^{-1}$ and that of N_2 is $-712\ kJ\ mol^{-1}$, the average bond enthalpy of N – H bond in NH_3 is **[2010]**

(a) $-964\ kJ\ mol^{-1}$ (b) $+352\ kJ\ mol^{-1}$

(c) $+1056\ kJ\ mol^{-1}$ (d) $-1102\ kJ\ mol^{-1}$

25. For a particular reversible reaction at temperature T, ΔH and ΔS were found to be both +ve. If T_e is the temperature at equilibrium, the reaction would be spontaneous when

(a) $T_e > T$ (b) $T > T_e$ **[2010]**

(c) T_e is 5 times T (d) $T = T_e$

26. The value of enthalpy change (ΔH) for the reaction

$$C_2H_5OH(l) + 3O_2(g) \rightarrow 2CO_2(g) + 3H_2O(l)$$

at 27 °C is $-1366.5\ kJ\ mol^{-1}$. The value of internal energy change for the above reaction at this temperature will be : **[2011RS]**

(a) –1369.0 kJ (b) –1364.0 kJ

(c) –1361.5 kJ (d) –1371.5 kJ

27. Consider the reaction :

$$4NO_2(g) + O_2(g) \rightarrow 2N_2O_5(g), \quad \Delta_r H = -111\ kJ.$$

If $N_2O_5(s)$ is formed instead of $N_2O_5(g)$ in the above reaction, the $\Delta_r H$ value will be :

(given, ΔH of sublimation for N_2O_5 is 54 kJ mol^{-1}) **[2011RS]**

(a) +54 kJ (b) +219 kJ

(c) –219 J (d) –165 kJ

28. The incorrect expression among the following is: **[2012]**

(a) $\frac{\Delta G_{system}}{\Delta S_{total}} = -T$

(b) In isothermal process,

$$w_{reversible} = -nRT \ln\frac{V_f}{V_i}$$

(c) $\ln K = \frac{\Delta H^o - T\Delta S^o}{RT}$

(d) $K = e^{-\Delta G^o/RT}$

29. A piston filled with 0.04 mol of an ideal gas expands reversibly from 50.0 mL to 375 mL at a constant temperature of 37.0ºC. As it does so, it absorbs 208 J of heat. The values of q and w for the process will be: **[2013]**

(R = 8.314 J/mol K) (ln 7.5 = 2.01)

(a) q = + 208 J, w = – 208 J

(b) q = – 208 J, w = – 208 J

(c) q = – 208 J, w = + 208 J

(d) q = + 208 J, w = + 208 J

30. For complete combustion of ethanol,

$C_2H_5OH(l) + 3O_2(g) \longrightarrow 2CO_2(g) + 3H_2O(l)$,

the amount of heat produced as measured in bomb calorimeter, is 1364.47 kJ mol^{-1} at 25ºC. Assuming ideality the enthalpy of combustion, $\Delta_c H$, for the reaction will be: **[2014]**

(R = 8.314 kJ mol^{-1})

(a) –1366.95 kJ mol^{-1}

(b) –1361.95 kJ mol^{-1}

(c) –1460.95 kJ mol^{-1}

(d) –1350.50 kJ mol^{-1}

31. The heats of combustion of carbon and carbon monoxide are –393.5 and –283.5 kJ mol^{-1}, respectively. The heat of formation (in kJ) of carbon monoxide per mole is : **[2016]**

(a) –676.5 (b) –110.5

(c) 110.5 (d) 676.5

32. ΔU is equal to **[2017]**

(a) Isochoric work (b) Isobaric work

(c) Adiabatic work (d) Isothermal work

33. Given **[2017]**

$C_{(graphite)} + O_2(g) \rightarrow CO_2(g)$;

$\Delta_r H^\circ = -393.5 \text{ kJ mol}^{-1}$

$H_2(g) + \frac{1}{2}O_2(g) \rightarrow H_2O(l)$;

$\Delta_r H^\circ = -285.8 \text{ kJ mol}^{-1}$

$CO_2(g) + 2H_2O(l) \rightarrow CH_4(g) + 2O_2(g)$;

$\Delta_r H^\circ = +890.3 \text{ kJ mol}^{-1}$

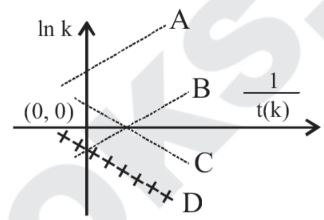
Based on the above thermochemical equations, the value of $\Delta_r H^\circ$ at 298 K for the reaction

$C_{(graphite)} + 2H_2(g) \rightarrow CH_4(g)$ will be :

(a) + 74.8 kJ mol^{-1} (b) + 144.0 kJ mol^{-1}

(c) – 74.8 kJ mol^{-1} (d) – 144.0 kJ mol^{-1}

34. Which of the following lines correctly show the temperature dependence of equilibrium constant, K, for an exothermic reaction? **[2018]**



(a) A and B (b) B and C

(c) C and D (d) A and D

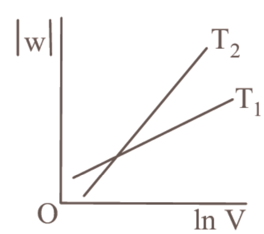
35. The combustion of benzene (*l*) gives CO_2 (g) and H_2O (*l*). Given that heat of combustion of benzene at constant volume is –3263.9 kJ mol^{-1} at 25°C; heat of combustion (in kJ mol^{-1}) of benzene at constant pressure will be : (R= 8.314 JK^{-1} mol^{-1}) **[2018]**

(a) 4152.6 (b) – 452.46

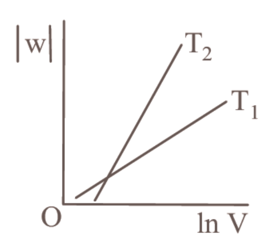
(c) 3260 (d) – 3267.6

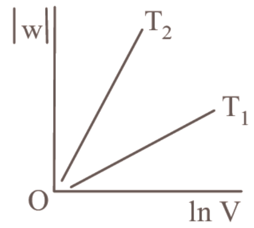
36. Consider the reversible isothermal expansion of an ideal gas in a closed system at two different temperatures T_1 and T_2 ($T_1 < T_2$). The correct graphical depiction of the dependence of work done (w) on the final volume (V) is: **[2019]**

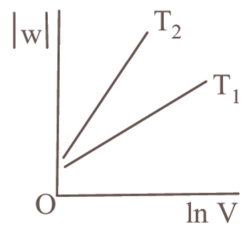
(a)



(b)



(c)


(d)


37. Among the following, the set of parameters that represents path functions, is: **[2019]**

(A) q + w (B) q

(C) w (D) H – TS

(a) (B) and (C) (b) (B), (C) and (D)

(c) (A) and (D) (d) (A), (B) and (C)

38. For the reaction ; **[2020]**

$A(l) \longrightarrow 2B(g)$

$\Delta U = 2.1$ kcal, $\Delta S = 20$ cal K^{-1} at 300 K.

Hence ΔG in kcal is________.

Answer Key

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
(b)	(a)	(d)	(b)	(c)	(c)	(a)	(c)	(b)	(c)	(d)	(b)	(N)	(c)	(c)
16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
(a)	(d)	(a)	(d)	(b)	(b)	(c)	(a)	(b)	(b)	(b)	(d)	(c)	(a)	(a)
31	32	33	34	35	36	37	38							
(b)	(c)	(c)	(a)	(d)	(b)	(a)	(–2.70)							

Solutions

1. **(b)** $\Delta G = \Delta H - T\Delta S$

For an endothermic reaction,

ΔH = +ve and at low temperature ΔS = +ve

Hence $\Delta G = (+)\Delta H - T(+)\Delta S$

and if $T\Delta S < \Delta H$ (at low temp)

ΔG = +ve (non spontaneous)

But at high temperature, reaction becomes spontaneous i.e. ΔG = –ve

because at higher temperature $T\Delta S > \Delta H$

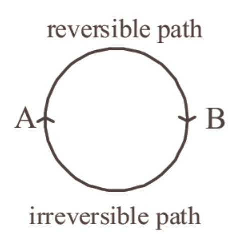
2. **(a)** According to first law of thermodynamics energy can neither be created nor destroyed although it can be converted from one form to another.

NOTE Carnot cycle is based upon this principle but during the conversion of heat into work some mechanical energy is always converted to other form of energy hence this data violates 1st law of thermodynamics.

3. **(d)** ΔH negative shows that the reaction is spontaneous. Higher negative value for Zn shows that the reaction is more feasible.

4. **(b)** The heat required to raise the temperature of body by 1K is called thermal capacity or heat capacity.

5. **(c)** For a cyclic process, the net change in the internal energy is zero because the change in internal energy does not depend on the path.



6. **(c)** $CH_2=CH_2(g)+H_2(g) \rightarrow CH_3-CH_3$
Enthalpy change = Bond energy of reactants – Bond energy of products.
$$\Delta H = 1(C=C)+4(C-H)+1(H-H)-1(C-C)-6(C-H)$$
$$=1(C=C)+1(H-H)-1(C-C)-2(C-H)$$
$$=615+435-347-2\times 414=1050-1175$$
$$=-125\text{ kJ.}$$

7. **(a)** For spontaneous reaction, dS > 0 and dG should be negative i.e. < 0.

8. **(c)** $\Delta G^o = -RT\ln K_c$ or $-\Delta G^o = RT\ln K_c$

9. **(b)** Enthalpy change for a reaction does not depend upon the nature of intermediate reaction steps.

10. **(c)** $w=-P\Delta V=-10^5[(1\times 10^{-2})-(1\times 10^{-3})]$
$= -900$ J

11. **(d)** (i) $C+O_2 \rightleftharpoons CO_2, \Delta H=-393.5\text{ kJmol}^{-1}$
(ii) $CO+\frac{1}{2}O_2 \rightleftharpoons CO_2,$
$\Delta H=-283.0\text{ kJmol}^{-1}$
Operating (i) - (ii), we have
$$C+\frac{1}{2}O_2 \rightarrow CO \quad \Delta H=-110.5\text{ kJmol}^{-1}$$

12. **(b)** $\Delta H=\Delta U+\Delta n_g RT$ for
$$N_2+3H_2 \longrightarrow 2NH_3$$
$\Delta n_g = 2-4=-2$
$\therefore\ \Delta H > \Delta U - 2RT$
or $\Delta U = \Delta H + 2RT \quad \therefore\ \Delta U > \Delta H$

13. **(N)** $\frac{1}{2}X_2+\frac{1}{2}Y_2 \longrightarrow XY, \Delta H=(-200)$ J
Let x be the bond dissociation energy of X_2. Then
$$\Delta H_f = -200 = \frac{1}{2}\Delta H_{x-x}+\frac{1}{2}\Delta H_{y-y} - \Delta H_{x-y}$$
$$-400 = x+0.5x-2x=-0.5x$$
or $x=\frac{400}{0.5}=800\text{ kJ mol}^{-1}$
(In the question paper, this option was not mentioned. So the answer has been marked 'N')

14. **(c)** According to first law of thermodynamics
$\Delta Q = \Delta U + \Delta W$
An isolated system is adiabatic. This means $\Delta Q=0$. The first law in this case yields
$0=\Delta U+\Delta W \Rightarrow \Delta W = -\Delta U$ ..... (i)
For expansion, ΔW is positive and hence ΔU is negative. This means T_f is less than T_i in both the cases.
For the same expansion of volume, the work done in irreversible process is greater than that in reversible one because the system has to work against friction etc. Thus
$\Delta W_{irreversible} > \Delta W_{reversible}$
$\Rightarrow -\Delta U_{irreversible} > -\Delta U_{reversible}$
[from equation (i)]
$\Rightarrow \Delta U_{irreversible} < \Delta U_{reversible}$
$\Rightarrow \Delta T_{irreversible} < \Delta T_{reversible}$
$\Rightarrow T_{f\ irreversible} > T_{f\ reversible}$

15. **(c)** The standard enthalpy of formation of CH_4 is given by the equation :
$$C(s)+2H_2(g) \longrightarrow CH_4(g)$$
Hence, dissociation energy of hydrogen and enthalpy of sublimation of carbon is required.

16. **(a)** $I_2(s)+Cl_2(g) \longrightarrow 2ICl(g)$
$$\Delta H=[\Delta H_{I_2(s)\rightarrow I_2(g)}+\Delta H_{I-I}+\Delta H_{Cl-Cl}] - 2[\Delta H_{I-Cl}]$$
$$=62.76+151.0+242.3-2\times 211.3=33.46$$
$$\Delta H_f^o(ICl)=\frac{33.46}{2}=16.73\text{ kJ/mol}$$

17. **(d)** For the reaction,
$$C(s)+\frac{1}{2}O_2(g) \longrightarrow CO(g)$$
$\Delta H=\Delta U+\Delta n_g RT$ or $\Delta H-\Delta U=\Delta n_g RT$
$\Delta n = 1-\frac{1}{2}=\frac{1}{2};$
$\Delta H-\Delta U=\frac{1}{2}\times 8.314\times 298$
$=1238.78\text{ J mol}^{-1}$

18. **(a)** $\Delta G° = \Delta H° - T\Delta S°$

For a spontaneous reaction $\Delta G° < 0$

or $\Delta H° - T\Delta S° < 0 \Rightarrow T > \frac{\Delta H°}{\Delta S°}$

$\Rightarrow T > \frac{179.3 \times 10^3}{160.2}$

$> 1117.9 K \approx 1118 K$

19. **(d)** Given $\Delta H = 41 \text{ kJ mol}^{-1} = 41000 \text{ J mol}^{-1}$

$T = 100 °C = 273 + 100 = 373 K$

$H_2O(l) \xrightarrow{\Delta} H_2O(g)$

$\Delta n_g = 1 - 0 = 1$

$\Delta U = \Delta H - \Delta nRT = 41000 - (1 \times 8.314 \times 373)$

$= 37898.88 \text{ J mol}^{-1} \simeq 37.9 \text{ kJmol}^{-1}$

20. **(b)** Spontaneity of reaction depends on tendency to acquire minimum energy state and maximum randomness. For a spontaneous process in an isolated system the change in entropy is positive.

21. **(b)** The energy involved in the conversion of $\frac{1}{2}Cl_2(g)$ to Cl^- (aq) is given by

$$\Delta H = \frac{1}{2}\Delta_{diss}H^{\circ}_{Cl_2} + \Delta_{eg}H^{\circ}_{Cl} + \Delta_{hyd}H^{\circ}_{Cl}$$

Substituting various values from given data, we get

$$\Delta H = \left(\frac{1}{2} \times 240\right) + (-349) + (-381)$$

$= (120 - 349 - 381) = -610 \text{ kJ mol}^{-1}$

22. **(c)** For a reaction to be at equilibrium $\Delta G° = 0$.

Since $\Delta G° = \Delta H° - T\Delta S°$ so at equilibrium

$\Delta H° - T\Delta S° = 0$

or $\Delta H° = T\Delta S°$

For the reaction

$\frac{1}{2}X_2 + \frac{3}{2}Y_2 \longrightarrow XY_3$; $\Delta H° = -30$ kJ

Calculating $\Delta S°$ for the above reaction, we get

$$\Delta S° = 50 - \left[\frac{1}{2} \times 60 + \frac{3}{2} \times 40\right]$$

$= 50 - (30 + 60) = -40 \text{ Jk}^{-1}$

At equilibrium, $T\Delta S° = \Delta H°$

$[\because \Delta G° = 0]$

$\therefore T \times (-40 \text{ Jk}^{-1}) = -30 \times 1000$ J

$[\because 1 \text{ kJ} = 1000 \text{J}]$

or $T = \frac{-30 \times 1000}{-40} = 750$ K

23. **(a)** H_2O (l) $\longrightarrow H^+$(aq.) + OH^- (aq.);

$\Delta H = 57.32$ kJ ... (i)

$H_2(g) + \frac{1}{2}O_2(g) \longrightarrow H_2O(l)$;

$\Delta H = -286.20$ kJ ... (ii)

By adding equation (i) & (ii) we get,

$H_2(g) + \frac{1}{2}(O_2) \longrightarrow H^+(aq) + OH^-(aq)$

$\Delta H = 57.32 - 286.2 = -228.88$ kJ

Here $\Delta_f H°$ of H^+(aq.) = 0

$\therefore \Delta_f H°$ of $OH^- = -228.88$ kJ

24. **(b)** $N_2 + 3H_2 \longrightarrow 2NH_3$

$\Delta H = 2 \times -46.0 \text{ kJ mol}^{-1}$

Let x be the bond enthalpy of N – H bond then

[**Note :** Enthalpy of formation or bond formation enthalpy is given which is negative but the given reaction involves bond breaking hence values should be taken as positive.]

$\Delta H = \Sigma$ Bond energies of reactants

$- \Sigma$ Bond energies of products

$2 \times -46 = 712 + 3 \times (436) - 6x$

$-92 = 2020 - 6x$

$6x = 2020 + 92$

$6x = 2112$

$x = +352$ kJ/mol

25. **(b)** At equilibrium $\Delta G = 0$

Hence, $\Delta G = \Delta H - T_e\Delta S = 0$

$\therefore \Delta H = T_e\Delta S$ or $T_e = \frac{\Delta H}{\Delta S}$

For a spontaneous reaction

ΔG must be negative which is possible only if $\Delta H < T\Delta S$

or $T > \frac{\Delta H}{\Delta S}$; $T_e < T$

26. (b) $C_2H_5OH(l) + 3O_2(g)$

$$\longrightarrow 2CO_2(g) + 3H_2O(l)$$

$$\Delta n_g = 2 - 3 = -1$$

$$\Delta U = \Delta H - \Delta n_g RT$$

$$= -1366.5 - [(-1) \times 8.314 \times 300]$$

$$= -1366.5 + \left[(1) \times \frac{8.314}{10^3} \times 300\right]$$

$$= -1366.5 + 0.8314 \times 3 = -1364 \text{ kJ}$$

27. (d)

$$4NO_2(g) + O_2(g) \rightarrow 2N_2O_5(g),\ \Delta_r H = -111 \text{ kJ}$$

$$\begin{array}{l} \downarrow \Delta_r H' \\ 2N_2O_5(s) \xrightarrow{54 \text{ kJ mol}^{-1}} 2N_2O_5(g) \end{array}$$

$$\Delta_r H' = \Delta_r H - 54$$
$$= -111 - 54$$
$$= -165 \text{ kJ}$$

28. (c) $\Delta G° = \Delta H° - T\Delta S°$

$$-RT \ln K = \Delta H° - T\Delta S°$$

$$\ln K = -\frac{\Delta H° - T\Delta S°}{RT}$$

29. (a) Process is isothermal reversible expansion, hence

$\Delta U = 0$, therefore $q = -w$.

Since $q = +208$ J, $w = -208$ J

30. (a) $C_2H_5OH(l) + 3O_2(g) \longrightarrow$

$$2CO_2(g) + 3H_2O(l)$$

Bomb calorimeter gives ΔU of the reaction

Given, $\Delta U = -1364.47 \text{ kJ mol}^{-1}$

$\Delta n_g = -1$

$$\Delta H = \Delta U + \Delta n_g RT$$

$$= -1364.47 - \frac{1 \times 8.314 \times 298}{1000}$$

$$= -1366.95 \text{ kJ mol}^{-1}$$

31. (b) Given

$$C(s) + O_2(g) \rightarrow CO_2(g);\ \Delta H = -393.5 \text{ kJ mol}^{-1} \quad \ldots(i)$$

$$CO(g) + \frac{1}{2}O_2(g) \rightarrow CO_2(g);$$

$$\Delta H = -283.5 \text{ kJ mol}^{-1} \quad \ldots(ii)$$

$\therefore$ Heat of formation of CO = eqn (i) – eqn (ii)

$$= -393.5 - (-283.5)$$
$$= -110 \text{ kJ}$$

32. (c) From 1st law of thermodynamics

$\Delta U = q + w$

For adiabatic process :

$q = 0$

$\therefore\ \Delta U = w$

33. (c) $C_{(graphite)} + O_2(g) \longrightarrow CO_2(g);$

$$\Delta_r H° = -393.5 \text{ kJ/mol}^{-1} \ldots(i)$$

$$H_2(g) + \frac{1}{2}O_2(g) \longrightarrow H_2O(l);$$

$$\Delta_r H° = -285.8 \text{ kJ/mol}^{-1} \ldots(ii)$$

$$CO_2(g) + 2H_2O(l) \longrightarrow CH_4(g) + 2O_2(g);$$

$$\Delta H_r° = +890.3 \text{ kJ/mol}^{-1} \ldots(iii)$$

$$C_{(graphite)} + 2H_2(g) \longrightarrow CH_4(g);$$

$$\Delta H = ? \quad \ldots(iv)$$

[Eq. (i) + Eq. (iii)] + [2 × Eq. (ii)] = Eq (iv)

$$\therefore\ [\Delta H_1 + \Delta H_3] + [2 \times \Delta H_2] = \Delta H_4$$

$$[(-393.5) + (890.3)] + [2(-285.8)]$$

$$= -74.8 \text{ kJ/mol}$$

34. (a) From thermodynamics relation.

$$\Delta G° = -RT \ln K$$

$$\Delta H° - T\Delta S° = RT \ln K$$

$$-\frac{\Delta H°}{RT} + \frac{T\Delta S°}{RT} = \ln K$$

$$\text{or } \ln K = -\frac{\Delta H°}{RT} + \frac{\Delta S°}{R}$$

For exothermic reaction, $\Delta H° = -ve$

$$\text{slope} = -\frac{\Delta H°}{R} = +ve$$

So from graph, lines should be A & B.

35. (d) $C_6H_6(l) + \frac{15}{2}O_2(g) \longrightarrow$

$$6CO_2(g) + 3H_2O(l)$$

$$\Delta n_g = 6 - \frac{15}{2} = -\frac{3}{2}$$

$$\Delta H = \Delta U + \Delta n_g RT$$

$$= -3263.9 + \left(-\frac{3}{2}\right) \times 8.314 \times 10^{-3} \times 298$$

$$= -3263.9 + (-3.71) = -3267.6 \text{ kJ mol}^{-1}$$

36. (b) For reversible isothermal expansion,

$$w = -nRT \ln \frac{V_2}{V_1}$$

$$\Rightarrow \quad |w| = nRT \ln \frac{V_2}{V_1}$$

$|w| = nRT(\ln V_2 - \ln V_1)$

$|w| = nRT \ln V_2 - nRT\, V_1$

$y = mx + c$

So, slope of curve 2 is more than curve 1 and intercept of curve 2 is more negative than curve 1.

37. (a) We know that heat and work are not state functions but $q + w = \Delta U$ is a state function. $H - TS$ (i.e. G) is also a state function.

38. (–2.70)

$\Delta U = 2.1 \text{ kcal} = 2.1 \times 10^3$ cal

$\Delta n_g = 2$

$$\begin{aligned}\Delta H &= \Delta U + \Delta n_g RT \\ &= 2.1 \times 10^3 + 2 \times 2 \times 300 \\ &= 2100 + 1200 \\ &= 3300 \text{ cal}\end{aligned}$$

$$\begin{aligned}\Delta G &= \Delta H - T\Delta S \\ &= 3300 - 300 \times 20 \\ &= 3300 - 6000 \\ &= -2700 \text{ cals} \\ &= -2.7 \text{ kcal}\end{aligned}$$

7 Equilibrium

1. 1 M NaCl and 1 M HCl are present in an aqueous solution. The solution is **[2002]**
   (a) not a buffer solution with pH < 7
   (b) not a buffer solution with pH > 7
   (c) a buffer solution with pH < 7
   (d) a buffer solution with pH > 7.
2. Species acting as both Bronsted acid and base is **[2002]**
   (a) HSO_4^- (b) Na_2CO_3
   (c) NH_3 (d) OH^-
3. Let the solubility of an aqueous solution of $Mg(OH)_2$ be x, then its K_{sp} is **[2002]**
   (a) $4x^3$ (b) $108x^5$
   (c) $27x^4$ (d) $9x$.
4. Change in volume of the system does not alter which of the following equilibria? **[2002]**
   (a) $N_2(g) + O_2(g) \rightleftharpoons 2NO(g)$
   (b) $PCl_5(g) \rightleftharpoons PCl_3(g) + Cl_2(g)$
   (c) $N_2(g) + 3H_2(g) \rightleftharpoons 2NH_3(g)$
   (d) $SO_2Cl_2(g) \rightleftharpoons SO_2(g) + Cl_2(g)$.
5. For the reaction $CO(g) + (1/2)\,O_2(g) \rightleftharpoons CO_2(g)$, K_p/K_c is **[2002]**
   (a) RT (b) $(RT)^{-1}$
   (c) $(RT)^{-1/2}$ (d) $(RT)^{1/2}$
6. Which one of the following statements is not true? **[2003]**
   (a) pH + pOH = 14 for all aqueous solutions
   (b) The pH of 1×10^{-8} M HCl is 8
   (c) 96,500 coulombs of electricity when passed through a $CuSO_4$ solution deposits 1 gram equivalent of copper at the cathode
   (d) The conjugate base of $H_2PO_4^-$ is HPO_4^{2-}
7. The solubility in water of a sparingly soluble salt AB_2 is 1.0×10^{-5} mol L^{-1}. Its solubility product will be **[2003]**
   (a) 4×10^{-10} (b) 1×10^{-15}
   (c) 1×10^{-10} (d) 4×10^{-15}
8. For the reaction equilibrium **[2003]**
   $N_2O_4(g) \rightleftharpoons 2NO_2(g)$
   the concentrations of N_2O_4 and NO_2 at equilibrium are 4.8×10^{-2} and 1.2×10^{-2} mol L^{-1} respectively. The value of K_c for the reaction is
   (a) 3×10^{-1} mol L^{-1} (b) 3×10^{-3} mol L^{-1}
   (c) 3×10^{3} mol L^{-1} (d) 3.3×10^{2} mol L^{-1}
9. Consider the reaction equilibrium **[2003]**
   $2\,SO_2(g) + O_2(g) \rightleftharpoons 2SO_3(g)$; $\Delta H° = -198$ kJ
   On the basis of Le Chatelier's principle, the condition favourable for the forward reaction is
   (a) increasing temperature as well as pressure
   (b) lowering the temperature and increasing the pressure
   (c) any value of temperature and pressure
   (d) lowering of temperature as well as pressure
10. When rain is accompanied by a thunderstorm, the collected rain water will have a pH value **[2003]**
    (a) slightly higher than that when the thunderstorm is not there
    (b) uninfluenced by occurrence of thunderstorm
    (c) that depends on the amount of dust in air
    (d) slightly lower than that of rain water without thunderstorm.
11. The conjugate base of $H_2PO_4^-$ is **[2004]**
    (a) H_3PO_4 (b) P_2O_5
    (c) PO_4^{3-} (d) HPO_4^{2-}

12. What is the equilibrium expression for the reaction $P_4(s)+5O_2(g) \rightleftharpoons P_4O_{10}(s)$? **[2004]**

(a) $K_c=[O_2]^5$

(b) $K_c=[P_4O_{10}]/5[P_4][O_2]$

(c) $K_c=[P_4O_{10}]/[P_4][O_2]^5$

(d) $K_c=1/[O_2]^5$

13. For the reaction,

$CO(g)+Cl_2(g) \rightleftharpoons COCl_2(g)$ the K_p/K_c is equal to **[2004]**

(a) $\sqrt{RT}$ (b) RT

(c) $1/RT$ (d) 1.0

14. The equilibrium constant for the reaction $N_2(g)+O_2(g) \rightleftharpoons 2NO(g)$ at temperature T is 4×10^{-4}. The value of K_c for the reaction

$NO(g) \rightleftharpoons \frac{1}{2}N_2(g)+\frac{1}{2}O_2(g)$ at the same temperature is **[2004, 2012]**

(a) 4×10^{-4} (b) 50

(c) 2.5×10^2 (d) 0.02

15. The molar solubility (in mol L^{-1}) of a sparingly soluble salt MX_4 is 's'. The corresponding solubility product is K_{sp}. 's' is given in term of K_{sp} by the relation : **[2004]**

(a) $s=(256\,K_{sp})^{1/5}$ (b) $s=(128\,K_{sp})^{1/4}$

(c) $s=(K_{sp}/128)^{1/4}$ (d) $s=(K_{sp}/256)^{1/5}$

16. The solubility product of a salt having general formula MX_2, in water, is : 4×10^{-12}. The concentration of M^{2+} ions in the aqueous solution of the salt is **[2005]**

(a) 4.0×10^{-10} M (b) 1.6×10^{-4} M

(c) 1.0×10^{-4} M (d) 2.0×10^{-6} M

17. The exothermic formation of ClF_3 is represented by the equation :

$Cl_2(g)+3F_2(g) \rightleftharpoons 2ClF_3(g); \Delta H=-329$ kJ

Which of the following will increase the quantity of ClF_3 in an equilibrium mixture of Cl_2, F_2 and ClF_3? **[2005]**

(a) Adding F_2

(b) Increasing the volume of the container

(c) Removing Cl_2

(d) Increasing the temperature

18. For the reaction : **[2005]**

$2NO_2(g) \rightleftharpoons 2NO(g) + O_2(g)$,

($K_c = 1.8\times10^{-6}$ at 184 °C)

[$R = 0.0831$ kJ/ (mol. K)]

When K_p and K_c are compared at 184°C, it is found that

(a) Whether K_p is greater than, less than or equal to K_c depends upon the total gas pressure

(b) $K_p = K_c$

(c) K_p is less than K_c

(d) K_p is greater than K_c

19. Hydrogen ion concentration in mol/L in a solution of pH = 5.4 will be : **[2005]**

(a) 3.98×10^{-6} (b) 3.68×10^{-6}

(c) 3.88×10^{6} (d) 3.98×10^{8}

20. What is the conjugate base of OH^- ? **[2005]**

(a) O^{2-} (b) O^-

(c) H_2O (d) O_2

21. An amount of solid NH_4HS is placed in a flask already containing ammonia gas at a certain temperature and 0.50 atm pressure. Ammonium hydrogen sulphide decomposes to yield NH_3 and H_2S gases in the flask. When the decomposition reaction reaches equilibrium, the total pressure in the flask rises to 0.84 atm? The equilibrium constant for NH_4HS decomposition at this temperature is **[2005]**

(a) 0.11 (b) 0.17

(c) 0.18 (d) 0.30

22. Phosphorus pentachloride dissociates as follows, in a closed reaction vessel **[2006]**

$PCl_5(g) \rightleftharpoons PCl_3(g)+Cl_2(g)$

If total pressure at equilibrium of the reaction mixture is P and degree of dissociation of PCl_5 is x, the partial pressure of PCl_3 will be

(a) $\left(\frac{x}{x-1}\right)P$ (b) $\left(\frac{x}{1-x}\right)P$

(c) $\left(\frac{x}{x+1}\right)P$ (d) $\left(\frac{2x}{1-x}\right)P$

23. The equilibrium constant for the reaction

$$SO_3(g) \rightleftharpoons SO_2(g) + \frac{1}{2}O_2(g)$$

is $K_c = 4.9 \times 10^{-2}$. The value of K_c for the reaction

$$2SO_2(g) + O_2(g) \rightleftharpoons 2SO_3(g)$$

will be **[2006]**

(a) 9.8×10^{-2} (b) 4.9×10^{-2}
(c) 416 (c) 2.40×10^{-3}

24. The first and second dissociation constants of an acid H_2A are 1.0×10^{-5} and 5.0×10^{-10} respectively. The overall dissociation constant of the acid will be **[2007]**

(a) 0.2×10^{5} (b) 5.0×10^{-5}
(c) 5.0×10^{15} (d) 5.0×10^{-15}.

25. The pK_a of a weak acid (HA) is 4.5. The pOH of an aqueous buffer solution of HA in which 50% of the acid is ionized is **[2007]**

(a) 7.0 (b) 4.5
(c) 2.5 (d) 9.5

26. In a saturated solution of the sparingly soluble strong electrolyte $AgIO_3$ (molecular mass = 283) the equilibrium which sets in is

$$AgIO_3(s) \rightleftharpoons Ag^+(aq) + IO_3^-(aq)$$

If the solubility product constant K_{sp} of $AgIO_3$ at a given temperature is 1.0×10^{-8}, what is the mass of $AgIO_3$ contained in 100 mL of its saturated solution? **[2007]**

(a) 1.0×10^{-4} g (b) 28.3×10^{-2} g
(c) 2.83×10^{-3} g (d) 1.0×10^{-7} g.

27. The equilibrium constants K_{p_1} and K_{p_2} for the reactions $X \rightleftharpoons 2Y$ and $Z \rightleftharpoons P + Q$, respectively are in the ratio of 1 : 9. If the degree of dissociation of X and Z be equal, then the ratio of total pressures at these equilibria is **[2008]**

(a) 1:36 (b) 1 : 1
(c) 1 : 3 (d) 1 : 9

28. For the following three reactions (i), (ii) and (iii), equilibrium constants are given: **[2008]**

(i) $CO(g) + H_2O(g) \rightleftharpoons CO_2(g) + H_2(g); K_1$

(ii) $CH_4(g) + H_2O(g) \rightleftharpoons CO(g) + 3H_2(g); K_2$

(iii) $CH_4(g) + 2H_2O(g) \rightleftharpoons CO_2(g) + 4H_2(g); K_3$

Which of the following is correct ?

(a) $K_1\sqrt{K_2} = K_3$ (b) $K_2K_3 = K_1$
(c) $K_3 = K_1K_2$ (d) $K_3.K_2^3 = K_1^2$

29. Four species are listed below: **[2008]**

(i) HCO_3^- (ii) H_3O^+
(iii) HSO_4^- (iv) HSO_3F

Which one of the following is the correct sequence of their acid strength?

(a) iv < ii < iii < i (b) ii < iii < i < iv
(c) i < iii < ii < iv (d) iii < i < iv < ii

30. The pK_a of a weak acid, HA, is 4.80. The pK_b of a weak base, BOH, is 4.78. The pH of an aqueous solution of the correspondng salt, BA, will be **[2008]**

(a) 9.58 (b) 4.79
(c) 7.01 (d) 9.22

31. Solid $Ba(NO_3)_2$ is gradually dissolved in a 1.0×10^{-4} M Na_2CO_3 solution. At what concentration of Ba^{2+} will a precipitate begin to form? (K_{sp} for for $BaCO_3 = 5.1 \times 10^{-9}$) **[2009]**

(a) 5.1×10^{-5} M (b) 8.1×10^{-8} M
(c) 8.1×10^{-7} M (d) 4.1×10^{-5} M

32. Three reactions involving $H_2PO_4^-$ are given below : **[2010]**

(i) $H_3PO_4 + H_2O \rightarrow H_3O^+ + H_2PO_4^-$

(ii) $H_2PO_4^- + H_2O \rightarrow HPO_4^{2-} + H_3O^+$

(iii) $H_2PO_4^- + OH^- \rightarrow H_3PO_4 + O^{2-}$

In which of the above does $H_2PO_4^-$ act as an acid ?

(a) (ii) only (b) (i) and (ii)
(c) (iii) only (d) (i) only

33. In aqueous solution the ionization constants for carbonic acid are

$K_1 = 4.2 \times 10^{-7}$ and $K_2 = 4.8 \times 10^{-11}$.

Select the correct statement for a saturated 0.034 M solution of the carbonic acid. **[2010]**

(a) The concentration of CO_3^{2-} is 0.034 M.

(b) The concentration of CO_3^{2-} is greater than that of HCO_3^-.

(c) The concentrations of H^+ and HCO_3^- are approximately equal.

(d) The concentration of H^+ is double that of CO_3^{2-}.

34. Solubility product of silver bromide is 5.0×10^{-13}. The quantity of potassium bromide (molar mass taken as 120 g mol^{-1}) to be added to 1 litre of 0.05 M solution of silver nitrate to start the precipitation of AgBr is **[2010]**

(a) 1.2×10^{-10} g (b) 1.2×10^{-9} g
(c) 6.2×10^{-5} g (d) 5.0×10^{-8} g

35. At 25 °C, the solubility product of $Mg(OH)_2$ is 1.0×10^{-11}. At which pH, will Mg^{2+} ions start precipitating in the form of $Mg(OH)_2$ from a solution of 0.001 M Mg^{2+} ions? **[2010]**

(a) 9 (b) 10
(c) 11 (d) 8

36. An acid HA ionises as

$$HA \rightleftharpoons H^+ + A^-$$

The pH of 1.0 M solution is 5. Its dissociation constant would be : **[2011RS]**

(a) 5 (b) 5×10^{-8}
(c) 1×10^{-5} (d) 1×10^{-10}

37. The K_{sp} for $Cr(OH)_3$ is 1.6×10^{-30}. The solubility of this compound in water is : **[2011RS]**

(a) $4\sqrt{1.6 \times 10^{-30}}$ (b) $4\sqrt{1.6 \times 10^{-30}/27}$
(c) $1.6 \times 10^{-30/27}$ (d) $2\sqrt{1.6 \times 10^{-30}}$

38. The pH of a 0.1 molar solution of the acid HQ is 3. The value of the ionization constant, K_a of the acid is : **[2012]**

(a) 3×10^{-1} (b) 1×10^{-3}
(c) 1×10^{-5} (d) 1×10^{-7}

39. How many litres of water must be added to 1 litre of an aqueous solution of HCl with a pH of 1 to create an aqueous solution with pH of 2 ? **[2013]**

(a) 0.1 L (b) 0.9 L
(c) 2.0 L (d) 9.0 L

40. For the reaction

$$SO_2(g) + \frac{1}{2}O_2(g) \rightleftharpoons SO_3(g)$$

if $K_p = K_c(RT)^x$ where the symbols have usual meaning then the value of x is (assuming ideality): **[2014]**

(a) –1 (b) $-\frac{1}{2}$
(c) $\frac{1}{2}$ (d) 1

41. The standard Gibbs energy change at 300 K for the reaction $2A \rightleftharpoons B + C$ is 2494.2 J. At a given time, the composition of the reaction mixture is $[A] = \frac{1}{2}$, $[B] = 2$ and $[C] = \frac{1}{2}$. The reaction proceeds in the : [R = 8.314 J/K/mol, $e = 2.718$] **[2015]**

(a) forward direction because $Q < K_c$
(b) reverse direction because $Q < K_c$
(c) forward direction because $Q > K_c$
(d) reverse direction because $Q > K_c$

42. The following reaction is performed at 298 K. **[2015]**

$$2NO(g) + O_2(g) \rightleftharpoons 2NO_2(g)$$

The standard free energy of formation of NO(g) is 86.6 kj/mol at 298 K. What is the standard free energy of formation of $NO_2(g)$ at 298 K? ($K_p = 1.6 \times 10^{12}$)

(a) $86600 - \dfrac{\ln(1.6 \times 10^{12})}{R\,(298)}$
(b) $0.5\,[2 \times 86{,}600 - R(298)\ln(1.6 \times 10^{12})]$
(c) $R(298)\ln(1.6 \times 10^{12}) - 86600$
(d) $86600 + R(298)\ln(1.6 \times 10^{12})$

43. The equilibrium constant at 298 K for a reaction $A + B \rightleftharpoons C + D$ is 100. If the initial concentration of all the four species were 1 M each, then equilibrium concentration of D (in mol L^{-1}) will be : **[2016]**

(a) 1.818 (b) 1.182
(c) 0.182 (d) 0.818

44. pK_a of a weak acid (HA) and pK_b of a weak base (BOH) are 3.2 and 3.4, respectively. The pH of their salt (AB) solution is **[2017]**

(a) 7.2 (b) 6.9
(c) 7.0 (d) 1.0

45. An aqueous solution contains 0.10 M H_2S and 0.20 M HCl. If the equilibrium constants for the formation of HS^- from H_2S is 1.0×10^{-7} and that of S^{2-} from HS^- ions is 1.2×10^{-13}, then the concentration of S^{2-} ions in aqueous solution is : **[2018]**

(a) 5×10^{-8} (b) 3×10^{-20}
(c) 6×10^{-21} (d) 5×10^{-19}

46. An aqueous solution contains an unknown concentration of Ba^{2+}. When 50 mL of a 1 M solution of Na_2SO_4 is added, $BaSO_4$ just begins to precipitate. The final volume is 500 mL. The solubility product of $BaSO_4$ is 1×10^{-10}. What is the original concentration of Ba^{2+}? **[2018]**

(a) 5×10^{-9} M (b) 2×10^{-9} M

(c) 1.1×10^{-9} M (d) 1.0×10^{-10} M

47. Which of the following salts is the most basic in aqueous solution? **[2018]**

(a) $Al(CN)_3$ (b) CH_3COOK

(c) $FeCl_3$ (d) $Pb(CH_3COO)_2$

48. 20 mL of 0.1 M H_2SO_4 solution is added to 30 mL of 0.2 M NH_4OH solution. The pH of the resultant mixture is: **[2019]**

[pK_b of $NH_4OH = 4.7$].

(a) 5.2 (b) 9.0

(c) 5.0 (d) 9.4

49. Which amongst the following is the strongest acid? **[2019]**

(a) $CHBr_3$ (b) CHI_3

(c) $CH(CN)_3$ (d) $CHCl_3$

50. Two solutions, A and B, each of 100 L was made by dissolving 4g of NaOH and 9.8 g of H_2SO_4 in water, respectively. The pH of the resultant solution obtained from mixing 40 L of solution A and 10 L of solution. B is____. **[2020]**

Answer Key

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
(a)	(a)	(a)	(a)	(c)	(b)	(d)	(b)	(b)	(d)	(d)	(d)	(c)	(b)	(d)
16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
(c)	(a)	(d)	(a)	(a)	(a)	(c)	(c)	(d)	(d)	(c)	(a)	(c)	(c)	(c)
31	32	33	34	35	36	37	38	39	40	41	42	43	44	45
(a)	(a)	(c)	(b)	(b)	(d)	(b)	(c)	(d)	(b)	(d)	(b)	(a)	(b)	(b)
46	47	48	49	50										
(c)	(b)	(b)	(c)	(10.60)										

Solutions

1. **(a)** A buffer is a solution of weak acid and its salt with strong base and vice versa. HCl is strong acid and NaCl is its salt with strong base. pH is less than 7 due to HCl.

2. **(a)** $(HSO_4)^-$ can accept and donate a proton

$(HSO_4)^- + H^+ \rightarrow H_2SO_4$ (acting as base)

$(HSO_4)^- - H^+ \rightarrow SO_4^{2-}$. (acting as acid)

3. **(a)** $Mg(OH)_2 \rightarrow \underset{x}{[Mg^{2+}]} + \underset{2x}{2[OH^-]}$

$K_{sp} = [Mg]\,[OH]^2 = [x][2x]^2 = x.4x^2 = 4x^3.$

4. **(a)** In reaction (a) the ratio of number of moles of reactants to products is same i.e. 2 : 2, hence change in volume will not alter the number of moles.

5. **(c)** $K_p = K_c (RT)^{\Delta n}$;

$$\Delta n = 1 - \left(1 + \frac{1}{2}\right) = 1 - \frac{3}{2} = -\frac{1}{2}.$$

$$\therefore \frac{K_p}{K_c} = (RT)^{-1/2}$$

6. **(b)** pH of an acidic solution should be less than 7. The reason is that from H_2O, $[H^+] = 10^{-7}$M which cannot be neglected in comparison to 10^{-8}M. The pH can be calculated as.

From acid, $[H^+] = 10^{-8}$M.

From H_2O, $[H^+] = 10^{-7}$M

$\therefore$ Total $[H^+] = 10^{-8} + 10^{-7}$

$= 10^{-8}(1 + 10) = 11 \times 10^{-8}$

$\therefore pH = -\log[H^+] = -\log 11\times10^{-8}$
$= -[\log 11 + 8\log 10]$
$= -[1.0414 - 8] = 6.9586$

7. **(d)** $AB_2 \rightleftharpoons A^{2+} + 2B^-$
$[A] = 1.0\times10^{-5}$, $[B] = [2.0\times10^{-5}]$
$K_{sp} = [B]^2[A] = [2\times10^{-5}]^2[1.0\times10^{-5}] = 4\times10^{-15}$

8. **(b)** $K_c = \dfrac{[NO_2]^2}{[N_2O_4]} = \dfrac{[1.2\times10^{-2}]^2}{[4.8\times10^{-2}]}$
$= 3\times10^{-3}$ mol/L

9. **(b)** Due to exothermic nature of reaction low or optimum temperature will be required. Since 3 moles are changing to 2 moles, therefore high pressure will be required.

10. **(d)** The rain water after thunderstorm contains dissolved acid and therefore the pH is less than rain water without thunderstorm.

11. **(d)** $\underset{\text{Acid}}{H_2PO_4^-} \xrightarrow{-H^+} \underset{\text{Conjugate base}}{HPO_4^{2-}}$

NOTE Conjugate acid-base differs by H^+

12. **(d)** For $P_4(s) + 5O_2(g) \rightleftharpoons P_4O_{10}(s)$
$K_c = \dfrac{1}{(O_2)^5}$.
Solids have concentration unity.

13. **(c)** $K_p = K_c(RT)^{\Delta n}$;
Here $\Delta n = 1 - 2 = -1$
$\therefore \dfrac{K_p}{K_c} = \dfrac{1}{RT}$

14. **(b)** $K_c = \dfrac{[NO]^2}{[N_2][O_2]} = 4\times10^{-4}$

$K'_c = \dfrac{[N_2]^{1/2}[O_2]^{1/2}}{[NO]} = \dfrac{1}{\sqrt{K_c}}$

$= \dfrac{1}{\sqrt{4\times10^{-4}}} = 50$

15. **(d)** $MX_4 \rightleftharpoons \underset{S}{M^{4+}} + \underset{4S}{4X^-}$
$K_{sp} = [s][4s]^4 = 256\, s^5$

$\therefore s = \left(\dfrac{K_{sp}}{256}\right)^{1/5}$

16. **(c)** $MX_2 \rightleftharpoons \underset{s}{M^{2+}} + \underset{2s}{2X^-}$
Where s is the solubility of MX_2
then $K_{sp} = 4s^3$
$4\times10^{-12} = 4s^3$
or $s = 1\times10^{-4}$

17. **(a)** The reaction given is an exothermic reaction thus accordingly to Le-Chatalier's principle lowering of temperature, addition of F_2 and or Cl_2 favour the forward direction and in hence the production of ClF_3.

18. **(d)** For the reaction:-
$2NO_2(g) \rightleftharpoons 2NO(g) + O_2(g)$
Given $K_c = 1.8\times10^{-6}$ at 184 ºC
R = 0.0831 kJ/mol. K
$K_p = K_c\times(RT)^{\Delta n}$
$K_p = 1.8\times10^{-6}\times0.0831\times457$
$= 6.836\times10^{-6}$
Hence it is clear that $K_p > K_c$

19. **(a)** $pH = -\log[H^+] = \log\dfrac{1}{[H^+]}$

$5.4 = \log\dfrac{1}{[H^+]}$

On solving, $[H^+] = 3.98\times10^{-6}$

20. **(a)** Conjugate acid-base pair differ by only one proton.
$OH^- \longrightarrow H^+ + O^{2-}$
Conjugate base of OH^- is O^{2-}

21. **(a)** $NH_4HS(s) \rightleftharpoons NH_3(s) + H_2S(g)$

	$NH_4HS(s)$	$NH_3(s)$	$H_2S(g)$
At start		0.5 atm	0 atm
At eqm.		$0.5 + x$ atm	x atm.

Then $0.5 + x + x = 2x + 0.5 = 0.84$ (given)
$x = 0.17$ atm.
$p_{NH_3} = 0.5 + 0.17 = 0.67$ atm;
$p_{H_2S} = 0.17$ atm
$K = p_{NH_3}\times p_{H_2S} = 0.67\times0.17\ atm^2$
$= 0.1139 = 0.11$

22. (c) $PCl_5(g) \rightleftharpoons PCl_3(g) + Cl_2(g)$

$\quad 1-x \qquad\qquad x \qquad\qquad x$

Total moles after dissociation

$1-x+x+x=1+x$

p_{PCl_3} = Mole fraction of PCl_3

$\times$ Total pressure $=\left(\dfrac{x}{1+x}\right)P$

23. (c) $SO_3(g) \rightleftharpoons SO_2(g)+\frac{1}{2}O_2(g)$

$$K_c=\frac{[SO_2][O_2]^{1/2}}{[SO_3]}=4.9\times10^{-2};$$

On taking the square of the above reaction

$$\frac{[SO_2]^2[O_2]}{[SO_3]^2}=24.01\times10^{-4}$$

Now for $2SO_2(g)+O_2(g) \rightleftharpoons 2SO_3$

$$K'_c=\frac{[SO_3]^2}{[SO_2]^2[O_2]}=\frac{1}{24.01\times10^{-4}}=416$$

24. (d) $H_2A \rightleftharpoons H^+ + HA^-$

$$\therefore\ K_1=1.0\times10^{-5}=\frac{[H^+][HA^-]}{[H_2A]}$$

$HA^- \rightleftharpoons H^+ + A^-$

$$\therefore\ K_2=5.0\times10^{-10}=\frac{[H^+][A^-]}{[HA^-]}$$

For the reaction,

$H_2A \rightleftharpoons 2H^+ + A^{2-}$

$$K=\frac{[H^+]^2[A^{2-}]}{[H_2A]}=K_1\times K_2$$

$=(1.0\times10^{-5})\times(5\times10^{-10})=5\times10^{-15}$

25. (d) For acidic buffer, $pH=pK_a+\log\left[\dfrac{salt}{acid}\right]$

$$pH=4.5+\log\frac{[salt]}{[acid]}$$

As HA is 50% ionized so [salt] = [acid]

$\therefore$ pH = 4.5

$\because$ pH + pOH = 14

pOH = 14 – pH = 14 – 4.5 = 9.5

26. (c) Let s = solubility

$AgIO_3 \rightleftharpoons \underset{s}{Ag^+} + \underset{s}{IO_3^-}$

$K_{sp}=[Ag^+][IO_3^-]=s\times s=s^2$

Given $K_{sp}=1\times10^{-8}$

$\therefore\ s=\sqrt{K_{sp}}=\sqrt{1\times10^{-8}}$

$=1.0\times10^{-4}$ mol/L $=1.0\times10^{-4}\times283$ g/L

($\because$ Molecular mass of $AgIO_3=283$)

$$=\frac{1.0\times10^{-4}\times283\times100}{1000}\text{ g/100 mL}$$

$=2.83\times10^{-3}$ g / 100 mL

27. (a) Let the initial moles of X be 'a' and that of Z be 'b' then for the given reactions,

	X	$\rightleftharpoons$ 2Y
Initial	a moles	0
At eqm. (moles)	a(1 – α)	2aα

Total no. of moles = a (1 – α) + 2aα

= a – aα + 2aα

= a (1 + α)

Now, $K_{p_1}=\dfrac{(n_y)^2}{n_x}\times\left(\dfrac{P_{T_1}}{\sum n}\right)^{\Delta n}$

or, $K_{p_1}=\dfrac{(2a\alpha)^2\cdot P_{T_1}}{[a(1-\alpha)][a(1+\alpha)]}$

	Z	$\rightleftharpoons$ P	+ Q
Initial	b moles	0	0
At eqm. (moles)	b(1 – α)	bα	bα

Total no . of moles = b(1 – α) + bα + bα

= b – bα + bα + bα

= b(1 + α)

Now $K_{p_2}=\dfrac{n_Q\times n_P}{n_z}\times\left[\dfrac{P_{T_2}}{\sum n}\right]^{\Delta n}$

or $K_{p_2}=\dfrac{(b\alpha)(b\alpha).P_{T_2}}{[b(1-\alpha)][b(1+\alpha)]}$

or $\dfrac{K_{p_1}}{K_{p_2}}=\dfrac{4\alpha^2.P_{T_1}}{(1-\alpha^2)}\times\dfrac{(1-\alpha)^2}{P_{T_2}.\alpha^2}=\dfrac{4P_{T_1}}{P_{T_2}}$

or $\frac{4P_{T_1}}{P_{T_2}} = \frac{1}{9}$ $\left[\because \frac{K_{p_1}}{K_{p_2}} = \frac{1}{9} \text{ given}\right]$

or $\frac{P_{T_1}}{P_{T_2}} = \frac{1}{36}$ or 1 : 36

28. (c) Reaction (iii) can be obtained by adding reactions (i) and (ii) therefore $K_3 = K_1 . K_2$ Hence (c) is the correct answer.

29. (c) The correct order of acidic strength of the given species is

$$\underset{(iv)}{HSO_3F} > \underset{(ii)}{H_3O^+} > \underset{(iii)}{HSO_4^-} > \underset{(i)}{HCO_3^-}$$

or (i) < (iii) < (ii) < (iv)

30. (c) In aqueous solution, BA(salt) hydrolyses to give

$$BA + H_2O \rightleftharpoons \underset{\text{base}}{BOH} + \underset{\text{acid}}{HA}$$

Now pH is given by

$$pH = \frac{1}{2}pK_w + \frac{1}{2}pK_a - \frac{1}{2}pK_b$$

Substituting given values, we get

$$pH = \frac{1}{2}(14 + 4.80 - 4.78) = 7.01$$

31. (a) $\underset{1\times10^{-4}M}{Na_2CO_3} \longrightarrow \underset{1\times10^{-4}M}{2Na^+} + \underset{1\times10^{-4}M}{CO_3^{2-}}$

$$K_{sp(BaCO_3)} = [Ba^{2+}][CO_3^{2-}]$$

$$[Ba^{2+}] = \frac{5.1\times10^{-9}}{1\times10^{-4}} = 5.1\times10^{-5}M$$

32. (a) (i) $\underset{acid_1}{H_3PO_4} + \underset{base_2}{H_2O} \longrightarrow \underset{acid_2}{H_3O^+} + \underset{base_1}{H_2PO_4^-}$

(ii) $\underset{acid_1}{H_2PO_4^-} + \underset{base_2}{H_2O} \longrightarrow \underset{base_1}{HPO_4^{2-}} + \underset{acid_2}{H_3O^+}$

(iii) $\underset{base_1}{H_2PO_4^-} + \underset{acid_2}{OH^-} \longrightarrow \underset{acid_1}{H_3PO_4} + \underset{base_2}{O^{2-}}$

Hence only in (ii) reaction, $H_2PO_4^-$ is acting as an acid.

33. (c) $H_3CO_3 \rightleftharpoons H^+ + HCO_3^-$ $K_1 = 4.2\times10^{-7}$

$HCO_3^- \rightleftharpoons H^+ + CO_3^{2-}$ $K_2 = 4.8\times10^{-11}$

Second dissociation constant (K_2) is much smaller than the first one (K_1). Just a small fraction of total HCO_3^- formed will undergo second stage of ionization. Hence in saturated solution

$[H^+] >>> [CO_3^{2-}]$; $[CO_3^{2-}] \neq 0.034$ M

$[HCO_3^{2-}] >> [CO_3^{2-}]$ and $[H^+] \approx [HCO_3^-]$.

34. (b) $AgBr \rightleftharpoons Ag^+ + Br^-$

$K_{sp} = [Ag^+][Br^-]$

For precipitation to occur

Ionic product > Solubility product

$$[Br^-] = \frac{K_{sp}}{[Ag^+]} = \frac{5\times10^{-13}}{0.05} = 10^{-11}$$

i.e., precipitation just starts when 10^{-11} moles of KBr is added to 1L $AgNO_3$ solution

$\therefore$ Number of moles of Br^- needed from KBr = 10^{-11}

$\therefore$ Mass of KBr = $10^{-11} \times 120 = 1.2 \times 10^{-9}$ g

35. (b) $Mg(OH)_2 \rightleftharpoons Mg^{2+} + 2OH^-$

$K_{sp} = [Mg^{2+}][OH^-]^2$

$1.0 \times 10^{-11} = 10^{-3} \times [OH^-]^2$

$$[OH^-] = \sqrt{\frac{10^{-11}}{10^{-3}}} = 10^{-4}$$

$\therefore$ pOH = 4

$\therefore$ pH + pOH = 14

$\therefore$ pH = 10

36. (d) pH = 5 means

$[H^+] = 10^{-5}$

	HA	$\rightleftharpoons$	H^+	+	A^-
$t = 0$	c		0		0
t_{eq}	c (1 – α)		cα		cα

$$K_a = \frac{[H^+][A^-]}{[HA]} = \frac{(c\alpha)^2}{c(1-\alpha)} = \frac{[H^+]^2}{c-[H^+]}$$

But, $[H^+] << c$

$\therefore$ $K_a = [H^+]^2 = (10^{-5})^2 = 10^{-10}$

37. (b) $Cr(OH)_3(s) \rightleftharpoons Cr^{3+}(aq.) + 3OH^-(aq.)$

$27s^4 = K_{sp}$

$$s = \left(\frac{K_{sp}}{27}\right)^{1/4} = \left(\frac{1.6\times10^{-30}}{27}\right)^{1/4}$$

38. (c) $H^+ = c\alpha; \alpha = \frac{[H^+]}{c}$

or $\alpha = \frac{10^{-3}}{0.1} = 10^{-2}$

$K_a = c\alpha^2 = 0.1\times10^{-2}\times10^{-2} = 10^{-5}$

39. (d) $\because$ pH = 1 ; $H^+ = 10^{-1} = 0.1$ M

pH = 2 ; $H^+ = 10^{-2} = 0.01$ M

$\therefore M_1 = 0.1 \quad V_1 = 1$

$M_2 = 0.01, \quad V_2 = ?$

From

$M_1V_1 = M_2V_2$

$0.1 \times 1 = 0.01 \times V_2$

$V_2 = 10$ litres

$\therefore$ Volume of water added = 10 – 1 = 9 litres

40. (b) $SO_2(g) + \frac{1}{2}O_2(g) \rightleftharpoons SO_3(g)$

$K_p = K_c(RT)^x$

where $x = \Delta n_g$ = number of gaseous moles in product

– number of gaseous moles in reactants

$$= 1 - \left(1 + \frac{1}{2}\right) = 1 - \frac{3}{2} = -\frac{1}{2}$$

41. (d) $\Delta G° = 2494.2$J

$2A \rightleftharpoons B + C.$

$[A] = \frac{1}{2}, [B] = 2, [C] = \frac{1}{2}$

$$Q = \frac{[B][C]}{[A]^2} = \frac{2\times1/2}{\left(\frac{1}{2}\right)^2} = 4$$

$\Delta G° = -2.303\ RT \log K_C.$

$2494.2\ J = -2.303 \times (8.314\ J/K/mol) \times (300K) \log K_C$

$$\Rightarrow \log K_C = -\frac{2494.2\ J}{2.303\times8.314\ J/K/mol\times300\ K}$$

$\Rightarrow \log K_C = -0.4341; K_C = 0.37; Q > K_C.$

42. (b) $\Delta G°_{rexn} = 2\Delta G°_f(NO_2) - 2\Delta G°_f(NO) - \Delta G°_f(O_2)$

$2\Delta G°_f(NO_2) = \Delta G°_{rexn} + 2\Delta G°_f(NO) + \Delta G°_f(O_2)$

$\because \quad \Delta G = \Delta G° + RT\ln K_p$

At equilibrium,

$\Delta G = 0, Q = K_p$

$\Delta G° = -RT\ln K_p$

$\Delta G°_f(O_2) = 0$

$\therefore \quad \Delta G°_f(NO_2)$

$$= \frac{1}{2}[2\times86600 - R(298)\ln(1.6\times10^{12})]$$

43. (a) Given,

	A	+	B	$\rightleftharpoons$	C	+	D
No. of moles initially	1		1		1		1
At equilibrium	1–a		1–a		1+a		1+a

$$K_c = \left(\frac{1+a}{1-a}\right)^2 = 100; \frac{1+a}{1-a} = 10$$

On solving; a = 0.81

$[D]_{At\ eq} = 1 + a = 1 + 0.81 = 1.81$

44. (b) The salt (AB) given is a salt of weak acid and weak base. Hence the pH can be calculated by the following formula

$$\therefore \quad pH = 7 + \frac{1}{2}pK_a - \frac{1}{2}pK_b$$

$$= 7 + \frac{1}{2}(3.2) - \frac{1}{2}(3.4) = 6.9$$

45. (b) $\underset{\text{At equb. } 0.10}{H_2S} \rightleftharpoons \underset{0.20}{2H^+} + \underset{?}{S^{2-}}, K_{a_1}.K_{a_2} = K_{eq}$

$$\therefore \quad \frac{[H^+]^2[S^{2-}]}{[H_2S]} = 1\times10^{-7}\times1.2\times10^{-13}$$

$$\frac{[0.2]^2[S^{2-}]}{[0.1]} = 1.2\times10^{-20}$$

$[S^{2-}] = 3\times10^{-20}$

46. (c) Concentration of SO_4^{2-} in $BaSO_4$ solution

$M_1V_1 = M_2V_2$

$1 \times 50 = M_2 \times 500$

$$M_2 = \frac{1}{10}$$

For just precipitation

Ionic product = K_{sp}

$[Ba^{2+}][SO_4^{2-}] = K_{sp}(BaSO_4)$

$$[Ba^{2+}] \times \frac{1}{10} = 10^{-10}$$

$[Ba^{2+}] = 10^{-9}$ M in 500 mL solution

Thus $[Ba^{2+}]$ in original solution

$(500 - 50 = 450 \text{ mL})$

$\Rightarrow M_1 \times 450 = 10^{-9} \times 500$

[where M_1 = Molarity of original solution]

$$M_1 = \frac{500}{450} \times 10^{-9} = 1.11 \times 10^{-9} \text{ M}$$

47. (b) CH_3COOK is a salt of weak acid (CH_3COOH) and strong base (KOH).

$FeCl_3$ is a salt of weak base $[Fe(OH)_3]$ and strong acid (HCl).

$Pb(CH_3COO)_2$ is a salt of weak base $Pb(OH)_2$ and weak acid (CH_3COOH)

$Al(CN)_3$ is a salt of weak base $[Al(OH)_3]$ and weak acid (HCN).

48. (b) m. mol of $H_2SO_4 = 20 \times 0.1 = 2$

m. mol of $NH_4OH = 30 \times 0.2 = 6$

	H_2SO_4 +	$2NH_4OH \rightarrow$	$(NH_4)_2SO_4 + 2H_2O$
Initial	2 m mol	6 m mol	0
Final	(2–2) = 0 m mol	(6–2×2) = 2 m mol	2 m mol

$[NH_4OH]_{left} = 2$ m mol

$[(NH_4)_2SO_4] = 2$ m mol

$[\overset{+}{N}H_4] = 2 \times 2 = 4$ m mol

Total Volume = 30 + 20 = 50 mL

$$pOH = pK_b + \log\left[\frac{\text{Salt}}{\text{Base}}\right]$$

$$= 4.7 + \log\frac{4/50}{2/50}$$

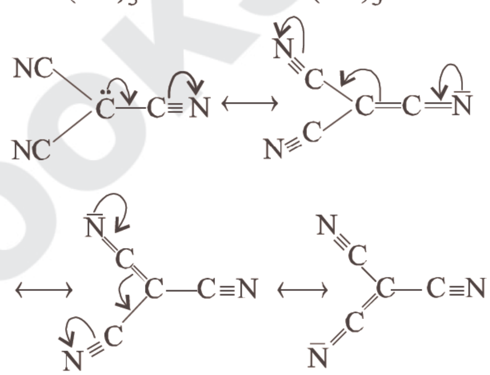
$= 4.7 + \log 2 = 5$

pH = 14 – pOH

pH = 14 – 5 = 9

49. (c) Due to the resonance stabilisation of the conjugate base, $CH(CN)_3$ is the strongest acid amongst the given compounds.

$$CH(CN)_3 \rightleftharpoons \overset{+}{H} + \overset{-}{C}(CN)_3$$



The conjugate bases of $CHBr_3$ and CHI_3 are stabilised by inductive effect of halogens. This is why, they are less stable. Also, the conjugate base of $CHCl_3$ involves back-bonding between $2p$ and $3p$ orbitals.

50. (10.60) $$M_{H_2SO_4} = \frac{9.8}{98 \times 100} = 10^{-3}\text{ M}$$

$$M_{NaOH} = \frac{4}{40 \times 100} = 10^{-3}\text{ M}$$

After neutralisation $[OH^-]$ can be calculated as

$$[OH^-] = \frac{(40 \times 10^{-3}) - (2 \times 10^{-3} \times 10)}{50}$$

$$= \frac{20}{50} \times 10^{-3}$$

$$[OH^-] = \frac{2}{5} \times 10^{-3}$$

pOH = 3.397

pH = 14 – pOH

= 14 – 3.397 = 10.603

8 Redox Reactions

1. Which of the following is a redox reaction? **[2002]**

(a) $NaCl + KNO_3 \rightarrow NaNO_3 + KCl$

(b) $CaC_2O_4 + 2HCl \rightarrow CaCl_2 + H_2C_2O_4$

(c) $Mg(OH)_2 + 2NH_4Cl \rightarrow MgCl_2 + 2NH_4OH$

(d) $Zn + 2AgCN \rightarrow 2Ag + Zn(CN)_2$.

2. Several blocks of magnesium are fixed to the bottom of a ship to **[2003]**

(a) make the ship lighter

(b) prevent action of water and salt

(c) prevent puncturing by under-sea rocks

(d) keep away the sharks

3. Which of the following chemical reactions depict the oxidizing beahviour of H_2SO_4? **[2006]**

(a) $NaCl + H_2SO_4 \longrightarrow NaHSO_4 + HCl$

(b) $2PCl_5 + H_2SO_4 \longrightarrow 2POCl_3 + 2HCl + SO_2Cl_2$

(c) $2HI + H_2SO_4 \longrightarrow I_2 + SO_2 + 2H_2O$

(d) $Ca(OH)_2 + H_2SO_4 \longrightarrow CaSO_4 + 2H_2O$

4. In the following balanced reaction,

$$X\,MnO_4^- + Y\,C_2O_4^{2-} + Z\,H^+ \rightleftharpoons X\,Mn^{2+} + 2Y\,CO_2 + \frac{Z}{2}H_2O$$

values of X, Y and Z respectively are **[2012, 2013]**

(a) 2, 5, 16 (b) 8, 2, 5

(c) 5, 2, 16 (d) 5, 8, 4

5. Which of the following reactions is an example of a redox reaction? **[2017]**

(a) $XeF_4 + O_2F_2 \rightarrow XeF_6 + O_2$

(b) $XeF_2 + PF_5 \rightarrow [XeF]^+ PF_6^-$

(c) $XeF_6 + H_2O \rightarrow XeOF_4 + 2HF$

(d) $XeF_6 + 2H_2O \rightarrow XeO_2F_2 + 4HF$

6. The oxidation states of Cr in $[Cr(H_2O)_6]Cl_3$, $[Cr(C_6H_6)_2]$, and $K_2[Cr(CN)_2(O)_2(O_2)(NH_3)]$ respectively are : **[2018]**

(a) +3, +4, and +6 (b) +3, +2, and +4

(c) +3, 0, and +6 (d) +3, 0, and + 4

7. Oxidation number of potassium in K_2O, K_2O_2 and KO_2, respectively, is: **[2020]**

(a) $+2, +1$ and $+\frac{1}{2}$ (b) $+1, +1$ and $+1$

(c) $+1, +4$ and $+2$ (d) $+1, +2$ and $+4$

Answer Key

1	2	3	4	5	6	7								
(d)	(b)	(c)	(a)	(a)	(c)	(b)								

Solutions

1. (d) $\overset{0}{Zn} + 2\overset{+1}{Ag}CN \longrightarrow \overset{0}{Ag} + \overset{+2}{Zn}(CN)_2$ ($Zn \to Zn$: $-2e^-$; $Ag \to Ag$: $+2e^-$)

The oxidation state shows a change only in (d)

2. (b) Magnesium provides cathodic protection and prevents rusting or corrosion.

3. (c) $2\overset{-1}{HI} + H_2\overset{+6}{S}O_4 \longrightarrow \overset{0}{I_2} + \overset{+4}{S}O_2 + 2H_2O$

in this reaction oxidation number of S is decreasing from +6 to +4 hence undergoing reduction and for HI oxidation number of I is increasing from –1 to 0 hence underegoing oxidation, therefore H_2SO_4 is acting as oxidising agent.

4. (a) $X\,MnO_4^- + Y\,C_2O_4^{2-} + Z\,H^+ \rightleftharpoons X\,Mn^{2+} + 2Y\,CO_2 + \frac{Z}{2}H_2O$

First half reaction

$MnO_4^- \longrightarrow Mn^{2+}$ .... (i)

On balancing

$MnO_4^- + 8H^+ + 5e^- \longrightarrow Mn^{2+} + 4H_2O$ .... (ii)

Second half reaction

$C_2O_4^{2-} \longrightarrow 2CO_2$ .... (iii)

On balancing

$C_2O_4^{2-} \longrightarrow 2CO_2 + 2e^-$ .... (iv)

On multiplying eqn. (ii) by 2 and (iv) by 5 and then adding we get

$$2MnO_4^- + 5C_2O_4^{2-} + 16H^+ \longrightarrow 2Mn^{2+} + 10CO_2 + 8H_2O$$

5. (a) In the reaction

$\overset{+4}{Xe}F_4 + \overset{+1}{O_2}F_2 \longrightarrow \overset{+6}{Xe}F_6 + \overset{0}{O_2}$ (Xe: Oxidation; O: Reduction)

6. (c) $[Cr(H_2O)_6]Cl_3$

$\Rightarrow x + 6 \times 0 + (-1) \times 3 = 0$

$\Rightarrow x = +3$

$[Cr(C_6H_6)_2]$

$x + 2 \times 0 = 0;\ x = 0$

$K_2[Cr(CN)_2(O)_2(O_2)(NH_3)]$

$2 \times 1 + x + 2 \times (-1) + 2 \times (-2) + (-2) + 0 = 0$

$x = +6$

7. (b) $K_2O : 2x - 2 = 0 \Rightarrow x = +1$

$K_2O_2 : 2x - 2 = 0 \Rightarrow x = +1$

$KO_2 : x - 1 = 0 \Rightarrow x = +1$

Thus, potassium shows +1 state in all its oxides, superoxides and peroxides.

9 Hydrogen

1. Which of the following species is diamagnetic in nature? **[2005]**
(a) H_2^- (b) H_2^+ (c) H_2 (d) He_2^+

2. Which of the following statements in relation to the hydrogen atom is correct ? **[2005]**
(a) 3*s*, 3*p* and 3*d* orbitals all have the same energy
(b) 3*s* and 3*p* orbitals are of lower energy than 3*d* orbital
(c) 3*p* orbital is lower in energy than 3*d* orbital
(d) 3*s* orbital is lower in energy than 3*p* orbital

3. In context with the industrial preparation of hydrogen from water gas ($CO + H_2$), which of the following is the correct statement? **[2008]**
(a) CO and H_2, are fractionally separated using differences in their densities
(b) CO is removed by absorption in aqueous Cu_2Cl_2 solution
(c) H_2 is removed through occlusion with pd
(d) CO is oxidised to CO_2 with steam in the presence of a catalyst followed by absorption of CO_2 in alkali

4. Very pure hydrogen (99.9) can be made by which of the following processes ? **[2012]**
(a) Reaction of methane with steam
(b) Mixing natural hydrocarbons of high molecular weight
(c) Electrolysis of water
(d) Reaction of salts like hydrides with water

5. In which of the following reactions H_2O_2 acts as a reducing agent? **[2014]**
(i) $H_2O_2 + 2H^+ + 2e^- \longrightarrow 2H_2O$
(ii) $H_2O_2 - 2e^- \longrightarrow O_2 + 2H^+$
(iii) $H_2O_2 + 2e^- \longrightarrow 2OH^-$
(iv) $H_2O_2 + 2OH^- - 2e^- \longrightarrow O_2 + 2H_2O$
(a) (i), (iii) (b) (ii), (iv)
(c) (i), (ii) (d) (iii), (iv)

6. From the following statements regarding H_2O_2, choose the incorrect statement : **[2015]**
(a) It has to be stored in plastic or wax lined glass bottles in dark
(b) It has to be kept away from dust
(c) It can act only as an oxidizing agent
(d) It decomposes on exposure to light

7. Which one of the following statements about water is **FALSE** ? **[2016]**
(a) There is extensive intramolecular hydrogen bonding in the condensed phase.
(b) Ice formed by heavy water sinks in normal water.
(c) Water is oxidized to oxygen during photosynthesis.
(d) Water can act both as an acid and as a base.

8. Hydrogen peroxide oxidises $[Fe(CN)_6]^{4-}$ to $[Fe(CN)_6]^{3-}$ in acidic medium but reduces $[Fe(CN)_6]^{3-}$ to $[Fe(CN)_6]^{4-}$ in alkaline medium. The other products formed are respectively:
(a) $(H_2O + O_2)$ and H_2O **[2018]**
(b) $(H_2O + O_2)$ and $(H_2O + OH^-)$
(c) H_2O and $(H_2O + O_2)$
(d) H_2O and $(H_2O + OH^-)$

9. The isotopes of hydrogen are: **[2019]**
(a) Tritium and protium only
(b) Protium and deuterium only
(c) Protium, deuterium and tritium
(d) Deuterium and tritium only

10. The number of water molecules(s) not coordinated to copper ion directly in $CuSO_4 \cdot 5H_2O$, is: **[2019]**

(a) 2 (b) 3 (c) 1 (d) 4

11. In comparison to the zeolite process for the removal of permanent hardness, the synthetic resins method is: **[2020]**

(a) less efficient as it exchanges only anions
(b) more efficient as it can exchange both cations as well as anions
(c) less efficient as the resins cannot be regenerated
(d) more efficient as it can exchange only cations

Answer Key

1	2	3	4	5	6	7	8	9	10	11				
(c)	(a)	(d)	(d)	(b)	(c)	(a)	(c)	(c)	(c)	(b)				

Solutions

1. **(c)** H_2 is diamagnetic as it contains all paired electrons

$$\underset{\text{(diamagnetic)}}{H_2 = \sigma_b^2}\ ,\quad \underset{\text{(paramagnetic)}}{H_2^+ = \sigma_b^1},\quad \underset{\text{(paramagnetic)}}{H_2^- = \sigma_b^2,\ \sigma_a^{*1}};$$

$$He_2^+ = \underset{\text{(paramagnetic)}}{\sigma_b^2, \sigma_a^{*1}}$$

2. **(a)** **NOTE** In one electron species, such as H-atom, the energy of orbital depends only on the principal quantum number, *n*.
i.e. is $< 2s = 2p < 3s = 3p = 3d < 4s$
$= 4p = 4d = 4f$

3. **(d)** On the industrial scale, hydrogen is prepared from water gas according to following reaction sequence

$$\underbrace{CO + H_2}_{\text{water gas}} + \underbrace{H_2O}_{\text{(steam)}} \xrightarrow{\text{catalyst}} CO_2 + 2H_2$$

$$\xrightarrow[\text{(alkali)}]{2NaOH} Na_2CO_3 + H_2O$$

4. **(d)** Very pure hydrogen can be prepared by the action of water on sodium hydride.
$$NaH + H_2O \longrightarrow NaOH + H_2\uparrow$$

5. **(b)** The reducing agent loses electron during redox reaction i.e. oxidised itself.

(i) $H_2\overset{-1}{O_2} + 2H^+ + 2e^- \longrightarrow 2H_2\overset{-2}{O}$ (Red.)

(ii) $H_2\overset{-1}{O_2} \longrightarrow \overset{0}{O_2} + 2H^+ + 2e^-$ (Ox.)

(iii) $H_2\overset{-1}{O_2} + 2e^- \longrightarrow 2\overset{-2}{O}H^-$ (Red.)

(iv) $H_2\overset{-1}{O_2} + 2OH^- \longrightarrow \overset{0}{O_2} + H_2O + 2e^-$ (Ox.)

6. **(c)** H_2O_2 has oxidizing and reducing properties both.

7. **(a)** There is extensive intermolecular hydrogen bonding in the condensed phase instead of intramolecular H-bonding.

8. **(c)** $[Fe(CN)_6]^{4-} + \frac{1}{2}H_2O_2 + H^+ \longrightarrow [Fe(CN)_6]^{3-} + H_2O$

$[Fe(CN)_6]^{3-} + \frac{1}{2}H_2O_2 + OH^- \longrightarrow [Fe(CN)_6]^{4-} + H_2O + \frac{1}{2}O_2$

9. **(c)** Hydrogen has three isotopes:
Protium ($_1H^1$), deuterium ($_1H^2$) and tritium ($_1H^3$).

10. **(c)** In $CuSO_4 \cdot 5H_2O$, four H_2O molecules are directly coordinated to the central metal ion while one H_2O molecule is hydrogen bonded.

11. **(b)** Synthetic resin method is more efficient than zeolite process as it can exchange both cations as well as anions.

10 The s-Block Elements

1. KO_2 (potassium super oxide) is used in oxygen cylinders in space and submarines because it **[2002]**
   (a) absorbs CO_2 and increases O_2 content
   (b) eliminates moisture
   (c) absorbs CO_2
   (d) produces ozone.
2. The metallic sodium disolves in liquid ammonia to form a deep blue coloured solution. The deep blue colour is due to formation of: **[2002]**
   (a) solvated electron, $e(NH_3)_x^-$
   (b) solvated atomic sodium, $Na(NH_3)_y$
   (c) $(Na^+ + Na^-)$
   (d) $NaNH_2 + H_2$
3. A metal M readily forms its sulphate MSO_4 which is water-soluble. It forms its oxide MO which becomes inert on heating. It forms an insoluble hyroxide $M(OH)_2$ which is soluble in NaOH solution. Then M is **[2002]**
   (a) Mg (b) Ba (c) Ca (d) Be.
4. In curing cement plasters water is sprinkled from time to time. This helps in **[2003]**
   (a) developing interlocking needle-like crystals of hydrated silicates
   (b) hydrating sand and gravel mixed with cement
   (c) converting sand into silicic acid
   (d) keeping it cool
5. The substance **not** likely to contain $CaCO_3$ is **[2003]**
   (a) calcined gypsum
   (b) sea shells
   (c) dolomite
   (d) a marble statue
6. The solubility of carbonates decreases down the magnesium group due to a decrease in **[2003]**
   (a) hydration energy of cations
   (b) inter-ionic attraction
   (c) entropy of solution formation
   (d) lattice energy of solids
7. Which one of the following processes will produce hard water ? **[2003]**
   (a) Saturation of water with $MgCO_3$
   (b) Saturation of water with $CaSO_4$
   (c) Addition of Na_2SO_4 to water
   (d) Saturation of water with $CaCO_3$
8. One mole of magnesium nitride on reaction with an excess of water gives **[2004]**
   (a) two moles of ammonia
   (b) one mole of nitric acid
   (c) one mole of ammonia
   (d) two moles of nitric acid
9. Based on lattice energy and other considerations, which one of the following alkali metal chlorides is expected to have the highest melting point ? **[2005]**
   (a) RbCl (b) KCl (c) NaCl (d) LiCl
10. The ionic mobility of alkali metal ions in aqueous solution is maximum for **[2006]**
    (a) Li^+ (b) Na^+ (c) K^+ (d) Rb^+

11. The products obtained on heating $LiNO_3$ will be : [2011RS]

(a) $Li_2O + NO_2 + O_2$ (b) $Li_3N + O_2$

(c) $Li_2O + NO + O_2$ (d) $LiNO_3 + O_2$

12. What is the best description of the change that occurs when $Na_2O(s)$ is dissolved in water ? [2011RS]

(a) Oxide ion accepts sharing a pair of electrons
(b) Oxide ion donates a pair of electrons
(c) Oxidation number of oxygen increases
(d) Oxidation number of sodium decreases

13. Which of the following on thermal decomposition yields a basic as well as acidic oxide ? [2012]

(a) $NaNO_3$ (b) $KClO_3$
(c) $CaCO_3$ (d) NH_4NO_3

14. The first ionisation potential of Na is 5.1 eV. The value of electron gain enthalpy of Na^+ will be [2013]

(a) –2.55 eV (b) – 5.1 eV
(c) –10.2 eV (d) +2.55 eV

15. Stability of the species Li_2, Li_2^- and Li_2^+ increases in the order of : [2013]

(a) $Li_2 < Li_2^+ < Li_2^-$ (b) $Li_2^- < Li_2^+ < Li_2$

(c) $Li_2 < Li_2^- < Li_2^+$ (d) $Li_2^- < Li_2 < Li_2^+$

16. Which one of the following alkaline earth metal sulphates has its hydration enthalpy greater than its lattice enthalpy ? [2015]

(a) $BaSO_4$ (b) $SrSO_4$
(c) $CaSO_4$ (d) $BeSO_4$

17. The main oxides formed on combustion of Li, Na and K in excess of air are, respectively: [2016]

(a) Li_2O_2, Na_2O_2 and KO_2
(b) Li_2O, Na_2O_2 and KO_2
(c) Li_2O, Na_2O and KO_2
(d) LiO_2, Na_2O_2 and K_2O

18. Both lithium and magnesium display several similar properties due to the diagonal relationship; however, the one which is incorrect is : [2017]

(a) Both form basic carbonates
(b) Both form soluble bicarbonates
(c) Both form nitrides
(d) Nitrates of both, Li and Mg, yield NO_2 and O_2 on heating

19. The alkaline earth metal nitrate that does not crystallise with water molecules, is: [2019]

(a) $Mg(NO_3)_2$ (b) $Sr(NO_3)_2$
(c) $Ca(NO_3)_2$ (d) $Ba(NO_3)_2$

20. Magnesium powder burns in air to give: [2019]

(a) $Mg(NO_3)_2$ and Mg_3N_2
(b) MgO and Mg_3N_2
(c) MgO only
(d) MgO and $Mg(NO_3)_2$

Answer Key

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
(a)	(a)	(d)	(a)	(a)	(a)	(b)	(a)	(c)	(d)	(a)	(b)	(c)	(b)	(b)
16	17	18	19	20										
(d)	(b)	(a)	(d)	(b)										

Solutions

1. **(a)** $2KO_2 + 2H_2O \rightarrow 2KOH + H_2O_2 + O_2$.
KO_2 is used as an oxidising agent. It is used as air purifier in space capsules, submarines and breathing masks as it **produces oxygen** and **removes carbon dioxide**.

2. **(a)** The alkali metals dissolve in liquid ammonia without evolution of hydrogen. The metal loses electrons and combine with ammonia molecule.
$M \longrightarrow M^+$ (in liquid ammonia) $+ e^-$ (ammoniated)
$M + (x+y)NH_3 \rightarrow [M(NH_3)_x]^+ + \underset{\text{Solvated electron}}{e^-(NH_3)_y}$
It is ammoniated electron which is responsible for colour.

3. **(d)** Sulphates of alkaline earth metals are sparingly soluble or almost not soluble in water whereas $BeSO_4$ is soluble in water due to high degree of solvation. $Be(OH)_2$ is insoluble in water but soluble in NaOH.
$BeO + 2NaOH \longrightarrow Na_2BeO_2 + H_2O$

4. **(a)** Setting of cement is exothermic process which develops interlocking crystals of hydrated silicates.

5. **(a)** Gypsum is $CaSO_4 \cdot 2H_2O$

6. **(a)** As we move down the group, the lattice energy of carbonates remains approximately the same. However, the hydration energy of the metal cations decrease from Be^{2+} to Ba^{2+}, hence the solubility of carbonates of the alkaline earth metals decreases down the group mainly due to decreasing hydration energy of the cations from Be^{2+} to Ba^{2+}.

7. **(b)** Permanent hardness of water is due to chlorides and sulphates of calcium and magnesium i.e $CaCl_2$, $CaSO_4$, $MgCl_2$ and $MgSO_4$.

8. **(a)** $Mg_3N_2 + 6H_2O \rightleftharpoons 3Mg(OH)_2 + 2NH_3$

9. **(c)** LiCl has partly covalent character. Other halides are ionic in nature. Lattice energy decreases with increase of ionic radius of cation, anion being the same. Larger is the lattice energy, higher will be the m. p., hence NaCl will have highest lattice energy.

10. **(d)** Smaller the size of cation, higher is its hydration energy and greater is its ionic mobility, hence the correct order is $Li^+ < Na^+ < K^+ < Rb^+$

11. **(a)** $4LiNO_3 \rightarrow 2Li_2O + 4NO_2 + O_2$

12. **(b)** $\underset{\text{Base}}{O^{2-}} + \underset{\text{Acid}}{H_2O} \longrightarrow \underset{\text{Conjugate base}}{OH^-} + \underset{\text{Conjugate acid}}{OH^-}$

13. **(c)** $CaCO_3 \xrightarrow{\Delta} \underset{\text{Basic oxide}}{CaO} + \underset{\text{Acidic oxide}}{CO_2\uparrow}$

14. **(b)** $\because$ For $Na \longrightarrow Na^+ + e^-$ $\quad IE_1 = 5.1$ eV
$\therefore$ For $Na^+ + e^- \longrightarrow Na$ $\quad EF = -5.1$ eV
(because the reaction is reverse)

15. **(b)** $Li_2(6) = \sigma 1s^2\, \sigma^* 1s^2\, \sigma 2s^2$
$\therefore$ Bond order $= \frac{1}{2}(4-2) = 1$
$Li_2^+(5) = \sigma 1s^2\, \sigma^* 1s^2\, \sigma 2s^1$
B.O. $= \frac{1}{2}(3-2) = 0.5$
$Li_2^-(7) = \sigma 1s^2\, \sigma^* 1s^2 \sigma 2s^2 \sigma^* 2s^1$
B.O. $= \frac{1}{2}(4-3) = 0.5$
The bond order of Li_2^+ and Li_2^- is same but Li_2^+ is more stable than Li_2^- because Li_2^+ is smaller in size and has 2 electrons in antibonding orbitals whereas Li_2^- has 3 electrons in antibonding orbitals. Hence Li_2^+ is more stable than Li_2^-.

16. **(d)** In alkaline earth metals, ionic size increases down the group. The lattice energy remains constant because sulphate ion is so large, that small change in cationic size does not make any difference. On moving down the group the degree of hydration of metal ions decreases very much leading to decrease in solubility.

17. **(b)** $4Li + O_2 \rightarrow 2Li_2O$ Lithium monoxide
$2Na + O_2 \rightarrow Na_2O_2$ Sodium peroxide
$K + O_2 \rightarrow KO_2$ Potassium superoxide

18. **(a)** Mg can form basic carbonate
$3MgCO_3 . Mg(OH)_2 . 3H_2O \downarrow$
While Li can form only carbonate (Li_2CO_3), not basic carbonate.

19. **(d)** The chances of formation of hydrate decreases with the decrease in the charge density down the group. Therefore, $Ba(NO_3)_2$ does not crystallise with water molecules.

20. **(b)** Mg burns in air and produces a mixture of nitride and oxide.

11 The p-Block Elements (Group-13 and 14)

1. Alum helps in purifying water by **[2002]**
   (a) forming Si complex with clay particles
   (b) sulphate part which combines with the dirt and removes it
   (c) coagulating the mud particles
   (d) making mud water soluble.
2. Graphite is a soft solid lubricant extremely difficult to melt. The reason for this anomalous behaviour is that graphite **[2003]**
   (a) is an allotropic form of diamond
   (b) has molecules of variable molecular masses like polymers
   (c) has carbon atoms arranged in large plates of rings of strongly bound carbon atoms with weak interplate bonds
   (d) is a non-crystalline substance
3. Glass is a **[2003]**
   (a) super-cooled liquid
   (b) gel
   (c) polymeric mixture
   (d) micro-crystalline solid
4. For making good quality mirrors, plates of float glass are used. These are obtained by floating molten glass over a liquid metal which does not solidify before glass. The metal used can be **[2003]**
   (a) tin (b) sodium
   (c) magnesium (d) mercury
5. Beryllium and aluminium exhibit many properties which are similar. But, the two elements differ in **[2004]**
   (a) forming covalent halides
   (b) forming polymeric hydrides
   (c) exhibiting maximum covalency in compounds
   (d) exhibiting amphoteric nature in their oxides
6. Aluminium chloride exists as dimer, Al_2Cl_6 in solid state as well as in solution of non-polar solvents such as benzene. When dissolved in water, it gives **[2004]**
   (a) $[Al(OH)_6]^{3-} + 3HCl$
   (b) $[Al(H_2O)_6]^{3+} + 3Cl^-$
   (c) $Al^{3+} + 3Cl^-$
   (d) $Al_2O_3 + 6HCl$
7. Heating an aqueous solution of aluminium chloride to dryness will give **[2005]**
   (a) $Al(OH)Cl_2$ (b) Al_2O_3
   (c) Al_2Cl_6 (d) $AlCl_3$
8. In silicon dioxide _______. **[2005]**
   (a) there are double bonds between silicon and oxygen atoms
   (b) silicon atom is bonded to two oxygen atoms
   (c) each silicon atom is surrounded by two oxygen atoms and each oxygen atom is bonded to two silicon atoms
   (d) each silicon atom is surrounded by four oxygen atoms and each oxygen atom is bonded to two silicon atoms.
9. The structure of diborane (B_2H_6) contains **[2005]**
   (a) four 2c-2e bonds and four 3c-2e bonds
   (b) two 2c-2e bonds and two 3c-3e bonds
   (c) two 2c-2e bonds and four 3c-2e bonds
   (d) four 2c-2e bonds and two 3c-2e bonds

10. A metal, M forms chlorides in its +2 and +4 oxidation states. Which of the following statements about these chlorides is correct? **[2006]**

(a) MCl_2 is more ionic than MCl_4
(b) MCl_2 is more easily hydrolysed than MCl_4
(c) MCl_2 is more volatile than MCl_4
(d) MCl_2 is more soluble in anhydrous ethanol than MCl_4

11. The stability of dihalides of Si, Ge, Sn and Pb increases steadily in the sequence **[2007]**

(a) $PbX_2 << SnX_2 << GeX_2 << SiX_2$
(b) $GeX_2 << SiX_2 << SnX_2 << PbX_2$
(c) $SiX_2 << GeX_2 << PbX_2 << SnX_2$
(d) $SiX_2 << GeX_2 << SnX_2 << PbX_2$

12. Which one of the following is the correct statement? **[2008]**

(a) Boric acid is a protonic acid
(b) Beryllium exhibits coordination number of six
(c) Chlorides of both beryllium and aluminium have bridged chloride structures in solid phase
(d) $B_2H_6.2NH_3$ is known as 'inorganic benzene'

13. Among the following substituted silanes, which one will give rise to cross linked silicone polymer on hydrolysis is **[2008]**

(a) R_4Si (b) R_2SiCl_2
(c) $RSiCl_3$ (d) R_3SiCl

14. In view of the signs of $\Delta_r G^\circ$ for the following reactions :

$$PbO_2 + Pb \rightarrow 2PbO, \quad \Delta_r G^\circ < 0$$

$$SnO_2 + Sn \rightarrow 2SnO, \quad \Delta_r G^\circ > 0$$

which oxidation states are more characteristics for lead and tin ? **[2011RS]**

(a) For lead + 2, for tin + 2
(b) For lead + 4, for tin + 4
(c) For lead + 2, for tin + 4
(d) For lead + 4, for tin + 2

15. Which of the following are Lewis acids? **[2018]**

(a) PH_3 and BCl_3 (b) $AlCl_3$ and $SiCl_4$
(c) PH_3 and $SiCl_4$ (d) BCl_3 and $AlCl_3$

16. When metal 'M' is treated with NaOH, a white gelatinous precipitate 'X' is obtained, which is soluble in excess of NaOH. Compound 'X' when heated strongly gives an oxide which is used in chromatography as an adsorbent. The metal 'M' is : **[2018]**

(a) Zn (b) Ca (c) Al (d) Fe

17. Correct statements among 'A' to 'D' regarding silicones are: **[2019]**

(A) They are polymers with hydrophobic character.
(B) They are biocompatible.
(C) In general, they have high thermal stability and low dielectric strength.
(D) Usually, they are resistant to oxidation and used as greases.

(a) (A), (B), (C) and (D)
(b) (A), (B) and (C) only
(c) (A) and (B) only
(d) (A), (B) and (D) only

18. Aluminium is usually found in +3 oxidation state. In contrast, thallium exists in +1 and + 3 oxidation states. This is due to: **[2019]**

(a) inert pair effect
(b) diagonal relationship
(c) lattice effect
(d) lanthanoid contraction

19. C_{60}, an allotrope of carbon cantains: **[2019]**

(a) 12 hexagons and 20 pentagons.
(b) 18 hexagons and 14 pentagons.
(c) 16 hexagons and 16 pentagons.
(d) 20 hexagons and 12 pentagons.

Answer Key

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
(c)	(c)	(a)	(d)	(c)	(b)	(b)	(d)	(d)	(a)	(d)	(c)	(c)	(c)	(b, d)
16	17	18	19											
(c)	(d)	(a)	(d)											

Solutions

1. **(c)** Alum furnishes Al^{3+} ions which bring about coagulation of negatively charged clay particles, bacteria etc.

2. **(c)** In graphite, carbon is sp^2 hybridized. Each carbon is thus linked to three other carbon atoms forming hexagonal rings. Since only three electrons of each carbon are used in making hexagonal ring, fourth electron of each carbon is free to move. This makes graphite a good conductor of heat and electricity.

   Further graphite has a two dimensional sheet like structure. These various sheets are held together by van der Waal's force of attraction. which makes it difficult to melt. Further due to these weak forces of attraction, one layer can slip over the other. Which makes graphite soft and a good lubricating agent.

3. **(a)** Glass is a translucent or transparent amorphous supercooled solid solution or we can say super cooled liquid of silicates and borats having a general formula R_2O. $MO . 6 SiO_2$. where R = Na or K and M = Ca, Ba, Zn or Pb.

4. **(d)** It is mercury because it exists as liquid at room temperature.

5. **(c)** The maximum valency of beryllium is +2, while that of aluminium it is +3.

6. **(b)** $Al_2Cl_6 + 12H_2O \rightleftharpoons 2[Al(H_2O)_6]^{3+} + 6Cl^-$

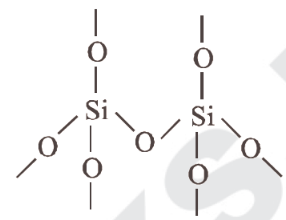
7. **(b)** The solution of aluminium chloride in water is acidic due to hydrolysis.

   $AlCl_3 + 3H_2O \longrightarrow Al(OH)_3 + 3HCl.$

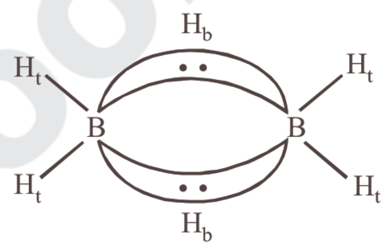
   On heating till dryness, $Al(OH)_3$ is converted into Al_2O_3.

   $$Al(OH)_3 \xrightarrow{\Delta} Al_2O_3 + 3H_2O$$

8. **(d)** In SiO_2 (quartz), each of O-atom is shared between two SiO_4^{4-} tetrahedra.



9. **(d)** In diborane structure of B_2H_6, there are four 2c–2e bonds and two 3c–2e bonds (see structure of diborane).



H_t = terminal hydrogen

H_b = bridging hydrogen

10. **(a)** Metal atom in the lower oxidation state forms ionic bond whereas in the higher oxidation state forms covalent bond, because higher oxidation state means small size and high polarizing power and hence greater the covalent character. So, MCl_2 is more ionic than MCl_4.

11. **(d)** Reluctance of valence shell electrons to participate in bonding is called inert pair effect. The stability of lower oxidation state (+2 for group 14 elements) increases on going down the group. So the correct order is $SiX_2 < GeX_2 < SnX_2 < PbX_2$

12. **(c)** The correct formula of inorganic benzene is $B_3N_3H_6$ so (d) is incorrect statement

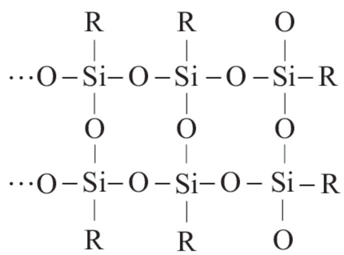
    Boric acid (H_3BO_3 or $\begin{array}{c} OH \\ | \\ B-OH \\ | \\ OH \end{array}$) is a Lewis acid so (a) is incorrect statement.

The coordination number exhibited by beryllium is 4 and not 6 so statement (b) is also incorrect.

Both $BeCl_2$ and $AlCl_3$ exhibit bridged structures in solid state, so (c) is correct statement.

13. (c) The cross linked polymers will be formed by $RSiCl_3$

$$nRSiCl_3 \xrightarrow[-3nHCl]{3nH_2O} nR-\underset{\substack{|\\OH}}{\overset{\substack{OH\\|}}{Si}}-OH \longrightarrow$$



(Cross linked polymer)

14. (c) Negative $\Delta_r G^\circ$ value indicates that + 2 oxidation state is more stable for Pb^{2+}. Also it is supported by inert pair effect that + 2 oxidation state is more stable for Pb and + 4 oxidation state is more stable for Sn.

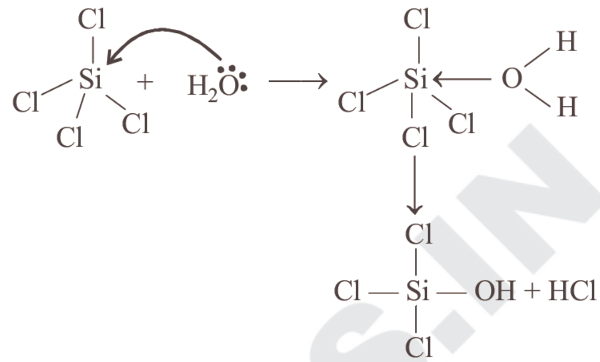
i.e. $Sn^{2+} < Pb^{2+}$, $Sn^{4+} > Pb^{4+}$

15. (b, d) BCl_3 and $AlCl_3$, both have vacant *p*-orbital and incomplete octet, thus they behave as Lewis acids.

$SiCl_4$ can accept lone pair of electron in *d*-orbital of silicon, hence it can act as Lewis acid.

Although the most suitable answer is (d). However, both options (b) and (d) can be considered as correct answers.

e.g. hydrolysis of $SiCl_4$



i.e., option (b) $AlCl_3$ and $SiCl_4$ is also correct.

16. (c)

$$Al + 3H_2O \xrightarrow{NaOH} \underset{\substack{(X)\\ \text{White gelatinous ppt.}}}{Al(OH)_3} \downarrow + 3/2H_2(g)$$

$$Al(OH)_3 \xrightarrow{\text{excess of NaOH}} \underset{\text{Soluble}}{Na[Al(OH)_4]}$$

$$\underset{(X)}{2Al(OH)_3} \xrightarrow{\Delta} Al_2O_3 + 3H_2O$$

Al_2O_3 is used as adsorbent in chromatography. Thus, metal 'M' is Al.

17. (d) Silicones are polymers containing Si—O—Si linkages with strong hydrophobic character.

Generally, they exhibit high thermal stability with high dielectric strength. Silicon greases are resistant to oxidation which are commonly used for greasing purposes.

18. (a) Due to the inert pair effect, thallium exists in more than one oxidation state. Also, for thallium + 1 oxidation state is more stable than +3 oxidation state.

19. (d) Fullerene (C_{60}) contains 20 hexagons (six membered) rings and 12 pentagons (five membered rings):

12 Organic Chemistry- Some Basic Principles & Techniques

1. Arrangement of $(CH_3)_3-C-$, $(CH_3)_2-CH-$, CH_3-CH_2- when attached to benzyl or an unsaturated group in increasing order of inductive effect is **[2002]**
   (a) $(CH_3)_3-C-<(CH_3)_2-CH-<CH_3-CH_2-$
   (b) $CH_3-CH_2-<(CH_3)_2-CH-<(CH_3)_3-C-$
   (c) $(CH_3)_2-CH-<(CH_3)_3-C-<CH_3-CH_2-$
   (d) $(CH_3)_3-C-<CH_3-CH_2-<(CH_3)_2-CH-$

2. A similarity between optical and geometrical isomerism is that **[2002]**
   (a) each forms equal number of isomers for a given compound
   (b) if in a compound one is present then so is the other
   (c) both are included in stereoisomerism
   (d) they have no similarity.

3. Which of the following does not show geometrical isomerism? **[2002]**
   (a) 1,2-dichloro-1-pentene
   (b) 1,3-dichloro-2-pentene
   (c) 1,1-dichloro-1-pentene
   (d) 1,4-dichloro-2-pentene

4. The functional group, which is found in amino acid is **[2002]**
   (a) $-COOH$ group (b) $-NH_2$ group
   (c) $-CH_3$ group (d) both (a) and (b).

5. Which of the following compounds has wrong IUPAC name? **[2002]**
   (a) $CH_3-CH_2-CH_2-COO-CH_2CH_3 \rightarrow$
   ethyl butanoate
   (b) $CH_3-\underset{\displaystyle CH_3}{\underset{|}{C}H}-CH_2-CHO \rightarrow$
   3-methyl-butanal
   (c) $CH_3-\underset{\displaystyle OH}{\underset{|}{C}H}-\underset{\displaystyle CH_3}{\underset{|}{C}H}-CH_3 \rightarrow$
   2-methyl-3-butanol
   (d) $CH_3-\underset{\displaystyle CH_3}{\underset{|}{C}H}-\overset{\displaystyle O}{\overset{\|}{C}}-CH_2-CH_3 \rightarrow$
   2-methyl-3-pentanone

6. The IUPAC name of $CH_3COCH(CH_3)_2$ is **[2002]**
   (a) 2-methyl-3-butanone
   (b) 4-methylisopropyl ketone
   (c) 3-methyl-2-butanone
   (d) isopropylmethyl ketone

7. In which of the following species is the underlined carbon having sp^3 hybridisation? **[2002]**
   (a) $CH_3\underline{C}OOH$ (b) $CH_3\underline{C}H_2OH$
   (c) $CH_3\underline{C}OCH_3$ (d) $CH_2=\underline{C}H-CH_3$

8. Racemic mixture is formed by mixing two **[2002]**
   (a) isomeric compounds
   (b) chiral compounds
   (c) meso compounds
   (d) enantiomers with chiral carbon.

9. Following types of compounds (as I, II) **[2002]**

   $CH_3CH=CHCH_3$ (I) $\quad$ $CH_3\underset{\displaystyle CH_2CH_3}{\underset{|}{C}}HOH$ (II)

   are studied in terms of isomerism in:
   (a) chain isomerism
   (b) position isomerism
   (c) conformers
   (d) stereoisomerism

10. The reaction: **[2002]**

$(CH_3)_3C-Br \xrightarrow{H_2O} (CH_3)_3-C-OH$

(a) elimination reaction
(b) substitution reaction
(c) free radical reaction
(d) addition reaction.

11. In the anion $HCOO^-$ the two carbon-oxygen bonds are found to be of equal length. What is the reason for it ? **[2003]**

(a) The $C = O$ bond is weaker than the $C — O$ bond
(b) The anion $HCOO^-$ has two resonating structures
(c) The anion is obtained by removal of a proton from the acid molecule
(d) Electronic orbitals of carbon atom are hybridised

12. The general formula $C_nH_{2n}O_2$ could be for open chain **[2003]**

(a) carboxylic acids (b) diols
(c) dialdehydes (d) diketones

13. Among the following four structures I to IV, **[2003]**

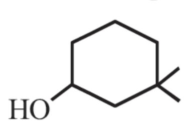
$C_2H_5-\underset{\text{(I)}}{\overset{\overset{\displaystyle CH_3}{|}}{CH}}-C_3H_7$, $CH_3-\overset{\overset{\displaystyle O}{\|}}{C}-\overset{\overset{\displaystyle CH_3}{|}}{CH}-C_2H_5$ (II),

$H-\underset{\underset{\displaystyle H}{|}}{\overset{\overset{\displaystyle H}{|}}{C}}^{\oplus}$ (III), $C_2H_5-\overset{\overset{\displaystyle CH_3}{|}}{CH}-C_2H_5$ (IV)

it is true that

(a) only I and II are chiral compounds
(b) only III is a chiral compound
(c) only II and IV are chiral compounds
(d) all four are chiral compounds

14. The IUPAC name of the compound is



(a) 3, 3-dimethyl - 1- cyclohexanol **[2004]**
(b) 1, 1-dimethyl-3-hydroxy cyclohexane
(c) 3, 3-dimethyl-1-hydroxy cyclohexane
(d) 1, 1-dimethyl-3-cyclohexanol

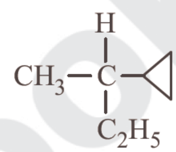
15. Which one of the following does not have sp^2 hybridised carbon ? **[2004]**

(a) Acetonitrile (b) Acetic acid
(c) Acetone (d) Acetamide

16. Which of the following will have a meso isomer also? **[2004]**

(a) 2, 3- Dichloropentane
(b) 2, 3-Dichlorobutane
(c) 2-Chlorobutane
(d) 2-Hydroxypropanoic acid

17. Amongst the following compounds, the optically active alkane having lowest molecular mass is **[2004]**

(a) 

(b) $CH_3-CH_2-\overset{\overset{\displaystyle CH_3}{|}}{C}H-CH_3$

(c) $CH_3-CH_2-CH_2-CH_3$

(d) $CH_3-CH_2-C\equiv CH$

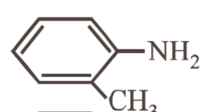
18. Consider the acidity of the carboxylic acids :

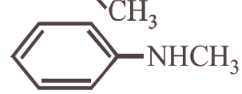
(1) PhCOOH
(2) o-$NO_2C_6H_4COOH$
(3) p-$NO_2C_6H_4COOH$
(4) m-$NO_2C_6H_4COOH$

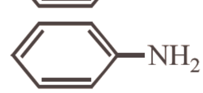
Which of the following order is correct? **[2004]**

(a) $2 > 4 > 1 > 3$ (b) $2 > 4 > 3 > 1$
(c) $1 > 2 > 3 > 4$ (d) $2 > 3 > 4 > 1$

19. Which of the following is the strongest base ? **[2004]**

(a) 

(b) 

(c) 

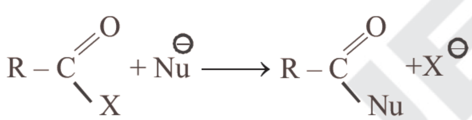
(d) 

20. Which of the following compounds is not chiral? **[2004]**
(a) 1-chloro-2-methyl pentane
(b) 2-chloropentane
(c) 1-chloropentane
(d) 3-chloro-2-methyl pentane

21. Due to the presence of an unpaired electron, free radicals are: **[2005]**
(a) cations
(b) anions
(c) chemically inactive
(d) chemically reactive

22. The decreasing order of nucleophilicity among the nucleophiles is **[2005]**
(A) CH_3COO^-
(B) CH_3O^-
(C) CN^-
(D)  is
(a) (C), (B), (A), (D) (b) (B), (C), (A), (D)
(c) (D), (C), (B), (A) (d) (A), (B), (C), (D)

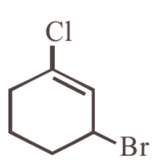
23. The reaction **[2004, 2005]**



is fastest when X is
(a) OCOR (b) OC_2H_5
(c) NH_2 (d) Cl

24. Which types of isomerism is shown by 2, 3-dichlorobutane? **[2005]**
(a) Structural (b) Geometric
(c) Optical (d) Diastereo

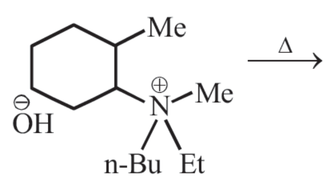
25. The IUPAC name of the compound shown below is : **[2006]**

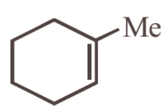
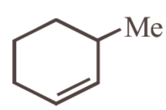


(a) 3-bromo-1-chlorocyclohexene
(b) 1-bromo-3-chlorocyclohexene
(c) 2-bromo-6-chlorocyclohex-1-ene
(d) 6-bromo-2-chlorocyclohexene

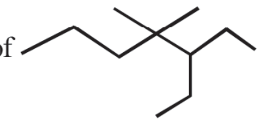
26. The increasing order of stability of the following free radicals is **[2006]**
(a) $(C_6H_5)_2\dot{C}H < (C_6H_5)_3\dot{C} < (CH_3)_3\dot{C} < (CH_3)_2\dot{C}H$
(b) $(CH_3)_2\dot{C}H < (CH_3)_3\dot{C} < (C_6H_5)_2\dot{C}H < (C_6H_5)_3\dot{C}$
(c) $(CH_3)_3\dot{C} < (CH_3)_2\dot{C}H < (C_6H_5)_2\dot{C}H < (C_6H_5)_3\dot{C}$
(d) $(C_6H_5)_3\dot{C} < (C_6H_5)_2\dot{C}H < (CH_3)_3\dot{C} < (CH_3)_2\dot{C}H$

27. $CH_3Br + Nu^- \longrightarrow CH_3 - Nu + Br^-$
The decreasing order of the rate of the above reaction with nucleophiles (Nu^-) A to D is **[2006]**
[Nu^- = (A) PhO^-, (B) AcO^-, (C) HO^-, (D) CH_3O^-]
(a) A > B > C > D (b) B > D > C > A
(c) D > C > A > B (d) D > C > B > A

28. 

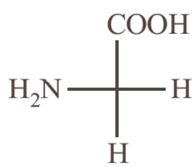
The alkene formed as a major product in the above elimination reaction is **[2006]**
(a)  (b) 
(c)  (d) $CH_2 = CH_2$

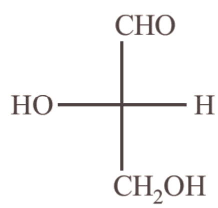
29. Increasing order of stability among the three main conformations (i.e. Eclipse, Anti, Gauche) of 2-fluoroethanol is **[2006]**
(a) Eclipse, Anti, Gauche
(b) Anti, Gauche, Eclipse
(c) Eclipse, Gauche, Anti
(d) Gauche, Eclipse, Anti

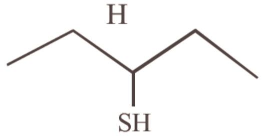
30. The IUPAC name of  is **[2007]**

(a) 3-ethyl-4,4-dimethylheptane
(b) 1, 1-diethyl-2,2-dimethylpentane
(c) 4, 4-dimethyl-5,5-diethylpentane
(d) 5, 5-diethyl-4,4-dimethylpentane.

31. Which of the following molecules is expected to rotate the plane of plane-polarised light? **[2007]**

(a) 

(b) 

(c) 

(d) 

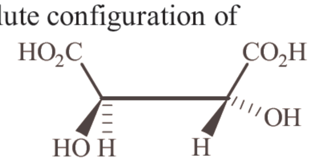
32. Presence of a nitro group in a benzene ring **[2007]**

(a) deactivates the ring towards electrophilic substitution
(b) activates the ring towards electrophilic substitution
(c) renders the ring basic
(d) deactivates the ring towards nucleophilic substitution.

33. Which one of the following conformation of cyclohexane is chiral? **[2007]**

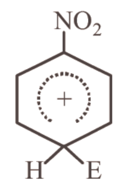
(a) Boat (b) Twist boat
(c) Rigid (d) Chair.

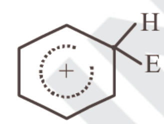
34. The absolute configuration of **[2008]**

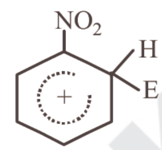


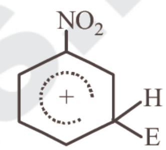
(a) S, S (b) R, R
(c) R, S (d) S, R

35. The electrophile, $E^{\oplus}$ attacks the benzene ring to generate the intermediate σ–complex. Of the following, which σ–complex is lowest energy? **[2008]**

(a) 

(b) 

(c) 

(d) 

36. The correct decreasing order of priority for the functional groups of organic compounds in the IUPAC system of nomenclature is **[2008]**

(a) $-COOH, -SO_3H, -CONH_2, -CHO$
(b) $-SO_3H, -COOH, -CONH_2, -CHO$
(c) $-CHO, -COOH, -SO_3H, -CONH_2$
(d) $-CONH_2, -CHO, -SO_3H, -COOH$

37. The IUPAC name of neopentane is **[2009]**

(a) 2, 2 dimethylpropane
(b) 2- methylpropane
(c) 2, 2-dimethylbutane
(d) 2- methylbutane

38. Arrange the carbanions, **[2009]**

$(CH_3)_3\overline{C}, \overline{C}Cl_3, (CH_3)_2\overline{C}H, C_6H_5\overline{C}H_2$

order of their decreasing stability is

(a) $(CH_3)_2\overline{C}H > \overline{C}Cl_3 > C_6H_5\overline{C}H_2 > (CH_3)_3\overline{C}$
(b) $\overline{C}Cl_3 > C_6H_5\overline{C}H_2 > (CH_3)_2\overline{C}H > (CH_3)_3\overline{C}$
(c) $(CH_3)_3\overline{C} > (CH_3)_2\overline{C}H > C_6H_5\overline{C}H_2 > \overline{C}Cl_3$
(d) $C_6H_5\overline{C}H_2 > \overline{C}Cl_3 > (CH_3)_3\overline{C} > (CH_3)_2\overline{C}H$

39. The alkene that exhibits geometrical isomerism is: **[2009]**

(a) 2- methyl propene
(b) 2-butene
(c) 2- methyl -2- butene
(d) propene

40. The number of stereoisomers possible for a compound of the molecular formula $CH_3-CH=CH-CH(OH)-Me$ is: **[2009]**

(b) 2 (c) 4
(d) 6 (d) 3

41. The correct order of increasing basicity of the given conjugate bases ($R = CH_3$) is **[2010]**

(a) $RCO\bar{O} < HC \equiv \bar{C} < \bar{R} < \bar{N}H_2$

(b) $\bar{R} < HC \equiv \bar{C} < RCO\bar{O} < \bar{N}H_2$

(c) $RCO\bar{O} < \bar{N}H_2 < HC \equiv \bar{C} < \bar{R}$

(d) $RCO\bar{O} < HC \equiv \bar{C} < \bar{N}H_2 < \bar{R}$

42. Out of the following, the alkene that exhibits optical isomerism is **[2010]**

(a) 3-methyl-2-pentene

(b) 4-methyl-1-pentene

(c) 3-methyl-1-pentene

(d) 2-methyl-2-pentene

43. The change in the optical rotation of freshly prepared solution of glucose is known as: **[2011RS]**

(a) racemisation (b) specific rotation

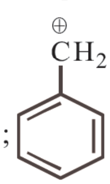
(c) mutarotation (d) tautomerism

44. A solution of (–) – 1 – chloro – 1– phenylethane in toluene racemises slowly in the presence of a small amount of $SbCl_5$, due to the formation of: **[2013]**

(a) carbanion (b) carbene

(c) carbocation (d) free radical

45. The order of stability of the following carbocations : **[2013]**

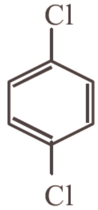
$\underset{\text{I}}{CH_2 = CH - \overset{\oplus}{C}H_2}$; $\underset{\text{II}}{CH_3 - CH_2 - \overset{\oplus}{C}H_2}$;  is :

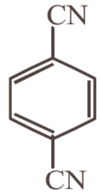
III

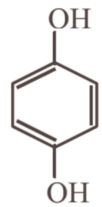
(a) III > II > I (b) II > III > I

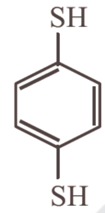
(c) I > II > III (d) III > I > II

46. For which of the following molecule significant $\mu \neq 0$? **[2014]**

(i) 

(ii) 

(iii) 

(iv) 

(a) Only (i) (b) (i) and (ii)

(c) Only (iii) (d) (iii) and (iv)

47. Which of the following compounds will exhibit geometrical isomerism ? **[2015]**

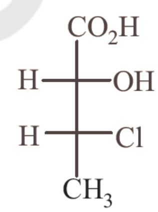
(a) 2 - Phenyl -1 - butene

(b) 1, 1 - Diphenyl - 1 - propene

(c) 1 - Phenyl - 2 - butene

(d) 3 - Phenyl -1 - butene

48. The absolute configuration of **[2016]**

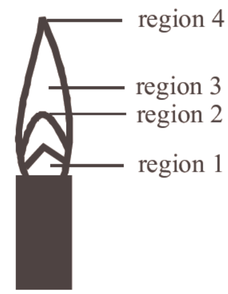


is :

(a) (2S, 3S) (b) (2R, 3R)

(c) (2R, 3S) (d) (2S, 3R)

49. The hottest region of Bunsen flame shown in the figure below is : **[2016]**



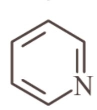
(a) region 3 (b) region 4

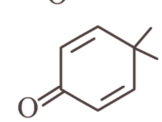
(c) region 1 (d) region 2

50. Which of the following molecules is least resonance stabilized? **[2017]**

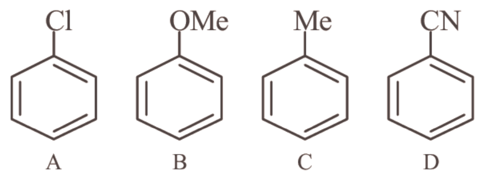
(a) 

(b) 

(c) 

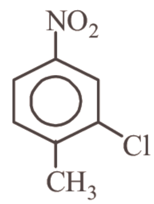
(d) 

51. The increasing order of reactivity of the following compounds towards aromatic electrophilic substitution reaction is: **[2019]**



(a) D < A < C < B (b) B < C < A < D
(c) A < B < C < D (d) D < B < A < C

52. The correct IUPAC name of the following compound is: **[2019]**



(a) 5-chloro-4-methyl-1-nitrobenzene
(b) 2-chloro-1-methyl-4-nitrobenzene
(c) 3-chloro-4-methyl-1-nitrobenzene
(d) 2-methyl-5-nitro-1-chlorobenzene

Answer Key

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
(b)	(c)	(c)	(d)	(c)	(c)	(b)	(d)	(d)	(b)	(b)	(a)	(a)	(a)	(a)
16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
(b)	(a)	(d)	(d)	(c)	(d)	(a)	(d)	(c)	(a)	(b)	(c)	(d)	(a)	(a)
31	32	33	34	35	36	37	38	39	40	41	42	43	44	45
(b)	(a)	(b)	(b)	(b)	(a)	(a)	(b)	(b)	(b)	(d)	(c)	(c)	(c)	(d)
46	47	48	49	50	51	52								
(d)	(c)	(d)	(d)	(d)	(a)	(b)								

Solutions

1. **(b)** $-CH_3$ group has +I effect, as number of $-CH_3$ group increases, the inductive effect increases.
Therefore the correct order is
$CH_3-CH_2- < (CH_3)_2-CH- < (CH_3)_3C-$

2. **(c)** Both differ in the arrangement of group in space, therefore grouped under sterio-isomerism.

3. **(c)** $\overset{1}{C}lCH=\overset{2}{C}(Cl)\,\overset{3}{C}H_2\overset{4}{C}H_2\overset{5}{C}H_3$
(a)

$Cl\overset{1}{C}H_2\overset{2}{C}H=\overset{3}{C}(Cl)-\overset{4}{C}H_2\overset{5}{C}H_3$
(b)

$Cl_2\overset{1}{C}=\overset{2}{C}H\overset{3}{C}H_2\overset{4}{C}H_2\overset{5}{C}H_3$
(c)

$Cl\overset{1}{C}H_2\overset{2}{C}H=\overset{3}{C}H\overset{4}{C}H(Cl)\overset{5}{C}H_3$
(d)

does not show geometrical isomerism due to presence of two similar atoms (Cl) on the doubly bonded C-atom.

4. **(d)** Amino acids contain $-NH_2$ and $-COOH$ groups, e.g glycine H_2NCH_2COOH.

5. **(c)** The correct name is 3 - methylbutan - 2 - ol

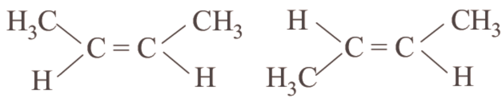
6. **(c)** $\overset{1}{C}H_3-\overset{2}{C}(=O)-\overset{3}{C}H(CH_3)-\overset{4}{C}H_3$
3-Methyl-2-butanone

7. **(b)** In molecules (a), (c) and (d), the carbon atom has a multiple bond, only (b) has sp^3 hybridisation.

8. **(d)** A mixture of equal amount of two enantiomers is called a racemic mixture.

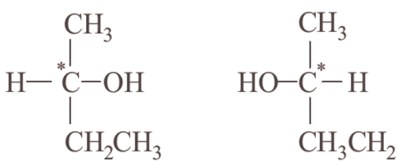
9. (d) Stereoisomerism, isomers differ in the arrangement of groups in space (Q2)

The two structures show sterioisomerism. Structure I shows geometrical isomerism as it contains two different atoms or groups H and CH_3 attached to each carbon containing double bond.



cis-butene *trans*-butene

Structure II shows optical isomerism as it contains a chiral carbon (attached to four different groups) atom.



Butyl alcohol (two enantiomers)

10. (b) The hydrolysis of *t*-butyl bromide is an example of S_N1 reaction.

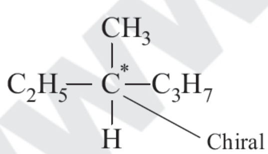
11. (b) $HCOO^-$ exists in following resonating structures

$$H-\overset{\overset{O}{\|}}{C}-O^- \longleftrightarrow H-\overset{\overset{O^{\ominus}}{|}}{C}=O$$

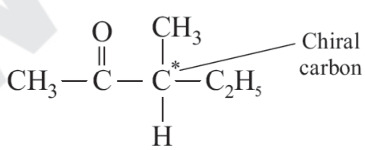
Hence in it both the carbon oxygen bonds are found equal.

12. (a) $C_nH_{2n}O_2$ is general formula for carboxylic acid

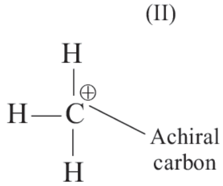
13. (a)



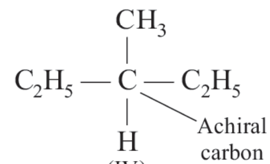
(I)



(II)

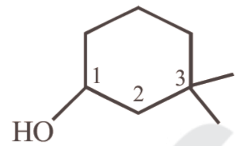


(III)



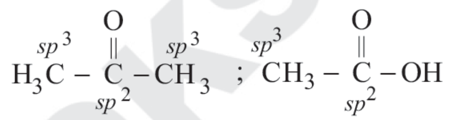
(IV)

14. (a)

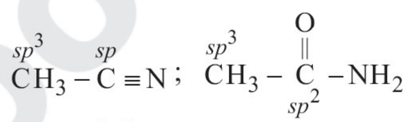


3, 3-Dimethyl -1 cyclohexanol

15. (a)

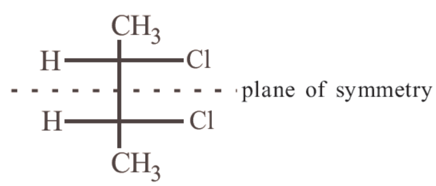


Acetone Acetic acid



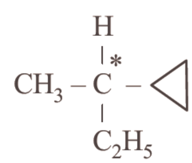
Acetonitrile Acetamide

16. (b) **NOTE** Compounds containing two similar chiral C-atoms have plane of symmetry and can exist in *meso* form too.



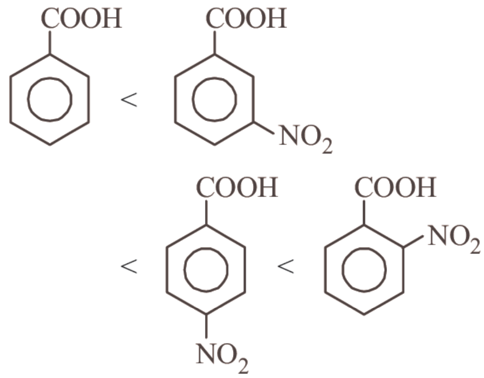
meso-2, 3-Dichlorobutane

17. (a) Only 2-cylcopropyl butane has a chiral centre.



18. (d) In carboxylic acids, presence of electron withdrawing substituent e.g. $-NO_2$ disperses the negative charge of the anion and stabilises it and hence increases the acidity of the parent acid.

Further *o*-isomer will have higher acidity than corresponding *m*-and *p*-isomers due to ortho and high inductive effect of $-NO_2$ group. Since nitro group at *p*-position has more pronounced electron withdrawing than $-NO_2$ group at *m*-position, hence the correct order is:



19. **(d)** Lone pair of electrons present on the nitrogen of benzyl amine is not involved in resonance.

20. **(c)** 1-chloropentane is not chiral while others are chiral in nature

$$Cl\overset{1}{C}H_2\overset{*2}{\underset{CH_3}{C}}H\overset{3}{C}H_2\overset{4}{C}H_2\overset{5}{C}H_3$$
(a)

$$H_3\overset{1}{C}\overset{*2}{\underset{Cl}{C}}H\overset{3}{C}H_2\overset{4}{C}H_2\overset{5}{C}H_3$$
(b)

$$Cl\overset{1}{C}H_2\overset{2}{C}H_2\overset{3}{C}H_2\overset{4}{C}H_2\overset{5}{C}H_3$$
(c)

$$\overset{1}{C}H_3\overset{2}{\underset{CH_3}{C}}H\overset{*3}{\underset{Cl}{C}}H\overset{4}{C}H_2\overset{5}{C}H_3$$
(d)

21. **(d)** Free radicals are electrically neutral, unstable and very reactive on account of the presence of odd electrons.

22. **(a)** In moving down a group, the basicity and nucleophilicity are inversely related, *i.e.* nucleophilicity increases while basicity decreases. In going from left to right across a period, the basicity and nucleophilicity are directly related. Both of the characteristics decrease as the electronegativity of the atom bearing lone pair of electrons increases. If the nucleophilic centre of two or more species is same, nucleophilicity parallels basicity, *i.e.* more basic the species, stronger is its nucleophilicity.

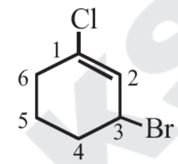
Hence based on the above facts, the correct order of nucleophilicity will be

$$\underset{(C)}{CN^-} > \underset{(B)}{CH_3O^-} > \underset{(A)}{CH_3COO^-} > \underset{(D)}{H_3CC_6H_4SO_3^-}$$

23. **(d)** —Cl is the best leaving group among the given options.

24. **(c)** $CH_3-\underset{H}{\overset{Cl}{C}}-\underset{H}{\overset{Cl}{C}}-CH_3$. 2, 3-Dichlorobutane exhibits optical isomerism due to the presence of two asymmetric carbon atoms.

25. **(a)**



3-Bromo-1-chlorocyclohexene

26. **(b)** The order of stability of free radicals

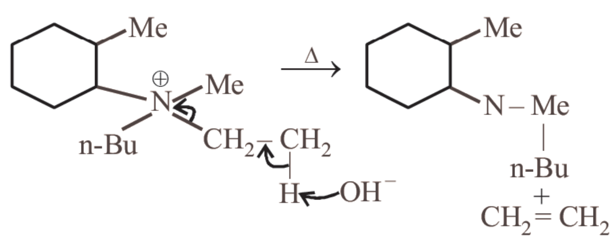
$$(C_6H_5)_3\dot{C} > (C_6H_5)_2\dot{C}H > (CH_3)_3\dot{C} > (CH_3)_2\dot{C}H$$

The stabilisation of first two is due to resonance and last two is due to hyper conjugation.

27. **(c)**

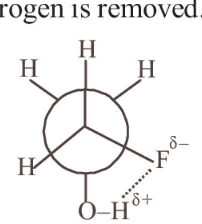
$$\underset{e^-s\text{ are delocalised}}{CH_3COO^-} < C_6H_5O^- < OH^- < \underset{\text{Max. }e^-\text{density on O}}{{}^-OCH_3}$$

28. **(d)** **Hofmann's rule :** When theoretically more than one type of alkenes are possible in eliminations reaction, the alkene containing least alkylated (least substituted) double bond is formed as major product. Hence



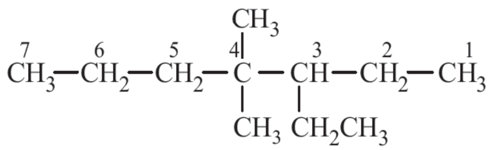
NOTE Therefore less sterically hundred β-hydrogen is removed.

29. **(a)**



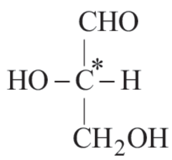
Due to hydrogen bonding between H & F gauche conformation is most stable hence the correct order is Eclipse, Anti, Gauche

30. **(a)**



3-Ethyl-4,4-dimethylheptane

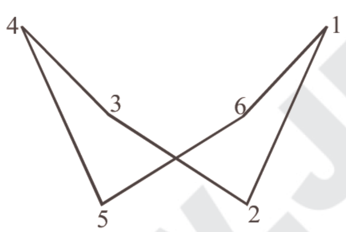
31. **(b)** The organic compounds which have chiral carbon atom (a carbon atom attached to four different groups or atoms) and do not have plane of symmetry rotate plane polarised light.



(* is asymmetric carbon)

32. **(a)** Nitro group is electron withdrawing group, so it deactivates the ring towards electrophilic substitution.

33. **(b)** Chiral conformation will not have plane of symmetry. Since twist boat does not have plane of symmetry, it is chiral.



34. **(b)** The absolute configuration is (R, R)

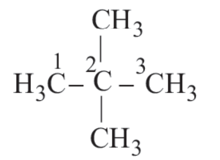
(use priority rules to get the absolute configuration)

35. **(b)** In option (b) the complex formed is with benzene whereas in other cases it is formed with nitrobenzene with $-NO_2$ group in different positions (*o*-, *m*-, *p*-). The complex formed with nitrobenzene in any position of $-NO_2$ group is less stable than the complex formed with benzene, so the most stable complex has lowest energy.

36. **(a)** The correct order of priority for the given functional group is

$$-COOH > -SO_3H > -\overset{O}{\overset{\|}{C}}-NH_2 > -\overset{O}{\overset{\|}{C}}-H$$

37. **(a)**



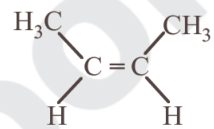
Neopentane
or 2,2- Dimethylpropane

38. **(b)**

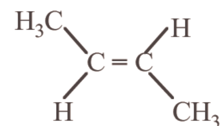
$$\overset{\ominus}{C}Cl_3 > C_6H_5\overset{-}{C}H_2 > (CH_3)_2\overset{-}{C}H > (CH_3)_3\overset{-}{C}$$

–ve charge highly dispersed due to – I effect; –M effect delocalises –ve charge; +I effect of CH_3 group intensifies the –ve charge

39. **(b)**



cis-2-butene



trans-2-butene

40. **(b)** $CH_3-CH=CH-\overset{*}{C}H(OH)CH_3$

It exhibits both geometrical as well as optical isomerism.

cis - R   *cis* - S

trans - R   *trans* - S

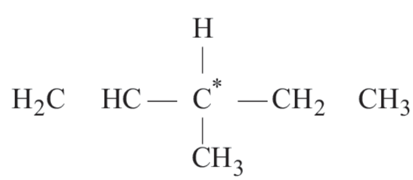
41. **(d)** The corresponding conjugate acid are

$$\underset{\text{Most acidic}}{CH_3COOH} > CH\equiv CH > NH_3 > \underset{\text{Least acidic}}{RH}$$

$\therefore$ the correct order of basicity is

$$RCOO^- < CH\equiv C^- < NH_2^- < R^-$$

42. **(c)**



3- Methyl-1-pentene

43. **(c)** When either of the two forms of glucose is dissolved in water, there is change in rotation till the equilibrium value of + 52.5º. This is known as mutarotation

$$\underset{+111.5^\circ}{\alpha\text{–D(+) Glucose}} \rightleftharpoons \underset{+52.5^\circ}{\text{Equilibrium mixture}} \rightleftharpoons \underset{+19.5^\circ}{\beta\text{–D–(+)Glucose}}$$

44. (c) Carbocations are planar hence can be attacked on either side to form racemic mixture.

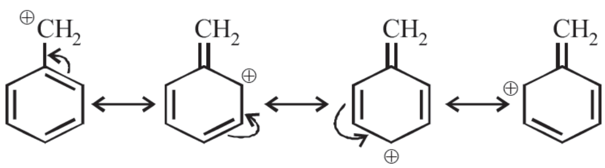
$$\underset{\underset{(-)}{\text{Ph}}}{\text{Cl}-\text{CH}-\text{CH}_3} \xrightarrow[\text{Toluene}]{\text{SbCl}_5} \underset{\text{(carbocation)}}{\text{Ph}-\overset{\oplus}{\text{C}}\text{H}-\text{CH}_3} + \text{SbCl}_6^- \longrightarrow$$

$$\underset{(d+\ell)\text{ mixture}}{\text{Ph}-\underset{\text{Cl}}{\text{CH}}-\text{CH}_3} + \text{SbCl}_5$$

45. (d) Higher stability of allyl and benzyl carbocations is due to dispersal of positive charge by resonance

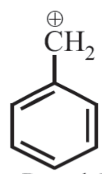
$$\text{CH}_2=\text{CH}-\overset{\oplus}{\text{C}}\text{H}_2 \longleftrightarrow \overset{\oplus}{\text{C}}\text{H}_2-\text{CH}=\text{CH}_2$$

Resonating structures of allyl carbocation



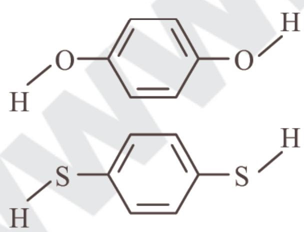
Resonating structures of benzyl carbocation

whereas in alkyl carbocations dispersal of positive charge on different hydrogen atoms is due to inductive effect. Hence the correct order of stability will be

 $> \text{CH}_2=\text{CH}-\overset{\oplus}{\text{C}}\text{H}_2 > \text{CH}_3-\text{CH}_2-\overset{\oplus}{\text{C}}\text{H}_2$

Benzyl, III   Allyl, I   Propyl, II

46. (d)

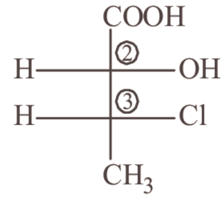


In both the molecules the bond moments are not cancelling with each other and hence the molecules has a resultant dipole.

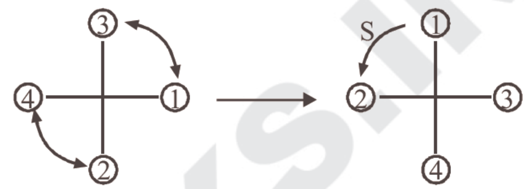
47. (c) $\text{H}_3\text{C}-\overset{\text{H}}{\overset{|}{\text{C}}}=\text{CH}-\text{CH}_2\text{Ph}$

In 1-phenyl-2-butene, the two groups around the doubly bonded carbons are different. This compound can show *cis*-and *trans*-isomerism.

48. (d)

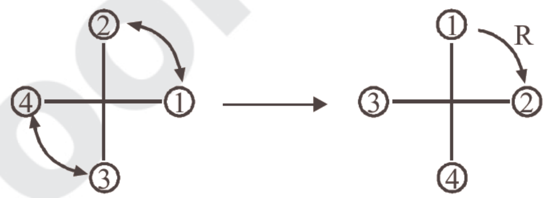


At (2),



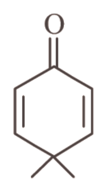
'S'configuration

At (3),

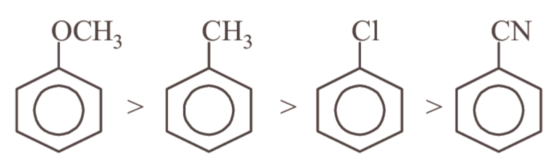


'p'configuration

49. (d) Region 2 (blue flame) will be the hottest region of Bunsen flame shown in given figure.

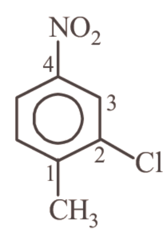
50. (d)  is nonaromatic and hence least reasonance stabilized, whereas other three are aromatic.

51. (a)



(+M) > (+I and +H) > (–I and +M) > (–M, –I)

52. (b)



2-Chloro-1-methyl-4-nitrobenzene

13 Hydrocarbons

1. Which of these will not react with acetylene? **[2002]**
 (a) dil.NaOH (b) ammonical $AgNO_3$
 (c) Na in liq.NH_3 (d) HCl

2. What is the product when acetylene reacts with hypochlorous acid? **[2002]**
 (a) CH_3COCl (b) $ClCH_2CHO$
 (c) Cl_2CHCHO (d) $ClCH_2COOH$.

3. On mixing a certain alkane with chlorine and irradiating it with ultraviolet light, it forms only one monochloroalkane. This alkane could be **[2003]**
 (a) pentane (b) isopentane
 (c) neopentane (d) propane

4. Butene-1 may be converted to butane by reaction with **[2003]**
 (a) $Sn - HCl$ (b) $Zn - Hg$
 (c) Pd/H_2 (d) $Zn - HCl$

5. Which one of the following has minimum boiling point? **[2004]**
 (a) 1 - Butene (b) 1 - Butyne
 (c) *n*- Butane (d) Isobutane

6. 2-Methylbutane on reacting with bromine in the presence of sunlight gives mainly **[2005]**
 (a) 1-bromo-3-methylbutane
 (b) 2-bromo-3-methylbutane
 (c) 2-bromo-2-methylbutane
 (d) 1-bromo-2-methylbutane

7. Reaction of one molecule of HBr with one molecule of 1, 3-butadiene at 40°C gives predominantly **[2005]**
 (a) 1-bromo-2-butene under kinetically controlled conditions
 (b) 3-bromobutene under thermodynamically controlled conditions
 (c) 1-bromo-2-butene under thermodynamically controlled conditions
 (d) 3-bromobutene under kinetically controlled conditions

8. Of the five isomeric hexanes, the isomer which can give two monochlorinated compounds is **[2005]**
 (a) 2-methylpentane
 (b) 2, 2-dimethylbutane
 (c) 2, 3-dimethylbutane
 (d) *n*-hexane

9. The compound formed as a result of oxidation of ethylbenzene by $KMnO_4$ is **[2007]**
 (a) benzyl alcohol (b) benzophenone
 (c) acetophenone (d) benzoic acid

10. Which of the following reactions will yield 2, 2-dibromopropane? **[2007]**
 (a) $CH_3 - CH = CH_2 + HBr \rightarrow$
 (b) $CH_3 - C \equiv CH + 2HBr \rightarrow$
 (c) $CH_3CH = CHBr + HBr \rightarrow$
 (d) $CH \equiv CH + 2HBr \rightarrow$

11. The reaction of toluene with Cl_2 in presence of $FeCl_3$ gives predominantly **[2007]**
 (a) *m*-chlorobenzene
 (b) benzoyl chloride
 (c) benzyl chloride
 (d) *o*- and *p*-chlorotoluenes.

12. Toluene is nitrated and the resulting product is reduced with tin and hydrochloric acid. The product so obtained is diazotised and then heated wth cuprous bromide. The reaction mixture so formed contains **[2008]**
 (a) mixture of *o*- and *p*-bromotoluenes
 (b) mixture of *o*- and *p*-dibromobenzenes
 (c) mixture of *o*- and *p*-bromoanilines
 (d) mixture of *o*- and *m*-bromotoluenes

13. In the following sequence of reactions, the alkene affords the compound 'B'

$$CH_3-CH=CH-CH_3 \xrightarrow{O_3} A \xrightarrow[Zn]{H_2O} B.$$

The compound B is **[2008]**

(a) CH_3CH_2CHO
(b) CH_3COCH_3
(c) $CH_3CH_2COCH_3$
(d) CH_3CHO

14. The hydrocarbon which can react with sodium in liquid ammonia is **[2008]**

(a) $CH_3CH_2CH_2C\equiv CCH_2CH_2CH_3$
(b) $CH_3CH_2C\equiv CH$
(c) $CH_3CH=CHCH_3$
(d) $CH_3CH_2C\equiv CCH_2CH_3$

15. The treatment of CH_3MgX with $CH_3C\equiv C-H$ produces **[2008]**

(a) $CH_3-CH=CH_2$
(b) $CH_3C\equiv C-CH_3$
(c) $CH_3-CH=CH-CH_3$
(d) CH_4

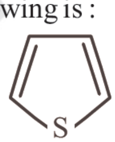
16. One mole of a symmetrical alkene on ozonolysis gives two moles of an aldehyde having a molecular mass of 44 u. The alkene is **[2010]**

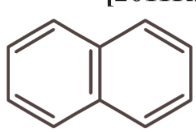
(a) propene (b) 1-butene
(c) 2-butene (d) ethene

17. Ozonolysis of an organic compound 'A' produces acetone and propionaldehyde in equimolar mixture. Identify 'A' from the following compounds: **[2011RS]**

(a) 1 – Pentene
(b) 2 – Pentene
(c) 2 – Methyl – 2 – pentene
(d) 2 – Methyl – 1 – pentene

18. The non aromatic compound among the following is : **[2011RS]**

(a) 

(b) 
(c) 

(d) 

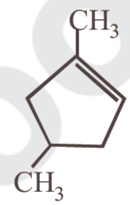
19. Which branched chain isomer of the hydrocarbon with molecular mass 72u gives only one isomer of mono substituted alkyl halide ? **[2012]**

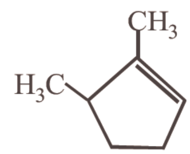
(a) Tertiary butyl chloride
(b) Neopentane
(c) Isohexane
(d) Neohexane

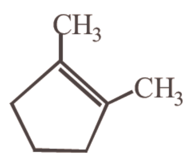
20. 2-Hexyne gives *trans*-2-hexene on treatment with : **[2012]**

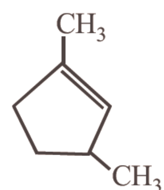
(a) Pt/H_2 (b) Li/NH_3
(c) $Pd/BaSO_4$ (d) $LiAlH_4$

21. Which compound would give 5-keto-2-methylhexanal upon ozonolysis ? **[2015]**

(a) 

(b) 

(c) 

(d) 

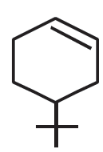
22. In the following sequence of reactions :

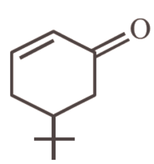
$$\text{Toluene} \xrightarrow{KMnO_4} A \xrightarrow{SOCl_2} B \xrightarrow[BaSO_4]{H_2/Pd} C$$

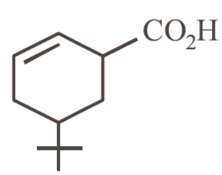
the product C is : **[2015]**

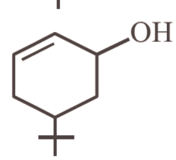
(a) $C_6H_5CH_2OH$ (b) C_6H_5CHO
(c) C_6H_5COOH (d) $C_6H_5CH_3$

23. The product of the reaction given below is:

 $\xrightarrow[2.\ H_2O/K_2CO_3]{1.\ NBS/h\nu} X$ **[2016]**

(a) 

(b) 

(c) 
(d) 

24. The reaction of propene with HOCl $(Cl_2 + H_2O)$ proceeds through the intermediate: **[2016]**

(a) $CH_3 - CH(OH) - CH_2^+$

(b) $CH_3 - CHCl - CH_2^+$

(c) $CH_3 - CH^+ - CH_2 - OH$

(d) $CH_3 - CH^+ - CH_2 - Cl$

25. 3-Methyl-pent-2-ene on reaction with HBr in presence of peroxide forms an addition product. The number of possible stereoisomers for the product is : **[2017]**

(a) Six (b) Zero
(c) Two (d) Four

26. The *trans*-alkenes are formed by the reduction of alkynes with: **[2018]**

(a) H_2-Pd/C, $BaSO_4$ (b) $NaBH_4$
(c) Na/liq. NH_3 (d) Sn - HCl

27. The major product of the following reaction is:

$$CH_3C \equiv CH \xrightarrow[\text{(ii) DI}]{\text{(i) DCl (1equiv.)}}$$ **[2019]**

(a) $CH_3CD(I)CHD(Cl)$

(b) $CH_3CD(Cl)CHD(I)$

(c) $CH_3CD_2CH(Cl)(I)$

(d) $CH_3C(I)(Cl)CHD_2$

Answer Key														
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
(a)	(c)	(c)	(c)	(d)	(c)	(c)	(c)	(d)	(b)	(d)	(a)	(d)	(b)	(d)
16	17	18	19	20	21	22	23	24	25	26	27			
(c)	(c)	(d)	(b)	(b)	(d)	(b)	(d)	(d)	(d)	(c)	(d)			

Solutions

1. (a) Acetylene reacts with the other three reagents as:

$$CH \equiv CNa \xleftarrow[\text{liq. } NH_3]{Na} CH \equiv CH \xrightarrow{+HCl} \begin{matrix} CH_2 \\ \| \\ CHCl \end{matrix}$$

$$\xrightarrow{+HCl} \begin{matrix} CH_3 \\ | \\ CHCl_2 \end{matrix}$$

$$CH \equiv CH \xrightarrow{[AgNO_3 + NH_4OH]} \underset{\text{white ppt.}}{AgC \equiv CAg} + NH_4NO_3$$

2. (c) $$CH \equiv CH + HOCl \longrightarrow \begin{matrix} CHOH \\ \| \\ CHCl \end{matrix}$$

$$\xrightarrow{HOCl} \left[\begin{matrix} CH(OH)_2 \\ | \\ CHCl_2 \end{matrix}\right] \xrightarrow{-H_2O} \underset{\text{Dichloroacetaldehyde}}{\begin{matrix} CHO \\ | \\ CHCl_2 \end{matrix}}$$

3. (c) In neopentane all H atoms are equivalent (1°).

$$\begin{matrix} & & CH_3 & & \\ & & | & & \\ CH_3 & - & C & - & CH_3 \\ & & | & & \\ & & CH_3 & & \end{matrix}$$

4. (c) Alkenes combine with hydrogen under pressure and in presence of a catalyst (Ni, Pt or Pd) and form alkanes.

$$\text{Butene - 1} \xrightarrow{H_2/Pd} \text{Butane}$$

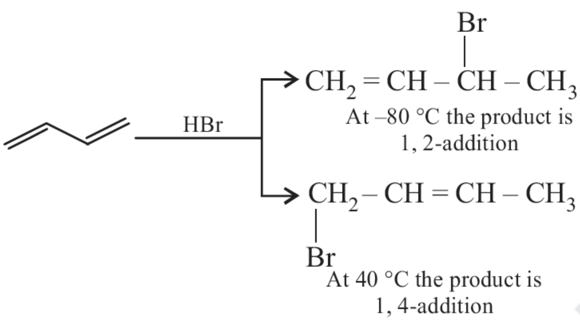
5. (d) **NOTE** Among isomeric alkanes, the straight chain isomer has higher boiling point than the branched chain isomer. The greater the branching of the chain, the lower is the boiling point. Further due to the presence of π electrons, these moleculs are slightly polar and hence have higher boiling points than the corresponding alkanes.
Thus B.pt. follows the order
alkynes > alkenes > alkanes (straight chain) > branched chain alkanes.

6. (c) $\underset{\text{2-Methylbutane}}{CH_3-\overset{\overset{CH_3}{|}}{C}H-CH_2-CH_3} \xrightarrow[\text{sun light}]{Br_2}$

$$\underset{\text{2-Bromo-2-methyl butane}}{CH_3-\overset{\overset{CH_3}{|}}{\underset{\underset{Br}{|}}{C}}-CH_2CH_3}$$

Ease of replacement of H-atoms $3° > 2° > 1°$.

7. (c)



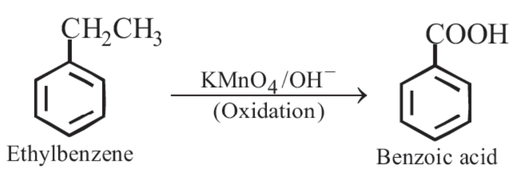
8. (c) $\overset{1°}{CH_3}-\underset{3°}{\overset{\overset{1°}{CH_3}}{\overset{|}{C}H}}-\underset{3°}{\overset{\overset{1°}{CH_3}}{\overset{|}{C}H}}-\overset{1°}{CH_3}$.

Since it contains only two types of H-atoms hence it will give only two monochlorinated compounds viz.

$$\underset{\text{1-Chloro-2, 3-dimethyl butane}}{Cl.CH_2-\overset{\overset{CH_3}{|}}{C}H-\overset{\overset{CH_3}{|}}{C}H-CH_3}$$

and $\underset{\text{2-Chloro-2,3-dimethylbutane}}{CH_3-\overset{\overset{CH_3}{|}}{\underset{\underset{Cl}{|}}{C}}-\overset{\overset{CH_3}{|}}{C}H-CH_3}$

9. (d) When alkyl benzenes are oxidised with alkaline $KMnO_4$ (strong oxidising agent), the entire alkyl group is oxidised to –COOH group regardless of length of side chain.

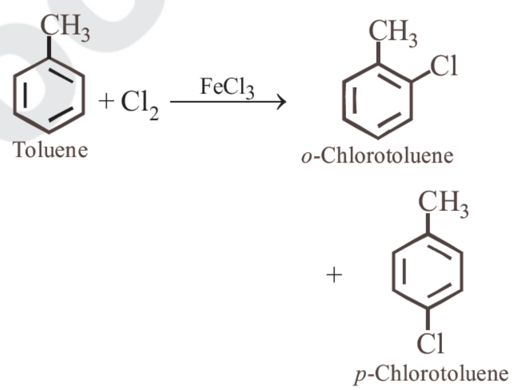


10. (b) The reaction follows Markownikoff rule which states that when unsymmetrical reagent adds across unsymmetrical double or triple bond, the negative part adds to carbon atom having lesser number of hydrogen atoms.

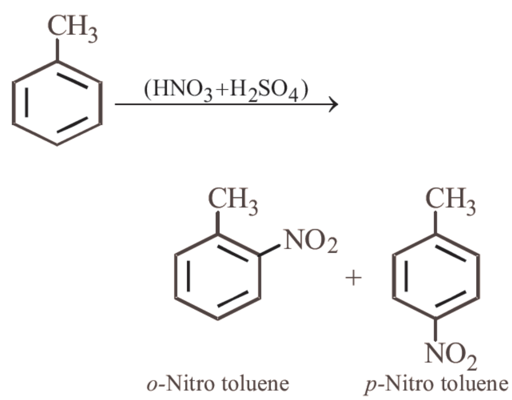
$$CH_3-C\equiv CH + HBr \longrightarrow$$

$$CH_3-\underset{\underset{Br}{|}}{C}=CH_2 \xrightarrow{HBr} \underset{\text{2,2-Dibromopropane}}{CH_3-\overset{\overset{Br}{|}}{\underset{\underset{Br}{|}}{C}}-CH_3}$$

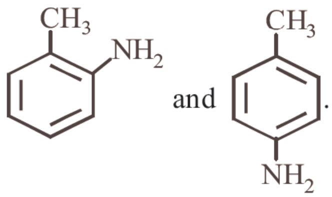
11. (d) $FeCl_3$ is Lewis acid. In presence of $FeCl_3$ and Cl_2 toluene undergoes electrophilic substitution in *o*- and *p*- positions.



12. (a) **NOTE** Toluene ($C_6H_5CH_3$) contains $-CH_3$ group which is *o*-, *p*- directing group so on nitration of toluene the $-NO_2$ group will occupy *o*-, *p*- positions.



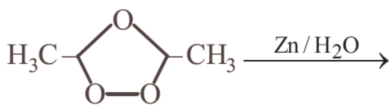
On reduction with Sn/HCl they will form corresponding anilines in which $-NO_2$ group changes to $-NH_2$. The mixture now contains



These anilines when diazotized and then treated with CuBr form *o*-, *p*- bromotoluenes (Sandmeyer reductions).

13. (d) Completing the sequence of given reactions,

$$CH_3-CH=CH-CH_3 \xrightarrow{O_3}$$



'A'
(ozonide)

$$2CH_3CHO + H_2O + ZnO$$
'B'

14. (b) Alkynes having terminal $-C\equiv H$ react with Na in liquid ammonia to yield H_2 gas

$$CH_3CH_2C\equiv CH \xrightarrow[\text{liquid } NH_3]{\text{Na in}} CH_3CH_2C\equiv C^-Na^+ + \frac{1}{2}H_2(g)$$

15. (d) $CH_3MgX + CH_3-C\equiv C-H \longrightarrow CH_3-C\equiv CMgX + CH_4(g)$

16. (c) $CH_3CH=CHCH_3 \xrightarrow{O_3}$
2-Butene

$$\xrightarrow{Zn/H_2O} 2CH_3CHO + H_2O_2$$

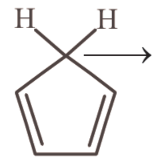
17. (c) From the products formed it is clear that the compound has 5 carbon atoms with a double bond and methyl group on 2nd carbon atom.

$$CH_3-\underset{}{\overset{CH_3}{\overset{|}{C}}}=CH-CH_2-CH_3$$
(2–Methyl–2–pentene)
(A)

$$\downarrow O_3/Zn,\ H_2O$$

$$CH_3-\overset{CH_3}{\overset{|}{C}}=O + CH_3-CH_2-CHO$$
Acetone Propionaldehyde

18. (d)



$\longrightarrow sp^3$ Carbon

Cyclopentadiene does not obey Huckel's Rule, as it has sp^3 carbon in the ring.

19. (b)

$$H_3C-\overset{CH_3}{\underset{CH_3}{\overset{|}{\underset{|}{C}}}}-CH_3 \xrightarrow[\text{monohalogenation}]{Cl_2/h\nu}$$
Neopentane

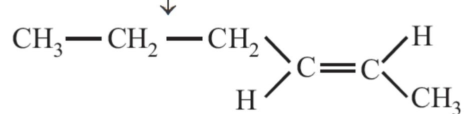
Single product

In neopentane, all hydrogen atoms are equivalent.

20. (b) Anti addition of hydrogen atoms to the triple bond occurs when alkynes are reduced with sodium (or lithium) metal in liq. ammonia, ethylamine, or alcohol at low temperatures. This reaction called, a dissolving metal reduction, produces an (E)- or *trans*-alkene. Sodium in liq. NH_3 is used as a source of electrons in the reduction of an alkyne to a *trans* alkene.

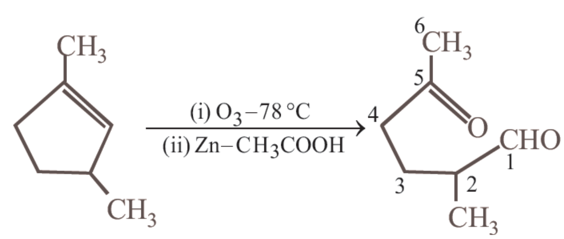
$$CH_3-CH_2-CH_2-C\equiv C-CH_3$$
2-Hexyne

$$\downarrow \text{Li}/NH_3 \text{ Birch reduction}$$



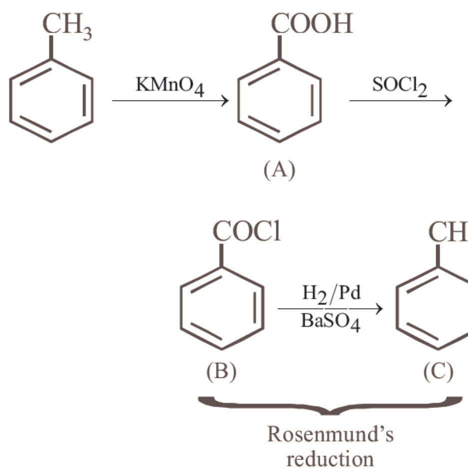
trans-2-Hexene

21. (d)

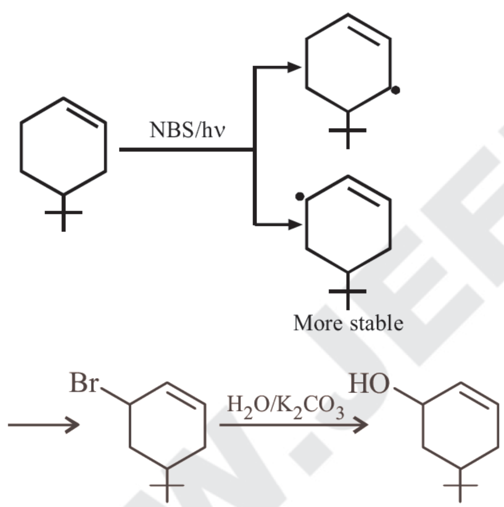


5-Keto-2-methylhexanal

22. **(b)**



23. **(d)** N – bromosuccinimide results into bromination at allylic and benzylic positions



24. **(d)**

$$CH_2 = CH - CH_3 + \overset{\oplus}{Cl} \rightarrow \begin{cases} CH_2(Cl) - \overset{\oplus}{C}H - CH_3 & \text{More stable intermediate} \\ \overset{\oplus}{C}H_2 - CH(Cl) - CH_3 \end{cases}$$

$$\underset{\displaystyle Cl}{CH_2} - \overset{\oplus}{C}H - CH_3 \xrightarrow{\overset{\ominus}{O}H} \underset{\displaystyle Cl}{CH_2} - \underset{\displaystyle OH}{CH} - CH_3$$

25. **(d)** Addition of HBr on 3-methylpent-2-ene in presence of peroxide, takes place in anti-Markownikov's rule.

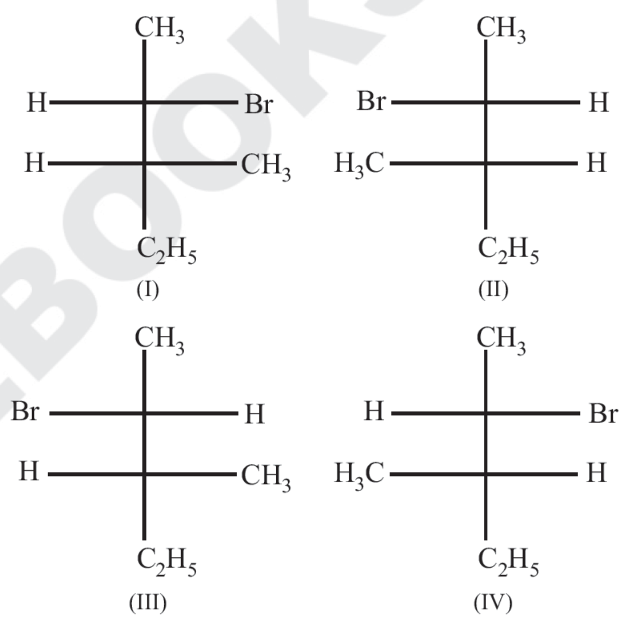
$$\overset{1}{C}H_3 - \overset{2}{C}H = \overset{3}{\underset{}{C}}(CH_3) - \overset{4}{C}H_2\overset{5}{C}H_3 \xrightarrow[-H_2O]{HBr}$$

3-Methyl pent-2-ene

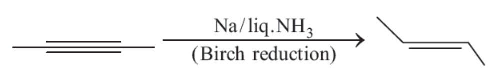
$$CH_3 - \overset{*}{C}H(Br) - \overset{*}{C}H(CH_3) - CH_2 - CH_3$$

2-Bromo-3-methyl pentane
(2 chiral centres)

Since two chiral centres are present in the product, four steremers (n^2) are possible.



26. **(c)**



27. **(d)** $CH_3 - C \equiv C - H \xrightarrow[1\ eq]{DCl} CH_3 - \underset{\displaystyle Cl}{C} = \underset{\displaystyle D}{CH}$

$$\downarrow DI$$

$$CH_3 - \overset{\displaystyle I}{\underset{\displaystyle Cl}{C}} - \overset{\displaystyle D}{\underset{\displaystyle D}{CH}}$$

Both additions follow Markovnikov's rule.

14 Environmental Chemistry

1. The smog is essentially caused by the presence of **[2004]**
(a) oxides of sulphur and nitrogen
(b) O_2 and N_2
(c) O_2 and O_3
(d) O_3 and N_2

2. Identify the wrong statement in the following: **[2008]**
(a) Chlorofluorocarbons are responsible for ozone layer depletion
(b) Greenhouse effect is responsible for global warming
(c) Ozone layer does not permit infrared radiation from the sun to reach the earth
(d) Acid rain is mostly because of oxides of nitrogen and sulphur

3. Identify the incorrect statement from the following: **[2011RS]**
(a) Ozone absorbs the intense ultraviolet radiation of the sun.
(b) Depletion of ozone layer is because of its chemical reactions with chlorofluoro-alkanes.
(c) Ozone absorbs infrared radiation.
(d) Oxides of nitrogen in the atmosphere can cause the depletion of ozone layer.

4. What is DDT among the following ? **[2012]**
(a) Greenhouse gas
(b) A fertilizer
(c) Biodegradable pollutant
(d) Non-biodegradable pollutant

5. The gas leaked from a storage tank of the Union Carbide plant in Bhopal gas tragedy was: **[2013]**
(a) Methyl isocyanate
(b) Methylamine
(c) Ammonia
(d) Phosgene

6. The concentration of fluoride, lead, nitrate and iron in a water sample from an underground lake was found to be 1000 ppb, 40 ppb, 100 ppm and 0.2 ppm, respectively. This water is unsuitable for drinking due to high concentration of : **[2016]**
(a) Nitrate (b) Iron
(c) Fluoride (d) Lead

7. A water sample has ppm level concentration of following anions **[2017]**
$F^- = 10$; $SO_4^{2-} = 100$; $NO_3^- = 50$
the anion/anions that make/makes the water sample unsuitable for drinking is/are :
(a) only NO_3^- (b) both SO_4^{2-} and NO_3^-
(c) only F^- (d) only SO_4^{2-}

8. The recommended concentration of fluoride ion in drinking water is up to 1ppm as fluoride ion is required to make teeth enamel harder by converting $[3Ca_3(PO_4)_2.\ Ca(OH)_2]$ to : **[2018]**
(a) $[CaF_2]$
(b) $[3(CaF_2).Ca(OH)_2]$
(c) $[3Ca_3(PO_4)_2.CaF_2]$
(d) $[3\{(Ca(OH)_2\}.CaF_2]$

9. A water sample has ppm level concentration of the following metals: Fe = 0.2; Mn = 5.0; Cu = 3.0; Zn = 5.0. The metal that makes the water sample unsuitable for drinking is: **[2019]**
(a) Cu (b) Mn (c) Fe (d) Zn

10. Excessive release of CO_2 into the atmosphere results in: **[2019]**
(a) global warming
(b) polar vortex
(c) formation of smog
(d) depletion of ozone

Answer Key														
1	2	3	4	5	6	7	8	9	10					
(a)	(c)	(c)	(d)	(a)	(a)	(c)	(c)	(b)	(a)					

Solutions

1. **(a)** Photochemical smog is caused by oxides of sulphur and nitrogen.

2. **(c)** **NOTE** Ozone layer acts as a shield and does not allow ultraviolet radiation from sun to reach earth. It does not prevent infra-red radiation from sun to reach earth.

3. **(c)** The ozone layer, existing between 20 to 35 km above the earth's surface, shield the earth from the harmful U. V. radiations from the sun.

Depletion of ozone is caused by oxides of nitrogen

$N_2O + h\upsilon \longrightarrow NO + N$

4. **(d)** DDT is a non-biodegradable pollutant.

5. **(a)**

6. **(a)** The maximum limit of nitrate in drinking water is 50 ppm. Excess nitrate in drinking water can cause disease such as methemoglobinemia ('blue baby' syndrome).

7. **(c)** Concentration of F^- in drinking water above 2ppm causes brown mottling of teeth.

8. **(c)**

$$\underset{\text{Hydroxyapatite}}{[3Ca_3(PO_4)_2.Ca(OH)_2]} + \underset{\substack{\text{(drinking}\\ \text{water}\\ \text{upto 1ppm)}}}{2F^-} \longrightarrow \underset{\substack{\text{Fluorapatite}\\ \text{(Harder teeth}\\ \text{enamel)}}}{[3Ca_3(PO_4)_2.CaF_2]} + 2OH^-$$

9. **(b)** The water sample containing Mn = 5 ppm is unsuitable for drinking as the prescribed level for Mn in drinking water is 0.5 ppm.

10. **(a)** Global warming is caused by the emission of green house gases. 72% of the totally emitted green house gases is CO_2. Therefore, excessive release of CO_2 is the main cause of global warming.

15 The Solid State

1. Na and Mg crystallize in bcc and fcc type crystals respectively, then the number of atoms of Na and Mg present in the unit cell of their respective crystal is **[2002]**
   (a) 4 and 2 (b) 9 and 14
   (c) 14 and 9 (d) 2 and 4
2. How many unit cells are present in a cube-shaped ideal crystal of NaCl of mass 1.00 g ? **[2003]**
   [Atomic masses : Na = 23, Cl = 35.5]
   (a) 5.14×10^{21} unit cells
   (b) 1.28×10^{21} unit cells
   (c) 1.71×10^{21} unit cells
   (d) 2.57×10^{21} unit cells
3. What type of crystal defect is indicated in the diagram below? **[2004]**

   $Na^+Cl^-Na^+Cl^-Na^+Cl^-$
   $Cl^-\square\ Cl^-Na^+\ \square\ Na^+$
   $Na^+Cl^-\ \square Cl^-Na^+Cl^-$
   $Cl^-Na^+Cl^-Na^+\ \square\ Na^+$

   (a) Interstitial defect
   (b) Schottky defect
   (c) Frenkel defect
   (d) Frenkel and Schottky defects
4. An ionic compound has a unit cell consisting of A ions at the corners of a cube and B ions on the centres of the faces of the cube. The empirical formula for this compound would be **[2005]**
   (a) A_3B (b) AB_3 (c) A_2B (d) AB
5. Total volume of atoms present in a face-centred cubic unit cell of a metal is (r is atomic radius) **[2006]**
   (a) $\frac{12}{3}\pi r^3$ (b) $\frac{16}{3}\pi r^3$
   (c) $\frac{20}{3}\pi r^3$ (d) $\frac{24}{3}\pi r^3$
6. In a compound, atoms of element Y form ccp lattice and those of element X occupy 2/3[rd] of tetrahedral voids. The formula of the compound will be **[2008]**
   (a) X_4Y_3 (b) X_2Y_3
   (c) X_2Y (d) X_3Y_4
7. Copper crystallises in *fcc* with a unit cell length of 361 pm. What is the radius of copper atom? **[2009]**
   (a) 127 pm (b) 157 pm
   (c) 181 pm (d) 108 pm
8. The edge length of a face centered cubic cell of an ionic substance is 508 pm. If the radius of the cation is 110 pm, the radius of the anion is **[2010]**
   (a) 288 pm (b) 398 pm
   (c) 618 pm (d) 144 pm
9. Percentages of free space in cubic close packed structure and in body centered packed structure are respectively **[2010]**
   (a) 30% and 26% (b) 26% and 32%
   (c) 32% and 48% (d) 48% and 26%
10. Copper crystallises in fcc lattice with a unit cell edge of 361 pm. The radius of copper atom is : **[2011RS]**
    (a) 108 pm (b) 128 pm
    (c) 157 pm (d) 181 pm

11. Lithium forms body centred cubic structure. The length of the side of its unit cell is 351 pm. Atomic radius of the lithium will be : **[2012]**

(a) 75 pm (b) 300 pm
(c) 240 pm (d) 152 pm

12. Which of the following exists as covalent crystals in the solid state ? **[2013]**

(a) Iodine (b) Silicon
(c) Sulphur (d) Phosphorus

13. Experimentally, it was found that a metal oxide has formula $M_{0.98}O$. Metal M, present as M^{2+} and M^{3+} in its oxide. Fraction of the metal which exists as M^{3+} would be : **[2013]**

(a) 7.01% (b) 4.08%
(c) 6.05% (d) 5.08%

14. CsCl crystallises in body centered cubic lattice. If '*a*' is its edge length then which of the following expressions is correct? **[2014]**

(a) $r_{Cs^+} + r_{Cl^-} = 3a$

(b) $r_{Cs^+} + r_{Cl^-} = \frac{3a}{2}$

(c) $r_{Cs^+} + r_{Cl^-} = \frac{\sqrt{3}}{2}a$

(d) $r_{Cs^+} + r_{Cl^-} = \sqrt{3}a$

15. The correct statement for the molecule, CsI_3 is:

(a) It is a covalent molecule. **[2014]**

(b) It contains Cs^+ and I_3^- ions.

(c) It contains Cs^{3+} and I^- ions.

(d) It contains Cs^+, I^- and lattice I_2 molecule.

16. Sodium metal crystallizes in a body centred cubic lattice with a unit cell edge of 4.29Å. The radius of sodium atom is approximately : **[2015]**

(a) 5.72Å (b) 0.93Å
(c) 1.86Å (d) 3.22Å

17. A metal crystallises in a face centred cubic structure. If the edge length of its unit cell is 'a', the closest approach between two atoms in metallic crystal will be : **[2017]**

(a) 2a (b) $2\sqrt{2}$ a

(c) $\sqrt{2}$ a (d) $\frac{a}{\sqrt{2}}$

18. Which type of 'defect' has the presence of cations in the interstitial sites? **[2018]**

(a) Schottky defect
(b) Vacancy defect
(c) Frenkel defect
(d) Metal deficiency defect

19. The one that is extensively used as a piezoelectric material is: **[2019]**

(a) tridymite
(b) amorphous silica
(c) quartz
(d) mica

Answer Key

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
(d)	(d)	(b)	(b)	(b)	(a)	(a)	(d)	(b)	(b)	(d)	(b)	(b)	(c)	(b)
16	17	18	19											
(c)	(d)	(c)	(c)											

Solutions

1. **(d)** In *bcc* - points are at corners and one in the centre of the unit cell.

Number of atoms per unit cell

$$= 8\times\frac{1}{8}+1=2$$

In *fcc* - points are at the corners and also centre of the six faces of each cell.

Number of atoms per unit cell

$$= 8\times\frac{1}{8}+6\times\frac{1}{2}=4$$

2. **(d)** Since in NaCl type of structure 4 formula units form a cell.

No. of unit cells present in a cubic crystal

$$= \frac{d\times a^3\times N_A}{M\times Z} = \frac{m\times N_A}{M\times Z}$$

$$\therefore \text{ units cells } = \frac{1.0\times 6.02\times 10^{23}}{58.5\times 4}$$

$$= 2.57\times 10^{21} \text{ unit cells.}$$

3. **(b)** When equal number of cations and anions are missing from their regular lattice positions, we have schottky defect.

This type of defects are more common in ionic compounds with high co-ordination number and where the size of positive and negative ions are almost equal e.g. NaCl KCl etc.

4. **(b)** Number of A ions in the unit cell.

$$= \frac{1}{8}\times 8=1$$

Number of B ions in the unit cell

$$= \frac{1}{2}\times 6=3$$

Hence, empirical formula of the compound $= AB_3$

5. **(b)** The face centered cubic unit cell contains 4 atoms

$$\therefore \text{ Total volume of atoms } = 4\times\frac{4}{3}\pi r^3$$

$$= \frac{16}{3}\pi r^3$$

6. **(a)** From the given data, we have

Number of Y atoms in a unit cell = 4

Number of X atoms in a unit cell $= 8\times\frac{2}{3}=\frac{16}{3}$

From the above we get the formula of the compound as $X_{16/3}Y_4$ or X_4Y_3

7. **(a)** For *fcc* unit cell, $4r=\sqrt{2}\,a$

$$r=\frac{\sqrt{2}\times 361}{4}=127 \text{ pm}$$

8. **(d)** For an *fcc* crystal

$$r_{cation}+r_{anion}=\frac{\text{edge length}}{2}$$

$$110+r_{anion}=\frac{508}{2}$$

$$r_{anion}=254-110=144 \text{ pm}$$

9. **(b)** Packing fraction is defined as the ratio of the volume of the unit cell that is occupied by the spheres to the volume of the unit cell.

P.F. for *ccp* and *bcc* are 0.74 and 0.68 respectively.

So, the free space in *ccp* and *bcc* are 26% & 32% respectively.

10. **(b)** For *fcc* lattice

$$a\sqrt{2}=4r$$

$a=361$ pm (given)

$$r=\frac{361\times\sqrt{2}}{4}=127.6\approx 128 \text{ pm}$$

11. **(d)** For *bcc* structure $\sqrt{3}a=4r$

$$r=\frac{\sqrt{3}}{4}a=\frac{\sqrt{3}}{4}\times 351=152 \text{ pm.}$$

12. **(b)**

13. **(b)** For one mole of the oxide

Moles of M = 0.98

Moles of O^{2-} = 1

Let moles of $M^{3+} = x$

$\therefore$ Moles of $M^{2+} = 0.98 - x$

On balancing charge

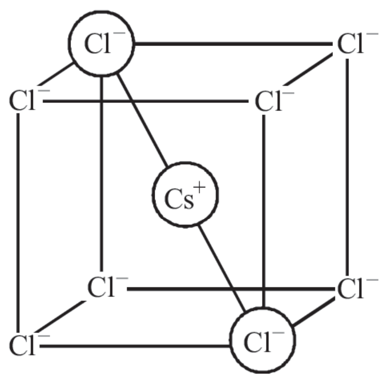
$(0.98 - x) \times 2 + 3(x) = 2$

$(0.98 - x) \times 2 + 3x - 2 = 0$

$x = 0.04$

$\therefore$ % of $M^{3+} = \dfrac{0.04}{0.98} \times 100 = 4.08\%$

14. **(c)**



Relation between radius of cation, anion and edge length of the cube

$$2r_{Cs^+} + 2r_{Cl^-} = \sqrt{3}a$$

$$r_{Cs^+} + r_{Cl^-} = \frac{\sqrt{3}a}{2}$$

15. **(b)** CsI_3 dissociates as $CsI_3 \rightarrow Cs^+ + I_3^-$

16. **(c)** In *bcc* the atoms touch along body diagonal

$\therefore \quad 2r + 2r = \sqrt{3}a$

$\therefore \quad r = \dfrac{\sqrt{3}a}{4} = \dfrac{\sqrt{3} \times 4.29}{4} = 1.857\text{Å}$

17. **(d)** For a *fcc* unit cell

$$r = \frac{\sqrt{2}\,a}{4}$$

$\therefore$ closest distance $(2r) = \dfrac{\sqrt{2}\,a}{4} = \dfrac{a}{\sqrt{2}}$

18. **(c)** In Frenkel defect some of ions (usually cations due to their small size) missing from their normal position, occupy position in interstitial sites.

19. **(c)** Quartz exhibits piezoelectricity and thus can be used as a piezoelectric material.

16 Solutions

1. Freezing point of an aqueous solution is -0.186 °C. Elevation of boiling point of the same solution is $K_b = 0.512$ °C, $K_f = 1.86$ °C, find the increase in boiling point. **[2002]**
   (a) 0.186 °C (b) 0.0512 °C
   (c) 0.092 °C (d) 0.2372 °C.

2. In mixture A and B components show -ve deviation as **[2002]**
   (a) $\Delta V_{mix} > 0, \Delta H_{mix} > 0$
   (b) $\Delta V_{mix} = 0, \Delta H_{mix} < 0$
   (c) $A - B$ interaction is weaker than $A - A$ and $B - B$ interaction
   (d) $A - B$ interaction is stronger than $A - A$ and $B - B$ interaction.

3. If liquids A and B form an ideal solution **[2003]**
   (a) the entropy of mixing is zero
   (b) the free energy of mixing is zero
   (c) the free energy as well as the entropy of mixing is zero
   (d) the enthalpy of mixing is zero

4. In a 0.2 molal aqueous solution of a weak acid HX, the degree of ionization is 0.3. Taking K_f for water as 1.85, the freezing point of the solution will be nearest to **[2003]**
   (a) -0.360 °C (b) -0.260 °C
   (c) $+0.480$ °C (d) -0.480 °C

5. A pressure cooker reduces cooking time for food because **[2003]**
   (a) boiling point of water involved in cooking is increased
   (b) the higher pressure inside the cooker crushes the food material
   (c) cooking involves chemical changes helped by a rise in temperature
   (d) heat is more evenly distributed in the cooking space

6. Which one of the following aqueous solutions will exihibit highest boiling point ? **[2004]**
   (a) 0.015 M urea (b) 0.01 M KNO_3
   (c) 0.01 M Na_2SO_4 (d) 0.015 M glucose

7. For which of the following parameters, the structural isomers C_2H_5OH and CH_3OCH_3 would be expected to have the same values?(Assume ideal behaviour). **[2004]**
   (a) Boiling points
   (b) Vapour pressure at the same temperature
   (c) Heat of vaporization
   (d) Gaseous densities at the same temperature and pressure

8. Which of the following liquid pairs shows a positive deviation from Raoult's law ? **[2004]**
   (a) Water - nitric acid
   (b) Benzene - methanol
   (c) Water - hydrochloric acid
   (d) Acetone - chloroform

9. Which one of the following statements is **FALSE?** **[2004]**
   (a) The correct order of osmotic pressure for 0.01 M aqueous solution of each compound is
   $BaCl_2 > KCl > CH_3COOH >$ sucrose
   (b) The osmotic pressure (π) of a solution is given by the equation $\pi = MRT$, where M is the molarity of the solution
   (c) Raoult's law states that the vapour pressure of a component over a solution is proportional to its mole fraction
   (d) Two sucrose solutions of same molality prepared in different solvents will have same freezing point depression

10. Benzene and toluene form nearly ideal solution. At 20 °C, the vapour pressure of benzene is 75 torr and that of toluene is 22 torr. The partial vapour pressure of benzene at 20°C for a solution containing 78 g of benzene and 46 g of toluene in torr is **[2005]**

(a) 53.5 (b) 37.5
(c) 25 (d) 50

11. Equimolar solutions in the same solvent have **[2005]**

(a) Different boiling and different freezing points
(b) Same boiling and same freezing points
(c) Same freezing point but different boiling points
(d) Same boiling point but different freezing points

12. If α is the degree of dissociation of Na_2SO_4, the vant Hoff's factor (i) used for calculating the molecular mass is

(a) $1-2\alpha$ (b) $1+2\alpha$ **[2005]**
(c) $1-\alpha$ (d) $1+\alpha$

13. Among the following mixtures, dipole-dipole as the major interaction, is present in **[2006]**

(a) KCl and water
(b) benzene and carbon tetrachloride
(c) benzene and ethanol
(d) acetonitrile and acetone

14. 18 g of glucose ($C_6H_{12}O_6$) is added to 178.2 g of water. The vapour pressure of water for this aqueous solution at 100 ºC is **[2006, 2016]**

(a) 76.00 Torr (b) 752.40 Torr
(c) 759.00 Torr (d) 7.60 Torr

15. A mixture of ethyl alcohol and propyl alcohol has a vapour pressure of 290 mm at 300 K. The vapour pressure of propyl alcohol is 200 mm. If the mole fraction of ethyl alcohol is 0.6, its vapour pressure (in mm) at the same temperature will be **[2007]**

(a) 360 (b) 350
(c) 300 (d) 700

16. Equal masses of methane and oxygen are mixed in an empty container at 25°C. The fraction of the total pressure exerted by oxygen is **[2007]**

(a) 1/2 (b) 2/3
(c) $\frac{1}{3}\times\frac{273}{298}$ (d) 1/3.

17. A 5.25% solution of a substance is isotonic with a 1.5% solution of urea (molar mass = 60 g mol^{-1}) in the same solvent. If the densities of both the solutions are assumed to be equal to 1.0 g cm^{-3}, molar mass of the substance will be **[2007]**

(a) 210.0 g mol^{-1} (b) 90.0 g mol^{-1}
(c) 115.0 g mol^{-1} (d) 105.0 g mol^{-1}.

18. At 80 °C, the vapour pressure of pure liquid 'A' is 520 mm Hg and that of pure liquid 'B' is 1000 mm Hg. If a mixture solution of 'A' and 'B' boils at 80 °C and at 1 atm pressure, the amount of 'A' in the mixture is (1 atm = 760 mm Hg) **[2008]**

(a) 52 mol percent (b) 34 mol percent
(c) 48 mol percent (d) 50 mol percent

19. The vapour pressure of water at 20 °C is 17.5 mm Hg. If 18 g of glucose ($C_6H_{12}O_6$) is added to 178.2 g of water at 20 °C, the vapour pressure of the resulting solution will be **[2008]**

(a) 17.325 mm Hg (b) 15.750 mm Hg
(c) 16.500 mm Hg (d) 17.500 mm Hg

20. A binary liquid solution is prepared by mixing *n*-heptane and ethanol. Which one of the following statements is correct regarding the behaviour of the solution? **[2009]**

(a) The solution is non-ideal, showing – ve deviation from Raoult's Law.
(b) The solution is non-ideal, showing + ve deviation from Raoult's Law.
(c) *n*-Heptane shows + ve deviation while ethanol shows – ve deviation from Raoult's Law.
(d) The solution formed is an ideal solution.

21. Two liquids X and Y form an ideal solution. At 300 K, vapour pressure of the solution containing 1 mol of X and 3 mol of Y is 550 mmHg. At the same temperature, if 1 mol of Y is further added to this solution, vapour pressure of the solution increases by 10 mmHg. Vapour pressure (in mmHg) of X and Y in their pure states will be, respectively: **[2009]**

(a) 300 and 400 (b) 400 and 600
(c) 500 and 600 (d) 200 and 300

22. If sodium sulphate is considered to be completely dissociated into cations and anions in aqueous solution, the change in freezing point of water (ΔT_f), when 0.01 mol of sodium sulphate is dissolved in 1 kg of water, is (K_f = 1.86 K kg mol^{-1}). **[2010]**

(a) 0.372 K (b) 0.0558 K
(c) 0.0744 K (d) 0.0186 K

23. On mixing, heptane and octane form an ideal solution. At 373 K, the vapour pressures of the two liquid components (heptane and octane) are 105 kPa and 45 kPa respectively. Vapour pressure of the solution obtained by mixing 25.0 g of heptane and 35 g of octane will be
(molar mass of heptane = 100 g mol^{-1} and of octane = 114 g mol^{-1}) **[2010]**

(a) 72.0 kPa (b) 36.1 kPa
(c) 96.2 kPa (d) 144.5 kPa

24. A 5% solution of cane sugar (molar mass 342) is isotonic with 1% of a solution of an unknown solute. The molar mass of unknown solute in g/mol is : **[2011RS]**

(a) 171.2 (b) 68.4
(c) 34.2 (d) 136.2

25. The density of a solution prepared by dissolving 120 g of urea (mol. mass = 60 u) in 1000 g of water is 1.15 g/mL. The molarity of this solution is : **[2012]**

(a) 0.50 M (b) 1.78 M
(c) 1.02 M (d) 2.05 M

26. K_f for water is 1.86 K kg mol^{-1}. If your automobile radiator holds 1.0 kg of water, how many grams of ethylene glycol ($C_2H_6O_2$) must you add to get the freezing point of the solution lowered to –2.8 °C ? **[2012]**

(a) 72 g (b) 93 g
(c) 39 g (d) 27 g

27. The molarity of a solution obtained by mixing 750 mL of 0.5(M) HCl with 250 mL of 2(M) HCl will be : **[2013]**

(a) 0.875 M (b) 1.00 M
(c) 1.75 M (d) 0.975 M

28. Consider separate solutions of 0.500 M $C_2H_5OH(aq)$, 0.100 M $Mg_3(PO_4)_2(aq)$, 0.250 M $KBr(aq)$ and 0.125 M $Na_3PO_4(aq)$ at 25 °C. Which statement is **true** about these solutions, assuming all salts to be strong electrolytes? **[2014]**

(a) They all have the same osmotic pressure.
(b) 0.100 M $Mg_3(PO_4)_2$(aq) has the highest osmotic pressure.
(c) 0.125 M Na_3PO_4(aq) has the highest osmotic pressure.
(d) 0.500 M C_2H_5OH(aq) has the highest osmotic pressure.

29. The vapour pressure of acetone at 20 °C is 185 torr. When 1.2 g of a non-volatile substance was dissolved in 100 g of acetone at 20 °C, its vapour pressure was 183 torr. The molar mass (g mol^{-1}) of the substance is : **[2015]**

(a) 128 (b) 488
(c) 32 (d) 64

30. The freezing point of benzene decreases by 0.45 °C when 0.2 g of acetic acid is added to 20 g of benzene. If acetic acid associates to form a dimer in benzene, percentage association of acetic acid in benzene will be : **[2017]**
(K_f for benzene = 5.12 K kg mol^{-1})

(a) 64.6% (b) 80.4%
(c) 74.6% (d) 94.6%

31. For 1 molal aqueous solution of the following compounds, which one will show the highest freezing point? **[2018]**

(a) $[Co(H_2O)_6]Cl_3$

(b) $[Co(H_2O)_5Cl]Cl_2.H_2O$

(c) $[Co(H_2O)_4Cl_2]Cl.2H_2O$

(d) $[Co(H_2O)_3Cl_3].3H_2O$

32. The osmotic pressure of a dilute solution of an ionic compound XY in water is four times that of a solution of 0.01 M $BaCl_2$ in water. Assuming complete dissociation of the given ionic compounds in water, the concentration of XY (in mol L^{-1}) in solution is: **[2019]**

(a) 4×10^{-2} (b) 6×10^{-2}

(c) 4×10^{-4} (d) 16×10^{-4}

33. A solution of sodium sulfate contains 92 g of Na^+ ions per kilogram of water. The molality of Na^+ ions in that solution in mol kg^{-1} is: **[2019]**

(a) 12 (b) 4

(c) 8 (d) 16

34. Which one of the following statements regarding Henry's law is not correct? **[2019]**

(a) Higher the value of K_H at a given pressure, higher is the solubility of the gas in liquids.

(b) Different gases have different K_H (Henry's law constant) values at the same temperature.

(c) The partial pressure of the gas in vapour phase is proportional to the mole fraction of the gas in the solution.

(d) The value of K_H increases with increase of temperature and K_H is function of the nature of the gas.

35. Liquid 'M' and liquid 'N' form an ideal solution. The vapour pressures of pure liquids 'M' and 'N' are 450 and 700 mmHg, respectively, at the same temperature. Then correct statement is: **[2019]**

(x_M = Mole fraction of 'M' in solution;
x_N = Mole fraction of 'N' in solution;
y_M = Mole fraction of 'M' in vapour phase;
y_N = Mole fraction of 'N' in vapour phase)

(a) $\frac{x_M}{x_N} = \frac{y_M}{y_N}$

(b) $(x_M - y_M) < (x_N - y_N)$

(c) $\frac{x_M}{x_N} < \frac{y_M}{y_N}$

(d) $\frac{x_M}{x_N} > \frac{y_M}{y_N}$

36. At 35 °C, the vapour pressure of CS_2 is 512 mm Hg and that of acetone is 344 mm Hg. A solution of CS_2 in acetone has a total vapour pressure of 600 mm Hg. The false statement amongst the following is: **[2020]**

(a) Raoult's law is not obeyed by this system

(b) a mixture of 100 mL CS_2 and 100 mL acetone has a volume < 200 mL

(c) CS_2 and acetone are less attracted to each other than to themselves

(d) heat must be absorbed in order to produce the solution at 35 °C

Answer Key

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
(b)	(d)	(d)	(d)	(a)	(c)	(d)	(b)	(d)	(d)	(b)	(b)	(d)	(b)	(b)
16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
(d)	(a)	(d)	(a)	(b)	(b)	(b)	(a)	(b)	(d)	(b)	(a)	(a)	(d)	(d)
31	32	33	34	35	36									
(d)	(b)	(b)	(a)	(d)	(b)									

Solutions

1. **(b)** $\Delta T_b = K_b \dfrac{W_B}{M_B \times W_A} \times 1000$;

$\Delta T_f = K_f \dfrac{W_B}{M_B \times W_A} \times 1000$;

$\dfrac{\Delta T_b}{\Delta T_f} = \dfrac{K_b}{K_f}; \dfrac{\Delta T_b}{0.186} = \dfrac{0.512}{1.86}$

$= 0.0512\,°C$.

2. **(d)** In solution containing A and B component showing negative deviation A–A and B–B interactions are weaker than that of A–B interactions. For such solutions,

$\Delta H = -ve$ and $\Delta V = -ve$

3. **(d)** When A and B form an ideal solution, $\Delta H_{mix} = 0$

4. **(d)** $\Delta T_f = i \times K_f \times m$;

$\Delta T_f = 1.85 \times 0.2 \times 1.3 = 0.480\,°C$

$(\underset{1-0.3}{HX} \rightleftharpoons \underset{0.3}{H^+} + \underset{0.3}{X^-}, \; i = 1.3)$

$\Delta T_f = T_f^{\circ} - T_f$

$\therefore \; T_f = 0 - 0.480\,°C = -0.480\,°C$

5. **(a)** **NOTE** On increasing pressure, the temperature is also increased. Thus in pressure cooker due to increase in pressure the b.p. of water increases.

6. **(c)** $\because \Delta T_b = T_b - T^o$

Where T_b = b.pt of solution

T_b° = b.pt of solvent or $T_b = T_b^{\circ} + \Delta T_b$

NOTE Elevation in boiling point is a colligative property, which depends upon the no. of particles.Thus greater the number of particles, greater is it elevation and hence greater will be its boiling point.

$Na_2SO_4 \rightleftharpoons 2Na^+ + SO_4^{2-}$

Since Na_2SO_4 has maximum number of particles (3), hence has maximum boiling point.

7. **(d)** Gaseous densities of ethanol and dimethyl ether would be same at same temperature and pressure. The heat of vaporisation, V.P. and b.pts will differ due to H-bonding in ethanol.

8. **(b)** **NOTE** Positive deviations are shown by such solutions in which solvent-solvent and solute-solute interactions are stronger than the solvent-solute interactions. In such solutions, the interactions among molecules becomes weaker. Therefore, their escaping tendency increases which results in the increase in their partial vapour pressure.

In a solution of methanol there exists intermolecular H–bonding.

$$\cdots\cdots \underset{\substack{|\\ CH_3}}{O} - H \cdots\cdots \underset{\substack{|\\ CH_3}}{O} — H \cdots\cdots \underset{\substack{|\\ CH_3}}{O} — H \cdots\cdots$$

In this solution benzene molecules come between methanol molecules which weaken intermolecular forces. This results in increase in vapour pressure.

9. **(d)** $\Delta T_f = i \times K_f \times m$. Since K_f has different values for different solvents, hence even if m is same, ΔT_f will be different.

10. **(d)** Vapour pressure of benzene = 75 torr

Vapour pressure of toluene = 22 torr

mass of benzene in = 78g

Moles of benzene $= \dfrac{78}{78} = 1$ mol

$\therefore$ (Mol.wt of benzene = 78)

Mass of toluene in solution = 46g

Hence moles of toluene $= \dfrac{46}{92} = 0.5$ mol

Now partial pressure of benzene

$= P^o_b . X_b = 75 \times \dfrac{1}{1+0.5} = 75 \times \dfrac{1}{1.5}$

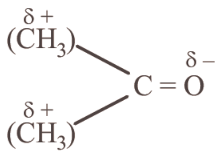
$= 75 \times \dfrac{2}{3} = 50$

11. **(b)** Equimolar solutions of normal solutes in the same solvent will have the same b. pts and same f. pts.

12. **(b)** $\underset{1-\alpha}{Na_2SO_4} \rightleftharpoons \underset{2\alpha}{2Na} + \underset{\alpha}{SO_4^{--}}$

vant Hoff's factor, $i = \dfrac{1-\alpha+2\alpha+\alpha}{1} = 1+2\alpha$

13. **(d)** Acetonitrile $(\overset{\delta+}{CH_3} - C \equiv \overset{\delta-}{N})$ and acetone



both are polar molecules, hence dipole-dipole interaction exists between them. Between KCl and water, ion-dipole interaction is found and in benzene-ethanol and benzene–carbon tetrachloride dispersion force is present.

14. **(b)** Moles of glucose $= \dfrac{18}{180} = 0.1$

Moles of water $= \dfrac{178.2}{18} = 9.9$

Total moles $= 0.1 + 9.9 = 10$

p_{H_2O} = Mole fraction $\times P_1^\circ$

$= \dfrac{9.9}{10} \times 760$

$= 752.4$ Torr

15. **(b)** $P_A^\circ = ?$, Given $P_B^\circ = 200\text{mm}$, $X_A = 0.6$, $X_B = 1 - 0.6 = 0.4$, $P = 290$

$P = P_A + P_B = P_A^\circ X_A + P_B^\circ X_B$

$290 = P_A^\circ \times 0.6 + 200 \times 0.4$

$\therefore\ P_A^\circ = 350\text{mm}$

16. **(d)** Let the mass of methane and oxygen = mg.

Mole fraction of O_2

$= \dfrac{\text{Moles of } O_2}{\text{Moles of } O_2 + \text{Moles of } CH_4}$

$= \dfrac{m/32}{m/32 + m/16} = \dfrac{m/32}{3m/32} = \dfrac{1}{3}$

Partial pressure of O_2

= Total pressure × mole fraction of O_2,

$p_{O_2} = P \times \dfrac{1}{3} = \dfrac{1}{3}P$

17. **(a)** Osmotic pressure (π) of isotonic solutions are equal.

For solution of unknown substance (π= CRT)

$C_1 = \dfrac{5.25/M}{V}$

For solution of urea,

$C_2 \text{ (concentration)} = \dfrac{1.5/60}{V}$

Given, $\pi_1 = \pi_2$

$\because \pi = CRT$

$\therefore\ C_1RT = C_2RT$ or $C_1 = C_2$

or $\dfrac{5.25/M}{V} = \dfrac{1.5/60}{V}$

$\therefore\ M = 210$ g/mol

18. **(d)** At 1 atmospheric pressure the boiling point of mixture is 80 °C.

At boiling point the vapour pressure of mixture, P_T = 1 atmosphere = 760 mm Hg.

Using the relation,

$P_T = P_A^\circ X_A + P_B^\circ X_B$, we get

$P_B^\circ = 1000$ mm Hg, $X_A + X_B = 1\}$

or $760 = 520X_A + 1000 - 1000X_A$

or $480X_A = 240$

or $X_A = \dfrac{240}{480} = \dfrac{1}{2}$ or 50 mol. percent

19. **(a)** **NOTE** On addition of glucose to water, vapour pressure of water will decrease. The vapour pressure of a solution of glucose in water can be calculated using the relation

$\dfrac{P^\circ - P_s}{P^\circ} = \dfrac{\text{Moles of glucose in solution}}{\text{Moles of water in solution}}$

or $\dfrac{17.5 - P_s}{17.5} = \dfrac{18/180}{178.2/18}$ $[\because P^\circ = 17.5]$

or $17.5 - P_s = \frac{0.1 \times 17.5}{9.9}$ or $P_s = 17.325$ mm Hg.

20. **(b)** For this solution intermolecular interactions between *n*-heptane and ethanol are weaker than *n*-heptane - *n*-heptane & ethanol-ethanol interactions, hence the solution of *n*-heptane and ethanol is non-ideal and shows positive deviation from Raoult's law.

21. **(b)** $P_{total} = P_x^{\circ} X_x + P_y^{\circ} X_y$

$$550 = P_x^{\circ} \times \frac{1}{4} + P_y^{\circ} \times \frac{3}{4}$$

$$P_x^{\circ} + 3P_y^{\circ} = 550 \times 4 \quad ...(i)$$

In second case,

$$P_{total} = P_x^{\circ} \times \frac{1}{5} + P_y^{\circ} \times \frac{4}{5}$$

$$P_x^{\circ} + 4P_y^{\circ} = 560 \times 5 \quad ...(ii)$$

Subtract (i) from (ii)

$\therefore P_y^{\circ} = 560 \times 5 - 550 \times 4 = 600$

$\because P_x^{\circ} = 400$

22. **(b)** Sodium sulphate dissociates as

$$Na_2SO_4(s) \longrightarrow 2Na^+ + SO_4^{2-}$$

Hence van't Hoff factor, i = 3

Now $\Delta T_f = i K_f . m$

$= 3 \times 1.86 \times 0.01 = 0.0558$ K

23. **(a)** $P_{Total} = P^{\circ}_A X_A + P^{\circ}_B X_B$

$= P^{\circ}_{Heptane} X_{Heptane} + P^{\circ}_{Octane} X_{Octane}$

$$= 105 \times \frac{25/100}{\frac{25}{100} + \frac{35}{114}} + 45 \times \frac{35/114}{\frac{25}{100} + \frac{35}{114}}$$

$$= 105 \times \frac{0.25}{0.25 + 0.3} + 45 \times \frac{0.3}{0.25 + 0.3}$$

$$= \frac{105 \times 0.25}{0.55} + \frac{45 \times 0.3}{0.55} = \frac{26.25 + 13.5}{0.55}$$

$= 72$ kPa

24. **(b)** For isotonic solutions

$\pi_1 = \pi_2$

$C_1 = C_2$

$$\frac{5/342}{0.1} = \frac{1/M}{0.1}$$

$$\frac{5}{342} = \frac{1}{M}$$

$$\Rightarrow M = \frac{342}{5} = 68.4 \text{ g/mol}$$

25. **(d)** $\text{Molarity} = \frac{\text{Moles of solute}}{\text{Volume of solution}(\ell)}$

Mass of solution = 1000 + 120 = 1120

$$d = \frac{M}{V}; V = \frac{M}{d} = \frac{1120}{1.15} \text{mL}$$

$$= \frac{120 \times 1.15}{60 \times 1120} \times 1000 = 2.05 \text{ M}$$

26. **(b)** $\Delta T_f = i \times K_f \times m$

Given $\Delta T_f = 2.8$, $K_f = 1.86$ K kg mol^{-1} i = 1

(ethylene glygol is a non- electrolyte)

Wt. of solvent = 1 kg

Let of wt of solute = x

Mol. wt of ethylene glycol = 62

$$2.8 = 1 \times 1.86 \times \frac{x}{62 \times 1}$$

$$\text{or} \quad x = \frac{2.8 \times 62}{1.86} = 93 \text{ g}$$

27. **(a)** From molarity equation :

$M_1V_1 + M_2V_2 = M \times V$

$$M = \frac{M_1V_1 + M_2V_2}{V} \text{ where V = total volume}$$

$$= \frac{750 \times 0.5 + 250 \times 2}{1000}$$

$= 0.875$ M

28. **(a)** $\pi = i\,CRT$

$\pi_{C_2H_5OH} = 1 \times 0.500 \times R \times T = 0.5RT$

$\pi_{Mg_3(PO_4)_2} = 5 \times 0.100 \times R \times T = 0.5RT$

$\pi_{KBr} = 2 \times 0.250 \times R \times T = 0.5RT$

$\pi_{Na_3PO_4} = 4 \times 0.125 \times RT = 0.5RT$

Since the osmotic pressure of all the given solutions is equal. Hence all are isotonic solutions.

29. (d) Using relation,

$$\frac{P^\circ - P_s}{P^\circ} = \frac{w_2 M_1}{w_1 M_2}$$

where w_1, M_1 = mass in g and mol. mass of solvent

w_2, M_2 = mass in g and mol. mass of solute

$P^\circ = 185$ torr, $P_s = 183$ torr

$$\frac{185-183}{185} = \frac{1.2 \times 58}{100 \times M_2}$$

(Mol. mass of acetone = 58)

$M_2 = 64.68 \approx 64$

$\therefore$ Molar mass of substance = 64

30. (d) In benzene,

$$\underset{1-\alpha}{2CH_3COOH} \rightleftharpoons \underset{\alpha/2}{(CH_3COOH)_2}$$

$$i = 1 - \alpha + \alpha/2 = 1 - \alpha/2$$

Here α is degree of association

$$\Delta T_f = i \times K_f \times m$$

$$0.45 = \left(1 - \frac{\alpha}{2}\right)(5.12)\frac{\left(\frac{0.2}{60}\right)}{\frac{20}{1000}}$$

$$1 - \frac{\alpha}{2} = 0.527$$

$$\alpha = 0.945$$

% degree of association = 94.6%

31. (d)

		Number of particles (i)
(a)	$[Co(H_2O)_6]Cl_3$	4
(b)	$[Co(H_2O)_5Cl]Cl_2 \cdot H_2O$	3
(c)	$[Co(H_2O)_4Cl_2]Cl \cdot 2H_2O$	2
(d)	$[Co(H_2O)_3Cl_3] \cdot 3H_2O$	1

$\Delta T_f \propto i$; where $\Delta T_f = (T_f - T_f')$

Remember, greater the no. of particles, lower will be the freezing point. Compound (d) will have the highest freezing point due to least number of particles.

32. (b) We know, $\pi = iCRT$; $\pi_{xy} = 4\pi_{BaCl_2}$

$\therefore 2[XY] = 4 \times (0.01) \times 3$

$[XY] = 0.06$

$= 6 \times 10^{-2}$ mol/L

33. (b) Number of moles in 92 g of $Na^+ = \frac{92}{23}$

= 4 moles

$$\text{Molality } (m) = \frac{\text{Number of moles}}{\text{Mass of solvent (in kg)}}$$

$$\therefore \quad m = \frac{4}{1} = 4 \text{ mol kg}^{-1}$$

34. (a) The solubility of the gas in liquids decreases with the increase in value of K_H at a given pressure.

35. (d) $P_M^\circ = 450$ mmHg, $P_N^\circ = 700$ mmHg

$P_M = P_M^\circ x_M = y_M P_T$

$$\Rightarrow \quad P_M^\circ = \frac{y_M}{x_M}(P_T)$$

Similarly, $P_N^\circ = \frac{y_N}{x_N}(P_T)$

Given, $P_M^\circ < P_N^\circ$

$$\Rightarrow \quad \frac{y_M}{x_M} < \frac{y_N}{x_N}$$

$$\Rightarrow \quad \frac{y_M}{y_N} < \frac{x_M}{x_N}$$

36. (b) Mixture of carbon disulphide and acetone will show positive deviation from Raoult's Law.

The dipolar interaction between solute (CS_2) solvent (acetone) molecules in solution are weaker. So the vapour pressure of solution will be greater than the individual vapoure pressure of pure components.

17 Electrochemistry

1. Conductivity (unit Siemen's S) is directly proportional to area of the vessel and the concentration of the solution in it and is inversely proportional to the length of the vessel then the unit of the constant of proportionality is **[2002]**
(a) $Sm\,mol^{-1}$ (b) $Sm^2\,mol^{-1}$
(c) $S^{-2}m^2\,mol$ (d) $S^2m^2\,mol^{-2}$.

2. EMF of a cell in terms of reduction potential of its left and right electrodes is **[2002]**
(a) $E = E_{left} - E_{right}$ (b) $E = E_{left} + E_{right}$
(c) $E = E_{right} - E_{left}$ (d) $E = -(E_{right} + E_{left})$.

3. What will be the emf for the given cell **[2002]**
$Pt \mid H_2(P_1) \mid H^+(aq) \mid\mid H_2(P_2) \mid Pt$
(a) $\frac{RT}{F}\log_e\frac{P_1}{P_2}$ (b) $\frac{RT}{2F}\log_e\frac{P_2}{P_1}$
(c) $\frac{RT}{F}\log_e\frac{P_2}{P_1}$ (d) None of these

4. Which of the following reaction is possible at anode? **[2002]**
(a) $2\,Cr^{3+} + 7H_2O \longrightarrow Cr_2O_7^{2-} + 14H^+$
(b) $F_2 \longrightarrow 2F^-$
(c) $(1/2)\,O_2 + 2H^+ \longrightarrow H_2O$
(d) None of these

5. When the sample of copper with zinc impurity is to be purified by electrolysis, the appropriate electrodes are **[2002]**

	Cathode	Anode
(a)	pure zinc	pure copper
(b)	impure sample	pure copper
(c)	impure zinc	impure sample
(d)	pure copper	impure sample

6. For a cell reaction involving a two-electron change, the standard e.m.f. of the cell is found to be 0.295 V at 25ºC. The equilibrium constant of the reaction at 25ºC will be **[2003]**
(a) 29.5×10^{-2} (b) 10
(c) 1×10^{10} (d) 1×10^{-10}

7. Standard reduction electrode potentials of three metals A, B & C are respectively +0.5 V, – 3.0 V & –1.2 V. The reducing powers of these metals are **[2003]**
(a) A > B > C (b) C > B > A
(c) A > C > B (d) B > C > A

8. When during electrolysis of a solution of $AgNO_3$ 9650 coulombs of charge pass through the electroplating bath, the mass of silver deposited on the cathode will be **[2003]**
(a) 10.8 g (b) 21.6 g
(c) 108 g (d) 1.08 g

9. For the redox reaction : **[2003]**
$Zn(s) + Cu^{2+}(0.1M) \rightarrow Zn^{2+}(1M) + Cu(s)$
taking place in a cell, E°_{cell} is 1.10 volt. E_{cell} for the cell will be $\left(2.303\frac{RT}{F} = 0.0591\right)$ **[2003]**
(a) 1.80 volt (b) 1.07 volt
(c) 0.82 volt (d) 2.14 volt

10. In a hydrogen-oxygen fuel cell, combustion of hydrogen occurs to **[2004]**
(a) produce high purity water
(b) create potential difference between two electrodes
(c) generate heat
(d) remove adsorbed oxygen from elctrode surfaces

11. Consider the following E° values

$E^o_{Fe^{3+}/Fe^{2+}} = +0.77V$; $E^o_{Sn^{2+}/Sn} = -0.14V$

Under standard conditions the potential for the reaction

$Sn(s) + 2Fe^{3+}(aq) \rightarrow 2Fe^{2+}(aq) + Sn^{2+}(aq)$ is **[2004]**

(a) 0.91 V (b) 1.40 V
(c) 1.68 V (d) 0.63 V

12. The standard e.m.f. of a cell involving one electron change is found to be 0.591 V at 25°C. The equilibrium constant of the reaction is ($F = 96{,}500\ C\ mol^{-1}$; $R = 8.314\ JK^{-1}\ mol^{-1}$) **[2004]**

(a) 1.0×10^{10} (b) 1.0×10^{5}
(c) 1.0×10^{1} (d) 1.0×10^{30}

13. The limiting molar conductivities $\Lambda°$ for NaCl, KBr and KCl are 126, 152 and 150 $S\ cm^2\ mol^{-1}$ respectively. The Λ^o for NaBr is **[2004]**

(a) $278\ S\ cm^2\ mol^{-1}$ (b) $176\ S\ cm^2\ mol^{-1}$
(c) $128\ S\ cm^2\ mol^{-1}$ (d) $302\ S\ cm^2\ mol^{-1}$

14. In a cell that utilises the reaction

$Zn(s) + 2H^+(aq) \rightarrow Zn^{2+}(aq) + H_2(g)$

addition of H_2SO_4 to cathode compartment, will **[2004]**

(a) increase the E and shift equilibrium to the right
(b) lower the E and shift equilibrium to the right
(c) lower the E and shift equlibrium to the left
(d) increase the E and shift equilibrium to the left

15. The $E^o_{M^{3+}/M^{2+}}$ values for Cr, Mn, Fe and Co are – 0.41, + 1.57, + 0.77 and + 1.97V respectively. For which one of these metals the change in oxidation state from +2 to +3 is easiest? **[2004]**

(a) Fe (b) Mn
(c) Cr (d) Co

16. For a spontaneous reaction the ΔG, equilibrium constant (K) and E°_{cell} will be respectively **[2005]**

(a) –ve, >1, –ve (b) –ve, <1, –ve
(c) +ve, >1, –ve (d) –ve, >1, +ve

17. The highest electrical conductivity of the following aqueous solutions is of **[2005]**

(a) 0.1 M difluoroacetic acid
(b) 0.1 M fluoroacetic acid
(c) 0.1 M chloroacetic acid
(d) 0.1 M acetic acid

18. Aluminium oxide may be electrolysed at 1000°C to furnish aluminium metal (At. Mass = 27 amu; 1 Faraday = 96,500 Coulombs). The cathode reaction is– $Al^{3+} + 3e^- \rightarrow Al$

To prepare 5.12 kg of aluminium metal by this method we require **[2005]**

(a) 5.49×10^1 C of electricity
(b) 5.49×10^4 C of electricity
(c) 1.83×10^7 C of electricity
(d) 5.49×10^7 C of electricity

19.

Electrolyte:	KCl	KNO_3	HCl	NaOAc	NaCl
$\Lambda^\infty (S\ cm^2\ mol^{-1})$:	149.9	145	426.2	91	126.5

Calculate Λ^∞_{HOAc} using appropriate molar conductances of the electrolytes listed above at infinite dilution in H_2O at 25 °C **[2005]**

(a) 217.5 (b) 390.7
(c) 552.7 (d) 517.2

20. The molar conductivities Λ^o_{NaOAc} and Λ^o_{HCl} at infinite dilution in water at 25°C are 91.0 and 426.2 S cm^2/mol respectively. To calculate Λ^o_{HOAc}, the additional value required is **[2006]**

(a) Λ^o_{NaOH} (b) Λ^o_{NaCl}
(c) $\Lambda^o_{H_2O}$ (d) Λ^o_{KCl}

21. Resistance of a conductivity cell filled with a solution of an electrolyte of concentration 0.1 M is 100 Ω. The conductivity of this solution is 1.29 S m^{-1}. Resistance of the same cell when filled with 0.2 M of the same solution is 520 Ω. The molar conductivity of 0.2 M solution of electrolyte will be **[2006]**

(a) $1.24 \times 10^{-4}\ S\ m^2\ mol^{-1}$
(b) $12.4 \times 10^{-4}\ S\ m^2\ mol^{-1}$
(c) $124 \times 10^{-4}\ S\ m^2\ mol^{-1}$
(d) $1240 \times 10^{-4}\ S\ m^2\ mol^{-1}$

22. Given the data at 25 ºC

$$Ag + I^- \longrightarrow AgI + e^- \quad E^0 = 0.152 \text{ V}$$
$$Ag \longrightarrow Ag^+ + e^- \quad E^0 = -0.800 \text{ V}$$

What is the value of log K_{sp} for AgI? (2.303 RT/F = 0.059 V) **[2006]**

(a) –37.83 (b) –16.13
(c) –8.12 (d) +8.612

23. The equivalent conductances of two strong electrolytes at infinite dilution in H_2O (where ions move freely through a solution) at 25°C are given below : **[2007]**

$$\Lambda^\circ_{CH_3COONa} = 91.0 \text{ S cm}^2 \text{ / equiv.}$$
$$\Lambda^\circ_{HCl} = 426.2 \text{ S cm}^2 \text{ / equiv.}$$

What additional information/ quantity one needs to calculate Λ° of an aqueous solution of acetic acid?

(a) Λ° of chloroacetic acid ($ClCH_2COOH$)
(b) Λ° of NaCl
(c) Λ of CH_3COOK
(d) the limiting equivalent coductance of $H^+ (\lambda^\circ_{H^+})$.

24. The cell,

$$Zn \mid Zn^{2+} (1 \text{ M}) \parallel Cu^{2+} (1 \text{ M}) \mid Cu \ (E^\circ_{cell} = 1.10 \text{ V})$$

was allowed to be completely discharged at 298 K. The relative concentration of Zn^{2+} to Cu^{2+} $\left(\frac{[Zn^{2+}]}{[Cu^{2+}]}\right)$ is **[2007]**

(a) 9.65×10^4 (b) antilog (24.08)
(c) 37.3 (d) $10^{37.3}$.

25. Given $E^o_{Cr^{3+}/Cr} = -0.72$ V,

$E^o_{Fe^{2+}/Fe} = -0.42$ V.

The potential for the cell

$Cr|Cr^{3+}(0.1M)||Fe^{2+}(0.01\ M)|Fe$ is **[2008]**

(a) 0.26 V (b) 0.336 V
(c) –0.339 (d) 0.26 V

26. In a fuel cell methanol is used as fuel and oxygen gas is used as an oxidizer. The reaction is

$$CH_3OH(l) + 3/2O_2(g) \longrightarrow CO_2(g) + 2H_2O(l)$$

At 298 K standard Gibb's energies of formation for $CH_3OH(l)$, $H_2O(l)$ and $CO_2(g)$ are –166.2 –237.2 and –394.4 kJ mol^{-1} respectively. If standard enthalpy of combustion of methanol is – 726 kJ mol^{-1}, efficiency of the fuel cell will be: **[2009]**

(a) 87% (b) 90%
(c) 97% (d) 80%

27. Given:

$E^\circ_{Fe^{3+}/Fe} = -0.036$ V,

$E^\circ_{Fe^{2+}/Fe} = -0.439$ V

The value of standard electrode potential for the change,

$Fe^{3+}(aq) + e^- \longrightarrow Fe^{2+}(aq)$ will be: **[2009]**

(a) 0.385 V (b) 0.770 V
(c) –0.270 V (d) –0.072 V

28. The Gibbs energy for the decomposition of Al_2O_3 at 500°C is as follows :

$$\frac{2}{3}Al_2O_3 \rightarrow \frac{4}{3}Al + O_2, \Delta_r G = +966 \text{ kJ mol}^{-1}$$

The potential difference needed for electrolytic reduction of Al_2O_3 at 500°C is at least **[2010]**

(a) 4.5 V (b) 3.0 V
(c) 2.5 V (d) 5.0 V

29. Resistance of 0.2 M solution of an electrolyte is 50 Ω. The specific conductance of the solution is 1.3 S m^{-1}. If resistance of the 0.4 M solution of the same electrolyte is 260 Ω, its molar conductivity is : **[2011RS]**

(a) 6.25×10^{-4} S m^2 mol^{-1}
(b) 625×10^{-4} S m^2 mol^{-1}
(c) 62.5 S m^2 mol^{-1}
(d) 6250 S m^2 mol^{-1}

30. The standard reduction potentials for Zn^{2+}/Zn, Ni^{2+}/Ni and Fe^{2+}/Fe are –0.76, –0.23 and –0.44 V respectively.

The reaction $X + Y^{2+} \longrightarrow X^{2+} + Y$ will be spontaneous when : **[2012]**

(a) X = Ni, Y = Fe (b) X = Ni, Y = Zn
(c) X= Fe, Y = Zn (d) X= Zn, Y = Ni

31. Given :

$E^o_{Cr^{3+}/Cr} = -0.74$ V; $E^o_{MnO_4^-/Mn^{2+}} = 1.51$ V

$E^o_{Cr_2O_7^{2-}/Cr^{3+}} = 1.33$ V; $E^o_{Cl/Cl^-} = 1.36$ V

Based on the data given above, strongest oxidising agent will be : **[2013]**

(a) Cl (b) Cr^{3+}

(c) Mn^{2+} (d) MnO_4^-

32. Resistance of 0.2 M solution of an electrolyte is 50 Ω. The specific conductance of the solution is 1.4 S m^{-1}. The resistance of 0.5 M solution of the same electrolyte is 280 Ω. The molar conductivity of 0.5 M solution of the electrolyte in S $m^2 mol^{-1}$ is: **[2014]**

(a) 5×10^{-4} (b) 5×10^{-3}

(c) 5×10^{3} (d) 5×10^{2}

33. The equivalent conductance of NaCl at concentration C and at infinite dilution are λ_C and λ_∞, respectively. The correct relationship between λ_C and λ_∞ is given as:
(Where the constant B is positive) **[2014]**

(a) $\lambda_C = \lambda_\infty + (B)C$

(b) $\lambda_C = \lambda_\infty - (B)C$

(c) $\lambda_C = \lambda_\infty - (B)\sqrt{C}$

(d) $\lambda_C = \lambda_\infty + (B)\sqrt{C}$

34. Given below are the half-cell reactions:

$Mn^{2+} + 2e^- \rightarrow Mn; E^o = -1.18$ V

$2\left(Mn^{3+} + e^- \rightarrow Mn^{2+}\right); E^o = +1.51$ V

The E^o for $3Mn^{2+} \longrightarrow Mn + 2Mn^{3+}$ will be: **[2014]**

(a) –2.69 V; the reaction will not occur

(b) –2.69 V; the reaction will occur

(c) –0.33 V; the reaction will not occur

(d) –0.33 V; the reaction will occur

35. Two Faraday of electricity is passed through a solution of $CuSO_4$. The mass of copper deposited at the cathode is
(at. mass of Cu = 63.5 amu) **[2015]**

(a) 2g (b) 127 g

(c) 0 g (d) 63.5 g

36. Given **[2017]**

$E^o_{Cl_2/Cl^-} = 1.36V, E^o_{Cr^{3+}/Cr} = -0.74V,$

$E^o_{Cr_2O_7^{2-}/Cr^{3+}} = 1.33V, E^o_{MnO_4^-/Mn^{2+}} = 1.51V.$

Among the following, the strongest reducing agent is

(a) Cr (b) Mn^{2+}

(c) Cr^{3+} (d) Cl^-

37. How long (approximate) should water be electrolysed by passing through 100 amperes current so that the oxygen released can completely burn 27.66 g of diborane? (Atomic weight of B = 10.8 u) **[2018]**

(a) 6.4 hours (b) 0.8 hours

(c) 3.2 hours (d) 1.6 hours

38. The anodic half-cell of lead-acid battery is recharged using electricity of 0.05 Faraday. The amount of $PbSO_4$ electrolyzed in g during the process is : (Molar mass of $PbSO_4$ = 303 g mol^{-1}) **[2019]**

(a) 22.8 (b) 15.2

(c) 7.6 (d) 11.4

39. The standard Gibbs energy for the given cell reaction in kJ mol^{-1} at 298 K is: **[2019]**

$Zn(s) + Cu^{2+}(aq) \longrightarrow Zn^{2+}(aq) + Cu(s),$

$E^\circ = 2$ V at 298 K

(Faraday's constant, F = 96000 C mol^{-1})

(a) –384 (b) 384

(c) 192 (d) –192

40. Given that the standard potentials (E^0) of Cu^{2+}/Cu and Cu^+/Cu are 0.34 V and 0.522 V respectively, the E^0 of Cu^{2+}/Cu^+ is: **[2020]**

(a) +0.182 V (b) +0.158 V

(c) –0.182 V (d) –0.158 V

Answer Key														
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
(b)	(c)	(d)	(a)	(d)	(c)	(d)	(a)	(b)	(b)	(a)	(a)	(c)	(a)	(c)
16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
(d)	(a)	(d)	(b)	(b)	(b)	(b)	(b)	(d)	(d)	(c)	(b)	(c)	(a)	(d)
31	32	33	34	35	36	37	38	39	40					
(d)	(a)	(c)	(a)	(d)	(a)	(c)	(c)	(a)	(b)					

Solutions

1. **(b)** Given conductivity $\propto \frac{\text{area} \times \text{conc.}}{\text{length}}$

$$= \frac{\text{Km}^2\text{mol}}{\text{m} \times \text{m}^3}$$

$\therefore \quad K = Sm^2 mol^{-1}$

2. **(c)** E_{cell} = Reduction potential of cathode (right) – reduction potential of anode (left)

$= E_{right} - E_{left}$

3. **(d)** Oxidation half call:-

$$\underset{P_1}{H_2(g)} \longrightarrow 2H^+(1M) + 2e^-$$

Reduction half cell

$$2H^+(1M) + 2e^- \longrightarrow \underset{P_2}{H_2(g)}$$

The net cell reaction

$$\underset{P_1}{H_2(g)} \longrightarrow \underset{P_2}{H_2(g)}$$

$E^o_{cell} = 0.00\ V;\ n = 2$

$$\therefore \quad E_{cell} = E^o_{cell} - \frac{RT}{nF}\log_e K$$

$$= 0 - \frac{RT}{nF}\log_e \frac{P_2}{P_1}$$

$$\text{or } E_{cell} = \frac{RT}{2F}\log_e \frac{P_1}{P_2}$$

4. **(a)** $2Cr^{3+} + 7H_2O \rightarrow Cr_2O_7^{2-} + 14H^+$

O.S. of Cr changes from +3 to +6 by loss of electrons. At anode oxidation takes place.

5. **(d)** Pure metal always deposits at cathode.

6. **(c)** The equilibrium constant is related to the standard emf of cell by the expression

$$\log K = E^o{}_{cell} \times \frac{n}{0.059} = 0.295 \times \frac{2}{0.059}$$

$$\log K = \frac{590}{59} = 10 \text{ or } K = 1 \times 10^{10}$$

7. **(d)**

A	B	C
+0.5V	–3.0V	–1.2V

NOTE The higher the negative value of reduction potential, the more is the reducing power.

Hence B > C > A.

8. **(a)** When 96500 coulomb of electricity is passed through the electroplating bath the amount of Ag deposited = 108g

$\therefore$ when 9650 coulomb of electricity is passed deposited Ag.

$$= \frac{108}{96500} \times 9650 = 10.8\text{ g}$$

9. **(b)** $E_{cell} = E^o_{cell} + \frac{0.059}{n}\log\frac{[Cu^{2+}]}{[Zn^{2+}]}$

$$= 1.10 + \frac{0.059}{2}\log[0.1]$$

$$= 1.10 - 0.0295 = 1.07\text{ V}$$

10. **(b)** In $H_2 - O_2$ fuel cell, the combustion of H_2 occurs to create potential difference between the two electrodes

11. **(a)** $Fe^{3+} + e^- \rightarrow Fe^{2+}$;

$Sn^{2+} + 2e^- \rightarrow Sn(s)$;

for $Sn(s) + 2Fe^{3+}(aq) \rightarrow 2Fe^{2+}(aq) + Sn^{2+}(aq)$

$\therefore$ Standard potential for the given reaction

or $E^\circ_{cell} = E^\circ_{Sn/Sn^{2+}} + E^\circ_{Fe^{3+}/Fe^{2+}}$

$= 0.14 + 0.77 = 0.91$ V

12. **(a)** $E_{cell} = E^\circ_{cell} - \frac{0.059}{n} \log K_c$

or $0 = 0.591 - \frac{0.0591}{1} \log K_c$

or $\log K_c = \frac{0.591}{0.0591} = 10$ or $K_c = 1 \times 10^{10}$

13. **(c)** $\Lambda^\circ_{NaCl} = \lambda^\circ_{Na^+} + \lambda_{Cl^-}$ ....(i)

$\Lambda^\circ_{KBr} = \lambda^\circ_{K^+} + \lambda^\circ_{Br^-}$ ....(ii)

$\Lambda^\circ_{KCl} = \lambda^\circ_{K^+} + \lambda_{Cl^-}$ ....(iii)

operating (i) + (ii) – (iii)

$\Lambda^\circ_{NaBr} = \lambda^\circ_{Na^+} + \lambda^\circ_{Br^-}$

$= 126 + 152 - 150 = 128 \text{ S cm}^2 \text{ mol}^{-1}$

14. **(a)** $Zn(s) + 2H^+(aq) \rightleftharpoons Zn^{2+}(aq) + H_2(g)$

$E_{cell} = E^\circ_{cell} - \frac{0.059}{2} \log \frac{[Zn^{2+}][H_2]}{[H^+]^2}$

Addition of H_2SO_4 will increase $[H^+]$ and E_{cell} will also increase and thus the equilibrium shifts towards RHS.

15. **(c)** The given values show that Cr has maximum oxidation potental, therefore its oxidation will be easiest. (Change the sign to get the oxidation values)

16. **(d)** **NOTE** For spontaneous reaction ΔG should be negative. Equilibrium constant should be more than one

($\Delta G = -2.303$ RT log K_c, If $K_c = 1$ then $\Delta G = 0$; If $K_c > 1$ then $\Delta G = -ve$).

Again $\Delta G = -nFE^\circ_{cell}$.

E°_{cell} must be +ve to have ΔG –ve.

17. **(a)** Difluoro acetic acid being strongest acid will furnish maximum number of ions showing highest electrical conductivity. The decreasing acidic strength of the carboxylic acids given is

difluoro acetic acid > fluoro acetic acid > chloro acetic acid > acetic acid.

18. **(d)** 1 mole of $e^- = 1F = 96500$ C

27g of Al is deposited by 3×96500 C

5120 g of Al will be deposited by

$= \frac{3 \times 96500 \times 5120}{27} = 5.49 \times 10^7$ C

ALTERNATE SOLUTION

We know, $Q = \frac{mFz}{M}$

$\therefore Q = \frac{5120 \times 96500 \times 3}{27} = 5.49 \times 10^7$ C

19. **(b)** $\Lambda^\infty_{HCl} = 426.2$ ....(i)

$\Lambda^\infty_{AcONa} = 91.0$ ....(ii)

$\Lambda^\infty_{NaCl} = 126.5$ ....(iii)

$\Lambda^\infty_{AcOH} = (i) + (ii) - (iii)$

$= [426.2 + 91.0 - 126.5] = 390.7$

20. **(b)** $\Lambda^\circ_{CH_3COOH}$ is given by the following equation

$$\Lambda^\circ_{CH_3COOH} = (\Lambda^\circ_{CH_3COONa} + \Lambda^\circ_{HCl}) - (\Lambda^\circ_{NaCl})$$

Hence Λ°_{NaCl} is required.

21. **(b)** $R = 100\,\Omega,\ \kappa = \frac{1}{R}\left(\frac{l}{a}\right),$

$\frac{l}{a}$ (cell constant) $= 1.29 \times 100\ m^{-1}$

Given, $R = 520\Omega$,

$C = 0.2\ M = 0.2 \times 1000\ mol\ m^{-3}$

Λ_m (molar conductivity) = ?

$\Lambda_m = K \times V$ (K can be calculated as cell constant is known.)

$$K = \frac{1}{R}\left(\frac{l}{a}\right)$$

Hence,

$$K = \frac{129 m^{-1}}{520\ \Omega} = 0.248\ Sm^{-1}$$

$$\Lambda_m = \frac{K}{C} = \frac{0.248\ Sm^{-1}}{0.2 \times 1000\ mol\ m^{-3}}$$

$$= 12.4 \times 10^{-4} Sm^2 mol^{-1}$$

22. **(b)** (i) $Ag \longrightarrow Ag^+ + e^- \quad E^0 = -0.800\ V$

(ii) $Ag + I^- \longrightarrow AgI + e^- \quad E^0 = 0.152\ V$

From (i) and (ii), we have

$$AgI \longrightarrow Ag^+ + I^- \quad E^0 = -0.952\ V$$

$$E^0_{cell} = \frac{0.059}{n} \log K$$

$$\therefore\ -0.952 = \frac{0.059}{1} \log [Ag^+][I^-]$$

$[\because K_{sp} = [Ag^+][I^-]]$

or $\quad -\frac{0.952}{0.059} = \log K_{sp}$

or $\quad -16.13 = \log K_{sp}$

23. **(b)** **NOTE** According to Kohlrausch's law, equivalent conductivity of weak electrolyte, acetic acid (CH_3COOH) can be calculated as follows:

$$\Lambda^\circ_{CH_3COOH} = \left(\Lambda^\circ_{CH_3COONa} + \Lambda^\circ_{HCl}\right) - \Lambda^\circ_{NaCl}$$

$\therefore$ Value of Λ°_{NaCl} should also be known for calculating value of $\Lambda^\circ_{CH_3COOH}$.

24. **(d)** $E_{cell} = 0$; when cell is completely discharged.

$$E_{cell} = E^\circ_{cell} - \frac{0.059}{2}\log\left(\frac{[Zn^{2+}]}{[Cu^{2+}]}\right)$$

$$\text{or } 0 = 1.1 - \frac{0.059}{2}\log\left(\frac{[Zn^{2+}]}{[Cu^{2+}]}\right)$$

$$\log\left(\frac{[Zn^{2+}]}{[Cu^{2+}]}\right) = \frac{2 \times 1.1}{0.059} = 37.3$$

$$\therefore \left(\frac{[Zn^{2+}]}{[Cu^{2+}]}\right) = 10^{37.3}$$

25. **(d)** From the given representation of the cell, E_{cell} can be found as follows.

$$E_{cell} = E^o_{Fe^{2+}/Fe} - E^o_{Cr^{3+}/Cr} - \frac{0.059}{6}\log\frac{[Cr^{3+}]^2}{[Fe^{2+}]^3}$$

[Nernst -Equ.]

$$= -0.42 - (-0.72) - \frac{0.059}{6}\log\frac{(0.1)^2}{(0.01)^3}$$

$$= -0.42 + 0.72 - \frac{0.059}{6}\log\frac{0.1 \times 0.1}{0.01 \times 0.01 \times 0.01}$$

$$= 0.3 - \frac{0.059}{6}\log\frac{10^{-2}}{10^{-6}} = 0.3 - \frac{0.059}{6} \times 4$$

$$= 0.30 - 0.0393 = 0.26\,V$$

26. **(c)** $CH_3OH\,(l) + \frac{3}{2}O_2\,(g) \rightarrow CO_2\,(g) + 2H_2O\,(l)$

$$\Delta G_r = \Delta G_f(CO_2, g) + 2\Delta G_f(H_2O, l) - \Delta G_f(CH_3OH, l) - \frac{3}{2}\Delta G_f(O_2, g)$$

$$= -394.4 + 2(-237.2) - (-166.2) - 0$$
$$= -394.4 - 474.4 + 166.2 = -702.6 \text{ kJ}$$

$$\% \text{ efficiency} = \frac{702.6}{726} \times 100 = 97\%$$

27. **(b)** Given

$Fe^{3+} + 3e^- \rightarrow Fe$,

$E^\circ_{Fe^{3+}/Fe} = -0.036\,V$ ...(i)

$Fe^{2+} + 2e^- \rightarrow Fe$,

$E^\circ_{Fe^{2+}/Fe} = -0.439\,V$ ...(ii)

we have to calculate

$Fe^{3+} + e^- \rightarrow Fe^{2+}$, $\Delta G^\circ = ?$

To obtain this equation subtract equ. (ii) from (i) we get

$Fe^{3+} + e^- \rightarrow Fe^{2+}$ ...(iii)

As we know that $\Delta G^\circ = -nFE^\circ$

Thus for reaction (iii)

$\Delta G_3 = \Delta G_1 - \Delta G_2$

$-nFE_3^\circ = -nFE_1^\circ - (-nFE_2^\circ)$

$-nFE_3^\circ = nFE_2^\circ - nFE_1^\circ$

$-1FE_3^\circ = 2 \times 0.439F - 3 \times 0.036\,F$

$-1FE_3^\circ = 0.770\,F$

$\therefore\ E_3^\circ = -0.770V$

28. **(c)** $\frac{2}{3}Al_2O_3 \rightarrow \frac{4}{3}Al + O_2$

The ionic reactions are

$\frac{2}{3} \times (2Al^{3+}) + 4e^- \rightarrow \frac{4}{3}Al$

$\frac{2}{3} \times (3O^{2-}) \rightarrow O_2 + 4e^-$

Thus, no. of electron transferred $\Rightarrow$ n = 4

$\Delta G = -nFE$

or $E = \frac{\Delta G}{-nF} = \frac{966 \times 10^3}{4 \times 96500} = -2.5\,V$

$\therefore$ The potential difference needed for the reduction = 2.5 V.

29. **(a)** $k = \frac{1}{R} \times \frac{l}{a}$

$1.3 = \frac{1}{50} \times \frac{l}{a}$

$\frac{l}{a} = 65\,m^{-1}$

$\Lambda_m = \frac{k \times 1000}{\text{molarity}}$

[molarity is in moles/litre but 1000 is used to convert litre into cm^3]

$$= \frac{\left(\frac{1}{260} \times 65\ m^{-1}\right) \times 1000\ cm^3}{0.4\ mol}$$

$$= \frac{65\ m^{-1}}{260 \times 0.4\ mol} \times \frac{1}{1000}\ m^3$$

$= 6.25 \times 10^{-4}\ S\,m^2\,mol^{-1}$

30. **(d)** For a spontaneous reaction ΔG° must be –ve

Since $\Delta G^\circ = -nFE^\circ$

Hence for ΔG° to be –ve, ΔE° has to be positive. Which is possible when X = Zn, Y = Ni

$Zn + Ni^{2+} \longrightarrow Zn^{2+} + Ni$

$E^\circ_{cell} = E^\circ_{Ni^{2+}/Ni} - E^\circ_{Zn^{2+}/Zn}$

$= -0.23 - (-0.76) = +0.53$ (positive)

31. **(d)** Higher the value of standard reduction potential, stronger is the oxidising agent, hence MnO_4^- is the strongest oxidising agent.

32. **(a)** Given for 0.2 M solution

$R = 50\,\Omega$

$\kappa = 1.4\ S\ m^{-1} = 1.4 \times 10^{-2}\ S\ cm^{-1}$

Now, $R = \rho\frac{l}{a} = \frac{1}{\kappa} \times \frac{l}{a}$

$\Rightarrow \frac{l}{a} = R \times \kappa = 50 \times 1.4 \times 10^{-2}\ cm^{-1}$

For 0.5 M solution

$R = 280\,\Omega$

$\kappa = ?$

$\frac{l}{a} = 50 \times 1.4 \times 10^{-2}\ cm^{-1}$

$\Rightarrow R = \rho\frac{l}{a} = \frac{1}{\kappa} \times \frac{l}{a}$

$\Rightarrow \kappa = \frac{1}{280} \times 50 \times 1.4 \times 10^{-2}$

$= \frac{1}{280} \times 70 \times 10^{-2}$

$= 2.5 \times 10^{-3}\ S\ cm^{-1}$

Now, $\Lambda_m = \frac{\kappa \times 1000}{M}$

$= \frac{2.5 \times 10^{-3}\ S\ cm^{-1} \times 1000\ cm^3/L}{0.5\ mol/L}$

$= 5\ S\ cm^2\ mol^{-1} = 5 \times 10^{-4}\ S\ m^2\ mol^{-1}$

33. **(c)** According to Debye Huckle onsager equation,

$\lambda_C = \lambda_\infty - B\sqrt{C}$

34. **(a)** (a) $Mn^{2+} + 2e^- \rightarrow Mn; E^o = -1.18V$; ...(i)

(b) $Mn^{3+} + e^- \rightarrow Mn^{2+}; E^o = -1.51V$; ...(ii)

Now multiplying equation (ii) by two and subtracting from equation (i)

$3Mn^{2+} \rightarrow Mn^+ + 2Mn^{3+}$;

$E^o = E_{Ox.} + E_{Red.}$

$= -1.18 + (-1.51) = -2.69\ V$

(–ve value of EMF (i.e. ΔG = +ve) shows that the reaction is non-spontaneous)

35. **(d)** $Cu^{2+} + 2e^- \longrightarrow Cu$

2F i.e. 2×96500 C deposit Cu = 1 mol

= 63.5g

36. **(a)** $E^\circ_{MnO_4^-/Mn^{2+}} = 1.51V$

$E^\circ_{Cl_2/Cl^-} = 1.36V$

$E^\circ_{C_2O_7^{2-}/Cr^{3+}} = 1.33V$

$E^\circ_{Cr^{3+}/Cr} = -0.74$

Since Cr^{3+} is having least reduction potential, so Cr is the best reducing agent.

37. **(c)** $B_2H_6 + 3O_2 \longrightarrow B_2O_3 + 3H_2O$

27.66 g of B_2H_6 (1 mole) requires 3 moles of oxygen (O_2) for complete burning.

Now the oxygen is produced by the electrolysis of H_2O.

On electrolysis : $2H_2O \xrightarrow{4F} 2H_2 + O_2$

1 mole O_2 is produced by 4F charge

$\therefore$ 3 mole O_2 will be produced by 12F charge.

$\because$ Q = It

$12 \times 96500\ C = I \times t$

$12 \times 96500\ C = 100 \times t$

$t = \frac{12 \times 96500}{100}$ second

$= \frac{12 \times 96500}{100 \times 3600}$ hour

= 3.2 hours

38. **(c)** Half cell reaction: $PbSO_4 \rightarrow Pb^{4+} + 2e^-$

According to the reaction:

$PbSO_4 \rightarrow Pb^{4+} + 2e^-$

We require 2F for the electrolysis of 1 mol or 303 g of $PbSO_4$

$\therefore$ Amount of $PbSO_4$ electrolysed by 0.05

$F = \dfrac{303}{2} \times 0.05 = 7.575\ g \approx 7.6\ g$

39. **(a)** $\Delta G° = -nFE°_{cell}$

$= -2 \times (96000) \times 2\ V = -384000\ J/mol$

$= -384\ kJ/mol$

40. **(b)** $Cu^{2+} + 2e^- \longrightarrow Cu,\ \Delta G_1^\circ = -2F(0.34)$ ...(i)

$Cu^+ + e^- \longrightarrow Cu,\ \Delta G_2^\circ = -F(0.522)$ ...(ii)

Subtract (ii) from (i)

$Cu^{2+} + e^- \longrightarrow Cu^+, \quad \Delta G_3^\circ = -FE°$

$\therefore \Delta G_1^\circ - \Delta G_2^\circ = \Delta G_3^\circ$

$\Rightarrow -FE° = -2F(0.34) + F(0.522)$

$\Rightarrow E° = 0.68 - 0.522 = 0.158\ V$

18 Chemical Kinetics

1. Units of rate constant of first and zero order reactions in terms of molarity M unit are respectively **[2002]**
(a) sec^{-1}, $Msec^{-1}$ (b) sec^{-1}, M
(c) $Msec^{-1}$, sec^{-1} (d) M, sec^{-1}.

2. For the reaction $A + 2B \rightarrow C$, rate is given by $R = [A]\,[B]^2$, then the order of the reaction is **[2002]**
(a) 3 (b) 6 (c) 5 (d) 7

3. The differential rate law for the reaction $H_2 + I_2 \longrightarrow 2HI$ is **[2002]**
(a) $-\frac{d[H_2]}{dt} = -\frac{d[I_2]}{dt} = -\frac{d[HI]}{dt}$
(b) $\frac{d[H_2]}{dt} = \frac{d[I_2]}{dt} = \frac{1}{2}\frac{d[HI]}{dt}$
(c) $\frac{1}{2}\frac{d[H_2]}{dt} = \frac{1}{2}\frac{d[I_2]}{dt} = -\frac{d[HI]}{dt}$
(d) $-2\frac{d[H_2]}{dt} = -2\frac{d[I_2]}{dt} = \frac{d[HI]}{dt}$

4. If half-life of a substance is 5 yrs, then the total amount of substance left after 15 years, when initial amount is 64 grams is **[2002]**
(a) 16 grams (b) 2 grams
(c) 32 grams (d) 8 grams.

5. The integrated rate equation is **[2002]**
$Rt = \log C_0 - \log C_t$.
The straight line graph is obtained by plotting
(a) time vs $\log C_t$ (b) $\frac{1}{\text{time}}$ vs C_t
(c) time vs C_t (d) $\frac{1}{\text{time}}$ vs $\frac{1}{C_t}$

6. The half-life of a radioactive isotope is three hours. If the initial mass of the isotope were 256 g, the mass of it remaining undecayed after 18 hours would be **[2003]**
(a) 8.0 g (b) 12.0 g
(c) 16.0 g (d) 4.0 g

7. In respect of the equation $k = Ae^{-E_a/RT}$ in chemical kinetics, which one of the following statements is correct ? **[2003]**
(a) A is adsorption factor
(b) E_a is energy of activation
(c) R is Rydberg's constant
(d) k is equilibrium constant

8. For the reaction system **[2003]**
$$2NO(g) + O_2(g) \rightarrow 2NO_2(g)$$
volume is suddenly reduced to half its value by increasing the pressure on it. If the reaction is of first order with respect to O_2 and second order with respect to NO, the rate of reaction will
(a) diminish to one-eighth of its initial value
(b) increase to eight times of its initial value
(c) increase to four times of its initial value
(d) diminish to one-fourth of its initial value

9. In a first order reaction, the concentration of the reactant decreases from 0.8 M to 0.4 M in 15 minutes. The time taken for the concentration to change from 0.1 M to 0.025 M is **[2004]**
(a) 7.5 minutes (b) 15 minutes
(c) 30 minutes (d) 60 minutes

10. The rate equation for the reaction $2A + B \rightarrow C$ is found to be: rate $= k[A][B]$. The correct statement in relation to this reaction is that the **[2004]**

(a) rate of formation of C is twice the rate of disappearance of A
(b) $t_{1/2}$ is a constant
(c) unit of k must be s^{-1}
(d) value of k is independent of the initial concentrations of A and B

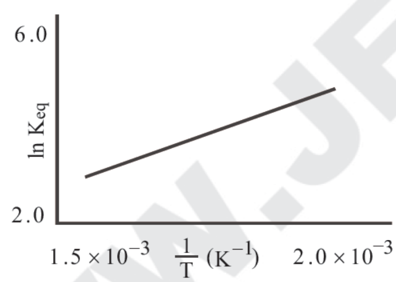
11. The half-life of a radioisotope is four hours. If the initial mass of the isotope was 200 g, the mass remaining after 24 hours undecayed is **[2004]**

(a) 3.125 g (b) 2.084 g
(c) 1.042 g (d) 4.167 g

12. A reaction involving two different reactants can never be **[2005]**

(a) bimolecular reaction
(b) second order reaction
(c) first order reaction
(d) unimolecular reaction

13. A schematic plot of $\ln K_{eq}$ versus inverse of temperature for a reaction is shown below **[2005]**



The reaction must be

(a) highly spontaneous at ordinary temperature
(b) one with negligible enthalpy change
(c) endothermic
(d) exothermic

14. $t_{\frac{1}{4}}$ can be taken as the time taken for the concentration of a reactant to drop to $\frac{3}{4}$ of its initial value. If the rate constant for a first order reaction is K, the $t_{\frac{1}{4}}$ can be written as **[2005]**

(a) 0.75/K (b) 0.69/K
(c) 0.29/K (d) 0.10/K

15. Consider an endothermic reaction $X \rightarrow Y$ with the activation energies E_b and E_f for the backward and forward reactions, respectively. In general **[2005]**

(a) there is no definite relation between E_b and E_f
(b) $E_b = E_f$
(c) $E_b > E_f$
(d) $E_b < E_f$

16. A reaction was found to be second order with respect to the concentration of carbon monoxide. If the concentration of carbon monoxide is doubled, with everything else kept the same, the rate of reaction will **[2006]**

(a) increase by a factor of 4
(b) double
(c) remain unchanged
(d) triple

17. Rate of a reaction can be expressed by Arrhenius equation as : **[2006]**

$$k = A\,e^{-E_a/RT}$$

In this equation, E_a represents

(a) the total energy of the reacting molecules at a temperature, T
(b) the fraction of molecules with energy greater than the activation energy of the reaction
(c) the energy above which all the colliding molecules will react
(d) the energy below which all the colliding molecules will react

18. The following mechanism has been proposed for the reaction of NO with Br_2 to form NOBr :

$$NO(g) + Br_2(g) \rightleftharpoons NOBr_2(g)$$
$$NOBr_2(g) + NO(g) \longrightarrow 2NOBr(g)$$

If the second step is the rate determining step, the order of the reaction with respect to NO(g) is **[2006]**

(a) 3 (b) 2
(c) 1 (d) 0

19. The energies of activation for forward and reverse reactions for $A_2 + B_2 \rightleftharpoons 2AB$ are 180 kJ mol^{-1} and 200 kJ mol^{-1} respectively. The presence of a catalyst lowers the activation energy of both (forward and reverse) reactions by 100 kJ mol^{-1}.

The enthalpy change of the reaction ($A_2 + B_2 \rightarrow 2AB$) in the presence of a catalyst will be (in kJ mol^{-1}) **[2007]**

(a) 20 (b) 300
(c) 120 (d) 280

20. Consider the reaction, $2A + B \rightarrow$ products. When concentration of B alone was doubled, the half-life did not change. When the concentration of A alone was doubled, the rate increased by two times. The unit of rate constant for this reaction is **[2007]**

(a) s^{-1} (b) L mol^{-1} s^{-1}
(c) no unit (d) mol L^{-1} s^{-1}.

21. A radioactive element gets spilled over the floor of a room. Its half-life period is 30 days. If the initial velocity is ten times the permissible value, after how many days will it be safe to enter the room? **[2007]**

(a) 100 days (b) 1000 days
(c) 300 days (d) 10 days.

22. For a reaction $\frac{1}{2}A \rightarrow 2B,$ rate of disappearance of 'A' is related to the rate of appearance of 'B' by the expression **[2008]**

(a) $-\frac{d[A]}{dt} = \frac{1}{2}\frac{d[B]}{dt}$ (b) $-\frac{d[A]}{dt} = \frac{1}{4}\frac{d[B]}{dt}$

(c) $-\frac{d[A]}{dt} = \frac{d[B]}{dt}$ (d) $-\frac{d[A]}{dt} = 4\frac{d[B]}{dt}$

23. The half life period of a first order chemical reaction is 6.93 minutes. The time required for the completion of 99% of the chemical reaction will be ($\log 2 = 0.301$) **[2009]**

(a) 23.03 minutes (b) 46.06 minutes
(c) 460.6 minutes (d) 230.03 minutes

24. The time for half life period of a certain reaction $A \longrightarrow$ Products is 1 hour. When the initial concentration of the reactant 'A' is 2.0 mol L^{-1}, how much time does it take for its concentration to come from 0.50 to 0.25 mol L^{-1} if it is a zero order reaction ? **[2010]**

(a) 4 h (b) 0.5 h
(c) 0.25 h (d) 1 h

25. Consider the reaction :

$$Cl_2(aq) + H_2S(aq) \rightarrow S(s) + 2H^+(aq) + 2Cl^-(aq)$$

The rate equation for this reaction is

rate = $k[Cl_2][H_2S]$

Which of these mechanisms is/are consistent with this rate equation? **[2010]**

A. $Cl_2 + H_2S \rightarrow H^+ + Cl^- + Cl^+ + HS^-$ (slow)

$Cl^+ + HS^- \rightarrow H^+ + Cl^- + S$ (fast)

B. $H_2S \rightleftharpoons H^+ + HS^-$ (fast equilibrium)

$Cl_2 + HS^- \rightarrow 2Cl^- + H^+ + S$ (Slow)

(a) B only (b) Both A and B
(c) Neither A nor B (d) A only

26. A reactant (A) froms two products : **[2011RS]**

$A \xrightarrow{k_1} B$, Activation energy Ea_1

$A \xrightarrow{k_2} C$, Activation energy Ea_2

If $Ea_2 = 2\,Ea_1$, then k_1 and k_2 are related as :

(a) $k_2 = k_1 e^{Ea_1/RT}$ (b) $k_2 = k_1 e^{Ea_2/RT}$

(c) $k_1 = Ak_2 e^{Ea_1/RT}$ (d) $k_1 = 2k_2 e^{Ea_2/RT}$

27. For a first order reaction (A) $\rightarrow$ products the concentration of A changes from 0.1 M to 0.025 M in 40 minutes.

The rate of reaction when the concentration of A is 0.01 M is : **[2012]**

(a) 1.73×10^{-5} M/min (b) 3.47×10^{-4} M/min
(c) 3.47×10^{-5} M/min (d) 1.73×10^{-4} M/min

28. The rate of a reaction doubles when its temperature changes from 300 K to 310 K. Activation energy of the a reaction will be : ($R = 8.314$ JK^{-1} mol^{-1} and $\log 2 = 0.301$) **[2013]**

(a) 53.6 kJ mol^{-1} (b) 48.6 kJ mol^{-1}
(c) 58.5 kJ mol^{-1} (d) 60.5 kJ mol^{-1}

29. For the non-stoichiometric reaction $2A + B \rightarrow C + D$, the following kinetic data were obtained in three separate experiments, all at 298 K.

Initial Concentration (A)	Initial Concentration (B)	Initial rate of formation of C (mol L^{-1} s^{-1})
0.1 M	0.1 M	1.2×10^{-3}
0.1 M	0.2 M	1.2×10^{-3}
0.2 M	0.1 M	2.4×10^{-3}

The rate law for the formation of C is: **[2014]**

(a) $\frac{dc}{dt} = k[A][B]$ (b) $\frac{dc}{dt} = k[A]^2[B]$

(c) $\frac{dc}{dt} = k[A][B]^2$ (d) $\frac{dc}{dt} = k[A]$

30. Higher order (>3) reactions are rare due to : **[2015]**
(a) shifting of equilibrium towards reactants due to elastic collisions
(b) loss of active species on collision
(c) low probability of simultaneous collision of all the reacting species
(d) increase in entropy and activation energy as more molecules are involved

31. Decomposition of H_2O_2 follows a first order reaction. In fifty minutes the concentration of H_2O_2 decreases from 0.5 to 0.125 M in one such decomposition. When the concentration of H_2O_2 reaches 0.05 M, the rate of formation of O_2 will be: **[2016]**
(a) 2.66 L min^{-1} at STP
(b) 1.34×10^{-2} mol min^{-1}
(c) 6.96×10^{-2} mol min^{-1}
(d) 6.93×10^{-4} mol min^{-1}

32. Two reactions R_1 and R_2 have identical pre-exponential factors. Activation energy of R_1 exceeds that of R_2 by 10 kJ mol^{-1}. If k_1 and k_2 are rate constants for reactions R_1 and R_2 respectively at 300 K, then $\ln(k_2/k_1)$ is equal to :
($R = 8.314 \text{ J mol}^{-1}\text{K}^{-1}$) **[2017]**
(a) 8 (b) 12
(c) 6 (d) 4

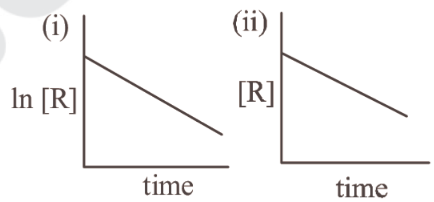
33. At 518 °C, the rate of decomposition of a sample of gaseous acetaldehyde, initially at a pressure of 363 Torr, was 1.00 Torr s^{-1} when 5% had reacted and 0.5 Torr s^{-1} when 33% had reacted. The order of the reaction is : **[2018]**
(a) 2 (b) 3
(c) 1 (d) 0

34. The following results were obtained during kinetic studies of the reaction; **[2019]**
$2A + B \rightarrow \text{Products}$

Experiment	[A] (in mol L^{-1})	[B] (in mol L^{-1})	Initial Rate of reaction (in mol L^{-1} min^{-1})
I	0.1	0.2	6.93×10^{-3}
II	0.1	0.25	6.93×10^{-3}
III	0.2	0.3	1.386×10^{-2}

The time (in minutes) required to consume half of A is:
(a) 5 (b) 10 (c) 1 (d) 100

35. The given plots represents the variation of the concentration of a reactant R with time for two different reactions (i) and (ii). The respective orders of the reactions are: **[2019]**



(a) 1, 0 (b) 1, 1
(c) 0, 1 (d) 0, 2

36. During the nuclear explosion, one of the products is ^{90}Sr with half life of 6.93 years. If 1 μg of ^{90}Sr was absorbed in the bones of a newly born baby in place of Ca, how much time, in years, is required to reduce it by 90% if it is not lost metabolically________. **[2020]**

Answer Key

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
(a)	(a)	(d)	(d)	(a)	(d)	(b)	(b)	(c)	(d)	(a)	(d)	(d)	(c)	(d)
16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
(a)	(c)	(b)	(a)	(b)	(a)	(b)	(b)	(c)	(d)	(c)	(b)	(a)	(d)	(c)
31	32	33	34	35	36									
(d)	(d)	(a)	(a)	(a)	(23.03)									

Solutions

1. **(a)** For a zero order reaction.
rate $=k[A]^o$ i.e. rate $= k$
hence unit of $k = \text{M.sec}^{-1}$
For a first order reaction.
rate $= k[A]$
$\therefore \quad k = \text{M.sec}^{-1}/\text{M} = \text{sec}^{-1}$

2. **(a)** **NOTE** Order is the sum of the power of the concentration terms in rate law expression.
Hence the order of reaction is $= 1 + 2 = 3$

3. **(d)** Rate of appearance of HI $= \frac{1}{2}\frac{d[HI]}{dt}$

Rate of disappearance of $H_2 = \frac{-d[H_2]}{dt}$

Rate of disappearance of $I_2 = \frac{-d[I_2]}{dt}$

hence $-\frac{d[H_2]}{dt} = -\frac{d[I_2]}{dt} = \frac{1}{2}\frac{d[HI]}{dt}$

or $-2\frac{d[H_2]}{dt} = -2\frac{d[I_2]}{dt} = \frac{d[HI]}{dt}$

4. **(d)** $t_{1/2} = 5$ years, T = 15 years.
Hence total number of half life periods $= \frac{15}{5} = 3$

$\therefore$ Amount left $= \frac{64}{(2)^3} = 8$ g

5. **(a)** $Rt = \log C_0 - \log C_t$
It is clear from the equation that if we plot a graph between $\log C_t$ and time, a straight line with a slope equal to $-\frac{k}{2.303}$ and intercept equal to $\log [C_0]$ will be obtained.

6. **(d)** $t_{1/2} = 3$ h, $T = 18$ h
$\because T = n \times t_{1/2}$

$\therefore n = \frac{18}{3} = 6$

Initial mass $(C_0) = 256$ g

$\therefore C_n = \frac{C_0}{2^n} = \frac{256}{(2)^6} = \frac{256}{64} = 4\,g$

7. **(b)** In equation $k = Ae^{-E_a/RT}$;
A = Frequency factor
k = velocity constant, R = gas constant and E_a = energy of activation

8. **(b)** $r = k\,[O_2][NO]^2$. When the volume is reduced to 1/2, the conc. will double
$\therefore$ New rate $= k[2O_2][2\,NO]^2 = 8\,k\,[O_2][NO]^2$
The new rate increases to eight times of its initial.

9. **(c)** As the concentration of reactant decreases from 0.8 to 0.4 in 15 minutes hence the $t_{1/2}$ is 15 minutes. To fall the concentration from 0.1 to 0.025 we need two half lives i.e., 30 minutes.

10. **(d)** The velocity constant depends on temperature only. It is independent of concentration of reactants.

11. **(a)** $N_t = N_0\left(\frac{1}{2}\right)^n$ where n is number of half life periods.

$n = \frac{\text{Total time}}{\text{half life}} = \frac{24}{4} = 6$

$\therefore N_t = 200\left(\frac{1}{2}\right)^6 = 3.125$ g

12. **(d)** The molecularity of a reaction is the number of reactant molecules taking part in a single step of the reaction.

NOTE The reaction involving two different reactants can never be unimolecular.

13. **(d)** The graph shows that reaction is exothermic.

$\log k = \frac{-\Delta H}{RT} + 1$

For exothermic reaction $\Delta H < 0$

$\therefore$ log k Vs $\frac{1}{T}$ would be negative straight line with positive slope.

14. **(c)** $t_{1/4} = \frac{2.303}{K}\log\frac{1}{3/4} = \frac{2.303}{K}\log\frac{4}{3}$

$= \frac{2.303}{K}(\log 4 - \log 3)$

$= \frac{2.303}{K}(2\log 2 - \log 3)$

$= \frac{2.303}{K}(2\times 0.301 - 0.4771) = \frac{0.29}{K}$

15. **(d)** Enthalpy of reaction $(\Delta H) = E_f - E_b$

For an endothermic reaction, ΔH = +ve hence for ΔH to be negative

$E_b < E_f$

16. **(a)** Since the reaction is 2nd order w.r.t CO. Thus, rate law is given as.

$r = k\,[CO]^2$

Let initial concentration of CO is a

i.e. $[CO] = a$

$\therefore \quad r_1 = k\,(a)^2 = ka^2$

When concentration becomes doubled, i.e. $[CO] = 2a$

$\therefore \quad r_2 = k\,(2a)^2 = 4ka^2 \quad \therefore \; r_2 = 4r_1$

So, the rate of reaction becomes 4 times.

17. **(c)** In Arrhenius equation, $k = A\,e^{-E_a/RT}$, E_a is the energy of activation, which is required by the colliding molecules to react resulting in the formation of products.

18. **(b)** (i) $NO(g) + Br_2(g) \rightleftharpoons NOBr_2(g)$

(ii) $NOBr_2(g) + NO(g) \longrightarrow 2NOBr(g)$

Rate law equation $= k[NOBr_2]\,[NO]$

But $NOBr_2$ is intermediate and must not appear in the rate law equation.

From Ist step, $K_c = \frac{[NOBr_2]}{[NO]\,[Br_2]}$

$\therefore \quad [NOBr_2] = K_c\,[NO]\,[Br_2]$

$\therefore$ Rate law equation $= k.K_c\,[NO]^2\,[Br_2]$

Hence order of reaction w.r.t. NO is 2.

19. **(a)** $\Delta H_R = E_f - E_b = 180 - 200 = -20$ kJ/mol

The nearest correct answer given in choices may be obtained by neglecting sign.

20. **(b)** For a first order reaction, $t_{1/2} = \frac{0.693}{K}$ i.e.

for a first order reaction $t_{1/2}$ does not depend up on the concentration. From the given data, we can say that order of reaction with respect to $B = 1$ because change in concentration of B does not change half life.

Order of reaction with respect to $A = 1$ because rate of reaction doubles when concentration of B is doubled keeping concentration of A constant.

$\therefore$ Order of reaction = 1 + 1 = 2 and units of second order reaction are L mol^{-1} sec^{-1}.

21. **(a)** Suppose activity of safe working = A

Given $A_0 = 10A$

$\lambda = \frac{0.693}{t_{1/2}} = \frac{0.693}{30}$

$t_{½} = \frac{2.303}{\lambda}\log\frac{A_0}{A} = \frac{2.303}{0.693/30}\log\frac{10A}{A}$

$= \frac{2.303\times 30}{0.693}\times\log 10 = 100$ days.

22. **(b)** The rate of reaction for the reaction

$\frac{1}{2}A \longrightarrow 2B$

can be written either as

$-2\frac{d}{dt}[A]$ with respect to 'A'

or $\frac{1}{2}\frac{d}{dt}[B]$ with respect to 'B'

From the above, we have

$-2\frac{d}{dt}[A] = \frac{1}{2}\frac{d}{dt}[B]$

or $-\frac{d}{dt}[A] = \frac{1}{4}\frac{d}{dt}[B]$

23. **(b)** For first order reaction,

$k = \frac{2.303}{t}\log\frac{100}{100-99}$

$\frac{0.693}{6.93} = \frac{2.303}{t}\log\frac{100}{1}$

$\frac{0.693}{6.93} = \frac{2.303\times 2}{t}$

$t = 46.06$ min

24. (c) For the reaction

A $\rightarrow$ Product

Given $t_{1/2} = 1$ hour

For a zero order reaction

$$t_{\text{completion}} = \frac{[A_0]}{k} = \frac{\text{Initial conc.}}{\text{Rate constant}}$$

$$\therefore\ t_{1/2} = \frac{[A_0]}{2k}$$

$$\text{or } k = \frac{[A_0]}{2t_{1/2}} = \frac{2}{2\times 1} = 1 \text{ mol lit}^{-1}\text{hr}^{-1}$$

Further for a zero order reaction

$$k = \frac{dx}{dt} = \frac{\text{change in concentration}}{\text{time}}$$

$$1 = \frac{0.50 - 0.25}{\text{time}}$$

$\therefore$ time = 0.25 hr.

25. (d) Since the slow step is the rate determining step, hence if we consider mechanism A. We find

Rate $= k[Cl_2][H_2S]$

Now, if we consider mechanism B. We find

Rate $= k[Cl_2][HS^-]$ ...(i)

From step 1 (fast equilibrium) of mechanism B.

$$k = \frac{[H^+][HS^-]}{H_2S}$$

$$\text{or } [HS^-] = \frac{k[H_2S]}{H^+}$$

Substituting this value in equation (i) we find

$$\text{Rate} = k[Cl_2]k\frac{[H_2S]}{H^+} = k'\frac{[Cl_2][H_2S]}{[H^+]}$$

Hence, only mechanism A is consistent with the given rate equation.

26. (c) $k_1 = A_1 e^{-Ea_1/RT}$ .....(i)

$k_2 = A_2 e^{-Ea_2/RT}$ .....(ii)

On dividing eqn. (i) by eqn. (ii)

$$\frac{k_1}{k_2} = \frac{A_1}{A_2}(E_{a_2} - E_{a_1})/RT \quad \text{.....(iii)}$$

Given $Ea_2 = 2Ea_1$

On substituting this value in eqn. (iii)

$$k_1 = k_2 A \times e^{Ea_1/RT}$$

27. (b) For a first order reaction

$$k = \frac{2.303}{t}\log\frac{a}{a-x} = \frac{2.303}{40}\log\frac{0.1}{0.025}$$

$$= \frac{2.303}{40}\log 4 = \frac{2.303\times 0.6020}{40}$$

$$= 3.47\times 10^{-2}$$

$R = k(A)^1 = 3.47 \times 10^{-2} \times 0.01$

$= 3.47 \times 10^{-4}$

28. (a) Activation energy can be calculated from the equation

$$\frac{\log k_2}{\log k_1} = \frac{-E_a}{2.303R}\left(\frac{1}{T_2} - \frac{1}{T_1}\right)$$

Given $\frac{k_2}{k_1} = 2$; $T_2 = 310\,K$; $T_1 = 300\,K$

$$\therefore\ \log 2 = \frac{-E_a}{2.303\times 8.314}\left(\frac{1}{310} - \frac{1}{300}\right)$$

$E_a = 53598.6$ J/mol $= 53.6$ kJ/mol.

29. (d) Let rate of reaction $= \frac{d[C]}{t} = k[A]^x[B]^y$

Now from the given data

$1.2 \times 10^{-3} = k[0.1]^x[0.1]^y$ .....(i)

$1.2 \times 10^{-3} = k[0.1]^x[0.2]^y$ .....(ii)

$2.4 \times 10^{-3} = k[0.2]^x[0.1]^y$ .....(iii)

Dividing equation (i) by (ii)

$$\Rightarrow \frac{1.2\times 10^{-3}}{1.2\times 10^{-3}} = \frac{k[0.1]^x[0.1]^y}{k[0.1]^x[0.2]^y}$$

We find, $y = 0$

Now dividing equation (i) by (iii)

$$\Rightarrow \frac{1.2\times 10^{-3}}{2.4\times 10^{-3}} = \frac{k[0.1]^x[0.1]^y}{k[0.2]^x[0.1]^y}$$

We find, $x = 1$

Hence, $\frac{d[C]}{dt} = k[A]^1[B]^0$

30. (c) Reactions of higher order (>3) are very rare due to very less chances of many molecules to undergo effective collisions.

31. (d) $H_2O_2\,(aq) \rightarrow H_2O\,(aq) + \frac{1}{2}O_2\,(g)$

For a first order reaction

$$k = \frac{2.303}{t}\log\frac{a}{(a-x)}$$

Given a = 0.5, (a – x) = 0.125, t = 50 min

$$\therefore \quad k = \frac{2.303}{50}\log\frac{0.5}{0.125}$$

$$= 2.78 \times 10^{-2}\ \text{min}^{-1}$$

$$r = k[H_2O_2] = 2.78 \times 10^{-2} \times 0.05$$

$$= 1.386 \times 10^{-3}\ \text{mol min}^{-1}$$

Now

$$-\frac{d[H_2O_2]}{dt} = \frac{d[H_2O]}{dt} = \frac{2d[O_2]}{dt}$$

$$\therefore \quad \frac{2d[O_2]}{dt} = -\frac{d[H_2O_2]}{dt}$$

$$\therefore \quad \frac{d[O_2]}{dt} = -\frac{1}{2}\times\frac{d[H_2O_2]}{dt}$$

$$= \frac{1.386\times10^{-3}}{2} = 6.93 \times 10^{-4}\ \text{mol min}^{-1}$$

32. (d) From Arrhenius equation,

$$k = A.e^{\frac{-Ea}{RT}}$$

so, $k_1 = A \cdot e^{-E_{a_1}/RT}$ .....(i)

$k_2 = A \cdot e^{-E_{a_2}/RT}$ .....(ii)

On dividing equation (ii) by (i)

$$\Rightarrow \frac{k_2}{k_1} = e^{\frac{(E_{a_1} - E_{a_2})}{RT}}$$

$$\ln\left(\frac{k_2}{k_1}\right) = \frac{E_{a_1} - E_{a_2}}{RT} = \frac{10,000}{8.314\times300} = 4$$

33. (a) $CH_3CHO \longrightarrow CH_4 + CO$

Generally $r \propto (a-x)^m$

m = order of reaction

a – x = unreacted

$r_1 = 1$ torr s^{-1}, when 5% reacted

$r_2 = 0.5$ torr s^{-1}, when 33 % reacted

$(a - x_1) = 0.95$(unreacted)

$(a - x_2) = 0.67$(unreacted)

$$\frac{r_1}{r_2} = \left[\frac{(a-x_1)}{(a-x_2)}\right]^m ; \frac{1}{0.5} = \left(\frac{0.95}{0.67}\right)^m$$

$$2 = (1.41)^m \Rightarrow 2 = (\sqrt{2})^m$$

$$\Rightarrow m = 2$$

34. (a) From experiment 1 and II, it is observed that order of reaction w.r.t. (c) is zero.

From experiment II and III, α can be calculated as:

$$\frac{1.386\times10^{-2}}{6.93\times10^{-3}} = \left(\frac{0.2}{0.1}\right)^{\alpha}$$

$\therefore \alpha = 1$

Now, Rate = $K[A]^1$

or, $6.93 \times 10^{-3} = K(0.1)$

$K = 6.93 \times 10^{-2}$

For the reaction, $2A + B \rightarrow$ Products

$$2Kt = \ln\frac{[A]_0}{[A]}$$

$$\therefore \quad t_{1/2} = \frac{0.693}{2K} = \frac{0.693}{06.93\times10^{-2}\times2}$$

$$t_{1/2} = 5$$

35. (a) In graph (i), ln [Reactant vs time is linear with positive intercept and negative slope. Hence it is 1^{st} order In graph (ii), [Reactant] vs time is linear with positive intercept and negative slope. Hence, it is zero order.

36. (23.03) $t_{1/2} = 6.93$ years,

$a = 10^{-6}$ g

$$t_{1/2} = \frac{0.693}{K}$$

$$\Rightarrow \quad K = \frac{0.693}{t_{1/2}} = \frac{0.693}{6.93} = 0.1$$

For I^{st} order reaction,

$$K = \frac{2.303}{t}\log\frac{a}{a-x}$$

$$t = \frac{2.303}{K}\log\frac{a}{a-x}$$

$$= \frac{2.303}{0.1}\log\frac{10^{-6}}{10^{-7}}$$

$$= \frac{2.303}{0.1} = 23.03 \text{ years}$$

19 Surface Chemistry

1. The formation of gas at the surface of tungsten due to adsorption is the reaction of order **[2002]**
   (a) 0 (b) 1
   (c) 2 (d) insufficient data.
2. Which one of the following characteristics is **not** correct for physical adsorption ? **[2003]**
   (a) Adsorption increases with increase in temperature
   (b) Adsorption is spontaneous
   (c) Both enthalpy and entropy of adsorption are negative
   (d) Adsorption on solids is reversible
3. Identify the correct statement regarding enzymes **[2004]**
   (a) Enzymes are specific biological catalysts that cannot be poisoned
   (b) Enzymes are normally heterogeneous catalysts that are very specific in their action
   (c) Enzymes are specific biological catalysts that can normally function at very high temperatures (T~1000K)
   (d) Enzymes are specific biological catalysts that possess well-defined active sites
4. The volume of a colloidal particle, V_c as compared to the volume of a solute particle in a true solution V_s, could be **[2005]**
   (a) $\frac{V_c}{V_s} \simeq 10^3$ (b) $\frac{V_c}{V_s} \simeq 10^{-3}$
   (c) $\frac{V_c}{V_s} \simeq 10^{23}$ (d) $\frac{V_c}{V_s} \simeq 1$
5. The disperse phase in colloidal iron (III) hydroxide and colloidal gold is positively and negatively charged, respectively. Which of the following statements is NOT correct ? **[2005]**
   (a) Coagulation in both sols can be brought about by electrophoresis
   (b) Mixing the sols has no effect
   (c) Sodium sulphate solution causes coagulation in both sols
   (d) Magnesium chloride solution coagulates the gold sol more readily than the iron (III) hydroxide sol
6. In Langmuir's model of adsorption of a gas on a solid surface **[2006]**
   (a) the mass of gas striking a given area of surface is proportional to the pressure of the gas
   (b) the mass of gas striking a given area of surface is independent of the pressure of the gas
   (c) the rate of dissociation of adsorbed molecules from the surface does not depend on the surface covered
   (d) the adsorption at a single site on the surface may involve multiple molecules at the same time
7. Gold numbers of protective colloids A, B, C and D are 0.50, 0.01, 0.10 amd 0.005, respectively. the correct order of their protective powers is **[2008]**
   (a) D < A < C < B (b) C < B < D < A
   (c) A < C < B < D (d) B < D < A < C

8. Which of the following statements is incorrect regarding physisorptions? **[2009]**

(a) More easily liquefiable gases are adsorbed readily.

(b) Under high pressure, it results into multi-molecular layer on adsorbent surface.

(c) Enthalpy of adsorption ($\Delta H_{adsorption}$) is low and positive.

(d) It occurs because of van der Waal's forces.

9. According to Freundlich adsorption isotherm which of the following is correct? **[2012]**

(a) $\frac{x}{m} \propto p^0$

(b) $\frac{x}{m} \propto p^1$

(c) $\frac{x}{m} \propto p^{1/n}$

(d) All the above are correct for different ranges of pressure

10. The coagulating power of electrolytes having ions Na^+, Al^{3+} and Ba^{2+} for arsenic sulphide sol increases in the order : **[2013]**

(a) $Al^{3+} < Ba^{2+} < Na^+$ (b) $Na^+ < Ba^{2+} < Al^{3+}$

(c) $Ba^{2+} < Na^+ < Al^{3+}$ (d) $Al^{3+} < Na^+ < Ba^{2+}$

11. For a linear plot of log (x/m) versus log p in a Freundlich adsorption isotherm, which of the following statements is correct? (k and n are constants). **[2016]**

(a) Only $1/n$ appears as the slope.

(b) $\log(1/n)$ appears as the intercept.

(c) Both k and $1/n$ appear in the slope term.

(d) $1/n$ appears as the intercept.

12. The Tyndall effect is observed only when following conditions are satisfied: **[2017]**

(i) The diameter of the dispersed particles is much smaller than the wavelength of the light used.

(ii) The diameter of the dispersed particle is not much smaller than the wavelength of the light used.

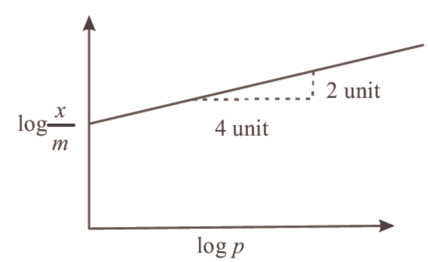
(iii) The refractive indices of the dispersed phase and dispersion medium are almost similar in magnitude.

(iv) The refractive indices of the dispersed phase and dispersion medium differ greatly in magnitude.

(a) (i) and (iv) (b) (ii) and (iv)

(c) (i) and (iii) (d) (ii) and (iii)

13. Adsorption of a gas follows Freundlich adsorption isotherm. In the given plot, x is the mass of the gas adsorbed on mass m of the adsorbent at pressure p. is proportional to: **[2019]**



(a) p^2 (b) $p^{1/4}$

(c) $p^{1/2}$ (d) p

14. The aerosol is a kind of colloid in which: **[2019]**

(a) solid is dispersed in gas

(b) gas is dispersed in solid

(c) gas is dispersed in liquid

(d) liquid is dispersed in water

Answer Key														
1	2	3	4	5	6	7	8	9	10	11	12	13	14	
(b)	(a)	(d)	(a)	(b)	(a)	(c)	(c)	(d)	(c)	(a)	(b)	(c)	(a)	

Solutions

1. **(b)** It is zero order reaction

   [NOTE Adsorption of gas on metal surface is of zero order]

2. **(a)** As adsorption is an exothermic process.

   ∴ Rise in temperature will decrease adsorption (according to Le-Chatelier principle).

3. **(d)** Enzymes are very specific biological catalysts possessing well - defined active sites

4. **(a)** Particle size of colloidal particle = 1mμ to 100 mμ. Let it be 10 mμ.

   $$V_c = \frac{4}{3}\pi r^3$$

   $$V_c = \frac{4}{3}\pi(10)^3$$

   Particle size of true solution particle = 1mμ

   $$V_s = \frac{4}{3}\pi(1)^3 ;$$

   $$\therefore \frac{V_c}{V_s} = 10^3$$

5. **(b)** When oppositely charged sols are mixed, their charges are neutralised. Both sols may be partially or completely precipitated.

6. **(a)** According to Langmuir's model of adsorption of a gas on a soild surface, the mass of gas adsorbed (x) per gram of the adsorbent (m) is directly proportional to the pressure of the gas (p) at constant temperature, i.e.

   $$\frac{x}{m} \propto p$$

7. **(c)** For a protective colloid, lesser the value of gold number better is the protective power.

   Thus the correct order of protective power of A, B, C and D is

	(A)	< (C)	< (B)	< (D)
Gold number	0.50	0.10	0.01	0.005

8. **(c)** Adsorption is an exothermic process, hence ΔH will always be negative.

9. **(d)** The Freundlich adsorption isotherm is mathematically represented as

   $$\frac{x}{m} = kp^{1/n}$$

   At high pressure, $1/n = 0$

   Hence, $x/m \propto p^\circ$

   At low pressure, $1/n = 1$

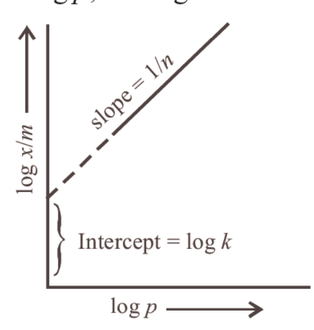
   Hence, $x/m \propto p^1$

10. **(c)** According to Hardy Schulze rule, greater the charge on cation, greater is its coagulating power for negatively charged sol (As_2S_3), hence the correct order of coagulating power : $Na^+ < Ba^{2+} < Al^{3+}$

11. **(a)** According to Freundlich adsorption isotherm

   $$\log\frac{x}{m} = \log k + \frac{1}{n}\log p$$

   Thus if a graph is plotted between $\log(x/m)$ and $\log p$, a straight line will be obtained



The slope of the line is equal to $1/n$ and the intercept on $\log x/m$ axis will correspond to $\log k$.

12. (b)

13. (c) In Freundlich adsorption isotherm the extent of adsorption (x/m) of a gas on the surface of a solid is related to the pressure of the gas (p) which can be formulated as:

$$\frac{x}{m} = k(p)^{1/n}$$

$$\Rightarrow \log\frac{x}{m} = \log k + \frac{1}{n}\log p$$

In the given plot, the slope between $\log\frac{x}{m}$ versus $\log p = \frac{2}{4} = \frac{1}{2}$

$$\therefore \frac{x}{m} \propto p^{1/2}$$

14. (a) In aerosol, the dispersion medium is gas while the dispersed phase can be both solid or liquid.

20 General Principles and Processes of Isolation of Elements

1. Aluminium is extracted by the electrolysis of **[2002]**
   (a) bauxite
   (b) alumina
   (c) alumina mixed with molten cryolite
   (d) molten cryolite.
2. The metal extracted by leaching with a cyanide is **[2002]**
   (a) Mg (b) Ag
   (c) Cu (d) Na.
3. Which one of the following ores is best concentrated by froth-floation method? **[2004]**
   (a) Galena (b) Cassiterite
   (c) Magnetite (d) Malachite
4. During the process of electrolytic refining of copper, some metals present as impurity settle as 'anode mud'. These are **[2005]**
   (a) Fe and Ni (b) Ag and Au
   (c) Pb and Zn (d) Sn and Ag
5. Which of the following factors is of ***no significance*** for roasting sulphide ores to oxides and not subjecting the sulphide ores to carbon reduction directly? **[2008]**
   (a) Metal sulphides are thermodynamically more stable than CS_2
   (b) CO_2 is thermodynamically more stable than CS_2
   (c) Metal sulphides are less stable than the corresponding oxides
   (d) CO_2 is more volatile than CS_2
6. Which method of purification is represented by the following equation ? **[2012]**

   $Ti(s) + 2I_2(g) \xrightarrow{523K}$

   $TiI_4(g) \xrightarrow{1700K} Ti(s) + 2I_2(g)$

   (a) Zone refining (b) Cupellation
   (c) Polling (d) van Arkel
7. The metal that cannot be obtained by electrolysis of an aqueous solution of its salts is: **[2014]**
   (a) Ag (b) Ca
   (c) Cu (d) Cr
8. In the context of the Hall - Heroult process for the extraction of Al, which of the following statements is **false** ? **[2015]**
   (a) Al^{3+} is reduced at the cathode to form Al
   (b) Na_3AlF_6 serves as the electrolyte
   (c) CO and CO_2 are produced in this process
   (d) Al_2O_3 is mixed with CaF_2 which lowers the melting point of the mixture and brings conductivity
9. Which one of the following ores is best concentrated by froth floatation method? **[2016]**
   (a) Galena (b) Malachite
   (c) Magnetite (d) Siderite
10. The ore that contains both iron and copper is: **[2019]**
    (a) copper pyrites (b) malachite
    (c) dolomite (d) azurite
11. The ore that contains the metal in the form of fluoride is: **[2019]**
    (a) cryolite (b) malachite
    (c) magnetite (d) sphalerite
12. The purest form of commercial iron is: **[2020]**
    (a) pig iron
    (b) wrought iron
    (c) cast iron
    (d) scrap iron and pig iron

Answer Key														
1	2	3	4	5	6	7	8	9	10	11	12			
(c)	(b)	(c)	(b)	(c)	(d)	(b)	(b)	(a)	(a)	(a)	(b)			

Solutions

1. **(c)** Pure aluminium can be obtained by electrolysis of a mixture containing alumina, crayolite and fluorspar in the ratio 20 : 24 : 20. The fusion temperature of this mixture is 900 ºC and it is a good conductor of electricity.

2. **(b)** Silver ore forms a soluble complex with NaCN from which silver is precipitated using scrap zinc.

$$Ag_2S + 2NaCN \rightarrow \underset{\substack{\text{sodioargentocynanide} \\ \text{(soluble)}}}{Na[Ag(CN)_2]} \xrightarrow{Zn} Na_2[Zn(CN)_4] + Ag\downarrow$$

3. **(c)** **NOTE** Galena is PbS and thus purified by froth floatation method.

Froth floatation method is used to concentrate sulphide ores. This method is based on the preferential wetting properties with the frothing agent and water.

4. **(b)** During the process of electrolytic refining Ag and Au are obtained as anode mud.

5. **(c)** **NOTE** The reduction of metal sulphides by carbon reduction process is not spontaneous because ΔG for such a process is positive. The reduction of metal oxide by carbon reduction process is spontaneous as ΔG for such a process is negative.

From this we find that on thermodynamic considerations CO_2 is more stable than CS_2 and the metal sulphides are more stable than corresponding oxides.

In view of above the factor listed in choice (c) is incorrect and so is of no significance. Hence the correct answer is (c)

6. **(d)** van Arkel : In this process an impure metal is converted into metal iodide which is easily decomposed to pure metal and free I_2.

The process is known as van Arkel method.

7. **(b)** On electrolysis of aqueous solution of *s*-block elements, H_2 gas is discharged at cathode.

$$\text{At cathode: } H_2O + e^- \longrightarrow \frac{1}{2}H_2 + OH^-$$

8. **(b)** In the metallurgy of aluminium, purified Al_2O_3 is mixed with Na_3AlF_6 or CaF_2 which lowers the melting point of the mix and brings conductivity.

9. **(a)** Froth floatation method is mainly applicable for sulphide ores.

(1) Malachite ore : $Cu(OH)_2 . CuCO_3$

(2) Magnetite ore : Fe_3O_4

(3) Siderite ore : $FeCO_3$

(4) Galena ore : PbS (Sulphide ore)

10. **(a)** Amongst the given ores, copper pyrite ($CuFeS_2$), dolomite ($MgCO_3.CaCO_3$), malachite [$CuCO_3.Cu(OH)_2$], azurite [$2CuCO_3.Cu(OH)_2$], copper pyrite contains both copper and iron.

11. **(a)**

Magnetite	Fe_3O_4
Sphalerite	ZnS
Cryolite	Na_3AlF_6
Malachite	$CuCO_3.Cu(OH)_2$

12. **(b)** Wrought iron is purest form of commercial iron.

21 The p-Block Elements (Group 15, 16, 17 & 18)

1. In XeF_2, XeF_4, XeF_6 the number of lone pairs on Xe are respectively **[2002]**
   (a) 2, 3, 1 (b) 1, 2, 3
   (c) 4, 1, 2 (d) 3, 2, 1.
2. In case of nitrogen, NCl_3 is possible but not NCl_5 while in case of phosphorus, PCl_3 as well as PCl_5 are possible. It is due to **[2002]**
   (a) availability of vacant *d* orbitals in P but not in N
   (b) lower electronegativity of P than N
   (c) lower tendency of H-bond formation in P than N
   (d) occurrence of P in solid while N in gaseous state at room temperature.
3. Number of sigma bonds in P_4O_{10} is **[2002]**
   (a) 6 (b) 7 (c) 17 (d) 16.
4. Oxidation number of Cl in $CaOCl_2$ (bleaching power) is: **[2002]**
   (a) zero, since it contains Cl_2
   (b) – 1, since it contains Cl^-
   (c) + 1, since it contains ClO^-
   (d) + 1 and – 1 since it contains ClO^- and Cl^-
5. What may be expected to happen when phosphine gas is mixed with chlorine gas ? **[2003]**
   (a) PCl_3 and HCl are formed and the mixture warms up
   (b) PCl_5 and HCl are formed and the mixture cools down
   (c) $PH_3 . Cl_2$ is formed with warming up
   (d) The mixture only cools down
6. Concentrated hydrochloric acid when kept in open air, sometimes produces a cloud of white fumes. The explanation for it is that **[2003]**
   (a) oxygen in air reacts with the emitted HCl gas to form a cloud of chlorine gas
   (b) strong affinity of HCl gas for moisture in air results in forming of droplets of liquid solution which appears like a cloudy smoke.
   (c) due to strong affinity for water, concentrated hydrochloric acid pulls moisture of air towards itself. This moisture forms droplets of water and hence the cloud.
   (d) concentrated hydrochloric acid emits strongly smelling HCl gas all the time.
7. Which one of the following substances has the highest proton affinity ? **[2003]**
   (a) H_2S (b) NH_3 (c) PH_3 (d) H_2O
8. Which among the following factors is the most important in making fluorine, the strongest oxidizing halogen ? **[2004]**
   (a) Hydration enthalpy
   (b) Ionization enthalpy
   (c) Electron affinity
   (d) Bond dissociation energy
9. Excess of KI reacts with $CuSO_4$ solution and then $Na_2S_2O_3$ solution is added to it. Which of the statements is **incorrect** for this reaction ? **[2004]**
   (a) $Na_2S_2O_3$ is oxidised
   (b) CuI_2 is formed
   (c) Cu_2I_2 is formed
   (d) Evolved I_2 is reduced

10. Which one of the following statement regarding helium is **incorrect** ? **[2004]**
(a) It is used to produce and sustain powerful superconducting magnets
(b) It is used as a cryogenic agent for carrying out experiments at low temperatures
(c) It is used to fill gas balloons instead of hydrogen because it is lighter and non-inflammable
(d) It is used in gas-cooled nuclear reactors

11. The number of hydrogen atom(s) attached to phosphorus atom in hypophosphorous acid is **[2005]**
(a) three (b) one
(c) two (d) zero

12. The correct order of the thermal stability of hydrogen halides (H–X) is **[2005]**
(a) $HI > HCl < HF > HBr$
(b) $HCl < HF > HBr < HI$
(c) $HF > HCl > HBr > HI$
(d) $HI < HBr > HCl < HF$

13. Which of the following statements is true? **[2006]**
(a) $HClO_4$ is a weaker acid than $HClO_3$
(b) HNO_3 is a stronger acid than HNO_2
(c) H_3PO_3 is a stronger acid than H_2SO_3
(d) In aqueous medium HF is a stronger acid than HCl

14. The increasing order of the first ionization enthalpies of the elements B, P, S and F (lowest first) is **[2006]**
(a) $B < P < S < F$ (b) $B < S < P < F$
(c) $F < S < P < B$ (d) $P < S < B < F$

15. What products are expected from the disproportionation reaction of hypochlorous acid? **[2006]**
(a) HCl and Cl_2O (b) HCl and $HClO_3$
(c) $HClO_3$ and Cl_2O (d) $HClO_2$ and $HClO_4$

16. Identify the incorrect statement among the following. **[2007]**
(a) Br_2 reacts with hot and strong NaOH solution to give NaBr and H_2O.
(b) Ozone reacts with SO_2 to give SO_3.
(c) Silicon reacts with $NaOH_{(aq)}$ in the presence of air to give Na_2SiO_3 and H_2O.
(d) Cl_2 reacts with excess of NH_3 to give N_2 and HCl.

17. Regular use of the following fertilizers increases the acidity of soil? **[2007]**
(a) Ammonium sulphate
(b) Potassium nitrate
(c) Urea
(d) Superphosphate of lime.

18. Which one of the following reactions of xenon compounds is not feasible? **[2009]**
(a) $3XeF_4 + 6H_2O \longrightarrow 2Xe + XeO_3 + 12HF + 1.5O_2$
(b) $2XeF_2 + 2H_2O \longrightarrow 2Xe + 4HF + O_2$
(c) $XeF_6 + RbF \longrightarrow Rb[XeF_7]$
(d) $XeO_3 + 6HF \longrightarrow XeF_6 + 3H_2O$

19. Which of the following has maximum number of lone pairs associated with Xe ? **[2011RS]**
(a) XeF_4 (b) XeF_6
(c) XeF_2 (d) XeO_3

20. The molecule having smallest bond angle is : **[2012]**
(a) NCl_3 (b) $AsCl_3$
(c) $SbCl_3$ (d) PCl_3

21. Among the following oxoacids, the correct decreasing order of acid strength is: **[2014]**
(a) $HOCl > HClO_2 > HClO_3 > HClO_4$
(b) $HClO_4 > HOCl > HClO_2 > HClO_3$
(c) $HClO_4 > HClO_3 > HClO_2 > HOCl$
(d) $HClO_2 > HClO_4 > HClO_3 > HOCl$

22. Which among the following is the most reactive ? **[2015]**
(a) I_2 (b) ICl (c) Cl_2 (d) Br_2

23. **Assertion:** Nitrogen and oxygen are the main components in the atmosphere but these do not react to form oxides of nitrogen.

Reason: The reaction between nitrogen and oxygen requires high temperature.

[2015]

(a) The assertion is incorrect, but the reason is correct
(b) Both the assertion and reason are incorrect
(c) Both assertion and reason are correct, and the reason is the correct explanation for the assertion
(d) Both assertion and reason are correct, but the reason is not the correct explanation for the assertion

24. Which one has the highest boiling point ? **[2015]**

(a) Kr (b) Xe (c) He (d) Ne

25. The pair in which phosphorus atoms have a formal oxidation state of + 3 is : **[2016]**

(a) Orthophosphorous and hypophosphoric acids
(b) Pyrophosphorous and pyrophosphoric acids
(c) Orthophosphorous and pyrophosphorous acids
(d) Pyrophosphorous and hypophosphoric acids

26. The products obtained when chlorine gas reacts with cold and dilute aqueous NaOH are : **[2017]**

(a) ClO^- and ClO_3^- (b) ClO_2^- and ClO_3^-
(c) Cl^- and ClO^- (d) Cl^- and ClO_2^-

27. The compound that **does not** produce nitrogen gas by the thermal decomposition is : **[2018]**

(a) $Ba(N_3)_2$ (b) $(NH_4)_2Cr_2O_7$
(c) NH_4NO_2 (d) $(NH_4)_2SO_4$

28. The correct order of the oxidation states of nitrogen in NO, N_2O, NO_2 and N_2O_3 is: **[2019]**

(a) $NO_2 < NO < N_2O_3 < N_2O$
(b) $NO_2 < N_2O_3 < NO < N_2O$
(c) $N_2O < N_2O_3 < NO < NO_2$
(d) $N_2O < NO < N_2O_3 < NO_2$

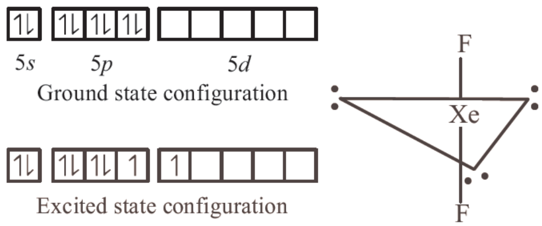
29. Chlorine reacts with hot and concentrated NaOH and produces compounds (*X*) and (*Y*). Compound (*X*) gives white precipitate with silver nitrate solution. The average bond order between Cl and O atoms in (*Y*) is ______. **[2020]**

Answer Key

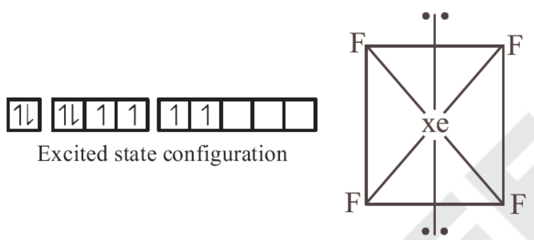
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
(d)	(a)	(d)	(d)	(b)	(a)	(b)	(d)	(b)	(c)	(c)	(c)	(b)	(b)	(b)
16	17	18	19	20	21	22	23	24	25	26	27	28	29	
(d)	(a)	(d)	(c)	(c)	(c)	(b)	(c)	(b)	(c)	(c)	(d)	(d)	(1.67)	

Solutions

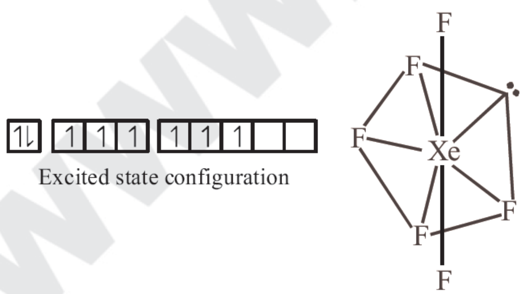
1. **(d)** In the formation of XeF_2, sp^3d hybridisation occurs which gives the molecule a trigonal bipyramidal structure.



In the formation of XeF_4, sp^3d^2 hybridization occurs which gives the molecule an octahedral structure.



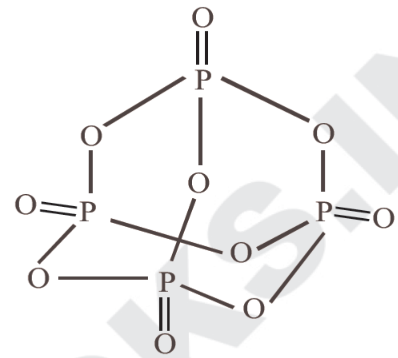
In the formation of XeF_6, sp^3d^3 hybridization occurs which gives the molecule a pentagonal bipyramidal structure.



2. **(a)** $_7N = 1s^2\,2s^2\,2p^3$; $_{15}P = 1s^2\,2s^2\,2p^6\,3s^2\,3p^3$

NOTE In phosphorus the 3*d*-orbitals are available. Hence phosphorus can form pentahalides but nitrogen can not form pentahalide due to absence of *d*-orbitals.

3. **(d)**



4. **(d)** $CaOCl_2$ can also be written as

$$\underset{x_1}{Ca\,(OCl)}\,\underset{x_2}{Cl}$$

Hence oxidation no of Cl in OCl^- is

$$-2 + x_1 = -1$$
$$x_1 = 2 - 1 = +1$$

Oxidation no. of another Cl is –1 as it is present as Cl^-.

5. **(b)** On mixing phosphine with chlorine gas, PCl_5 and HCl are formed. The mixture cools down.

$$PH_3 + 4Cl_2 \longrightarrow PCl_5 + 3HCl$$

6. **(a)** $$4HCl + \underset{\text{air}}{O_2} \longrightarrow \underset{\text{cloud of white fumes}}{2Cl_2\uparrow + 2H_2O}$$

7. **(b)** Among the given compounds, the $\ddot{N}H_3$ is most basic. Hence it has highest proton affinity.

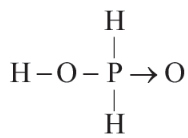
8. **(d)** Fluorine has low dissociation energy of F – F bond, and reaction of atomic fluorine is exothermic in nature.

9. **(b)** $$4\overset{-1}{KI} + 2CuSO_4 \rightarrow \overset{0}{I_2} + Cu_2I_2 + 2K_2SO_4$$

$$\overset{0}{I_2} + 2Na_2\overset{2+}{S_2}O_3 \rightarrow Na_2\overset{+2.5}{S_4}O_6 + 2Na\overset{-1}{I}$$

In this CuI_2 is **not** formed.

10. **(c)** Helium is heavier than hydrogen although it is non-inflammable

11. **(c)** Hypophosphorous acid is 

Two H-atoms are attached to P atom.

12. **(c)** The H–X bond strength decreases from HF to HI. i.e. HF > HCl > HBr > HI. Thus HF is most stable while HI is least stable. The decreasing stability of the hydrogen halide is also reflected in the values of dissociation energy of the H–X bond

H – F	H – Cl	H – Br	H – I
135kcal mol^{-1}	103kcal mol^{-1}	87kcal mol^{-1}	71kcal mol^{-1}

13. **(b)** The $H\overset{+5}{N}O_3$ is stronger than $H\overset{+3}{N}O_2$. The more the oxidation state of N, the more is the acid character.

14. **(b)** The correct order of ionisation enthalpies is

$$F > P > S > B$$

NOTE On moving along a period ionization enthalapy increases from left to right and decreases from top to bottom in a group. But this trend breaks up in case of such element having fully or half filled stable orbitals.

In this case, P has a stable half filled electronic configuration, hence its ionisation enthalapy is greater in comparision to S. Therefore the correct order is B < S < P<F.

15. **(b)** During disproportionation same compound undergoes simultaneous oxidation and reduction.

Oxidation

$$3H\overset{-}{O}\overset{+}{Cl} \longrightarrow 2H\overset{-1}{Cl} + H\overset{+5}{Cl}O_3$$

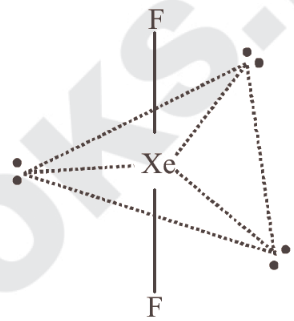
Reduction

16. **(d)** Chlorine reacts with excess of ammonia to produce ammonium chloride and nitrogen.

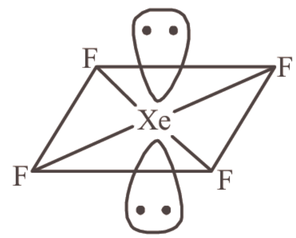
17. **(a)** $(NH_4)_2SO_4 + 2H_2O \longrightarrow 2H_2SO_4 + NH_4OH$
H_2SO_4 is strong acid and increases the acidity of soil.

18. **(d)** The products of the concerned reaction react each other forming back the reactants.

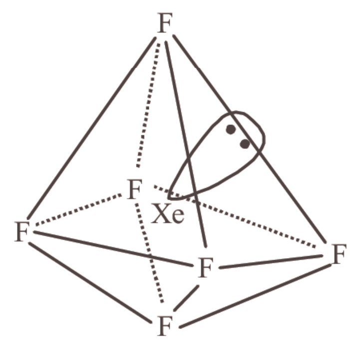
$$XeF_6 + 3H_2O \longrightarrow XeO_3 + 6HF \cdot$$

19. **(c)** XeF_2: 

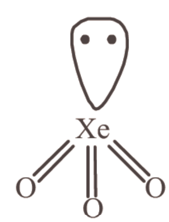
3 *lp*

XeF_4: 

2 *lp*

XeF_6: 

1 *lp*

XeO_3: 

1 *lp*

20. **(c)** All the members form volatile halides of the type AX_3. All halides are pyramidal in shape. The bond angle decreases on moving down the group (from NCl_3 to $SbCl_3$) due to decrease in bond pair-bond pair repulsion or increase in lone pair-bond pair repulsion.

21. **(c)** Acidic strength increases as the oxidation number of central atom increases.

Hence acidic strength order is

$$\overset{(+7)}{HClO_4} > \overset{(+5)}{HClO_3} > \overset{(+3)}{HClO_2} > \overset{(+1)}{HClO}$$

22. **(b)** Order of reactivity of halogens

$$Cl_2 > Br_2 > I_2$$

But the interhalogen compounds are generally more reactive than halogens (except F_2), since the bond between two dissimilar electronegative elements is weaker than the bond between two similar atoms i.e, $X-X$

23. **(c)** Nitrogen and oxgen in air do not react to form oxides of nitrogen in atmosphere because the reaction between nitrogen and oxygen requires high temperature.

24. **(b)** Xe. As we move down the group, the melting and boiling points show a regular increase due to corresponding increase in the magnitude of their van der waal forces of attraction as the size of the atom increases.

25. **(c)** Phosphorous acids contain P in +3 oxidation state.

Acid	Formula	Oxidation state of phosphorus
Pyrophosphorous acid	$H_4P_2O_5$	+3
Pyrophosphoric acid	$H_4P_2O_7$	+5
Orthophosphorous acid	H_3PO_3	+3
Hypophosphoric acid	$H_4P_2O_6$	+4

26. **(c)** $\underset{\text{[cold and dilute]}}{Cl_2 + NaOH} \rightarrow NaCl + NaClO + H_2O$

27. **(d)**

(a) $Ba(N_3)_2 \xrightarrow{\Delta} Ba + 3N_2$

(b) $(NH_4)_2Cr_2O_7 \xrightarrow{\Delta} Cr_2O_3 + N_2 + 4H_2O$

(c) $NH_4NO_2 \xrightarrow{\Delta} N_2 + 2H_2O$

(d) $(NH_4)_2SO_4 \xrightarrow{\Delta} 2NH_3 + H_2SO_4$

NH_3 is evolved in reaction (d).

28. **(d)**

(Oxide)	(Oxidation state)
N_2O	+1
NO	+2
N_2O_3	+3
NO_2	+4

So, $N_2O < NO < N_2O_3 < NO_2$

29. **(1.67)**

$$3Cl_2 + \underset{\text{Hot \& conc.}}{6NaOH} \longrightarrow \underset{(X)}{5NaCl} + \underset{(Y)}{NaClO_3} + 3H_2O$$

$$\underset{(X)}{NaCl} + AgNO_3 \longrightarrow \underset{\text{(white ppt.)}}{AgCl} + NaNO_3$$

Average bond order between Cl and O atom in $NaClO_3 = \frac{5}{3} = 1.67$

22 The d-and f-Block Elements

1. Most common oxidation states of Ce (cerium) are **[2002]**
   (a) +2, +3 (b) +2, +4
   (c) +3, +4 (d) +3, +5.
2. Arrange Ce^{3+}, La^{3+}, Pm^{3+} and Yb^{3+} in increasing order of their ionic radii. **[2002]**
   (a) $Yb^{3+} < Pm^{3+} < Ce^{3+} < La^{3+}$
   (b) $Ce^{3+} < Yb^{3+} < Pm^{3+} < La^{3+}$
   (c) $Yb^{3+} < Pm^{3+} < La^{3+} < Ce^{3+}$
   (d) $Pm^{3+} < La^{3+} < Ce^{3+} < Yb^{3+}$
3. Which of the following ions has the maximum magnetic moment? **[2002]**
   (a) Mn^{2+} (b) Fe^{2+} (c) Ti^{2+} (d) Cr^{2+}
4. The most stable ion is **[2002]**
   (a) $[Fe(OH)_5]^{3-}$ (b) $[Fe(Cl)_6]^{3-}$
   (c) $[Fe(CN)_6]^{3-}$ (d) $[Fe(H_2O)_6]^{3+}$.
5. When $KMnO_4$ acts as an oxidising agent and ultimately forms $[MnO_4]^{-2}$, MnO_2, Mn_2O_3, Mn^{+2} then the number of electrons transferred in each case respectively is **[2002]**
   (a) 4, 3, 1, 5 (b) 1, 5, 3, 7
   (c) 1, 3, 4, 5 (d) 3, 5, 7, 1.
6. The radius of La^{3+} (Atomic number of La = 57) is 1.06Å. Which one of the following given values will be closest to the radius of Lu^{3+} (Atomic number of Lu = 71) ? **[2003]**
   (a) 1.40 Å (b) 1.06 Å
   (c) 0.85 Å (d) 1.60 Å
7. Ammonia forms the complex ion $[Cu(NH_3)_4]^{2+}$ with copper ions in alkaline solutions but not in acidic solutions. What is the reason for it ? **[2003]**
   (a) In acidic solutions protons, coordinate with ammonia molecules forming NH_4^+ ions and thus NH_3 molecules are not available
   (b) In alkaline solutions insoluble $Cu(OH)_2$ is precipitated which is soluble in excess of any alkali
   (c) Copper hydroxide is an amphoteric substance
   (d) In acidic solutions hydration protects copper ions
8. A red solid is insoluble in water. However it becomes soluble if some KI is added to water. Heating the red solid in a test tube results in liberation of some violet coloured fumes and droplets of a metal appear on the cooler parts of the test tube. The red solid is **[2003]**
   (a) HgI_2 (b) HgO
   (c) Pb_3O_4 (d) $(NH_4)_2Cr_2O_7$
9. A reduction in atomic size with increase in atomic number is a characteristic of elements of **[2003]**
   (a) *d*-block (b) *f*-block
   (c) radioactive series (d) high atomic masses
10. What would happen when a solution of potassium chromate is treated with an excess of dilute nitric acid? **[2003]**
    (a) $Cr_2O_7^{2-}$ and H_2O are formed
    (b) CrO_4^{2-} is reduced to +3 state of Cr
    (c) CrO_4^{2-} is oxidized to +7 state of Cr
    (d) Cr^{3+} and $Cr_2O_7^{2-}$ are formed

11. Which one of the following nitrates will leave behind a metal on strong heating ? **[2003]**
(a) Copper nitrate (b) Manganese nitrate
(c) Silver nitrate (d) Ferric nitrate

12. Of the following outer electronic configurations of atoms, the highest oxidation state is achieved by which one of them ? **[2004]**
(a) $(n-1)d^3\, ns^2$ (b) $(n-1)d^5\, ns^1$
(c) $(n-1)d^8\, ns^2$ (d) $(n-1)d^5\, ns^2$

13. The soldiers of Napolean army while at Alps during freezing winter suffered a serious problem as regards to the tin buttons of their uniforms. Gray metallic tin buttons got converted to white powder. This transformation is related to **[2004]**
(a) a change in the partial pressure of oxygen in the air
(b) a change in the crystalline structure of tin
(c) an interaction with nitrogen of the air at very low temperature
(d) an interaction with water vapour contained in the humid air

14. Among the properties : (a) reducing, (b) oxidising and (c) complexing, the set of properties shown by CN^- ion towards metal species is **[2004]**
(a) c, a (b) b, c
(c) a, b (d) a, b, c

15. Cerium (Z = 58) is an important member of the lanthanoids. Which of the following statements about cerium is **incorrect**? **[2004]**
(a) The +4 oxidation state of cerium is not known in solutions
(b) The +3 oxidation state of cerium is more stable than the +4 oxidation state
(c) The common oxidation states of cerium are +3 and +4
(d) Cerium (IV) acts as an oxidizing agent

16. The correct order of magnetic moments (spin only values in B.M.) among the following is **[2004]**
(a) $[Fe(CN)_6]^{4-} > [MnCl_4]^{2-} > [CoCl_4]^{2-}$
(b) $[MnCl_4]^{2-} > [Fe(CN)_6]^{4-} > [CoCl_4]^{2-}$
(c) $[MnCl_4]^{2-} > [CoCl_4]^{2-} > [Fe(CN)_6]^{4-}$
(d) $[Fe(CN)_6]^{4-} > [CoCl_4]^{2-} > [MnCl_4]^{2-}$
(Atomic nos. : Mn = 25, Fe = 26, Co = 27)

17. Heating mixture of Cu_2O and Cu_2S will give **[2005]**
(a) Cu_2SO_3 (b) $CuO + CuS$
(c) $Cu + SO_3$ (d) $Cu + SO_2$

18. The oxidation state of chromium in the final product formed by the reaction between KI and acidified potassium dichromate solution is: **[2005]**
(a) +3 (b) +2
(c) +6 (d) +4

19. Calomel (Hg_2Cl_2) on reaction with ammonium hydroxide gives **[2005]**
(a) HgO
(b) Hg_2O
(c) $NH_2 - Hg - Hg - Cl$
(d) $HgNH_2Cl$

20. The lanthanide contraction is responsible for the fact that **[2005]**
(a) Zr and Zn have the same oxidation state
(b) Zr and Hf have about the same radius
(c) Zr and Nb have similar oxidation state
(d) Zr and Y have about the same radius

21. The value of the 'spin only' magnetic moment for one of the following configurations is 2.84 BM. The correct one is **[2005]**
(a) d^5 (in strong ligand field)
(b) d^3 (in weak as well as in strong fields)
(c) d^4 (in weak ligand fields)
(d) d^4 (in strong ligand fields)

22. Which of the following factors may be regarded as the main cause of lanthanide contraction? **[2005]**
(a) Greater shielding of 5*d* electrons by 4*f* electrons
(b) Poorer shielding of 5*d* electrons by 4*f* electrons
(c) Effective shielding of one of the 4*f* electrons by another in the subshell
(d) Poor shielding of one of the 4*f* electrons by another in the subshell

23. A metal, M forms chlorides in its +2 and +4 oxidation states. Which of the following statements about these chlorides is correct? **[2006]**

(a) MCl_2 is more ionic than MCl_4
(b) MCl_2 is more easily hydrolysed than MCl_4
(c) MCl_2 is more volatile than MCl_4
(d) MCl_2 is more soluble in anhydrous ethanol than MCl_4

24. Lanthanoid contraction is caused due to **[2006]**

(a) the same effective nuclear charge from Ce to Lu
(b) the imperfect shielding on outer electrons by 4*f* electrons from the nuclear charge
(c) the appreciable shielding on outer electrons by 4*f* electrons from the nuclear charge
(d) the appreciable shielding on outer electrons by 5*d* electrons from the nuclear charge

25. The "spin-only" magnetic moment [in units of Bohr magneton, (μ_B)] of Ni^{2+} in aqueous solution would be (At. No. Ni = 28) **[2006]**

(a) 6 (b) 1.73
(c) 2.84 (d) 4.90

26. Identify the incorrect statement among the following: **[2007]**

(a) 4*f* and 5*f* orbitals are equally shielded.
(b) *d*-Block elements show irregular and erratic chemical properties among themselves.
(c) La and Lu have partially filled *d*-orbitals and no other partially filled orbitals.
(d) The chemistry of various lanthanoids is very similar.

27. The actinoids exhibit more number of oxidation states in general than the lanthanoids. This is because **[2007]**

(a) the 5*f* orbitals extend further from the nucleus than the 4*f* orbitals
(b) the 5*f* orbitals are more buried than the 4*f* orbitals
(c) there is a similarity between 4*f* and 5*f* orbitals in their angular part of the wave function
(d) the actinoids are more reactive than the lanthanoids.

28. Larger number of oxidation states are exhibited by the actinoids than those by the lanthanoids, the main reason being **[2008]**

(a) 4*f* orbitals more diffused than the 5*f* orbitals
(b) lesser energy difference between 5*f* and 6*d* than between 4*f* and 5*d* orbitals
(c) more energy difference between 5*f* and 6*d* than between 4*f* and 5*d* orbitals
(d) more reactive nature of the actionids than the lanthanoids

29. Amount of oxalic acid present in a solution can be determined by its titration with $KMnO_4$ solution in the presence of H_2SO_4. The titration gives unsatisfactory result when carried out in the presence of HCl, because HCl **[2008]**

(a) gets oxidised by oxalic acid to chlorine
(b) furnishes H^+ ions in addition to those from oxalic acid
(c) reduces permanganate to Mn^{2+}
(d) oxidises oxalic acid to carbon dioxide and water

30. Knowing that the chemistry of lanthanoids(Ln) is dominated by its + 3 oxidation state, which of the following statements is incorrect? **[2009]**

(a) The ionic size of Ln (III) decreases in general with increasing atomic number
(b) Ln (III) compounds are generally colourless.
(c) Ln (III) hydroxides are mainly basic in character.
(d) Because of the large size of the Ln (III) ions the bonding in its compounds is predominantly ionic in character.

31. The correct order of $E^\circ_{M^{2+}/M}$ values with negative sign for the four successive elements Cr, Mn, Fe and Co is **[2010]**

(a) Mn > Cr > Fe > Co (b) Cr < Fe > Mn > Co
(c) Fe > Mn > Cr > Co (d) Cr > Mn > Fe > Co

32. Iron exhibits +2 and + 3 oxidation states. Which of the following statements about iron is incorrect ? **[2012]**

(a) Ferrous oxide is more basic in nature than the ferric oxide.
(b) Ferrous compounds are relatively more ionic than the corresponding ferric compounds.
(c) Ferrous compounds are less volatile than the corresponding ferric compounds.
(d) Ferrous compounds are more easily hydrolysed than the corresponding ferric compounds.

33. Which of the following arrangements does not represent the correct order of the property stated against it ? **[2013]**

(a) $V^{2+} < Cr^{2+} < Mn^{2+} < Fe^{2+}$: paramagnetic behaviour
(b) $Ni^{2+} < Co^{2+} < Fe^{2+} < Mn^{2+}$: ionic size
(c) $Co^{3+} < Fe^{3+} < Cr^{3+} < Sc^{3+}$: stability in aqueous solution
(d) $Sc < Ti < Cr < Mn$: number of oxidation states

34. Four successive members of the first row transition elements are listed below with atomic numbers. Which one of them is expected to have the highest $E^{o}_{M^{3+}/M^{2+}}$ value ? **[2013]**

(a) Cr(Z = 24) (b) Mn(Z = 25)
(c) Fe(Z = 26) (d) Co(Z = 27)

35. Which series of reactions correctly represents chemical reactions related to iron and its compound? **[2014]**

(a) $Fe \xrightarrow{\text{dil. } H_2SO_4} FeSO_4 \xrightarrow{H_2SO_4,\, O_2} Fe_2(SO_4)_3 \xrightarrow{\text{heat}} Fe$

(b) $Fe \xrightarrow{O_2,\text{ heat}} FeO \xrightarrow{\text{dil. } H_2SO_4} FeSO_4 \xrightarrow{\text{heat}} Fe$

(c) $Fe \xrightarrow{Cl_2,\text{ heat}} FeCl_3 \xrightarrow{\text{heat, air}} FeCl_2 \xrightarrow{Zn} Fe$

(d) $Fe \xrightarrow{O_2,\text{ heat}} Fe_3O_4 \xrightarrow{CO,\ 600\ °C} FeO \xrightarrow{CO,\ 700\ °C} Fe$

36. The equation which is balanced and represents the correct product(s) is: **[2014]**

(a) $Li_2O + 2KCl \longrightarrow 2LiCl + K_2O$

(b) $\left[CoCl(NH_3)_5\right]^{+} + 5H^{+} \longrightarrow Co^{2+} + 5NH_4^{+} + Cl^{-}$

(c) $\left[Mg(H_2O)_6\right]^{2+} + (EDTA)^{4-} \xrightarrow{\text{excess NaOH}} \left[Mg(EDTA)\right]^{2+} + 6H_2O$

(d) $CuSO_4 + 4KCN \longrightarrow K_2\left[Cu(CN)_4\right] + K_2SO_4$

37. Match the catalysts to the correct processes : **[2015]**

	Catalyst		Process
(A)	$TiCl_4$	(i)	Wacker process
(B)	$PdCl_2$	(ii)	Ziegler - Natta polymerization
(C)	$CuCl_2$	(iii)	Contact process
(D)	V_2O_5	(iv)	Deacon's process

(a) (A) - (ii), (B) - (iii), (C) - (iv), (D) - (i)
(b) (A) - (iii), (B) - (i), (C) - (ii), (D) - (iv)
(c) (A) - (iii), (B) - (ii), (C) - (iv), (D) - (i)
(d) (A) - (ii), (B) - (i), (C) - (iv), (D) - (iii)

38. The colour of $KMnO_4$ is due to : **[2015]**

(a) $L \rightarrow M$ charge transfer transition
(b) σ - σ^* transition
(c) $M \rightarrow L$ charge transfer transition
(d) $d-d$ transition

39. The reaction of zinc with dilute and concentrated nitric acid, respectively, produces: **[2016]**

(a) NO and N_2O (b) NO_2 and N_2O
(c) N_2O and NO_2 (d) NO_2 and NO

40. Which of the following compounds is metallic and ferromagnetic? **[2016]**

(a) VO_2 (b) MnO_2
(c) TiO_2 (d) CrO_2

41. In the following reactions, ZnO is respectively acting as a/an: **[2017]**

(i) $ZnO + Na_2O \longrightarrow Na_2ZnO_2$
(ii) $ZnO + CO_2 \longrightarrow ZnCO_3$

(a) base and acid (b) base and base
(c) acid and acid (d) acid and base

42. Match the catalysts **(Column I)** with products **(Column II)**. **[2019]**

Column I	Column II
Catalyst	**Product**
(A) V_2O_5	(i) Polyethylene
(B) $TiCl_4/Al(Me)_3$	(ii) ethanol
(C) $PdCl_2$	(iii) H_2SO_4
(D) Iron Oxide	(iv) NH_3

(a) (A)-(iii); (B)-(iv); (C)-(i); (D)-(ii)
(b) (A)-(ii); (B)-(iii); (C)-(i); (D)-(iv)
(c) (A)-(iii); (B)-(i); (C)-(ii); (D)-(iv)
(d) (A)-(iv); (B)-(iii); (C)-(ii); (D)-(i)

43. The atomic radius of Ag is closest to: **[2020]**

(a) Au (b) Ni (c) Cu (d) Hg

Answer Key

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
(c)	(a)	(a)	(c)	(c)	(c)	(a)	(a)	(b)	(a)	(c)	(d)	(b)	(a)	(a)
16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
(c)	(d)	(a)	(d)	(b)	(d)	(d)	(a)	(b)	(c)	(a)	(a)	(b)	(c)	(b)
31	32	33	34	35	36	37	38	39	40	41	42	43		
(a)	(d)	(a)	(d)	(d)	(b)	(d)	(a)	(c)	(d)	(d)	(c)	(a)		

Solutions

1. **(c)** Common oxidation states of Ce(cerium) are + 3 and + 4.

2. **(a)** In lanthanides there is a regular decrease in the atomic radii as well as ionic radii of trivalent ions as the atomic number increases from Ce to Lu. This decrease in size of atoms and ions is known as **lanthanide contraction**. Although the atomic radii do show some irregularities but ionic radii decrease from La to Lu. Thus the correct order is.

$$\underset{86.8\,pm}{Yb^{3+}} < \underset{97\,pm}{Pm^{3+}} < \underset{102\,pm}{Ce^{3+}} < \underset{103\,pm}{La^{3+}}$$

3. **(a)** Mn^{2+} –5 unpaired electrons

Fe^{2+} – 4 unpaired electrons

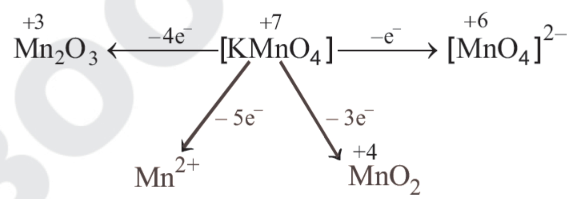
Ti^{2+} – 2 unpaired electrons

Cr^{2+} – 4 unpaired electrons

NOTE Magnetic moment $\propto$ Number of unpaired electrons

4. **(c)** The cyano and hydroxo complexes are far more stable than those formed by halide ion. This is due to the fact that CN^- and OH^- are strong Lewis bases (nucleophiles). Further $[Fe(OH)_5]^{3-}$ is not formed. hence most stable ion is $[Fe(CN)_6]^{3-}$

5. **(c)**



6. **(c)** Ionic radii $\propto \frac{1}{z}$

Thus, $\frac{z_2}{z_1} = \frac{71}{57} = \frac{1.06}{(\text{Ionic radii of } Lu^{3+})}$

$\therefore$ Ionic radii of $Lu^{3+} = 0.85$ Å

7. **(a)** $\ddot{N}H_3 + H^+ \text{ (acid medium)} \rightleftharpoons \overset{+}{N}H_4$

8. **(a)** When KI is added to mercuric iodide it disssolves in it and forms complex.

$$\underset{\substack{\text{red solid}\\ \text{(insoluble in water)}}}{HgI_2} + KI \rightarrow \underset{\text{(soluble)}}{K_2[HgI_4]}$$

On heating, HgI_2 decomposes as

$$HgI_2 \rightleftharpoons Hg + \underset{\text{(violet vapours)}}{I_2 \uparrow}$$

9. **(b)** *f*-Block elements show a regular decrease in atomic size due to lanthanide/actinide contraction.

10. **(a)** $2K_2CrO_4 + 2HNO_3 \longrightarrow K_2Cr_2O_7 + 2KNO_3 + H_2O$

11. (c) $AgNO_3$ on heating till red **hot** decomposes as follows:

$$AgNO_3 \rightarrow Ag + NO_2 + \frac{1}{2}O_2$$

12. (d) $(n-1)d^5ns^2$ attains the maximum O.S. of + 7

13. (b) Grey tin $\rightleftharpoons$ White tin

Grey tin is brittle and crumbles down to powder in very cold climate.

The conversion of grey tin to white tin is acompained by increase in volume., This is knwon as **tin plaque** or **tin disease**.

14. (a) CN^- ion acts good complexing as well as reducing agent.

15. (a) The +4 oxidation state of cerium is also known in solution.

16. (c) $[Fe(CN)_6]^{4-} \rightarrow \boxed{\uparrow\downarrow}\boxed{\uparrow\downarrow}\boxed{\uparrow\downarrow}\boxed{:}\boxed{:}\ \boxed{:}\ \boxed{:}\boxed{:}\boxed{:}$

No of unpaired electron = 0

$[MnCl_4]^{2-} \rightarrow \boxed{\uparrow}\boxed{\uparrow}\boxed{\uparrow}\boxed{\uparrow}\boxed{\uparrow}\ \boxed{:}\ \boxed{:}\boxed{:}\boxed{:}$

No of unpaired electrons = 5

$[CoCl_4]^{2-} \rightarrow \boxed{\uparrow\downarrow}\boxed{\uparrow\downarrow}\boxed{\uparrow}\boxed{\uparrow}\boxed{\uparrow}\ \boxed{:}\ \boxed{:}\boxed{:}\boxed{:}$

No of unpaired electrons = 3

NOTE The greater the number of unpaired electrons, greater the magnitude of magnetic moment. Hence the correct order will be

$[MnCl_4]^{2-} > [CoCl_4]^{2-} > [Fe(CN)_6]^{4-}$

17. (d) $2Cu_2O + Cu_2S \longrightarrow 6Cu + SO_2$

self reduction.

18. (a) $Cr_2O_7^{2-} + 6I^- + 14H^+ \longrightarrow 2Cr^{3+} + 7H_2O + 3I_2$

19. (d) $Hg_2Cl_2 + 2NH_4OH \longrightarrow Hg + HgNH_2Cl + NH_4Cl + 2H_2O$

20. (b) **NOTE** In vertical columns of transition elements, there is an increase in size from first member to second member as expected but from second member to third member, there is very small chang in size and some times sizes are same. This is due to lanthanide contraction this is the reason for Zr and Hf to have same radius.

21. (d) d^5 —— strong ligand field

$\boxed{\uparrow\downarrow}\boxed{\uparrow\downarrow}\boxed{\uparrow}\boxed{}\boxed{}$

$\mu = \sqrt{n(n+2)} = \sqrt{1(1+2)} = \sqrt{3} = 1.73BM$

d^3— in weak as well as in strong field

$\boxed{\uparrow}\boxed{\uparrow}\boxed{\uparrow}\boxed{}\boxed{}$

$\mu = \sqrt{3(3+2)} = \sqrt{15} = 3.87$ B.M.

d^4– in weak ligand field

$\boxed{\uparrow}\boxed{\uparrow}\boxed{\uparrow}\boxed{\uparrow}\boxed{}$

$\mu = \sqrt{4(4+2)} = \sqrt{24} = 4.89$

d^4– in strong ligand field

$\boxed{\uparrow\downarrow}\boxed{\uparrow}\boxed{\uparrow}\boxed{}\boxed{}$

$\mu = \sqrt{2(2+2)} = \sqrt{8} = 2.82$

22. (d) Lanthanide contraction is associated with the intervention of the 4*f* orbitals which must be filled before the 5*d* series of elements begin. The filling of 7*f* before 5*d* orbitals results in a regular decrease in atomic radii called lanthanoid contraction which essentially compensates for the expected increase in atomic size with increasing atomic number. The factor responsible for the lanthanoid contraction is the imperfect shelding of one electron by another in the same set of orbitals.

23. (a) Metal atom in the lower oxidation state forms the ionic bond and in the higher oxidation state the covalent bond because higher oxidation state means small size and great polarizing power and hence greater covalent character. Hence MCl_2 is more ionic than MCl_4.

24. (b) The configuration of lanthanides shows that the additional electron enters the 4*f* subshell. The shielding of one 4 *f* electron by another is very little or imperfect. The imperfect shielding of *f* electrons is due to

the shape of f orbitals which is very much diffused. Thus as the atomic number increases, the nuclear charge increases by unity at each step, while no comparable increase in the mutual shielding effect of $4f$ occurs. This causes a contraction in the size of the $4f$ subshell as a result of which atomic and ionic radii decrease gradually from La to Lu.

25. (c) The number of unpaired electrons in $Ni^{2+}(aq) = 2$. Water is a weak ligand, hence no pairing will take place

$\therefore$ spin magnetic moment

$= \sqrt{n(n+2)} = \sqrt{2(2+2)} = \sqrt{8} = 2.82$

26. (a) $4f$ Orbital is nearer to nucleus as compared to $5f$ orbital therefore, shielding of $4f$ is more than $5f$.

27. (a) **NOTE** More the distance between nucleus and outer orbitals, lesser will be force of attraction on them. Distance between nucleus and $5f$ orbitals is more as compared to distance between $4f$ orbital and nucleus. So actinoids exhibit more number of oxidation states in general than the lanthanoids.

28. (b) **NOTE** The main reason for exhibiting larger number of oxidation states by actinoids as compared to lanthanoids is lesser energy difference between $5f$ and $6d$ orbitals as compared to that between $4f$ and $5d$ orbitals.

In case of actinoids we can remove electrons from $5f$ as well as from $6d$ and due to this actinoids exhibit larger number of oxidation state than lanthanoids.

29. (c) The titration of oxalic acid with $KMnO_4$ in presence of HCl gives unsatisfactory result because of the fact that $KMnO_4$ can also oxidise HCl along with oxalic acid. HCl on oxidation gives Cl_2 and HCl reduces $KMnO_4$ to Mn^{2+}, thus the correct answer is (c).

30. (b) Most of the Ln^{3+} compounds except La^{3+} and Lu^{3+} are coloured due to the presence of unpaired f-electrons.

31. (a) Across the first transition series, the negative values for standard electrode potential decrease except for Mn due to the stable d^5 configuration.

Thus, correct order is

$$Mn^{2+} > Cr^{2+} > Fe^{2+} > Co^{2+}.$$

32. (d) Fe^{3+} is easily hydrolysed than Fe^{2+} due to more positive charge.

33. (a)

(a) $V = 3d^3\ 4s^2$; $V^{2+} = 3d^3 = 3$ unpaired electrons

$Cr = 3d^5\ 4s^1$; $Cr^{2+} = 3d^4 = 4$ unpaired electrons

$Mn = 3d^5\ 4s^2$; $Mn^{2+} = 3d^5 = 5$ unpaired electrons

$Fe = 3d^6\ 4s^2$; $Fe^{2+} = 3d^6 = 4$ unpaired electrons

Hence the correct order of paramagnetic behaviour

$V^{2+} < Cr^{2+} = Fe^{2+} < Mn^{2+}$

(b) For the same oxidation state, the ionic radii generally decreases as the atomic number increases in a particular transition series, hence the order is

$Mn^{2+} > Fe^{2+} > Co^{2+} > Ni^{2+}$

(c) Larger size, least hydrated more stable in aqueous solution. As we move across the period ($Sc^{3+} \rightarrow Cr^{3+} \rightarrow Fe^{3+} \rightarrow Co^{3+}$), the ionic size usually decreases. Sc^{3+} with the large size as least hydrated and hence more stable.

(d) Sc – (+ 2), (+ 3)

Ti – (+ 2), (+ 3), (+ 4)

Cr – (+ 2), (+ 3), (+ 4), (+ 5), (+ 6)

Mn – (+ 2), (+ 3), (+ 4), (+ 5), (+ 6), (+ 7)

i.e. Sc < Ti < Cr < Mn

34. **(d)** $E^\circ_{Cr^{3+}/Cr^{2+}} = -0.41\,V,$

$E^\circ_{Fe^{3+}/Fe^{2+}} = +0.77\,V$

$E^\circ_{Mn^{3+}/Mn^{2+}} = +1.57\,V,$

$E^\circ_{Co^{3+}/Co^{2+}} = +1.97\,V$

35. **(d)** In equation (a) $Fe_2(SO_4)_3$, and in equation (b) $FeSO_4$, on decomposition will form oxide instead of Fe. In equation (c) $FeCl_3$ cannot be reduced when heated in air. Hence equation (d) is correct.

36. **(b)** The equation in option (b) is correct since both charges as well as atoms are balanced. For the rest,

(a) Given reaction in unfavourable in the forward direction (K_2O is unstable, while Li_2O is stable)

(c) Given reaction is not balanced w.r.t. charge.

(d) Given reaction will give $K_3[Cu(CN)_4]$ as product instead of $K_2[Cu(CN)_4]$.

37. **(d)** (A) - (ii), (B) - (i), (C) - (iv), (D) - (iii)

38. **(a)** $L \rightarrow M$ charge transfer spectra. $KMnO_4$ is coloured because it absorbs light in the visible range of electromagnetic radiation. The permanganate ion is the source of colour, as a ligand to metal ($L \rightarrow M$) charge transfer takes place between oxygen's *p* orbitals and the empty *d*-orbitals on the metal. This charge transfer takes place when a photon of light is absorbed which leads to the purple colour of the compound.

39. **(c)** Reaction of Zn with different concentration of HNO_3 are as follows :

$$Zn + 4HNO_3(60\%) \longrightarrow Zn(NO_3)_2 + 2NO_2\uparrow + 2H_2O$$

$$3Zn + 8HNO_3(30\%) \longrightarrow 3Zn(NO_3)_2 + 2NO\uparrow + 4H_2O$$

$$4Zn + 10HNO_3(20\%) \longrightarrow 4Zn(NO_3)_2 + N_2O\uparrow + 5H_2O$$

$$5Zn + 12HNO_3(10\%) \longrightarrow 5Zn(NO_3)_2 + N_2\uparrow + 6H_2O$$

$$4Zn + 10HNO_3(3\%) \longrightarrow 4Zn(NO_3)_2 + NH_4NO_3 + 3H_2O$$

Hence option (c) is correct.

40. **(d)** Out of all the four given metallic oxides, CrO_2 is attracted by magnetic field very strongly. The effect persists even when the magnetic field is removed. Thus CrO_2 is metallic and ferromagnetic in nature.

41. **(d)** (i) $\underset{\text{acid}}{ZnO} + \underset{\text{base}}{Na_2O} \rightarrow \underset{\text{salt}}{Na_2ZnO_2}$

(ii) $\underset{\text{base}}{ZnO} + \underset{\text{acid}}{CO_2} \rightarrow \underset{\text{salt}}{ZnCO_3}$

42. **(c)**

(A) $V_2O_5 \rightarrow$ Preparation of H_2SO_4 in contact process

(B) $TiCl_4 + Al(Me)_3 \rightarrow$ Polyethylene (Ziegler-Natta catalyst)

(C) $PdCl_2 \rightarrow$ Ethanal (Wacker's process)

(D) Iron oxide $\rightarrow NH_3$ in (Haber's process)

43. **(a)** Atomic size of elements of 4*d* and 5*d* transition series are nearly same due to lanthanide contraction.

23 Co-ordination Compounds

1. A square planar complex is formed by hybridisation of which atomic orbitals? **[2002]**
   (a) s, p_x, p_y, d_{yz} (b) $s, p_x, p_y, d_{x^2-y^2}$
   (c) s, p_x, p_y, d_{z^2} (d) s, p_y, p_z, d_{xy}

2. The type of isomerism present in nitropentamminechromium (III) chloride is **[2002]**
   (a) optical (b) linkage
   (c) ionization (d) polymerisation.

3. $CH_3 - Mg - Br$ is an organometallic compound due to **[2002]**
   (a) Mg – Br bond
   (b) C – Mg bond
   (c) C – Br bond
   (d) C – H bond.

4. One mole of the complex compound $Co(NH_3)_5Cl_3$, gives 3 moles of ions on dissolution in water. One mole of the same complex reacts with two moles of $AgNO_3$ solution to yield two moles of AgCl (s). The structure of the complex is **[2003]**
   (a) $[Co(NH_3)_3Cl_3]. 2 NH_3$
   (b) $[Co(NH_3)_4Cl_2] Cl . NH_3$
   (c) $[Co(NH_3)_4Cl] Cl_2. NH_3$
   (d) $[Co(NH_3)_5Cl] Cl_2$

5. In the coordination compound, $K_4[Ni(CN)_4]$, the oxidation state of nickel is **[2003]**
   (a) 0 (b) +1
   (c) +2 (d) –1

6. The coordination number of a central metal atom in a complex is determined by **[2004]**
   (a) the number of ligands around a metal ion bonded by sigma and pi-bonds both
   (b) the number of ligands around a metal ion bonded by pi-bonds
   (c) the number of ligands around a metal ion bonded by sigma bonds
   (d) the number of only anionic ligands bonded to the metal ion.

7. Which one of the following complexes is an outer orbital complex ? **[2004]**
   (a) $[Co(NH_3)_6]^{3+}$
   (b) $[Mn(CN)_6]^{4-}$
   (c) $[Fe(CN)_6]^{4-}$
   (d) $[Ni(NH_3)_6]^{2+}$
   (Atomic nos. : Mn = 25; Fe = 26; Co = 27, Ni = 28)

8. Coordination compounds have great importance in biological systems. In this context which of the following statements is **incorrect** ? **[2004]**
   (a) Cyanocobalamin is B_{12} and contains cobalt
   (b) Haemoglobin is the red pigment of blood and contains iron
   (c) Chlorophylls are green pigments in plants and contain calcium
   (d) Carboxypeptidase – A is an enzyme and contains zinc.

9. Which one of the following has largest number of isomers? **[2004]**

(a) $[Ir(PR_3)_2H(CO)]^{2+}$

(b) $[Co(NH_3)_5Cl]^{2+}$

(c) $[Ru(NH_3)_4Cl_2]^+$

(d) $[Co(en)_2Cl_2]^+$

(R = alkyl group, en = ethylenediamine)

10. The oxidation state Cr in $[Cr(NH_3)_4Cl_2]^+$ is **[2005]**

(a) 0 (b) +1

(c) +2 (d) +3

11. The IUPAC name of the coordination compound $K_3[Fe(CN)_6]$ is **[2005]**

(a) Tripotassium hexacyanoiron (II)

(b) Potassium hexacyanoiron (II)

(c) Potassium hexacyanoferrate (III)

(d) Potassium hexacyanoferrate (II)

12. Which of the following compounds shows optical isomerism? **[2005]**

(a) $[Co(CN)_6]^{3-}$ (b) $[Cr(C_2O_4)_3]^{3-}$

(c) $[ZnCl_4]^{2-}$ (d) $[Cu(NH_3)_4]^{2+}$

13. Which one of the following cyano complexes would exhibit the lowest value of paramagnetic behaviour ? **[2005]**

(a) $[Co(CN)_6]^{3-}$ (b) $[Fe(CN)_6]^{3-}$

(c) $[Mn(CN)_6]^{3-}$ (d) $[Cr(CN)_6]^{3-}$

(At. Nos : Cr = 24, Mn = 25, Fe = 26, Co = 27)

14. The IUPAC name for the complex $[Co(NO_2)(NH_3)_5]Cl_2$ is : **[2006]**

(a) pentaammine nitrito-N-cobalt(II) chloride

(b) pentaammine nitrito-N-cobalt(III) chloride

(c) nitrito-N-pentaamminecobalt(III) chloride

(d) nitrito-N-pentaamminecobalt(II) chloride

15. Nickel (Z = 28) combines with a uninegative monodentate ligand X^- to form a paramagnetic complex $[NiX_4]^{2-}$. The number of unpaired electron(s) in the nickel and geometry of this complex ion are, respectively : **[2006]**

(a) one, square planar

(b) two, square planar

(c) one, tetrahedral

(d) two, tetrahedral

16. In $Fe(CO)_5$, the Fe – C bond possesses **[2006]**

(a) ionic character

(b) σ-character only

(c) π-character

(d) both σ and π characters

17. How many EDTA (ethylenediaminetetraacetic acid) molecules are required to make an octahedral complex with a Ca^{2+} ion? **[2006]**

(a) One (b) Two

(c) Six (d) Three

18. Which of the following has a square planar geometry? **[2007]**

(a) $[PtCl_4]^{2-}$ (b) $[CoCl_4]^{2-}$

(c) $[FeCl_4]^{2-}$ (d) $[NiCl_4]^{2-}$

(At. nos.: Fe = 26, Co = 27, Ni = 28, Pt = 78)

19. The coordination number and the oxidation state of the element 'E' in the complex $[E(en)_2(C_2O_4)]NO_2$ (where (en) is ethylene diamine) are, respectively, **[2008]**

(a) 6 and 2 (b) 4 and 2

(c) 4 and 3 (d) 6 and 3

20. In which of the following complexes of the Co (at. no. 27), will the magnitude of Δ_0 be the highest? **[2008]**

(a) $[Co(CN)_6]^{3-}$

(b) $[Co(C_2O_4)_3]^{3-}$

(c) $[Co(H_2O)_6]^{3+}$

(d) $[Co(NH_3)_6]^{3+}$

21. Which of the following shows optical isomerism ? **[2009]**

(a) $[Co(en)(NH_3)_2]^{2+}$

(b) $[Co(H_2O)_4(en)]^{3+}$

(c) $[Co(en)_2(NH_3)_2]^{3+}$

(d) $[Co(NH_3)_3Cl]^+$

22. Which of the following pairs represent linkage isomers? **[2009]**

(a) $[Pd(PPh_3)_2(NCS)_2]$ and $[Pd(PPh_3)_2(SCN)_2]$

(b) $[Co(NH_3)_5NO_3]SO_4$ and $[Co(NH_3)_5SO_4]NO_3$

(c) $[PtCl_2(NH_3)_4]Br_2$ and $[PtBr_2(NH_3)_4]Cl_2$

(d) $[Cu(NH_3)_4][PtCl_4]$ and $[Pt(NH_3)_4][CuCl_4]$

23. A solution containing 2.675 g of $CoCl_3 . 6NH_3$ (molar mass = 267.5 g mol^{-1}) is passed through a cation exchanger. The chloride ions obtained in solution were treated with excess of $AgNO_3$ to give 4.78 g of AgCl (molar mass = 143.5 g mol^{-1}). The formula of the complex is (At. mass of Ag = 108 u) **[2010]**

(a) $[Co(NH_3)_6]Cl_3$

(b) $[CoCl_2(NH_3)_4]Cl$

(c) $[CoCl_3(NH_3)_3]$

(d) $[CoCl(NH_3)_5]Cl_2$

24. Which one of the following has an optical isomer? **[2010]**

(a) $[Zn(en)(NH_3)_2]^{2+}$ (b) $[Co(en)_3]^{3+}$

(c) $[Co(H_2O)_4(en)]^{3+}$ (d) $[Zn(en)_2]^{2+}$

(en = ethylenediamine)

25. Which one of the following complex ions has geometrical isomers ? **[2011]**

(a) $[Ni(NH_3)_5Br]^+$

(b) $[Co(NH_3)_2(en)_2]^{3+}$

(c) $[Cr(NH_3)_4(en)_2]^{3+}$

(d) $[Co(en)_3]^{3+}$

(en = ethylenediamine)

26. Which among the following will be named as dibromido bis(ethylenediamine) chromium (III) bromide? **[2012]**

(a) $[Cr(en)_3]Br_3$

(b) $[Cr(en)_2Br_2]Br$

(c) $[Cr(en)Br_4]^-$

(d) $[Cr(en)Br_2]Br$

27. Which of the following complex species is not expected to exhibit optical isomerism ? **[2013]**

(a) $[Co(en)_3]^{3+}$

(b) $[Co(en)_2Cl_2]^+$

(c) $[Co(NH_3)_3Cl_3]$

(d) $[Co(en)(NH_3)_2Cl_2]^+$

28. The octahedral complex of a metal ion M^{3+} with four monodentate ligands L_1, L_2, L_3 and L_4 absorb wavelengths in the region of red, green, yellow and blue, respectively. The increasing order of ligand strength of the four ligands is: **[2014]**

(a) $L_4 < L_3 < L_2 < L_1$

(b) $L_1 < L_3 < L_2 < L_4$

(c) $L_3 < L_2 < L_4 < L_1$

(d) $L_1 < L_2 < L_4 < L_3$

29. Which of the following compounds is not colored yellow ? **[2015]**

(a) $(NH_4)_3[As(Mo_3O_{10})_4]$

(b) $BaCrO_4$

(c) $Zn_2[Fe(CN)_6]$

(d) $K_3[Co(NO_2)_6]$

30. The number of geometric isomers that can exist for square planar complex $[Pt(Cl)(py)(NH_3)(NH_2OH)]^+$ is (py = pyridine) : **[2015]**

(a) 4 (b) 6

(c) 2 (d) 3

31. Which one of the following complexes shows optical isomerism? **[2016]**

(a) *trans* $[Co(en)_2Cl_2]Cl$

(b) $[Co(NH_3)_4Cl_2]Cl$

(c) $[Co(NH_3)_3Cl_3]$

(d) *cis*$[Co(en)_2Cl_2]Cl$

(en = ethylenediamine)

32. The pair having the same magnetic moment is:
[At. No.: Cr = 24, Mn = 25, Fe = 26, Co = 27] **[2016]**

(a) $[Mn(H_2O)_6]^{2+}$ and $[Cr(H_2O)_6]^{2+}$

(b) $[CoCl_4]^{2-}$ and $[Fe(H_2O)_6]^{2+}$

(c) $[Cr(H_2O)_6]^{2+}$ and $[CoCl_4]^{2-}$

(d) $[Cr(H_2O)_6]^{2+}$ and $[Fe(H_2O)_6]^{2+}$

33. On treatment of 100 mL of 0.1 M solution of $CoCl_3 . 6H_2O$ with excess $AgNO_3$; 1.2×10^{22} ions are precipitated. The complex is : **[2017]**

(a) $[Co(H_2O)_4Cl_2]Cl.2H_2O$

(b) $[Co(H_2O)_3Cl_3].3H_2O$

(c) $[Co(H_2O)_6]Cl_3$

(d) $[Co(H_2O)_5Cl]Cl_2.H_2O$

34. Consider the following reaction and statements:
$[Co(NH_3)_4Br_2]^+ + Br^- \rightarrow [Co(NH_3)_3Br_3] + NH_3$

(I) Two isomers are produced if the reactant complex ion is a *cis*-isomer.

(II) Two isomers are produced if the reactant complex ion is a *trans* -isomer

(III) Only one isomer is produced if the reactant complex ion is a *trans* -isomer

(IV) Only one isomer is produced if the reactant complex ion is a *cis*-isomer.

The correct statements are: **[2018]**

(a) (I) and (II) (b) (I) and (III)

(c) (III) and (IV) (d) (II) and (IV)

35. Two complexes $[Cr(H_2O)_6]Cl_3$ (A) and $[Cr(NH_3)_6]Cl_3$ (B) are violet and yellow coloured, respectively. The incorrect statement regarding them is: **[2019]**

(a) Δ_0 values of (A) and (B) are calculated from the energies of violet and yellow light, respectively.

(b) both are paramagnetic with three unpaired electrons.

(c) both absorb energies corresponding to their complementary colors.

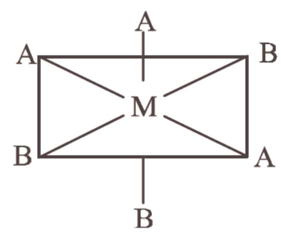
(d) Δ_0 value for (A) is less than that of (B).

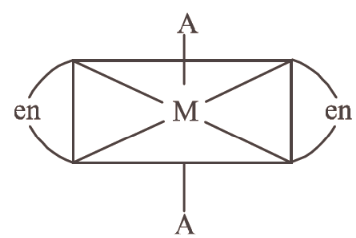
36. The highest value of the calculated spin only magnetic moment (in BM) among all the transition metal complexes is : **[2019]**

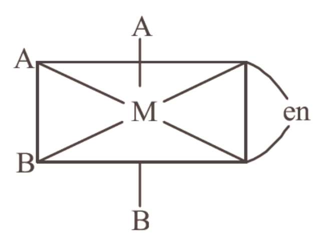
(a) 5.92 (b) 6.93

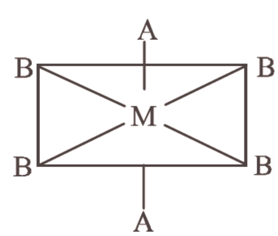
(c) 3.87 (d) 4.90

37. The one that will show optical activity is:
(en = ethane 1, 2-diamine) **[2019]**

(a)


(b)


(c)


(d)


38. The degenerate orbitals of $[Cr(H_2O)_6]^{3+}$ are: **[2019]**

(a) d_{xz} and d_{yz}

(b) d_{yz} and dz^2

(c) d_{z^2} and d_{xz}

(d) $d_{x^2-y^2}$ and d_{xy}

39. The IUPAC name of the complex $[Pt(NH_3)_2Cl(NH_2CH_3)]Cl$ is: **[2020]**

(a) Diamminechlorido (methanamine) platinum (II)chloride

(b) Diammine (methanamine) chlorido platinum (II)chloride

(c) Diamminechlorido (aminomethane) platinum (II)chloride

(d) Bisammine (methanamine) chlorido platinum (II)chloride

Answer Key

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
(b)	(b)	(b)	(d)	(a)	(c)	(d)	(c)	(d)	(d)	(c)	(b)	(a)	(b)	(d)
16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
(d)	(a)	(a)	(d)	(a)	(c)	(a)	(a)	(b)	(b)	(b)	(c)	(b)	(c)	(d)
31	32	33	34	35	36	37	38	39						
(d)	(d)	(d)	(b)	(a)	(a)	(c)	(a)	(a)						

Solutions

1. (b) A square planar complex is formed by hybridisation of s, p_x, p_y and $d_{x^2-y^2}$ atomic orbitals

2. (b) The chemical formula of nitropentammine chromium (III) chloride is

$$\left[Cr\left(NH_3\right)_5 NO_2\right]Cl_2$$

It can exist in following two structures

$\left[Cr\left(NH_3\right)_5 NO_2\right]Cl_2$ and

Nitropentamminechromium (III) chloride

$\left[Cr\left(NH_3\right)_5 ONO\right]Cl_2$

Nitritopentamminechromium (III) chloride

Therefore the type of isomerism found in this compound is linkage isomerism as NO_2 group is linked through N as $-NO_2$ or through O as $-ONO$.

3. (b) Compounds that contain carbon-metal bond are known as organometallic compounds. In CH_3-Mg-Br (Grignard's reagent), a bond is present between carbon and Mg (metal), hence it is an organometallic compound.

4. (d) $Co(NH_3)_5Cl_3 \rightleftharpoons [Co(NH_3)_5Cl]^{+2} + 2Cl^-$

$\therefore$ Structure is $[Co(NH_3)_5Cl]Cl_2$.

Now $[Co(NH_3)_5Cl]Cl_2 + 2AgNO_3$

$\longrightarrow [Co(NH_3)_5Cl](NO_3)_2 + 2AgCl$

5. (a) Let the O. N. of Ni in $K_4[Ni(CN)_4]$ be $= x$ then

$$4(+1) + x + (-1) \times 4 = 0$$

$$\Rightarrow 4 + x - 4 = 0$$

$$x = 0$$

6. (c) The coordination number of central metal atom in a complex is equal to number of monovalent ligands, twice the number of

bidentate ligands and so on, around the metal ion bonded by coordinate bonds. Hence coordination number = No. of σ bonds formed by metals with ligands.

7. (d) $\underset{d^2sp^3}{[Fe(CN)_6]^{4-}}, \underset{d^2sp^3}{[Mn(CN)_6]^{4-}},$

$\underset{d^2sp^3}{[Co(NH_3)_6]^{3+}}, \underset{sp^3d^2}{[Ni(NH_3)_6]^{2+}}$

Hence $[Ni(NH_3)_6]^{2+}$ is outer orbital complex.

8. (c) The chlorophyll molecule plays an important role in photosynthesis. It has porphyrin rings and the metal Mg, not Ca.

9. (d) Isomers

$\underset{cis \text{ and } trans}{[Ru(NH_3)_4Cl_2]^+}, \underset{\text{none}}{[Co(NH_3)_5Cl]^{2+}},$

$\underset{cis \text{ and } trans}{[Ir(PR_3)_2H(CO)]^{2+}}, \underset{cis \text{ and } trans \text{ and optical isomers}}{[Co(en)_2Cl_2]^+}$

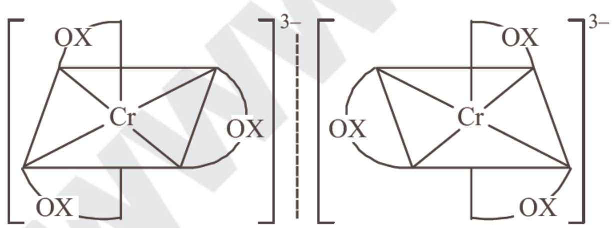
10. (d) Oxidation state of Cr in $[Cr(NH_3)_4Cl_2]^+$.

Let it be x, $1 \times x + 4 \times 0 + 2(-1) = 1$

Therefore $x = 3$.

11. (c) $K_3[Fe(CN)_6]$ is potassium hexacyanoferrate (III).

12. (b)



Non-superimposable mirror images, hence optical isomers.

13. (a)

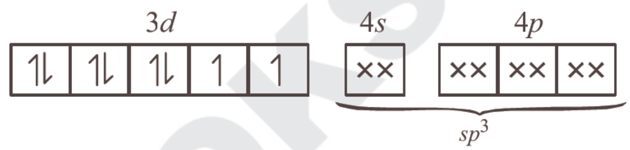
	Metal ion	Unpaired electrons	Magnetic moment
(a)	Co^{3+}	0	0
(b)	Fe^{3+}	1	$\sqrt{3}$ B.M.
(c)	Mn^{3+}	2	$\sqrt{8}$ B.M.
(d)	Cr^{3+}	3	$\sqrt{18}$ B.M.

The effective magnetic moment is given by the number of unpaired electrons in a substance, the lesser the number of unpaired electrons lower is its magnetic moment in Bohr – Magneton and lower shall be its paramagnetism.

14. (b) $[Co(NO)_2(NH_3)_5]Cl_2$

Pentaamminenitrito-N-cobalt (III) chloride

15. (d) $[Ni\ X_4]^{2-}$, the electronic configuration of Ni^{2+} is



It contains two unpaired electrons and the hybridisation is sp^3 (tetrahedral).

16. (d) Due to some backbonding by sideways overlapping between d-orbitals of metal and p-orbital of carbon, the Fe–C bond in $Fe(CO)_5$ has both σ and π character.

17. (a) EDTA is hexadentate, four donor O atoms and 2 donor N atoms, and for the formation of octahedral complex one molecule is required.

18. (a) Complexes with dsp^2 hybridisation are square planar. All the complexes of Pt^{2+} are square planar including those with weak field ligand such as halide ions. Thus option (a) is correct.

19. (d) In the given complex we have two bidentate ligands (i.e en and C_2O_4), so coordination number of E is 6

$(2 \times 2 + 1 \times 2 = 6)$

Let the oxidation state of E in complex be x, then

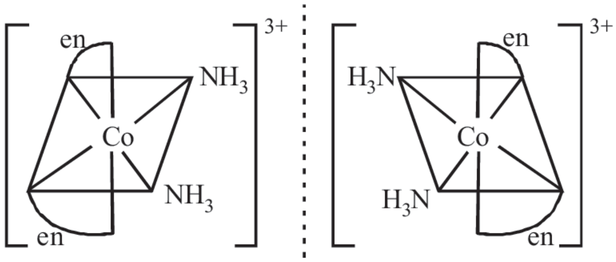
$[x + (-2) = 1]$ or $x - 2 = 1$

or $x = +3$, so its oxidation state is + 3

Thus option (d) is correct.

20. (a) In octahedral complex, the magnitude of Δ_0 will be highest in a complex having strongest ligand. Of the given ligands, CN^- is strongest, so Δ_0 will be highest for $[Co(CN)_6]^{3-}$. Thus option (a) is correct.

21. (c)



Enantiomers of *cis*-$\left[Co(en)_2(NH_3)_2\right]^{3+}$

22. (a) The SCN^- ion can coordinate through S or N atom giving rise to linkage isomerism

$M \leftarrow SCN$ thiocyanato

$M \leftarrow NCS$ isothiocyanato.

23. (a) $\underset{2.675\text{ g}}{CoCl_3.6NH_3} \longrightarrow xCl^-$

$xCl^- + AgNO_3 \longrightarrow \underset{4.78\text{ g}}{xAgCl\downarrow}$

Number of moles of the complex

$= \dfrac{2.675}{267.5} = 0.01$ mol

Number of moles of AgCl obtained

$= \dfrac{4.78}{143.5} = 0.03$ mol

$\therefore$ No. of moles of AgCl obtained

$= 3 \times$ No. of moles of complex

$\therefore n = \dfrac{0.03}{0.01} = 3$

24. (b) For a substance to be optical isomer following conditions should be fulfiled

(a) A coordination compound which can rotate the plane of polarised light is said to be optically active.

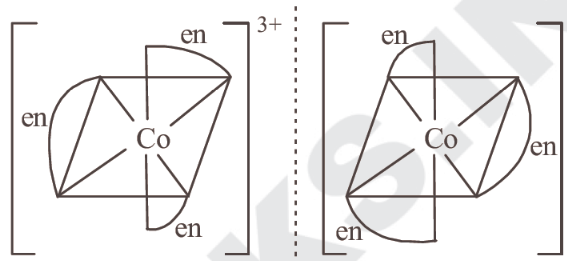
(b) When the coordination compounds have same formula but differ in their abilities to rotate directions of the plane of polarised light are said to exhibit optical isomerism and the molecules are optical isomers. The optical isomers are pair of molecules which are non-superimposable mirror images of each other.

(c) This is due to the absence of elements of symmetry in the complex.

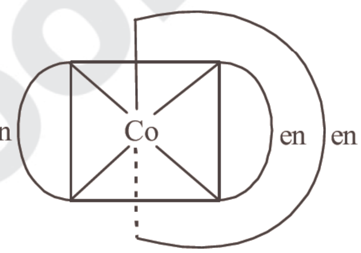
(d) Optical isomerism is expected in tetrahedral complexes of the type Mabcd.

Based on this only option (b) shows optical isomerism

$[Co(en)_3]^{3+}$



d-form Mirror *l*-form

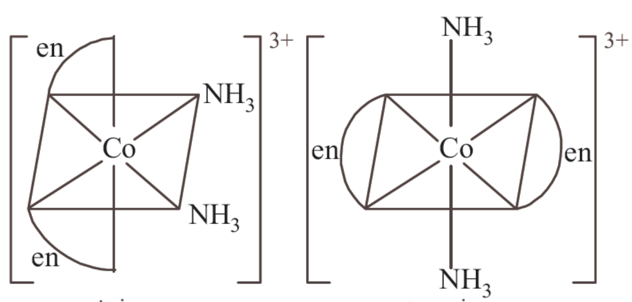


'Meso' or optically inactive form

Complexes of Zn^{2+} cannot show optical isomerism as they are tetrahedral complexes with plane of symmetry.

$[CO(H_2O)_4(en)]^{3+}$ has two planes of symmetry hence it is also optically inactive.

25. (b)

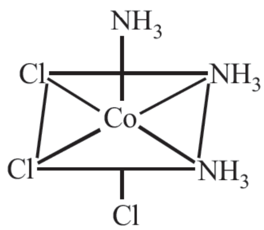


cis-isomer *trans*-isomer

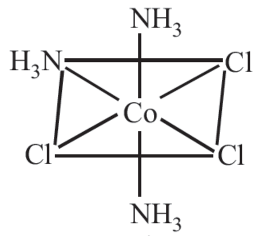
26. (b) $[Cr(en)_2Br_2]Br$

dibromido bis(ethylenediamine) chromium (III) Bromide.

27. (c) Octahedral coordination entities of the type Ma_3b_3 exhibit geometrical isomerism. The compound exists both as facial and meridional isomers, both contain plane of symmetry

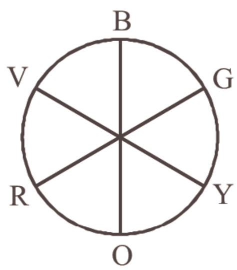


fac-isomer



mer-isomer

28. **(b)**



For a given metal ion, weak field ligands create a complex with smaller Δ_0, which will absorbs light of longer λ and thus lower frequency. Conservely, stronger field ligands create a larger Δ_0, absorb light of shorter λ and thus higher v i.e. higher energy.

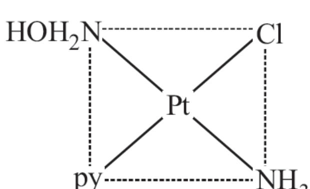
$$\underset{\lambda = 650\text{ nm}}{\text{Red}} < \underset{570\text{ nm}}{\text{Yellow}} < \underset{490\text{ nm}}{\text{Green}} < \underset{450\text{ nm}}{\text{Blue}}$$

So order of ligand strength is

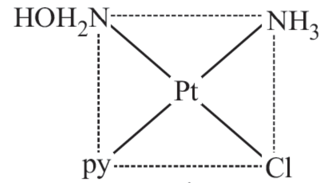
$$L_1 < L_3 < L_2 < L_4$$

29. **(c)**

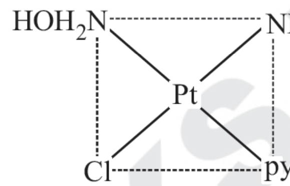
30. **(d)** Square planar complexes of type M[ABCD] form three isomers. Their position may be obtained by fixing the position of one ligand and placing at the *trans* position any one of the remaining three ligands one by one.



trans

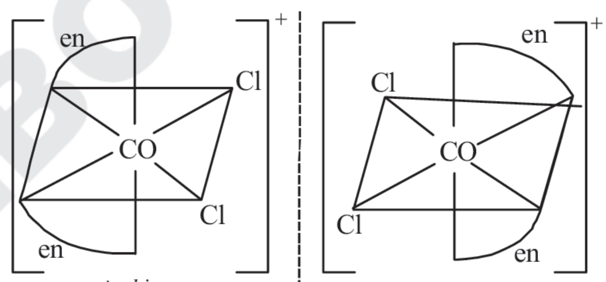


cis



cis

31. **(d)** Optical isomerism occurs when a molecule is non-super imposable with its mirror image hence the complex *cis*-$[Co(en)_2Cl_2]Cl$ is optically active.



cis-d-isomer *cis-ℓ*-isomer

32. **(d)**

Metal ion		Unpaired electron	Magnetic moment
(i)	Cr^{2+}	4	$\sqrt{24}$ B.M.
(ii)	Fe^{2+}	4	$\sqrt{24}$ B.M.
(iii)	Co^{2+}	3	$\sqrt{15}$ B.M.
(iv)	Mn^{2+}	5	$\sqrt{35}$ B.M.

Since (i) and (ii), each has 4 unpaired electron, so they will exhibit same magnetic moment. Thus option (d) is correct.

33. **(d)** Moles of complex

$$= \frac{\text{Molarity} \times \text{Volume(mL)}}{1000}$$

$$= \frac{100 \times 0.1}{1000} = 0.01 \text{ mole}$$

Moles of ions precipitated with excess of

$$AgNO_3 = \frac{1.2 \times 10^{22}}{6.02 \times 10^{23}} = 0.02 \text{ moles}$$

$\therefore$ Number of Cl^- present in ionisation sphere

$$= \frac{\text{Number of moles of ions precipitated}}{\text{Number of moles of complex}}$$

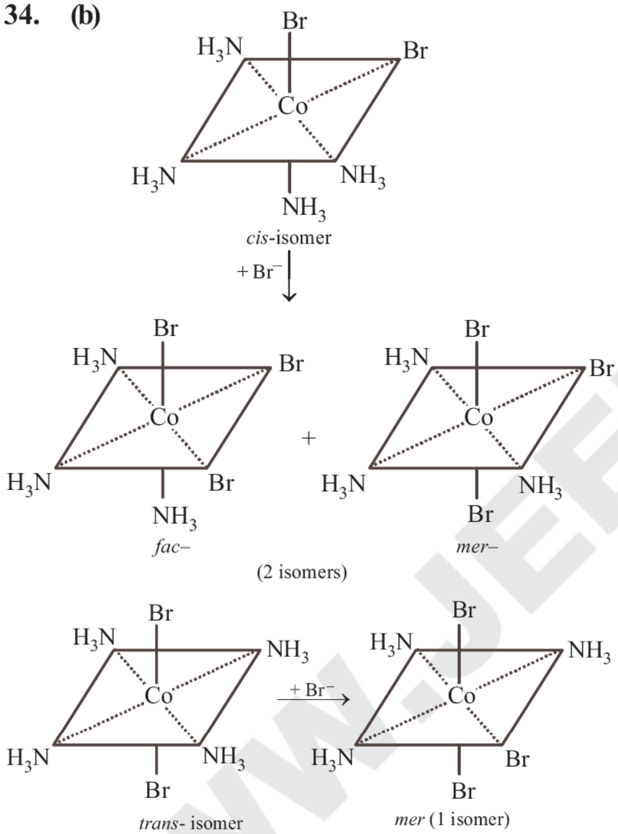
$$= \frac{0.02}{0.01} = 2$$

It means $2Cl^-$ ions present in ionization sphere.

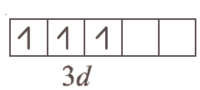
Thus formula of the complex is

$[Co(H_2O)_5Cl]Cl_2.H_2O$

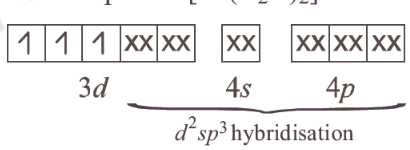
34. (b)



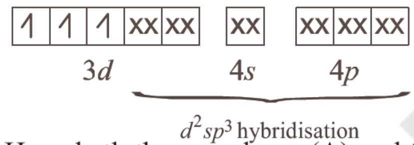
35. (a) E.C. of Cr^{3+} $(3d^3)$:



For complex A $[Cr(H_2O)_2]^{3+}$:



For complex B $[Cr(NH_3)_6]^{3+}$:



Here, both the complexes (A) and (B) are paramagnetic with 3 unpaired electrons each. Also H_2O is a weak field ligand which causes lesser splitting than NH_3 which is comparatively stronger field ligand. Hence, the (Δ_0) value of (A) and (B) are calculated from the wavelengths of light absorbed and not from the wavelengths of light emitted.

36. (a) Magnetic moment, $\mu = \sqrt{n(n+2)}$ BM

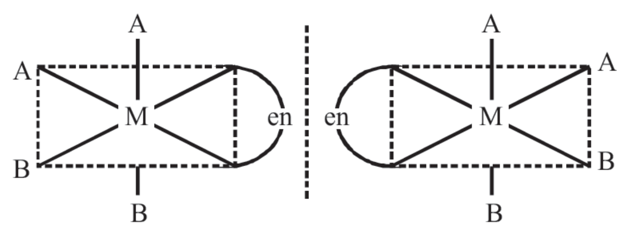
(where, n = no. of unpaired electrons)

As transition metal atom/ion in a complex may have unpaired electrons ranging from zero to 5. So, maximum number of unpaired electrons that may be present in a complex is 5.

$\therefore$ Maximum value of magnetic moment among all the transition metal complexes is

$$= \sqrt{5(5+2)} = \sqrt{35} = 5.92 \text{ BM}$$

37. (c)



No plane of symmetry or centre of symmetry Hence it is optically active.

38. (a) Cr^{3+} has d^3 configuration and forms an octahedral inner orbitals complex.

The set of degenerate orbitals are (d_{xy}, d_{yz} and d_{xz}) and ($d_{x^2-y^2}$ and d_{z^2}).

39. (a) $[Pt(NH_3)_2Cl(NH_2CH_3)]Cl$

Diamminechlorido (methanamine) platinum (II) chloride.

24 Haloalkanes and Haloarenes

1. Bottles containing C_6H_5I and $C_6H_5CH_2I$ lost their original labels. They were labelled A and B for testing. A and B were separately taken in test tubes and boiled with NaOH solution. The end solution in each tube was made acidic with dilute HNO_3 and then some $AgNO_3$ solution was added. Substance B gave a yellow precipitate. Which one of the following statements is true for this experiment? **[2003]**
   (a) A is $C_6H_5CH_2I$
   (b) B is C_6H_5I
   (c) Addition of HNO_3 was unnecessary
   (d) A is C_6H_5I

2. The compound formed on heating chlorobenzene with chloral in the presence of concentrated sulphuric acid, is **[2004]**
   (a) freon (b) DDT
   (c) gammexene (d) hexachloroethane

3. Tertiary alkyl halides are practically inert to substitution by S_N2 mechanism because of **[2005]**
   (a) steric hindrance (b) inductive effect
   (c) instability (d) insolubility

4. Alkyl halides react with dialkyl copper reagents to give **[2005]**
   (a) alkenyl halides
   (b) alkanes
   (c) alkyl copper halides
   (d) alkenes

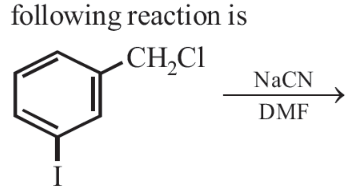
5. Elimination of bromine from 2-bromobutane results in the formation of – **[2005]**
   (a) Predominantly 2-butyne
   (b) Predominantly 1-butene
   (c) Predominantly 2-butene
   (d) equimolar mixture of 1-and 2-butenes

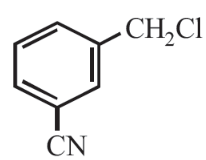
6. Phenyl magnesium bromide reacts with methanol to give **[2005]**
   (a) a mixture of toluene and Mg(OH)Br
   (b) a mixture of phenol and Mg(Me)Br
   (c) a mixture of anisole and Mg(OH)Br
   (d) a mixture of benzene and Mg(OMe)Br

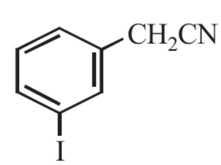
7. Fluorobenzene (C_6H_5F) can be synthesized in the laboratory **[2006]**
   (a) by direct fluorination of benzene with F_2 gas
   (b) by reacting bromobenzene with NaF solution
   (c) by heating phenol with HF and KF
   (d) from aniline by diazotisation followed by heating the diazonium salt with HBF_4

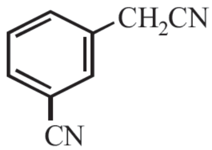
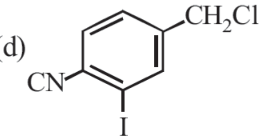
8. Reaction of *trans* 2-phenyl-1 bromocyclopentane on reaction with alcoholic KOH produces
   (a) 1-phenylcyclopentene **[2006]**
   (b) 3-phenylcyclopentene
   (c) 4-phenylcyclopentene
   (d) 2-phenylcyclopentene

9. The structure of the major product formed in the following reaction is **[2006]**



(a) 

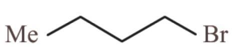
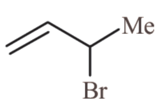
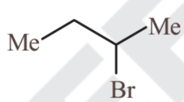
(b) 

(c) 
(d) 

10. Which of the following is the correct order of decreasing S_N2 reactivity? **[2007]**
(a) $R_2CH\,X > R_3C\,X > RCH_2\,X$
(b) $RCH_2\,X > R_3C\,X > R_2CH\,X$
(c) $RCH_2\,X > R_2CH\,X > R_3C\,X$
(d) $R_3C\,X > R_2CH\,X > RCH_2\,X$.
(X is a halogen)

11. The organic chloro compound, which shows complete sterochemical inversion during a S_N2 reaction, is **[2008]**
(a) $(C_2H_5)_2CHCl$ (b) $(CH_3)_3CCl$
(c) $(CH_3)_2\,CHCl$ (d) CH_3Cl

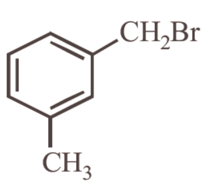
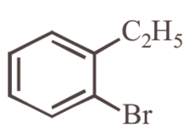
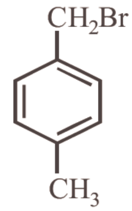
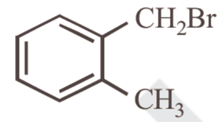
12. Consider the following bromides :


(A)

(B)

(C)

The correct order of S_N1 reactivity is **[2010]**
(a) B > C > A (b) B > A > C
(c) C > B > A (d) A > B > C

13. How many chiral compounds are possible on monochlorination of 2- methyl butane ? **[2012]**
(a) 8 (b) 2
(c) 4 (d) 6

14. Compound (A), C_8H_9Br, gives a yellow precipitate when warmed with alcoholic $AgNO_3$. Oxidation of (A) gives an acid (B), $C_8H_6O_4$. (B) easily forms anhydride on heating. Identify the compound (A). **[2013]**

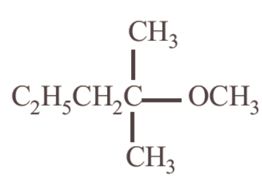
(a) 
(b) 
(c) 
(d) 

15. In S_N2 reactions, the correct order of reactivity for the following compounds: **[2014]**
CH_3Cl, CH_3CH_2Cl, $(CH_3)_2CHCl$ and $(CH_3)_3CCl$ is:
(a) $CH_3Cl > (CH_3)_2\,CHCl > CH_3CH_2Cl > (CH_3)_3\,CCl$
(b) $CH_3Cl > CH_3CH_2Cl > (CH_3)_2\,CHCl > (CH_3)_3\,CCl$
(c) $CH_3CH_2Cl > CH_3Cl > (CH_3)_2\,CHCl > (CH_3)_3\,CCl$
(d) $(CH_3)_2\,CHCl > CH_3CH_2Cl > CH_3Cl > (CH_3)_3\,CCl$

16. The major organic compound formed by the reaction of 1, 1, 1-trichloroethane with silver powder is: **[2014]**
(a) Acetylene (b) Ethene
(c) 2 - Butyne (d) 2 - Butene

17. The synthesis of alkyl fluorides is best accomplished by : **[2015]**
(a) Finkelstein reaction
(b) Swarts reaction
(c) Free radical fluorination
(d) Sandmeyer's reaction

18. 2-Chloro-2-methylpentane on reaction with sodium methoxide in methanol yields: **[2016]**

(i) 

(ii) $C_2H_5CH_2\underset{\displaystyle CH_3}{\underset{|}{C}}=CH_2$

(iii) $C_2H_5CH=\underset{\displaystyle CH_3}{\underset{|}{C}}-CH_3$

(a) (iii) only (b) (i) and (ii)
(c) All of these (d) (i) and (iii)

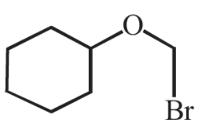
19. The increasing order of the reactivity of the following halides for the S_N1 reaction is **[2017]**

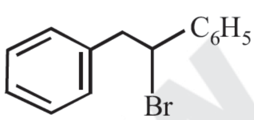
$CH_3CH(Cl)CH_2CH_3$ (I) $CH_3CH_2CH_2Cl$ (II)

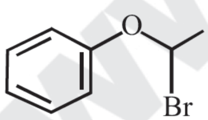
p-$H_3CO–C_6H_4–CH_2Cl$ (III)

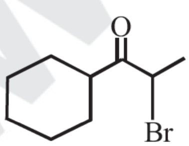
(a) (III) < (II) < (I) (b) (II) < (I) < (III)
(c) (I) < (III) < (II) (d) (II) < (III) < (I)

20. Which of the following , upon treatment with tert-BuONa followed by addition of bromine water, fails to decolourize the colour of bromine? **[2017]**

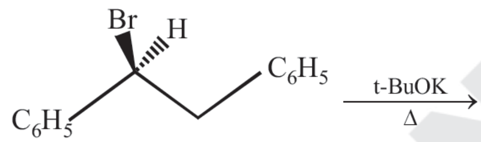
(a) 

(b) 

(c) 

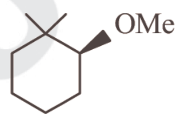
(d) 

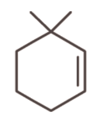
21. The major product obtained in the following reaction is : **[2017]**

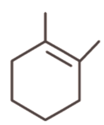
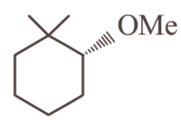


(a) $(\pm)C_6H_5CH(O^tBu)CH_2C_6H_5$
(b) $C_6H_5CH=CHC_6H_5$
(c) $(+)C_6H_5CH(O^tBu)CH_2C_6H_5$
(d) $(-)C_6H_5CH(O^tBu)CH_2C_6H_5$

22. The major product of the following reaction is : **[2018]**

(a) 

(b) 

(c) 
(d) 

23. Consider the following reactions: **[2020]**

(a) $(CH_3)_3CCH(OH)CH_3 \xrightarrow{\text{conc.}H_2SO_4}$

(b) $(CH_3)_2CHCH(Br)CH_3 \xrightarrow{\text{alc.KOH}}$

(c) $(CH_3)_2CHCH(Br)CH_3 \xrightarrow{(CH_3)_3O^-K^{\oplus}}$

(d) $(CH_3)_2\underset{\displaystyle H}{\underset{|}{C}}-CH_2-CHO \xrightarrow{\Delta}$

Which of these reaction(s) will not produce Saytzeff product?

(a) (a), (c) and (d) (b) (d) only
(c) (c) only (d) (b) and (d)

Answer Key

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
(d)	(b)	(a)	(b)	(c)	(d)	(d)	(a)	(b)	(c)	(d)	(a)	(c)	(d)	(b)
16	17	18	19	20	21	22	23							
(c)	(b)	(a)	(b)	(a)	(b)	(b)	(c)							

Solutions

1. **(d)** $\underset{(A)}{C_6H_5I} \xrightarrow{NaOH} C_6H_5ONa \xrightarrow{HNO_3/H^+}$

$C_6H_5OH \xrightarrow{AgNO_3}$ No yellow ppt.

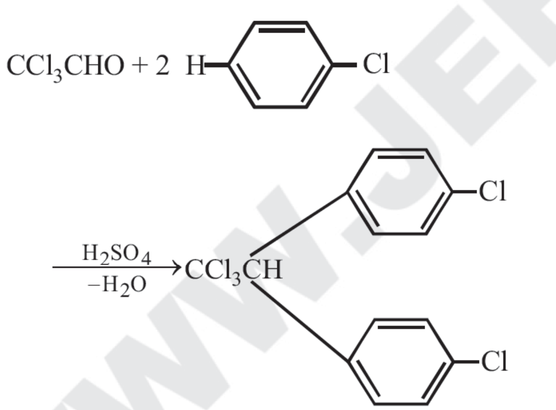
$\underset{(B)}{C_6H_5CH_2I} \xrightarrow{NaOH} C_6H_5CH_2ONa$

$\xrightarrow{HNO_3/H^+} C_6H_5CH_2OH$

$\xrightarrow{AgNO_3}$ yellow ppt.

Since benzyl iodide gives yellow ppt. hence this is compound B and A is phenyl iodide (C_6H_5I).

2. **(b)** DDT is prepared by heating chlorbenzene and chloral with concentrated sulphuric acid



1,1,1-Trichloro–2,2 bis (*p*-chlorophenyl) ethane or DDT

3. **(a)** Due to steric hindrance tertiary alkyl halides do not react by S_N2 mechanism, they react by S_N1 mechanism. S_N2 mechanism is followed in case of primary and secondary alkyl halides.

4. **(b)** In Corey House synthesis of alkanes alkyl halides react with lithium dialkyl cuprate

$$R'X + LiR_2Cu \longrightarrow R'-R + RCu + LiX$$

5. **(c)** $CH_3-\underset{}{\overset{Br}{\overset{|}{C}H}}-CH_2-CH_3 \xrightarrow{Alc.\ KOH}$

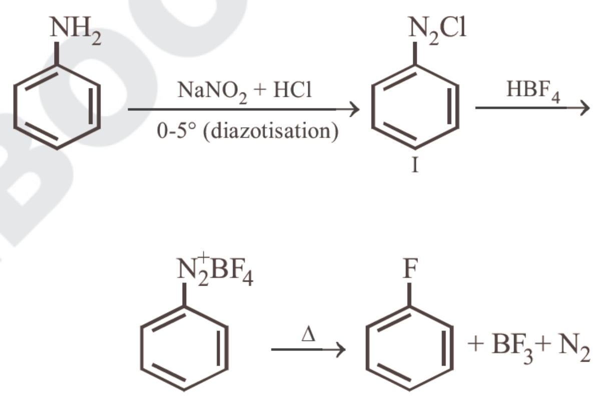
$CH_3-CH=CH-CH_3 + HBr$

The formation of 2-butene is in accordance to **Saytzeff's rule** (more substituted alkene is formed).

6. **(d)** $CH_3OH + C_6H_5MgBr \longrightarrow$

$CH_3OMgBr + C_6H_6$

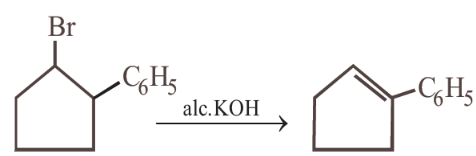
7. **(d)**



Benzene diazonium tetrafluoroborate

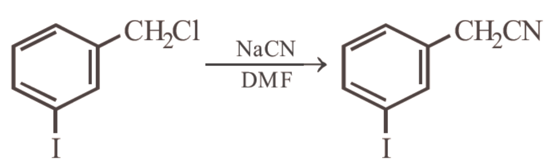
Conversion of I to II is known as Balz-schilmann reaction.

8. **(a)** The reaction is dehydrohalogenation



1-Phenyl cyclopentene

9. **(b)**



Nuclear substitution will not take place.

10. (c) In S_N2 mechanism transition state is pentavelent. For bulky alkyl group it will have sterical hinderance and smaller alkyl group will favour the S_N2 mechanism. So the decreasing order of reactivity of alkyl halide towards S_N2 mechanism is

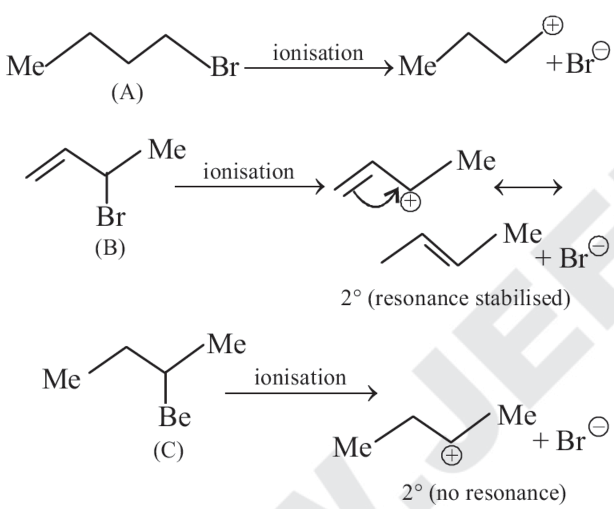
$$RCH_2X > R_2CHX > R_3CX$$

11. (d) S_N2 reaction is favoured by small groups on the carbon atom attached to halogen.

So, the order of reactivity is

$$CH_3Cl > (CH_3)_2CHCl > (CH_3)_3CCl > (C_2H_5)_2CHCl$$

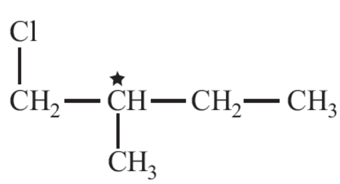
12. (a)



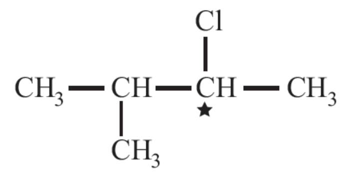
Since S_N1 reactions involve the formation of carbocation as intermediate in the rate determining step, **more is the stability of carbocation higher will be reactivity of alkyl halides towards S_N1 route.** Now we know that stability of carbocations follows the order : 3° > 2° > 1°, so S_N1 reactivity should also follow the same order.

3° > 2° > 1° > Methyl **(S_N1 reactivity)**

13. (c)



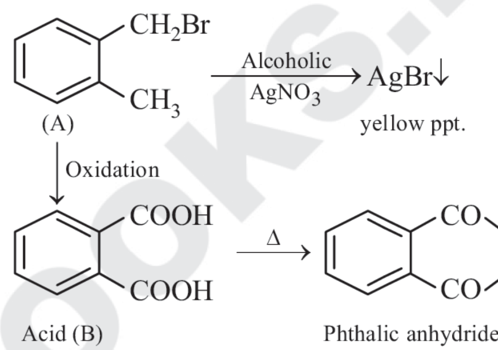
(R + S)



(R + S)

Four monochloro derivatives are chiral.

14. (d)



15. (b) Steric hindrance around the carbon atom having Cl will slow down the S_N2 reaction, hence lesser the hindrance, faster will be the reaction. So, the order of reactivity is

$CH_3Cl > (CH_3)CH_2-Cl > (CH_3)_2CH-Cl > (CH_3)_3CCl$

16. (c) $2Cl-\underset{Cl}{\overset{Cl}{\underset{|}{\overset{|}{C}}}}-CH_3 + 6Ag \longrightarrow$

1, 1, 1-trichloroethane

$$\underset{\text{2-butyne}}{CH_3C\equiv CCH_3} + 6AgCl$$

17. (b) Alkyl fluorides are more conveniently prepared by heating suitable chloro – or bromo-alkanes with organic fluorides such as AsF_3, SbF_3, CoF_2, AgF, Hg_2F_2 etc. This reaction is called **Swarts reaction**.

$$CH_3Br + AgF \longrightarrow CH_3F + AgBr$$

$$2CH_3CH_2Cl + Hg_2F_2 \longrightarrow 2CH_3CH_2F + Hg_2Cl_2$$

18. (a) When *tert*-alkyl halides are used in Williamson synthesis elimination occurs rather than substitution resulting into formation of alkene. Here alkoxide ion abstracts one of the β-hydrogen atoms along with it acting as a nucleophile.

$$CH_3-CH_2-CH_2-\underset{\underset{Cl}{|}}{\overset{\overset{CH_3}{|}}{C}}-CH_3+CH_3\overset{-}{O}\overset{+}{Na}\xrightarrow{CH_3OH}$$

2-Chloro-2-methylpentane

$$\underset{\text{2-Methyl-pent-2-ene}}{CH_3CH_2-\overset{\overset{H}{|}}{C}=\overset{\overset{CH_3}{|}}{C}-CH_3}+CH_3OH+NaBr$$

19. (b) Since S_N1 reactions involve the formation of carbocation as intermediate in the rate determining step, more the stability of carbocation higher will be the reactivity of alkyl halides towards S_N1 route.

Since stability of carbocations follows order.

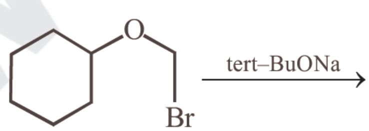
$$\underset{1°\ \text{carbocation}}{CH_3-CH_2-\overset{\oplus}{C}H_2}$$

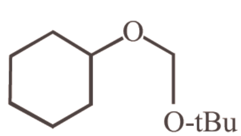
$$<\underset{2°\ \text{carbocation}}{CH_3-\overset{\oplus}{C}H-CH_2-CH_3}$$

$$<p-H_3CO-C_6H_4-\overset{\oplus}{C}H_2$$

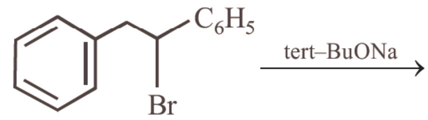
Max. stable due to +M effect of $-OCH_3$ group.

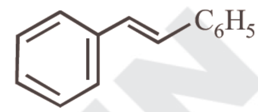
20. (a)

(a) 

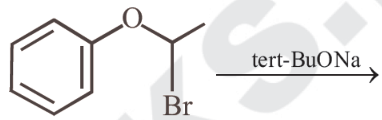


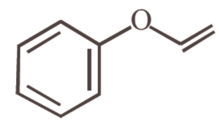
(fails to decolorise the colour of bromine due to absence of unsaturation)

(b) 

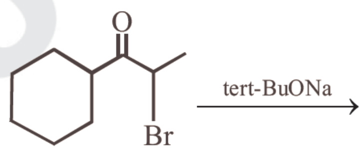


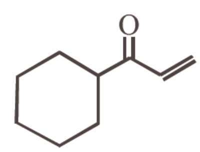
(it decolorises bromine solution)

(c) 



(it decolorises bromine solution)

(d) 



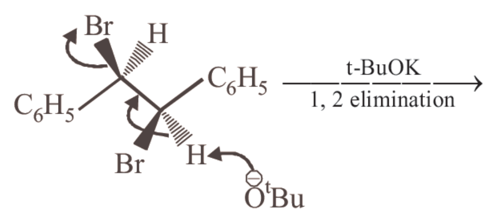
(it decolorises bromine solution)

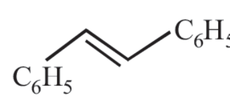
21. (b) Elimination reaction is highly favoured if

(a) Bulkier base is used

(b) Higher temperature is used

Hence in given reaction biomolecular elimination reaction provides major product.



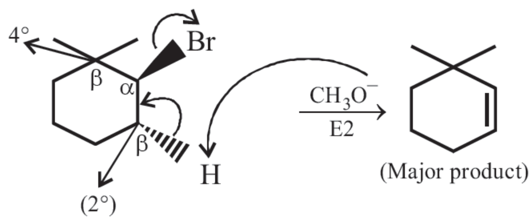


$+ t\text{-BuOH} + Br^-$

22. **(b)** CH_3O^- is a strong base and strong nucleophile, so favourable condition is $S_N2/E2$.

The given alkyl halide is 2° and β carbons are 4° and 2°, so sufficiently hindered, thus E2 dominates over S_N2.

Also, polarity of CH_3OH (solvent) is not as high as H_2O, so E1 is also dominated by E2.



23. **(c)**

(a) $(CH_3)_2CCH(OH)CH_3 \xrightarrow{\text{conc.}H_2SO_4}$

$$CH_3-\overset{\overset{CH_3}{|}}{C}=\underset{\underset{CH_3}{|}}{C}-CH_3$$

(b) $(CH_3)_2CHCH(Br)CH_3 \xrightarrow{\text{alc.KOH}}$

$$CH_3-\overset{\overset{CH_3}{|}}{C}=CH-CH_3$$

(c) $(CH_3)_2CHCH(Br)CH_3 \xrightarrow{t\text{-}BuO^-K^+}$

$$CH_3-\overset{\overset{CH_3}{|}}{CH}-CH=CH_2$$

Due to bulky nature of tertiary butoxide, the least hindered hydrogen is eliminated. Therefore, Hoffman product is formed.

(d) $(CH_3)_2\underset{\underset{OH}{|}}{C}-CH_2-CHO \xrightarrow{\Delta}$

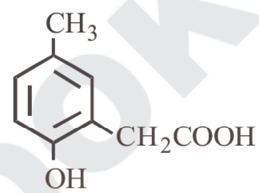
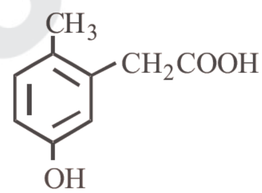
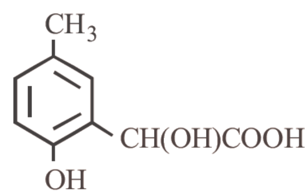
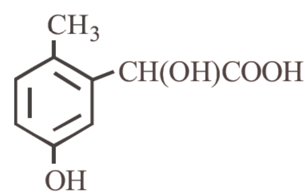
$$CH_3-\overset{\overset{CH_3}{|}}{C}=CH-CHO$$

25 Alcohols, Phenols and Ethers

1. During dehydration of alcohols to alkenes by heating with conc. H_2SO_4, the initiation step is **[2003]**
   (a) formation of carbocation
   (b) elimination of water
   (c) formation of an ester
   (d) protonation of alcohol molecule

2. Among the following compounds which can be dehydrated very easily ? **[2004]**
   (a) $CH_3CH_2-\overset{\overset{CH_3}{|}}{\underset{\underset{OH}{|}}{C}}-CH_2CH_3$
   (b) $CH_3CH_2CH_2\overset{\overset{OH}{|}}{C}HCH_3$
   (c) $CH_3CH_2CH_2CH_2CH_2OH$
   (d) $CH_3CH_2\underset{\underset{CH_3}{|}}{C}HCH_2CH_2OH$

3. The best reagent to convert pent-3-en-2-ol into pent-3-en-2-one is **[2005]**
   (a) Pyridinium chlorochromate
   (b) Chromic anhydride in glacial acetic acid
   (c) acidic dichromate
   (d) Acidic permanganate

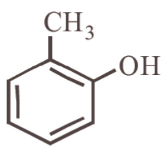
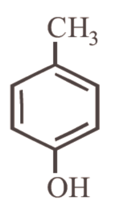
4. *p*-Cresol reacts with chloroform in alkaline medium to give the compound A which adds hydrogen cyanide to form, the compound B. The latter on acidic hydrolysis gives chiral carboxylic acid. The structure of the carboxylic acid is **[2005]**
   (a) 
   (b) 
   (c) 
   (d) 

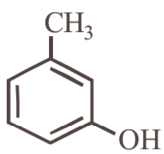
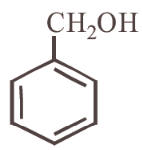
5. HBr reacts with $CH_2 = CH - OCH_3$ under anhydrous conditions at room temperature to give **[2006]**
   (a) $BrCH_2 - CH_2 - OCH_3$
   (b) $H_3C - CHBr - OCH_3$
   (c) CH_3CHO and CH_3Br
   (d) $BrCH_2CHO$ and CH_3OH

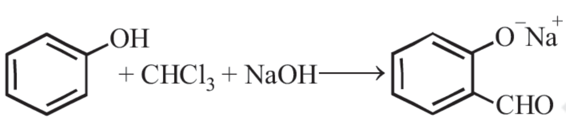
6. Among the following the one that gives positive iodoform test upon reaction with I_2 and NaOH is **[2006]**
   (a) $CH_3-\overset{\overset{CH_3}{|}}{C}HCH_2OH$

(b) $PhCHOHCH_3$
(c) $CH_3CH_2CH(OH)CH_2CH_3$
(d) $C_6H_5CH_2CH_2OH$

7. The structure of the compound that gives a tribromo derivative on treatment with bromine water is **[2006]**

(a)  (b) 

(c)  (d) 

8. 

The electrophile involved in the above reaction is **[2006]**

(a) trichloromethyl anion ($\overset{\ominus}{C}Cl_3$)
(b) formyl cation ($\overset{\oplus}{C}HO$)
(c) dichloromethyl cation ($\overset{\oplus}{C}HCl_2$)
(d) dichlorocarbene ($:CCl_2$)

9. In the following sequence of reactions,

$$CH_3CH_2OH \xrightarrow{P+I_2} A \xrightarrow[\text{ether}]{Mg} B \xrightarrow{HCHO} C \xrightarrow{H_2O} D$$

the compound D is **[2007]**

(a) propanal (b) butanal
(c) *n*-butyl alcohol (d) *n*-propyl alcohol

10. Phenol, when it reacts first with concentrated sulphuric acid and then with concentrated nitric acid, gives **[2008]**

(a) 2, 4, 6-trinitrobenzene
(b) *o*-nitrophenol
(c) *p*-nitrophenol
(d) nitrobenzene

11. The major product obtained on interaction of phenol with sodium hydroxide and carbon dioxide is **[2009]**

(a) salicylaldehyde (b) salicylic acid
(c) phthalic acid (d) benzoic acid

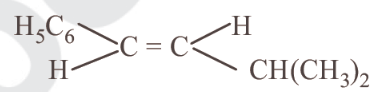
12. From amongst the following alcohols, the one that would react fastest with conc. HCl and anhydrous $ZnCl_2$, is **[2010]**

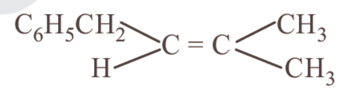
(a) 2-Butanol
(b) 2- Methylpropan-2-ol
(c) 2-Methylpropanol
(d) 1- Butanol

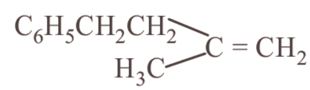
13. The main product of the following reaction is

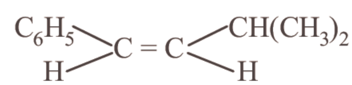
$$C_6H_5CH_2CH(OH)CH(CH_3)_2 \xrightarrow{conc.H_2SO_4} ?$$

[2010]

(a) 

(b) 

(c) 

(d) 

14. Consider thiol anion ($RS^{\ominus}$) and alkoxy anion ($RO^{\ominus}$). Which of the following statements is correct ? **[2011RS]**

(a) $RS^{\ominus}$ is less basic but more nucleophilic than $RO^{\ominus}$
(b) $RS^{\ominus}$ is more basic and more nucleophilic than $RO^{\ominus}$
(c) $RS^{\ominus}$ is more basic but less nucleophilic than $RO^{\ominus}$
(d) $RS^{\ominus}$ is less basic and less nucleophilic than $RO^{\ominus}$

15. The correct order of acid strength of the following compounds : **[2011RS]**

(A) Phenol (B) *p*–Cresol
(C) *m*–Nitrophenol (D) *p*–Nitrophenol
(a) D > C > A > B (b) B > D > A > C
(c) A > B > D > C (d) C > B > A > D

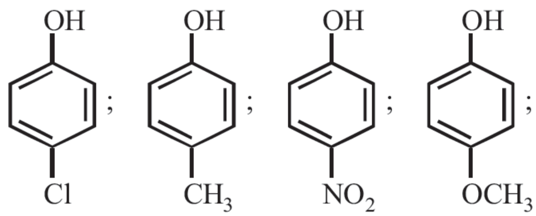
16. Consider the following reaction :

$C_2H_5OH + H_2SO_4 \longrightarrow$ Product

Among the following, which one cannot be formed as a product under any conditions ? **[2011RS]**

(a) Ethylene (b) Acetylene
(c) Diethyl ether (d) Ethylhydrogen sulphate

17. Arrange the following compounds in order of decreasing acidity : **[2013]**



(I) (II) (III) (IV)

(a) II > IV > I > III (b) I > II > III > IV
(c) III > I > II > IV (d) IV > III > I > II

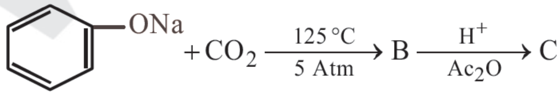
18. An unknown alochol is treated with the "Lucas reagent" to determine whether the alcohol is primary, secondary or tertiary. Which alcohol reacts fastest and by what mechanism : **[2013]**

(a) secondary alcohol by S_N1
(b) tertiary alcohol by S_N1
(c) secondary alcohol by S_N2
(d) tertiary alcohol by S_N2

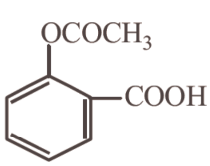
19. The most suitable reagent for the conversion of $R-CH_2-OH \rightarrow R-CHO$ is: **[2014]**

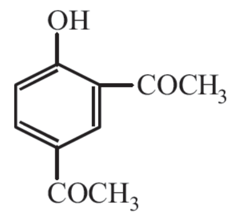
(a) $KMnO_4$
(b) $K_2Cr_2O_7$
(c) CrO_3
(d) PCC (Pyridinium chlorochromate)

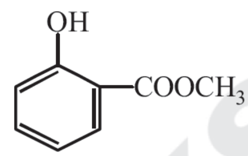
20. Sodium phenoxide when heated with CO_2 under pressure at 125 ºC yields a product which on acetylation produces C **[2014]**

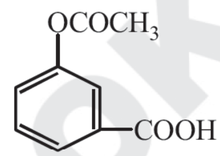


The major product C would be

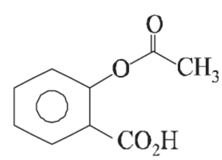
(a) 

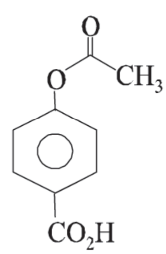
(b) 

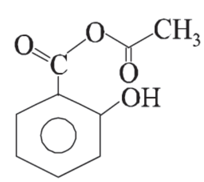
(c) 

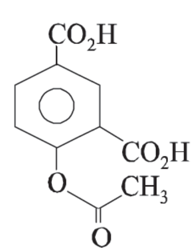
(d) 

21. Phenol on treatment with CO_2 in the presence of NaOH followed by acidification produces compound X as the major product. X on treatment with $(CH_3CO)_2O$ in the presence of catalytic amount of H_2SO_4 produces : **[2018]**

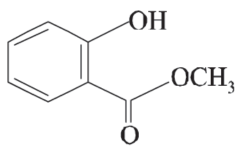
(a) 

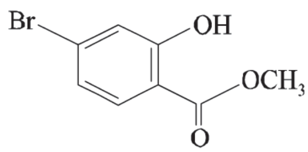
(b) 

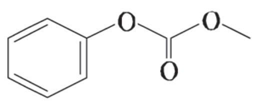
(c) 

(d) 

22. Phenol reacts with methyl chloroformate in the presence of NaOH to form product A. A reacts with Br_2 to form product B. A and B are respectively : **[2018]**

(a) 

and


(b) 

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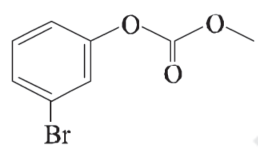
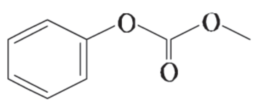


(c) 

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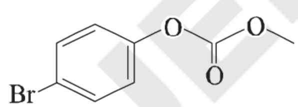
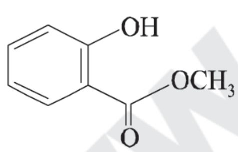


(d) 

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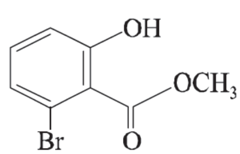
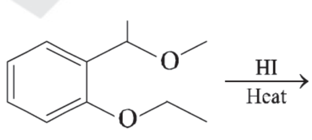
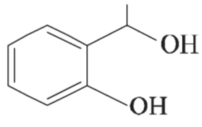
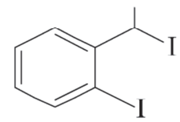
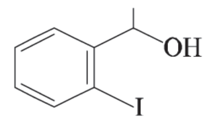


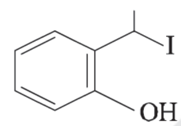
23. The major product formed in the following reaction is : **[2018]**



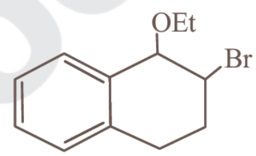
(a) 

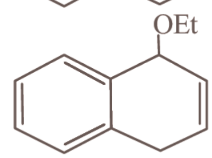
(b) 

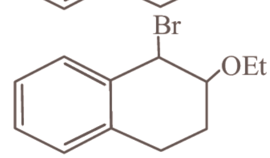
(c) 

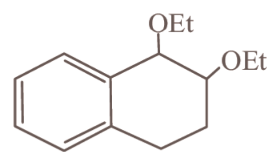
(d) 

24. The major product of the following reaction is: **[2019]**

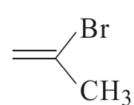
(a) 

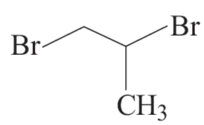
(b) 

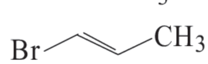
(c) 

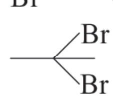
(d) 

25. 1-methyl ethylene oxide when treated with an excess of HBr produces: **[2020]**

(a) 

(b) 

(c) 

(d) 

Answer Key														
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
(d)	(a)	(a)	(c)	(b)	(b)	(c)	(d)	(d)	(b)	(b)	(b)	(a)	(a)	(a)
16	17	18	19	20	21	22	23	24	25					
(b)	(c)	(b)	(d)	(a)	(a)	(c)	(d)	(a)	(b)					

Solutions

1. **(d)** The dehydration of alcohol to form alkene occurs in following three steps. Step (1) is initiation step.

Step (1) Formation of protonated alcohol.

$$CH_3-CH_2-\ddot{\underset{\cdot\cdot}{O}}-H + H^+ \rightleftharpoons CH_3CH_2-\overset{+}{\underset{\cdot\cdot}{O}}\begin{matrix}H\\H\end{matrix}$$

(Protonated ethanol)

Step (2) Formation of carbocation

$$CH_3-CH_2-\overset{+}{\underset{\cdot\cdot}{O}}\begin{matrix}H\\H\end{matrix} \xrightarrow{\text{Slow}} CH_3-\overset{+}{C}H_2 + H_2O$$

Ethyl carbocation

Step (3) Elimination of a proton to form alkene

$$H-CH_2-\overset{+}{C}H_2 \underset{}{\overset{\text{Fast}}{\rightleftharpoons}} CH_2=CH_2 + H^+$$

Ethene

2. **(a)** 3-Methyl pentan-3-ol will be dehydrated most readily since it produces a very stable, tertiary, carbonium ion as intermediate.

$$CH_3-CH_2-\underset{OH}{\overset{CH_3}{C}}-CH_2-CH_3$$

$$\xrightarrow{H^+} CH_3-CH_2-\underset{\oplus}{\overset{CH_3}{C}}-CH_2-CH_3$$

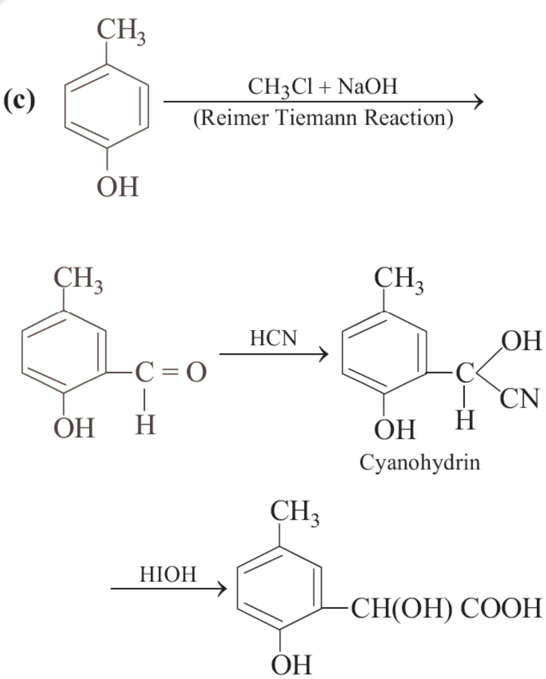
ter-Carbocation

3. **(a)** $CH_3-\overset{OH}{CH}-CH=CH-CH_3 \longrightarrow$

$$CH_3-\overset{O}{\overset{\|}{C}}-CH=CH-CH_3$$

Pyridinium chlorochromate (PCC) oxidises 1° and 2° alcholos to aldehydes and ketones.

4. **(c)**

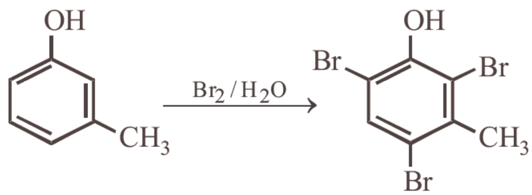


5. **(b)** Methyl vinyl ether under anhydrous condition at room temperature undergoes addition reaction.

$$CH_2=CH-OCH_3 \xrightarrow{HBr} CH_3-\underset{Br}{CH}-O-CH_3$$

6. **(b)** Ethanol and only those 2° alcohols which contain – $CHOHCH_3$ group undergo haloform reaction.

7. **(c)** In (c) both groups are activativating and undergo electrophilic substitution in the same positions.



8. **(d)** This is Riemer-Tiemann reaction and the electrophile is dichlorocarbene.

$$H-\underset{Cl}{\overset{Cl}{C}}-Cl + NaOH \xrightarrow[(\alpha\text{-elimination})]{} Cl-\ddot{C}-Cl + NaCl + H_2O$$

dichlorocarbene

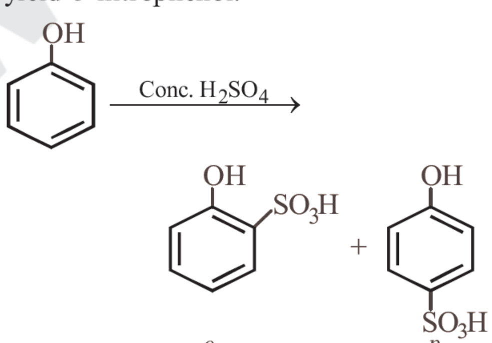
9. **(d)** $CH_3CH_2OH \xrightarrow{P+I_2} \underset{(A)}{CH_3CH_2I}$

$$\xrightarrow[Ether]{Mg} \underset{(B)}{CH_3CH_2MgI} \xrightarrow{HCHO}$$

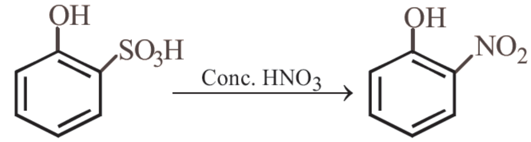
$$\underset{(C)}{H-\underset{H}{\overset{CH_2CH_3}{C}}-OMgI} \xrightarrow{H_2O} \underset{(D)}{H-\underset{H}{\overset{CH_2CH_3}{C}}-OH}$$

n-Propylalcohol

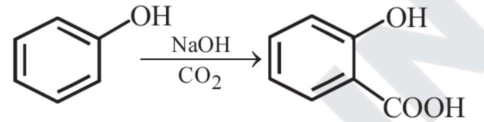
10. **(b)** Phenol on reaction with conc. H_2SO_4 gives a mixture of *o*- and *p*- products (i.e., $-SO_3H$ group occupies *o*-, *p*- position). At room temperature, *o*-product is more stable, which on treatment with conc. HNO_3 will yield *o*-nitrophenol.



o- *p*-



o-nitrophenol

11. **(b)** 

12. **(b)** Tertiary alcohols react fastest with conc. HCl and anhydrous $ZnCl_2$ (Lucas reagent) as its mechanism proceeds through the formation of stable tertiary carbocation.

Mechanism :

Step 1 : $CH_3-\underset{CH_3}{\overset{CH_3}{C}}-OH \xrightarrow{HCl}$

2 Methylpropane-2-ol

$$\rightleftharpoons (CH_3)_3C-\overset{+}{O}H_2 + Cl^-$$

Step 2 :

$$(CH_3)_3C-\overset{+}{O}H_2 \rightleftharpoons \underset{3°\ \text{Carbocation}}{(CH_3)_3C^+} + H_2O$$

Step 3 :

$$(CH_3)_3C^+ + Cl^- \rightleftharpoons \underset{t-\text{Butylchloride}}{(CH_3)_3C-Cl}$$

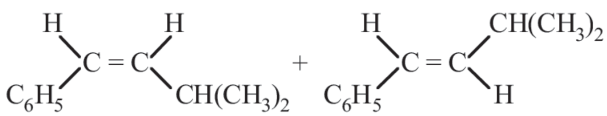
13. **(a)** Whenever dehydration can produce two different alkenes, major product is formed according to **Saytzeff rule** *i.e.* more substituted alkene (alkene having lesser number of hydrogen atoms on the two doubly bonded carbon atoms) is the major product.

Such reactions which can produce two or more structural isomers but one of them in greater amounts than the other are called **regioselective** *;* in case a reaction is 100% regioselective, it is termed as **regiospecific.**

In addition to being regioselective, alcohol dehydrations are **stereoselective** (*a reaction in which a single starting material can yield two or more*

stereoisomeric products, but gives one of them in greater amount than any other).

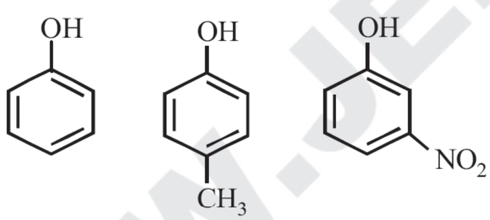
$$C_6H_5-CH_2-\underset{|}{\underset{OH}{CH}}-\underset{|}{\underset{CH_3}{CH}}-CH_3 \xrightarrow{\text{Conc. } H_2SO_4}$$



cis (minor) + *trans* (major)

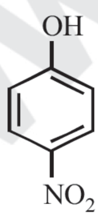
14. (a) On moving down a group, the basicity & nucleophilicity are inversely related, i.e. nucleophilicity increases while basicity decreases. i.e $RS^{\ominus}$ is more nucleophilic but less basic than $RO^{\ominus}$. This opposite behaviour is because of the fact that basicity and nucleophilicity depends upon different factors. Basicity is directly related to the strength of the H–element bond, while nucleophilicity is indirectly related to the electronegativity of the atom to which proton is attached.

15. (a)



+I effect (B); – I effect (C)

(A) (B) (C)



– M, – I effect

(D)

Electron withdrawing substituents increase the acidity of phenols; while electron releasing substituents decrease acidity. Thus the correct order is

$$D > C > A > B$$

16. (b)

$C_2H_5-OH + H_2SO_4$

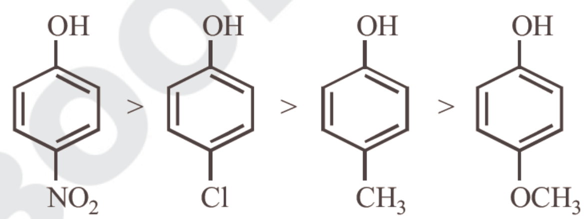
$\xrightarrow{433\,K} CH_2=CH_2$ (ethylene)

$\xrightarrow{413\,K} CH_3-CH_2-O-CH_2-CH_3$ (diethyl ether)

$\xrightarrow{383\,K} CH_3CH_2HSO_4 + H_2O$ (ethyl hydrogen sulphate)

Acetylene is not formed under any condition.

17. (c) Electron withdrawing substituents like $-NO_2$, –Cl increase the acidity of phenol while electron releasing substituents like $-CH_3$, $-OCH_3$ decrease acidity. hence the correct order of acidity will be



III (–M, –I) > I (–I > +M) > II (+I, +HC) > IV (+M)

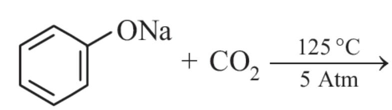
18. (b) Reaction of alcohols with Lucas reagent proceeds through carbocation formation.

Further 3° carbocations (from tertiary alcohols) are highly stable thus reaction proceeds through S_N1 mechanism.

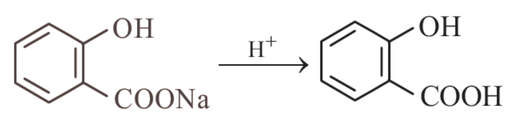
19. (d) An excellent reagent for oxidation of 1° alcohols to aldehydes is PCC (consult Q.3 also).

$$R-CH_2-OH \xrightarrow{PCC} R-CHO$$

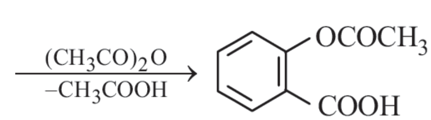
20. (a)



Sodium Phenoxide



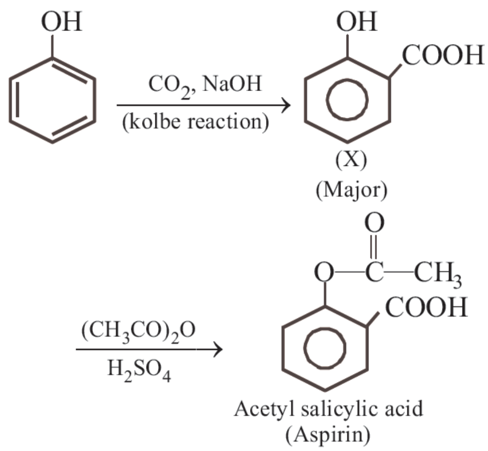
Salicyelic acid



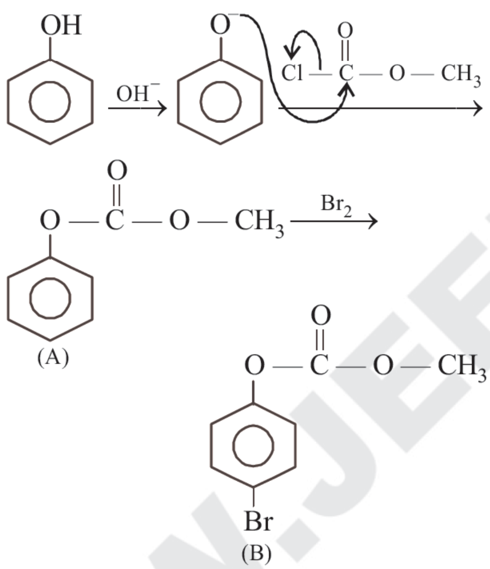
Aspirin

(Acetyl salicyelic acid)

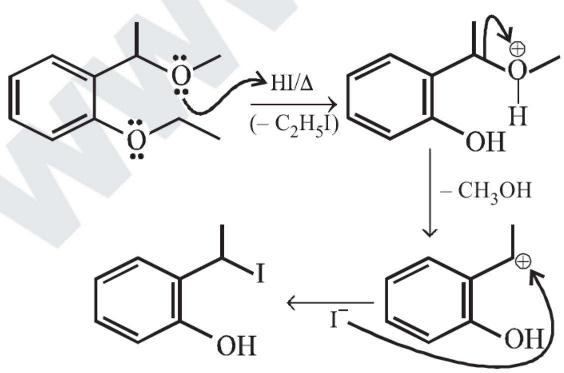
21. (a)



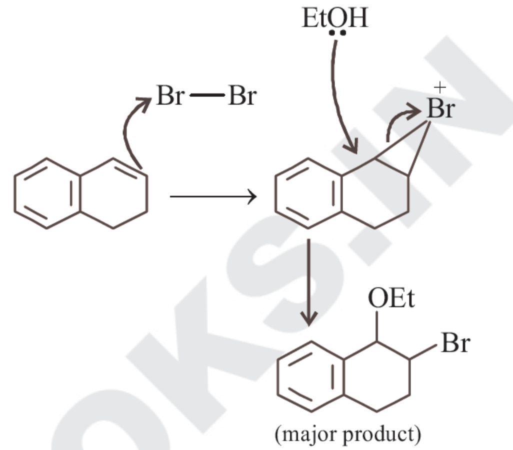
22. (c)



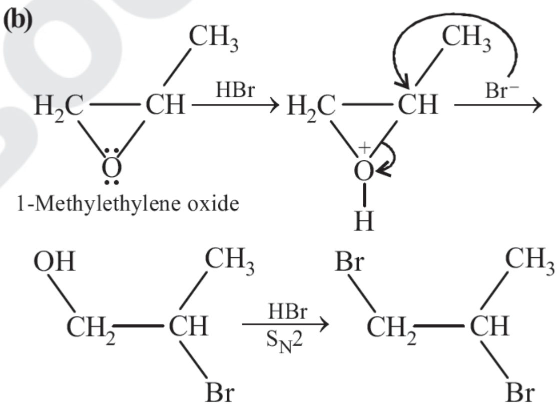
23. (d)



24. (a) Mechanism involved for the given reaction is:



25. (b)



26 Aldehydes, Ketones and Carboxylic Acids

1. $CH_3CH_2COOH \xrightarrow[\text{red P}]{Cl_2} A \xrightarrow{\text{alc. KOH}} B.$

What is B? **[2002]**

(a) CH_3CH_2COCl
(b) CH_3CH_2CHO
(c) $CH_2=CHCOOH$
(d) $ClCH_2CH_2COOH.$

2. On vigorous oxidation by permanganate solution.
$(CH_3)_2C=CH-CH_2-CHO$ gives **[2002]**

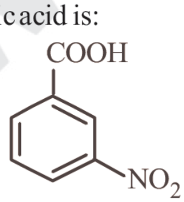
(a) $CH_3-\underset{\substack{|\\CH_3}}{\overset{\substack{OH\\|}}{C}}-\overset{\substack{OH\\|}}{CH}-CH_2CHO$

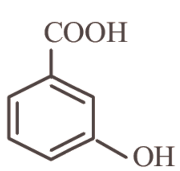
(b) $(CH_3)_2C=O+HOOCCH_2COOH$

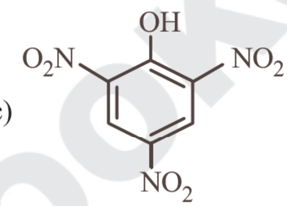
(c) $\begin{matrix}CH_3\diagdown\\ \quad C=O\\ CH_3\diagup\end{matrix} + OHCCH_2COOH$

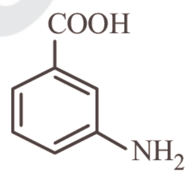
(d) $\begin{matrix}CH_3\diagdown\\ \quad C=O\\ CH_3\diagup\end{matrix} + OHCCH_2CHO$

3. Picric acid is: **[2002]**

(a) 

(b) 

(c) 

(d) 

4. When $CH_2=CH—COOH$ is reduced with $LiAlH_4$, the compound obtained will be **[2003]**

(a) $CH_2=CH—CH_2OH$
(b) $CH_3—CH_2—CH_2OH$
(c) $CH_3—CH_2—CHO$
(d) $CH_3—CH_2—COOH$

5. On mixing ethyl acetate with aqueous sodium chloride, the composition of the resultant solution is **[2004]**

(a) $CH_3COCl+C_2H_5OH+NaOH$
(b) $CH_3COONa+C_2H_5OH$
(c) $CH_3COOC_2H_5+NaCl$
(d) $CH_3Cl+C_2H_5COONa$

6. Acetyl bromide reacts with excess of CH_3MgI followed by treatment with a saturated solution of NH_4Cl gives **[2004]**

(a) 2-methyl-2-propanol
(b) acetamide
(c) acetone
(d) acetyl iodide

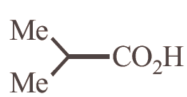
7. Which one of the following is reduced with zinc analgum and hydrochloric acid to give the corresponding hydrocarbon? **[2004]**
(a) Acetamide (b) Acetic acid
(c) Ethyl acetate (d) Butan-2-one

8. Which one of the following undergoes reaction with 50% sodium hydroxide solution to give the corresponding alcohol and acid? **[2004]**
(a) Butanal (b) Benzaldehyde
(c) Phenol (d) Benzoic acid

9. Among the following acids which has the lowest pK_a value? **[2005]**
(a) CH_3CH_2COOH
(b) $(CH_3)_2CH-COOH$
(c) HCOOH
(d) CH_3COOH

10. Reaction of cyclohexanone with dimethylamine in the presence of catalytic amount of an acid forms a compound if water during the reaction is continuously removed. The compound formed is generally known as **[2005]**
(a) an amine (b) an imine
(c) an enamine (d) a Schiff's base

11. The increasing order of the rate of HCN addition to compound A to D is **[2006]**
(A) HCHO (B) CH_3COCH_3
(C) $PhCOCH_3$ (D) PhCOPh
(a) D < C < B < A (b) C < D < B < A
(c) A < B < C < D (d) D < B < C < A

12. The correct order of increasing acid strength of the compounds **[2006]**
(A) CH_3CO_2H (B) $MeOCH_2CO_2H$
(C) CF_3CO_2H (D) 

is
(a) D < A < B < C (b) A < D < B < C
(c) B < D < A < C (d) D < A < C < B

13. A liquid was mixed with ethanol and a drop of concentrated H_2SO_4 was added. A compound with a fruity smell was formed. The liquid was : **[2009]**
(a) HCHO (b) CH_3COCH_3
(c) CH_3COOH (d) CH_3OH

14. Which of the following on heating with aqueous KOH, produces acetaldehyde? **[2009]**
(a) CH_3CH_2Cl (b) CH_2ClCH_2Cl
(c) CH_3CHCl_2 (d) CH_3COCl

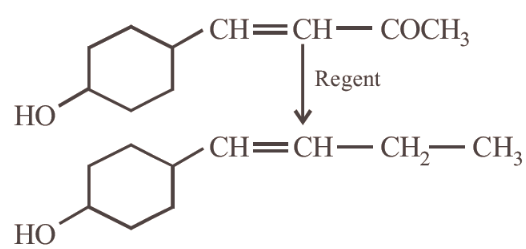
15. In Cannizzaro reaction given below

$$2PhCHO \xrightarrow{:OH^-} PhCH_2OH + PhCO_2^-$$

the slowest step is : **[2009]**
(a) the transfer of hydride to the carbonyl group
(b) the abstraction of proton from the carboxylic group
(c) the deprotonation of Ph CH_2OH
(d) the attack of : OH^- at the carboxyl group

16. Iodoform can be prepared from all except: **[2012]**
(a) Ethyl methyl ketone
(b) Isopropyl alcohol
(c) 3-Methyl 2-butanone
(d) Isobutyl alcohol

17. In the given transformation, which of the following is the most appropriate reagent ? **[2012]**



(a) $NH_2NH_2, \bar{O}H$ (b) $Zn-Hg/HCl$
(c) Na, Liq NH_3 (d) $NaBH_4$

18. An organic compound A upon reacting with NH_3 gives B. On heating B gives C. C in presence of KOH reacts with Br_2 to give $CH_3CH_2NH_2$. A is : **[2013]**
(a) CH_3COOH
(b) $CH_3CH_2CH_2COOH$
(c) $CH_3-\underset{\displaystyle CH_3}{\underset{|}{CH}}-COOH$
(d) CH_3CH_2COOH

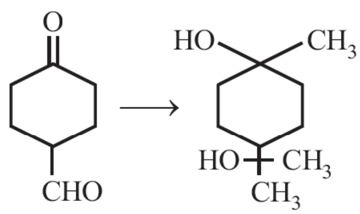
19. In the reaction,

$CH_3COOH \xrightarrow{LiAlH_4} A \xrightarrow{PCl_5} B \xrightarrow{Alc.KOH} C$,

the product C is: **[2014]**

(a) Acetaldehyde
(b) Acetylene
(c) Ethylene
(d) Acetyl chloride

20. The correct sequence of reagents for the following conversion will be : **[2017]**

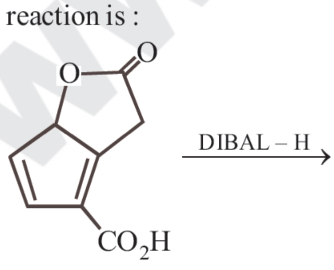


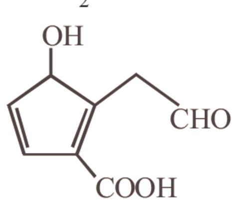
(a) $[Ag(NH_3)_2]^+ OH^-$, H^+/CH_3OH, CH_3MgBr
(b) CH_3MgBr, H^+/CH_3OH, $[Ag(NH_3)_2]^+ OH^-$
(c) CH_3MgBr, $[Ag(NH_3)_2]^+ OH^-$, H^+/CH_3OH
(d) $[Ag(NH_3)_2]^+ OH^-$, CH_3MgBr, H^+/CH_3OH

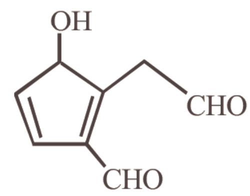
21. Sodium salt of an organic acid 'X' produces effervescence with conc. H_2SO_4. 'X' reacts with the acidified aqueous $CaCl_2$ solution to give a white precipitate which decolourises acidic solution of $KMnO_4$. 'X' is : **[2017]**

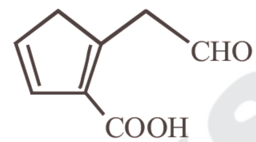
(a) C_6H_5COONa (b) $HCOONa$
(c) CH_3COONa (d) $Na_2C_2O_4$

22. The major product obtained in the following reaction is : **[2017]**



(a)


(b)


(c)


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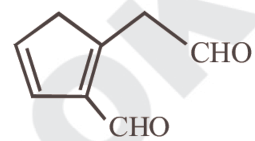


23. The correct decreasing order for acid strength is: **[2019]**

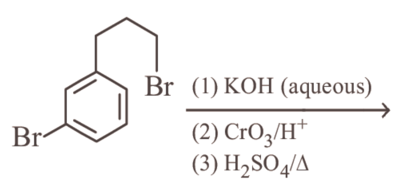
(a) $NO_2CH_2COOH > FCH_2COOH > CNCH_2COOH > ClCH_2COOH$
(b) $FCH_2COOH > CNCH_2COOH > NO_2CH_2COOH > ClCH_2COOH$
(c) $CNCH_2COOH > NO_2CH_2COOH > FCH_2COOH > ClCH_2COOH$
(d) $NO_2CH_2COOH > CNCH_2COOH > FCH_2COOH > ClCH_2COOH$

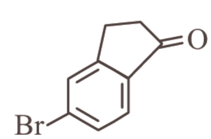
24. The major product of following reaction is:

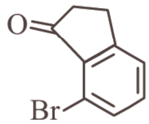
$$R - C \equiv N \xrightarrow[\text{(ii) } H_2O]{\text{(i) AlH (i–Bu)}_2}$$ **[2019]**

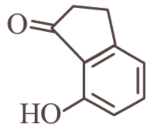
(a) RCOOH (b) $RCONH_2$
(c) RCHO (d) RCH_2NH_2

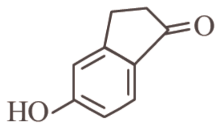
25. The major product of the following reaction is: **[2019]**



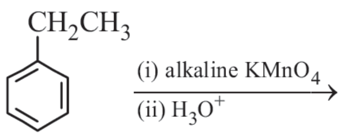
(a)


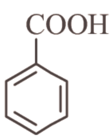
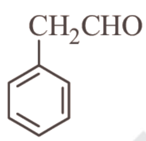
(b) 

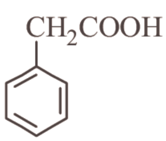
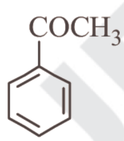
(c) 

(d) 

26. The major product of the following reaction is: **[2019]**



(a)  (b) 

(c)  (d) 

27. The major product of the following reaction is:

$CH_3CH=CHCO_2CH_3 \xrightarrow{LiAlH_4}$ **[2019]**

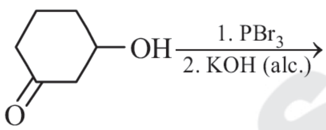
(a) $CH_3CH_2CH_2CO_2CH_3$

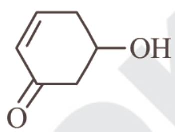
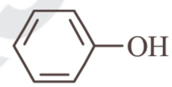
(b) $CH_3CH=CHCH_2OH$

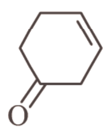
(c) $CH_3CH_2CH_2CH_2OH$

(d) $CH_3CH_2CH_2CHO$

28. The major product of the following reaction is: **[2019]**



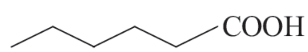
(a)  (b) 

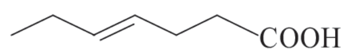
(c)  (d) 

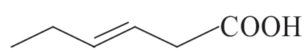
29. What is the product of following reaction?

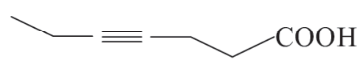
$$\text{Hex-3-ynal} \xrightarrow[\text{(ii) } PBr_3 \text{ (iii) Mg/ether (iv) } CO_2/H_3O^+]{\text{(i) } NaBH_4} ?$$

[2020]

(a) 

(b) 

(c) 

(d) 

Answer Key

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
(c)	(b)	(c)	(a)	(c)	(a)	(d)	(b)	(c)	(c)	(a)	(a)	(c)	(c)	(a)
16	17	18	19	20	21	22	23	24	25	26	27	28	29	
(d)	(a)	(d)	(c)	(a)	(d)	(b)	(d)	(c)	(a)	(a)	(b)	(c)	(d)	

Solutions

1. (c) $CH_3CH_2COOH \xrightarrow[\text{red P}]{Cl_2} CH_3CHClCOOH \xrightarrow[-HCl]{\text{alc.KOH}} \underset{\text{Acrylic acid}}{CH_2=CHCOOH}$

2. (b) Double bond is leaved and oxidised to –COOH, –CHO is also oxidised to –COOH

$(CH_3)_2C=CHCH_2CHO \xrightarrow{KMnO_4} (CH_3)_2CO + HOOCCH_2COOH$

3. (c) 2,4,6-Trinitrophenol is also known as picric acid.

4. (a) $LiAlH_4$ can reduce COOH group but not the double bond.

$CH_2=CH-COOH \xrightarrow{LiAlH_4} CH_2=CH-CH_2OH$

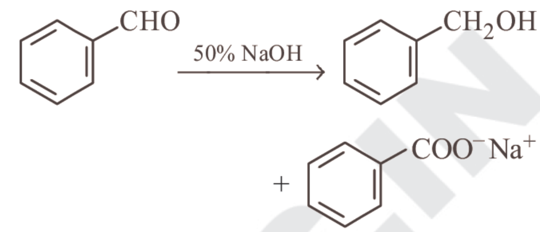
5. (c) There is no reaction hence the resultant mixture contains $CH_3COOC_2H_5 + NaCl$.

6. (a) $CH_3-\overset{\overset{\displaystyle O}{\|}}{C}-Br \xrightarrow[\text{(ii) Saturated } NH_4Cl]{\text{(i) } 2CH_3MgI} \underset{\text{2-Methyl-2-propanol}}{CH_3-\overset{\overset{\displaystyle CH_3}{|}}{\underset{\underset{\displaystyle CH_3}{|}}{C}}-OH}$

7. (d) It is Clemmensen's reduction

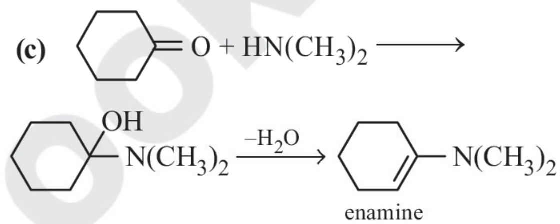
$\underset{\text{Butane–2–one}}{CH_3-\overset{\overset{\displaystyle O}{\|}}{C}-CH_2-CH_3} \xrightarrow[\text{Conc.HCl}]{Zn-Hg} \underset{\text{Butane}}{CH_3CH_2-CH_2CH_3}$

8. (b) This reaction is known as cannizzaro's reaction. In this reaction benzaldehyde in presence of 50%. NaOH undergoes disproportionation reaction and form one mol of benzyl alcohol (reduced product) and one mole of sod. benzoate (oxidation product)



9. (c) $pK_a = -\log K_a$; HCOOH is the strongest acid and hence it has the highest K_a or lowest pK_a value.

10. (c)



11. (a) **NOTE** Addition of HCN to carbonyl compounds is nucleophilic addition reaction. The order of reactivity of carbonyl compounds is

Aldehydes (smaller to higher) > Ketones (smaller to higher). Therefore,

$HCHO > CH_3COCH_3 > PhCOCH_3 > PhCOPh$

NOTE The lower reactivity of ketones is due to presence of two alkyl group which show +I effect. The reactivity of ketones decreases as the size of alkyl group increases.

12. (a) The correct order of increasing acid strength is

$(Me)_2CHCOOH < CH_3COOH < MeOCH_2COOH < CF_3COOH$

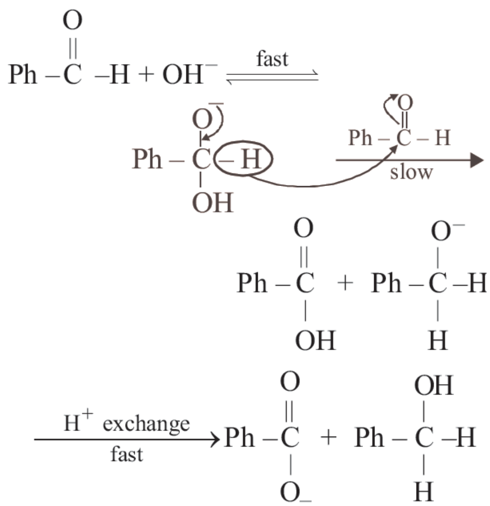
[**NOTE** Electron withdrawing groups increase the acid strength whereas electron donating groups decrease the acid strength.]

13. (c) Fruity smell is due to ester formation which is formed between ethanol and acid.

$CH_3COOH + C_2H_5OH \xrightarrow{\text{Conc. } H_2SO_4} CH_3COOC_2H_5 + H_2O$

14. **(c)** $CH_3CHCl_2 \xrightarrow{aq.KOH} CH_3CH(OH)_2 \xrightarrow{-H_2O} CH_3CHO$

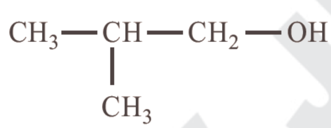
15. **(a)**



16. **(d)** Iodoform test is given by methyl ketones, acetaldehyde and methyl secondary alcohols.

Isobutyl alcohol is a primary alcohol except ethanol, C_2H_5OH, primary alcohols do not give haloform test.

Hence does not give positive iodoform test.



17. **(a)** Aldehydes and ketones can be reduced to hydrocarbons by the action (i) of amalgamated zinc and concentrated hydrochloric acid (Clemmenson reduction), or (ii) of hydrazine (NH_2NH_2) and a strong base like NaOH, KOH or potassium *tert*-butoxide in a high-boiling alcohol like ethylene glycol or triethylene glycol (Wolf-Kishner reduction) –OH group is acid-sensitive, so Clemmenson reduction can not be used.

18. **(d)** $A \xrightarrow[(I)]{NH_3} B \xrightarrow[II]{\Delta} C \xrightarrow[KOH,(III)]{Br_2} CH_3CH_2NH_2$

Reaction (III) is a Hofmann bromamide reaction. Hence, C should be $CH_3CH_2CONH_2$ which can be obtained from $CH_3CH_2COO^-NH_4^+$ (B).

Thus (A) should be CH_3CH_2COOH

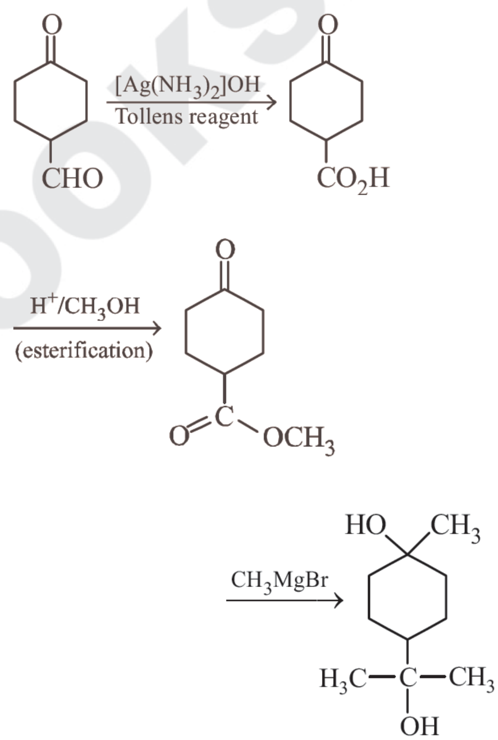
$$\underset{(A)}{CH_3CH_2-COOH} \xrightarrow{NH_3} \underset{(B)}{CH_3CH_2COO^-NH_4^+}$$

$$\xrightarrow{\Delta} \underset{(C)}{CH_3CH_2CONH_2} \xrightarrow[KOH]{Br_2} CH_3CH_2NH_2$$

19. **(c)** $CH_3COOH \xrightarrow{LiAlH_4} \underset{(A)}{CH_3CH_2OH}$

$$\xrightarrow{PCl_5} \underset{(B)}{CH_3CH_2Cl} \xrightarrow{Alc.\ KOH} \underset{(C)}{CH_2=CH_2}$$

20. **(a)**



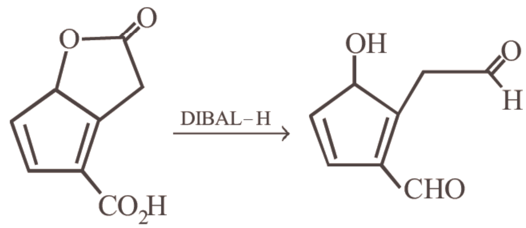
21. **(d)** $\underset{'X'}{Na_2C_2O_4} + \underset{(conc.)}{H_2SO_4} \rightarrow Na_2SO_4 + CO\uparrow + CO_2\uparrow + H_2O$

$$\underset{'X'}{Na_2C_2O_4} + CaCl_2 \rightarrow \underset{(white\ ppt.)}{CaC_2O_4\downarrow} + 2NaCl$$

$$5CaC_2O_4\downarrow + \underset{(purple)}{2KMnO_4} + 8H_2SO_4 \rightarrow$$

$$K_2SO_4 + 5CaSO_4 + \underset{(colourless)}{2MnSO_4} + 10CO_2 + 8H_2O$$

22. **(b)** DIBAL-H is a reducing agent. It reduces both ester and carboxylic group into an aldehyde at low temperature.



23. (d) The acidic strength of a compound or an acid depends on the inductive effect (–I). Higher the (–I) effect of a substituent higher will be acidic strength. Now, the decreasing order of (–I) effect of the given substituents is $NO_2 > CN > F > Cl$.

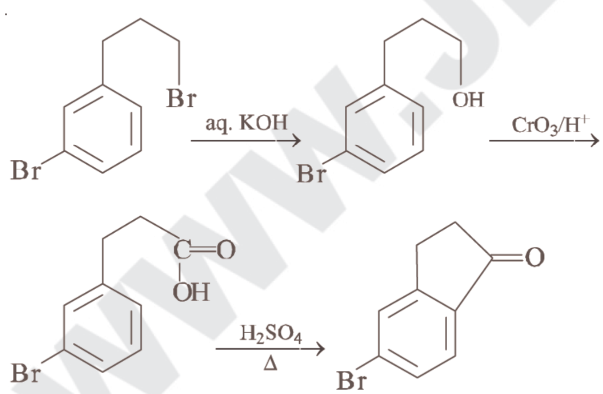
$\therefore$ The correct decreasing order of acidic strength amongst the given carboxylic acids is:

$$NO_2CH_2COOH > CNCH_2COOH > FCH_2COOH > ClCH_2COOH$$

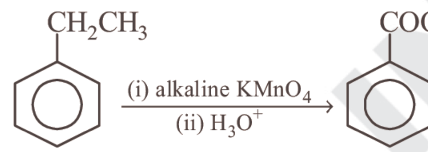
24. (c) $R-C\equiv N \xrightarrow[\text{(ii) } H_2O]{\text{(i) } AlH(i\text{-}Bu)_2} R-CHO$

The reduction of nitriles to aldehydes can be done using DIBAL-H[$AlH(i\text{-}Bu)_2$].

25. (a) For the given reaction condition, the major product is:



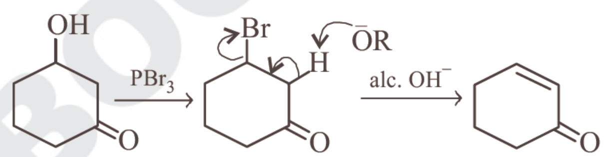
26. (a) Alkaline $KMnO_4$ converts  with a benzylic hydrogen into benzoic acid.



27. (b) $LiAlH_4$ reduces esters to alcohols but does not reduce C = C.

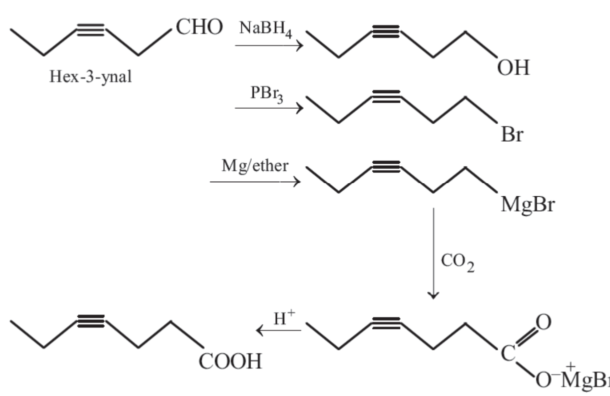
$$CH_3CH=CH-CO_2Me \xrightarrow{LiAlH_4} CH_3CH=CHCH_2OH$$

28. (c)

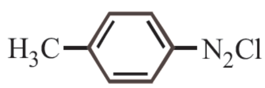
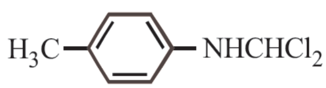
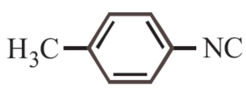
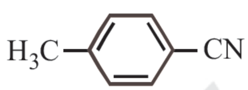
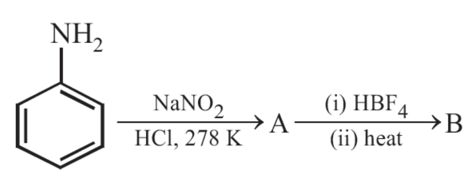


more stable due to conjugation

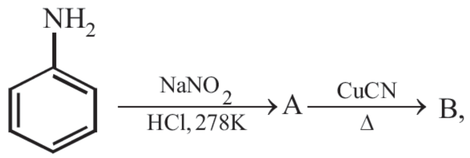
29. (d)



27 Amines

1. When a primary amine reacts with chloroform in ethanolic KOH then the product is **[2002]**
   (a) an isocyanide (b) an aldehyde
   (c) a cyanide (d) an alcohol.
2. The reaction of chloroform with alcoholic KOH and *p*-toluidine forms **[2003]**
   (a) 
   (b) 
   (c) 
   (d) 
3. The correct order of increasing basic nature for the bases NH_3, CH_3NH_2 and $(CH_3)_2NH$ is **[2003]**
   (a) $(CH_3)_2NH < NH_3 < CH_3NH_2$
   (b) $NH_3 < CH_3NH_2 < (CH_3)_2NH$
   (c) $CH_3NH_2 < (CH_3)_2NH < NH_3$
   (d) $CH_3NH_2 < NH_3 < (CH_3)_2NH$
4. Ethyl isocyanide on hydrolysis in acidic medium generates **[2003]**
   (a) propanoic acid and ammonium salt
   (b) ethanoic acid and ammonium salt
   (c) methylamine salt and ethanoic acid
   (d) ethylamine salt and methanoic acid
5. Which one of the following methods is neither meant for the synthesis nor for separation of amines? **[2005]**
   (a) Curtius reaction
   (b) Wurtz reaction
   (c) Hofmann method
   (d) Hinsberg method
6. Amongst the following the most basic compound is **[2005]**
   (a) *p*-nitroaniline (b) acetanilide
   (c) aniline (d) benzylamine
7. Which one of the following is the strongest base in aqueous solution ? **[2007]**
   (a) Methylamine
   (b) Trimethylamine
   (c) Aniline
   (d) Dimethylamine
8. In the chemical reaction,
   $CH_3CH_2NH_2 + CHCl_3 + 3KOH \longrightarrow$
   $(A) + (B) + 3H_2O$, the compounds (A) and (B) are respectively **[2007]**
   (a) C_2H_5NC and 3KCl
   (b) C_2H_5CN and 3KCl
   (c) $CH_3CH_2CONH_2$ and 3KCl
   (d) C_2H_5NC and K_2CO_3.
9. In the chemical reactions,
   
   the compounds 'A' and 'B' respectively are **[2010]**
   (a) nitrobenzene and fluorobenzene
   (b) phenol and benzene
   (c) benzene diazonium chloride and fluorobenzene
   (d) nitrobenzene and chlorobenzene

10. In the chemical reactions : **[2011RS]**



the compounds A and B respectively are :

(a) Benzene diazonium chloride and benzonitrile

(b) Nitrobenzene and chlorobenzene

(c) Phenol and bromobenzene

(d) Fluorobenzene and phenol

11. A compound with molecular mass 180 is acylated with CH_3COCl to get a compound with molecular mass 390. The number of amino groups present per molecule of the former compound is : **[2013]**

(a) 2 (b) 5

(c) 4 (d) 6

12. On heating an aliphatic primary amine with chloroform and ethanolic potassium hydroxide, the organic compound formed is: **[2014]**

(a) an alkanol (b) an alkanediol

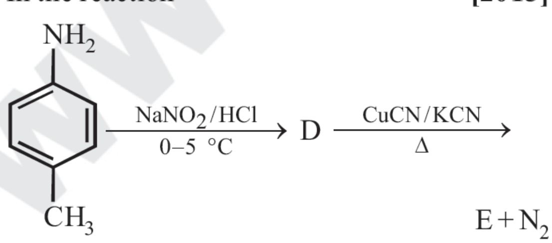
(c) an alkyl cyanide (d) an alkyl isocyanide

13. Considering the basic strength of amines in aqueous solution, which one has the smallest pK_b value? **[2014]**

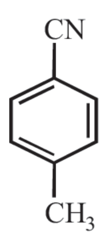
(a) $(CH_3)_2NH$ (b) CH_3NH_2

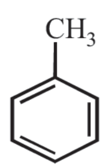
(c) $(CH_3)_3N$ (d) $C_6H_5NH_2$

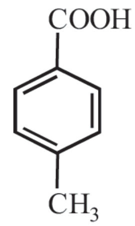
14. In the reaction **[2015]**

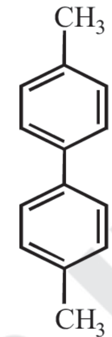


the product E is :

(a) 

(b) 

(c) 

(d) 

15. In the Hofmann bromamide degradation reaction, the number of moles of NaOH and Br_2 used per mole of amine produced are : **[2016]**

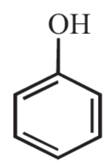
(a) Two moles of NaOH and two moles of Br_2.

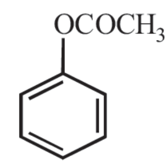
(b) Four moles of NaOH and one mole of Br_2.

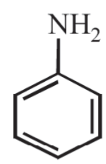
(c) One mole of NaOH and one mole of Br_2.

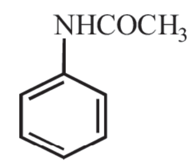
(d) Four moles of NaOH and two moles of Br_2.

16. Which of the following compounds will form significant amount of meta product during mono-nitration reaction ? **[2017]**

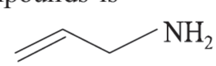
(a) 

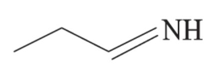
(b) 

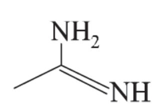
(c) 

(d) 

17. The increasing order of basicity of the following compounds is **[2018]**

(i) 

(ii) 

(iii) 

(iv) 

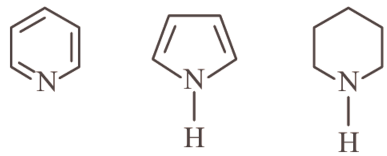
(a) (i) < (ii) < (iii) < (iv)

(b) (ii) < (i) < (iii) < (iv)

(c) (ii) < (i) < (iv) < (iii)

(d) (iv) < (ii) < (i) < (iii)

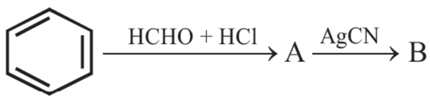
18. Arrange the following amines in the decreasing order of basicity : **[2019]**



I II III

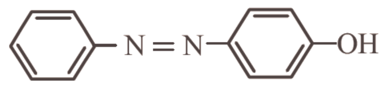
(a) I > II > III (b) III > I > II
(c) III > II > I (d) I > III > II

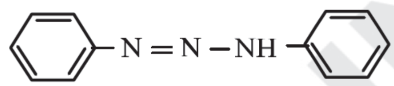
19. The compounds A and B in the following reaction are, respectively : **[2019]**

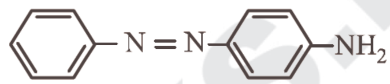


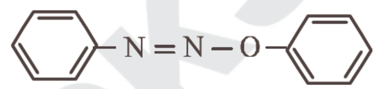
(a) A = Benzyl alcohol, B = Benzyl cyanide
(b) A = Benzyl chloride, B = Benzyl cyanide
(c) A = Benzyl alcohol, B = Benzyl isocyanide
(d) A = Benzyl chloride, B = Benzyl isocyanide

20. Aniline dissolved in dilute HCl is reacted with sodium nitrate at 0°C. This solution was added dropwise to a solution containing equimolar mixture of aniline and phenol in dil. HCl. The structure of the major product is: **[2019]**

(a) 

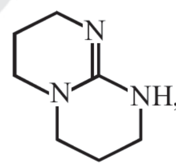
(b) 

(c) 

(d) 

21. The increasing order of pK_b for the following compounds will be: **[2020]**

$NH_2 - CH = NH$, (A)



(B)

CH_3NHCH_3 (C)

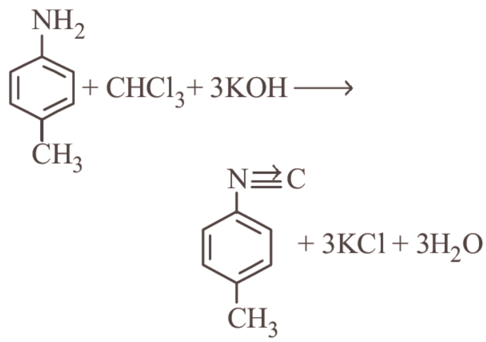
(a) (B) < (C) < (A) (b) (A) < (B) < (C)
(c) (C) < (A) < (B) (d) (B) < (A) < (C)

Answer Key

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
(a)	(c)	(b)	(d)	(b)	(d)	(d)	(a)	(c)	(a)	(b)	(d)	(a)	(a)	(b)
16	17	18	19	20	21									
(c)	(c)	(b)	(d)	(c)	(d)									

Solutions

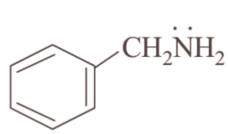
1. **(a)** $C_2H_5NH_2 + CHCl_3 + 3KOH$
$\longrightarrow C_2H_5N \rightleftarrows C + 3KCl + 3HCl$
(Ethyl isocyanide)

2. **(c)**


3. **(b)** The alkyl groups are electron releasing (+ I) groups, thus increase the electron density around the nitrogen thereby increasing the availability of the lone pair of electrons to proton or Lewis acid and making the amine more basic. Hence more the number of alkyl groups, more basic is the amine. Therefore, the correct order is
$NH_3 < CH_3NH_2 < (CH_3)_2NH$

4. **(d)** Ethyl isocyanide on hydrolysis form primary amines.
$CH_3CH_2N \rightleftarrows C + H_2O \xrightarrow{H^+}$
$CH_3CH_2NH_2 + HCOOH$

5. **(b)** Wurtz reaction is used for the preparation of hydrocarbons from alkyl halides.
$RX + 2Na + XR \longrightarrow R-R + 2NaX$

6. **(d)** Benzylamine


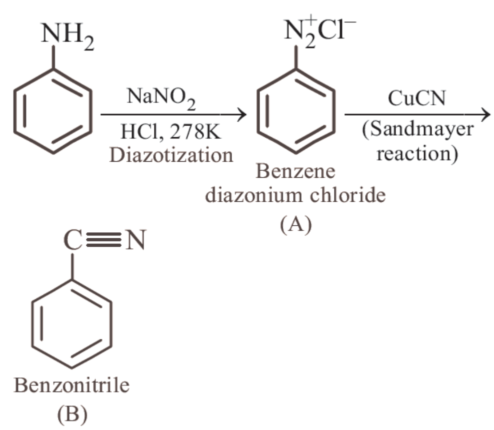
is most basic. In others the basic character is suppressed due to resonance (see applications of resonance).

7. **(d)** **NOTE** Aromatic amines (e.g. aniline) are less basic than aliphatic amines due to delocalisation of electrons. Among aliphatic amines the order of basicity in aq. solution is 2° > 1° > 3°. The electron density is decreased in 3° amine due to crowding of alkyl group over N atom which makes the approach and bonding by a proton relatively difficult. Therefore the basicity decreases.
$\therefore$ The correct order of basic strength is Dimethylamine > Methyl amine > Trimethyl amine > Aniline.

8. **(a)** This is carbylamine reaction.
$CH_3CH_2NH_2 + CHCl_3 + 3KOH$
$\longrightarrow C_2H_5NC + 3KCl + 3H_2O$

9. **(c)** $\underset{\text{1° Aromatic amine}}{ArNH_2} + NaNO_2 + 2HX \xrightarrow{\text{cold}}$
$\underset{\text{Arene diazonium salt}}{Ar-N \equiv N^+X^-} + NaX + 2H_2O$
The diazonium group can be replaced by fluorine by treating the diazonium salt with fluoroboric acid (HBF_4). The precipitated diazonium fluoroborate is isolated, dried and heated until decomposition occurs to yield the aryl fluoride. This reaction is known as **Balz-Schiemann reaction.**

$Ar-N_2^+X^- \xrightarrow{HBF_4} Ar-N_2^+BF_4^- \downarrow$
$\xrightarrow{\text{heat}} Ar-F + BF_3 + N_2$

10. **(a)**


11. (b) $R-\underbrace{NH_2}_{\text{Mol.Mass}=16} + CH_3-\overset{\overset{\displaystyle O}{||}}{C}-Cl \xrightarrow{-HCl}$

$$\underbrace{R-NH-\overset{\overset{\displaystyle O}{||}}{C}-CH_3}_{\text{Mol.mass}=58}$$

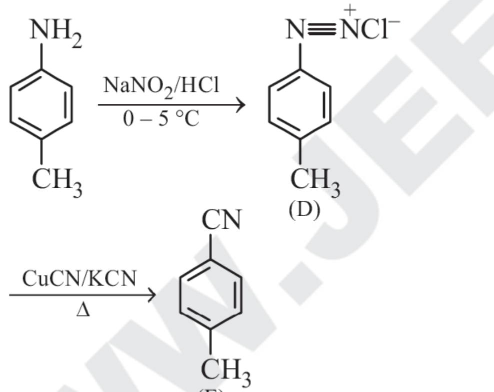
Now since the molecular mass increases by 42 unit as a result of the reaction of one mole of CH_3COCl with one-NH_2 group and the given increase in mass is (390 – 180) 210. Hence the number of – NH_2 groups $= 210/42 = 5$.

12. (d) $R-CH_2-NH_2 + CHCl_3 + 3KOH\,(alc)$

$$\xrightarrow{\text{(Carbyl amine reaction)}} \underbrace{R-CH_2-NC}_{\text{Alkyl isocyanide}} + 3KCl + 3H_2O$$

13. (a)

14. (a)



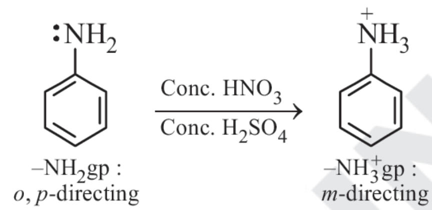
(D) (E)

15. (b) $RCONH_2 + Br_2 + 4NaOH \longrightarrow$

$$R\text{–}NH_2 + K_2CO_3 + 2NaBr + 2H_2O$$

16. (c) Nitration takes place in presence of conc. HNO_3 + conc. H_2SO_4.

In strongly acidic medium, amine is converted into anilinium ion (– NH_3^+); substitution is thus controlled not by –NH_2 group but by –NH_3^+ group which, because of its positive charge, directs the entering group to the meta-position instead of ortho- and para.

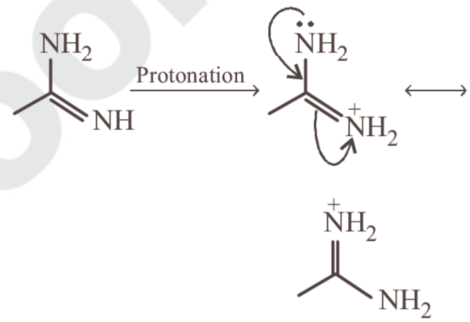


17. (c)

(i)

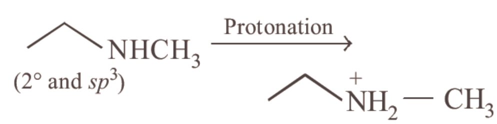
(ii)

(iii)

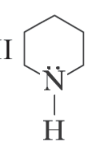


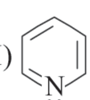
[Equivalent resonance]

(iv)

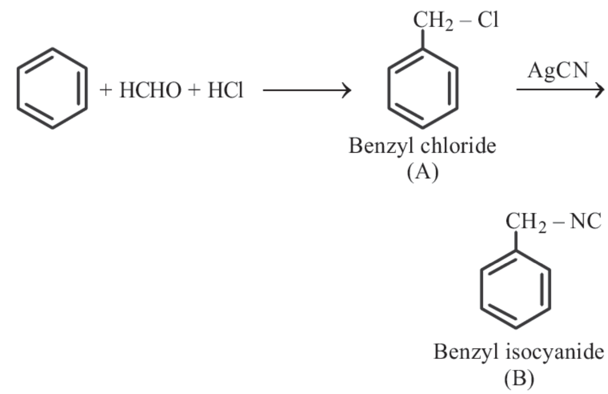


Hence, correct order of basicity will be : (ii) < (i) < (iv) < (iii).

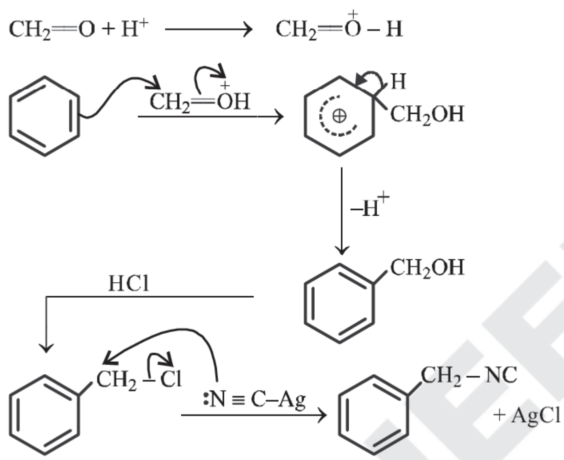
18. (b) Compound, III  is most basic as the lone pair of nitrogen is easily available for the donation.

In case of compound (I)  lone pair is not involved in resonance but nitrogen atom is sp^2 hybridised whereas in compound II the lone pair of nitrogen is involved in aromaticity which makes it least basic.

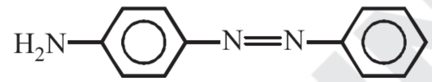
19. (d)



Mechanism:



20. (c) In acidic medium aniline is more reactive than phenol that's why electrophilic aromatic substitution of $Ph\overset{+}{N}_2$ takes place with aniline.



21. (d) Conjugate acid of guanadine(B) is resonance stabilised and have 2 resonance structure.
Similarly conjugate acid of (A) is also resonance stabilised and have one resonance structure. (C) does not exhibit resonance structure.
therefore the basic order is,

$k_b : (B) > (A) > (C)$

$\therefore\ pk_b : (B) < (A) < (C)$

28 Biomolecules

1. RNA is different from DNA because RNA contains **[2002]**
   (a) ribose sugar and thymine
   (b) ribose sugar and uracil
   (c) deoxyribose sugar and thymine
   (d) deoxyribose sugar and uracil.
2. Complete hydrolysis of cellulose gives **[2003]**
   (a) D-ribose (b) D-glucose
   (c) L-glucose (d) D-fructose
3. The reason for double helical structure of DNA is operation of **[2003]**
   (a) dipole-dipole interaction
   (b) hydrogen bonding
   (c) electrostatic attractions
   (d) van der Waals' forces
4. Which base is present in RNA but not in DNA ?
   (a) Guanine (b) Cytosine **[2004]**
   (c) Uracil (d) Thymine
5. Insulin production and its action in human body are responsible for the level of diabetes. This compound belongs to which of the following categories ? **[2004]**
   (a) An enzyme (b) A hormone
   (c) A co-enzyme (d) An antibiotic
6. Which of the following is a polyamide? **[2005]**
   (a) Bakelite (b) Terylene
   (c) Nylon-66 (d) Teflon
7. In both DNA and RNA, heterocyclic base and phosphate ester linkages are at – **[2005]**
   (a) C_5' and C_1' respectively of the sugar molecule
   (b) C_1' and C_5' respectively of the sugar molecule
   (c) C_2' and C_5' respectively of the sugar molecule
   (d) C_5' and C_2' respectively of the sugar molecule
8. The term anomers of glucose refers to **[2006]**
   (a) enantiomers of glucose
   (b) isomers of glucose that differ in configuration at carbon one (C-1)
   (c) isomers of glucose that differ in configurations at carbons one and four (C-1 and C-4)
   (d) a mixture of (D)-glucose and (L)-glucose
9. The pyrimidine bases present in DNA are **[2006]**
   (a) cytosine and thymine
   (b) cytosine and uracil
   (c) cytosine and adenine
   (d) cytosine and guanine
10. The secondary structure of a protein refers to **[2007]**
    (a) fixed configuration of the polypeptide backbone
    (b) $\alpha-$helical backbone
    (c) hydrophobic interactions
    (d) sequence of $\alpha-$amino acids.
11. α - D-(+)-glucose and β-D-(+)-glucose are **[2008]**
    (a) conformers (b) epimers
    (c) anomers (d) enatiomers
12. The two functional groups present in a typical carbohydrate are: **[2009]**
    (a) $-CHO$ and $-COOH$
    (b) $>C=O$ and $-OH$
    (c) $-OH$ and $-CHO$
    (d) $-OH$ and $-COOH$
13. Biuret test is **not** given by **[2010]**
    (a) carbohydrates (b) polypeptides
    (c) urea (d) proteins

14. Which of the following compounds can be detected by Molisch's test ? **[2012]**

(a) Nitro compounds
(b) Sugars
(c) Amines
(d) Primary alcohols

15. Which one of the following statements is correct? **[2012]**

(a) All amino acids except lysine are optically active
(b) All amino acids are optically active
(c) All amino acids except glycine are optically active
(d) All amino acids except glutamic acids are optically active

16. Synthesis of each molecule of glucose in photosynthesis involves : **[2013]**

(a) 18 molecules of ATP
(b) 10 molecules of ATP
(c) 8 molecules of ATP
(d) 6 molecules of ATP

17. Which one of the following bases is **not** present in DNA? **[2014]**

(a) Quinoline (b) Adenine
(c) Cytosine (d) Thymine

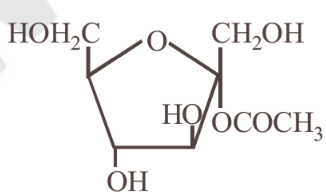
18. Which of the vitamins given below is water soluble ? **[2015]**

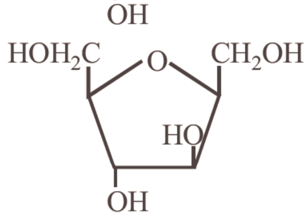
(a) Vitamin E (b) Vitamin K
(c) Vitamin C (d) Vitamin D

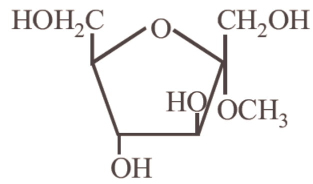
19. Thiol group is present in : **[2016]**

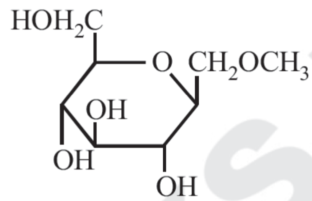
(a) Cysteine (b) Methionine
(c) Cytosine (d) Cystine

20. Which of the following compounds will behave as a reducing sugar in an aqueous KOH solution? **[2017]**

(a)


(b)


(c)


(d)


21. Glucose on prolonged heating with HI gives : **[2018]**

(a) *n*-Hexane
(b) 1- Hexene
(c) Hexanoic acid
(d) 6-Iodohexanal

22. The increasing order of pKa of the following amino acids in aqueous solution is: **[2019]**

Gly Asp Lys Arg

(a) Asp < Gly < Arg < Lys
(b) Gly < Asp < Arg < Lys
(c) Asp < Gly < Lys < Arg
(d) Arg < Lys < Gly < Asp

23. Which of the following statements is not true about sucrose? **[2019]**

(a) It is a non reducing sugar.
(b) The glycosidic linkage is present between C_1 of α-glucose and C_1 of β-fructose
(c) It is also named as invert sugar.
(d) On hydrolysis, it produces glucose and fructose.

24. Match the following: **[2020]**

(i) Riboflavin	(a) Beriberi
(ii) Thiamine	(b) Scurvy
(iii) Pyridoxine	(c) Cheilosis
(iv) Ascorbic acid	(d) Convulsions

(a) (i) – (a), (ii) – (d), (iii) – (c), (iv) – (b)
(b) (i) – (c), (ii) – (d), (iii) – (a), (iv) – (b)
(c) (i) – (c), (ii) – (a), (iii) – (d), (iv) – (b)
(d) (i) – (d), (ii) – (b), (iii) – (a), (iv) – (c)

Answer Key														
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
(b)	(b)	(b)	(c)	(b)	(c)	(b)	(b)	(a)	(b)	(c)	(c)	(a)	(b)	(c)
16	17	18	19	20	21	22	23	24						
(a)	(a)	(c)	(a)	(a)	(a)	(c)	(b)	(c)						

Solutions

1. **(b)** In RNA, the sugar is D–ribose and base is uracil, whereas in DNA, the sugar is D-2 deoxyribose and the nitrogenous base is thymine.

2. **(b)** Cellulose is a linear polymer of β–D– glucose in which C_1 of one glucose unit is connected to C_4 of the other through β–D glucosidic linkage. It does not undergo hydrolysis easily. However on heating with dilute H_2SO_4 under pressure, it undergoes hydrolysis to give only D–glucose.

$$(C_6H_{10}O_5)_n + nH_2O \xrightarrow{H^+} \underset{\text{D-Glucose}}{nC_6H_{12}O_6}$$

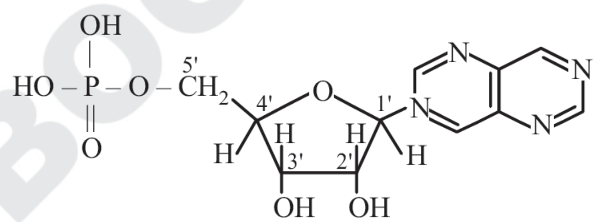
3. **(b)** DNA consists of two polynucleotide chains, each chain forms a right handed spiral with ten bases in one turn of the spiral. The two chains coil to double helix and run in opposite direction held together by hydrogen bonding.

4. **(c)** RNA contains cytosine and uracil as pyrimidine bases while DNA has cytosine and thymine.
Both have the same purine bases i.e., guanine and adenine.

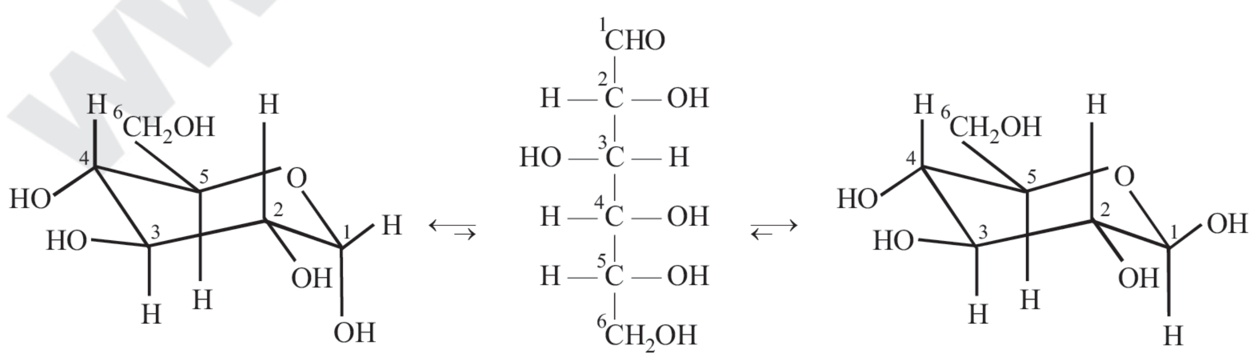
5. **(b)** Insulin is a biochemically active peptide hormone secreted by pancreas.

6. **(c)** Nylon is a general name for all polyamide synthetic fiber.

7. **(b)** In DNA and RNA heterocyclic base and phosphate ester are at C_1' and C_5' respectively of the sugar molecule.



8. **(b)** Cyclization of the open chain structure of D-(+)-glucose has created a new stereocenter at C_1 which explains the existence of two cyclic forms of D-(+)-glucose, namely α– and β–. These two cyclic forms are *diasteromers, such diastereomers which differ only in the configuration of chiral carbon developed on hemiacetal formation* (it is C_1 in glucose and C_2 in fructose) *are called* **anomers** and the hemiacetal carbon (C_1 or C_2) is called the **anomeric carbon.**



α-D-(+)-Glucopyranose
m.p. 126 °C ; $[\alpha]_0 = + 112.2°$
(36% at equilibrium)

Open chain form
of D-(+)-glucose
(negligible % at equlibrium)

β-D-(+)-Glucopyranose
m.p. 150 °C ; $[\alpha]_0 = + 18.7°$
(64% at equilibrium)

9. **(a)** The pyrimidine bases present in DNA are cytosine and thymine.

10. **(b)** The secondary structure of a protein refers to the shape in which a long peptide chain can exist. There are two different conformations of the peptide linkage present in protein, these are α-helix and β-conformation. The α-helix always has a right handed arrangement. In β-conformation all peptide chains are streched out to nearly maximum extension and then laid side by side and held together by intermolecular hydrogen bonds. The structure resembles the pleated folds of draping and therefore is known as β-pleated sheet.

11. **(c)**

12. **(c)** **NOTE** Glucose is considered as a typical carbohydrate which contains –CHO and –OH groups.

13. **(a)** Biuret test produces violet colour on addition of dilute $CuSO_4$ to alkaline solution of a compound containing peptide linkage.

Polypeptides, proteins and urea have $-\underset{\underset{O}{\|}}{C}-NH-$ (peptide) linkage, while carbohydrates have glycosidic llinkages. So, test of carbohydtrates should be different from that of other three.

14. **(b)** **Molisch's test :** This is a general test for carbohydrates. One or two drops of alcoholic solution of α-naphthol is added to 2 mL glucose solution, 1 mL of conc. H_2SO_4 solution is added carefully along the sides of the test-tube. The formation of a violet ring at the junction of two liquids confirms the presence of a carbohydrate or sugar.

15. **(c)** With the exception of glycine all other amino acids have different functional groups (atom) on the central tetrahedral alpha carbon.

$$H-\underset{\underset{NH_2}{|}}{\overset{\overset{H}{|}}{C}}-COOH$$

Glycine

16. **(a)** $6CO_2 + 12NADPH + 18ATP \longrightarrow C_6H_{12}O_6 + 12NADP + 18ADP$

17. **(a)** DNA contains ATGC bases

Quinoline is not present in DNA or RNA.

18. **(c)** Water soluble vitamins dissolve in water and are not stored by the body. The water soluble vitamins include the vitamin B-complex group and vitamin C.

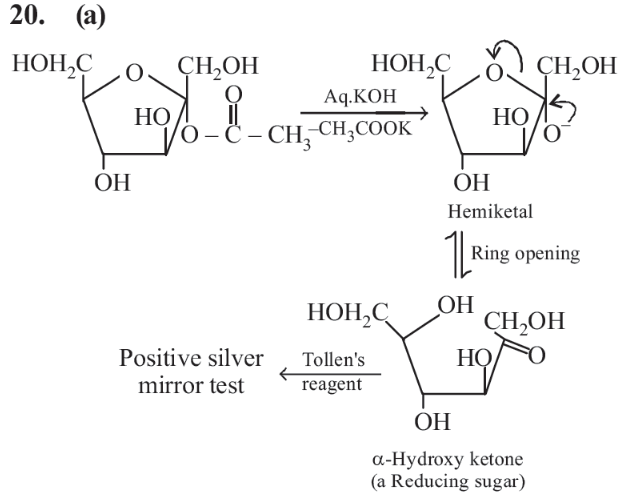
19. **(a)** Among 20 naturally occuring amino acids "cysteine" has '– SH' or thiol functional group.

$\Rightarrow$ General formula of amino acid

$$H_2N-\underset{\underset{R}{|}}{CH}-COOH$$

$\Rightarrow$ Value of $R = -CH_2-SH$ in cysteine.

20. **(a)**



21. (a)

$$\underset{\displaystyle CH_2 - OH}{\overset{\displaystyle CHO}{(CH - OH)_4}} \xrightarrow{HI,\ \Delta} \underset{n\text{-Hexane}}{CH_3CH_2CH_2CH_2CH_2CH_3}$$

22. (c) Structure of the given α-amino acids are:

$$\underset{\text{Glycine (Gly)}}{HN_3-CH_2-COO^-}$$

$$\underset{\text{Aspartic acid (Asp)}}{HOOC-CH_2-\underset{\overset{+|}{NH_3}}{CH}-COO^-}$$

$$\underset{\text{Lysine (Lys)}}{H_2N-(CH_2)_4-\underset{\overset{+|}{NH_3}}{CH}-COO^-}$$

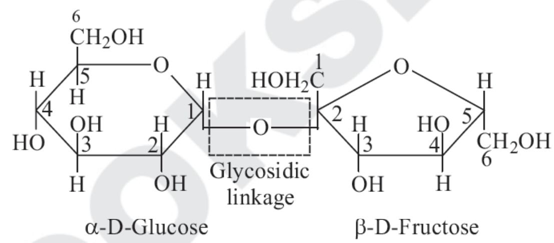
$$\underset{\text{Arginine (Arg)}}{H_2N-\overset{\overset{\displaystyle NH}{\|}}{C}-NH-(CH_2)_3-\underset{\overset{+|}{NH_3}}{CH}-COO^-}$$

Here, aspartic acid is an acidic and glycine is a neutral amino acid while lysine and arginine are basic amino acids. Also, arginine is more basic due to the stronger basic functional groups.

The order of *pKa* value is directly proportional to the basic strength of amino acids, i.e. Arg > Lys > Gly > Asp.

23. (b) Sucrose contains glycosidic linkage between C_1 of α-D glucose and C_2 of β-D-fructose.

$$C_{12}H_{22}O_{11} + H_2O \rightarrow \text{Glucose} + \text{Fructose}$$



Sucrose

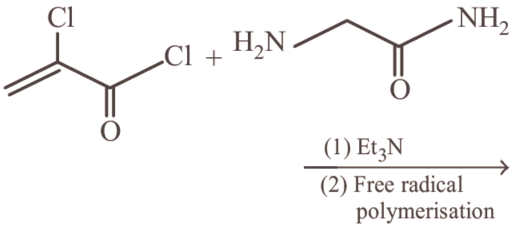
24. (c)

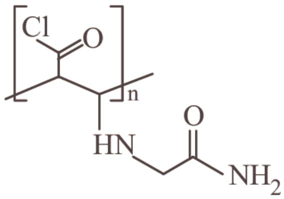
Vitamins	Deficiency Diseases
Vitamin B_1 (thiamine)	Beri-beri
Vitamin B_2 (riboflavin)	Cheilosis
Vitamin B_6 (pyridoxine)	Convulsions
Vitamin C (ascorbic acid)	Scurvy

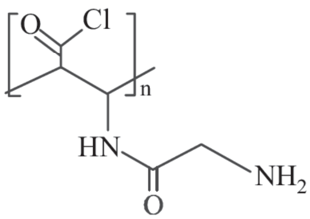
29 Polymers

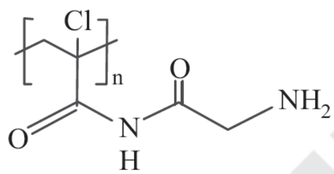
1. Polymer formation from monomers starts by **[2002]**
   (a) condensation reaction between monomers
   (b) coordinate reaction between monomers
   (c) conversion of monomer to monomer ions by protons
   (d) hydrolysis of monomers.
2. Nylon threads are made of **[2003]**
   (a) polyester polymer
   (b) polyamide polymer
   (c) polyethylene polymer
   (d) polyvinyl polymer
3. Which of the following is fully fluorinated polymer? **[2005]**
   (a) PVC (b) Thiokol
   (c) Teflon (d) Neoprene
4. Bakelite is obtained from phenol by reacting with **[2008]**
   (a) $(CH_2OH)_2$ (b) CH_3CHO
   (c) CH_3COCH_3 (d) HCHO
5. Buna-N synthetic rubber is a copolymer of : **[2009]**
   (a) $H_2C=CH-CH=CH_2$ and $H_5C_6-CH=CH_2$
   (b) $H_2C=CH-CN$ and $H_2C=CH-CH=CH_2$
   (c) $H_2C=CH-CN$ and $H_2C=CH-\underset{\displaystyle CH_3}{\underset{|}{C}}=CH_2$
   (d) $H_2C=CH-\overset{\displaystyle Cl}{\overset{|}{C}}=CH_2$ and $H_2C=CH-CH=CH_2$
6. The polymer containing strong intermolecular forces e.g. hydrogen bonding, is **[2010]**
   (a) teflon
   (b) nylon 6, 6
   (c) polystyrene
   (d) natural rubber
7. Thermosetting polymer, bakelite is formed by the reaction of phenol with : **[2011RS]**
   (a) CH_3CHO (b) HCHO
   (c) HCOOH (d) CH_3CH_2CHO
8. The species which can best serve as an initiator for the cationic polymerization is : **[2012]**
   (a) $LiAlH_4$ (b) HNO_3
   (c) $AlCl_3$ (d) BaLi
9. Which one is classified as a condensation polymer? **[2014]**
   (a) Dacron (b) Neoprene
   (c) Teflon (d) Acrylonitrile
10. Which polymer is used in the manufacture of paints and lacquers ? **[2015]**
    (a) Polypropene (b) Polyvinyl chloride
    (c) Bakelite (d) Glyptal
11. Which of the following statements about low density polythene is **FALSE**? **[2016]**
    (a) Its synthesis requires dioxygen or a peroxide initiator as a catalyst.
    (b) It is used in the manufacture of buckets, dust-bins etc.
    (c) Its synthesis requires high pressure.
    (d) It is a poor conductor of electricity.
12. The formation of which of the following polymers involves hydrolysis reaction? **[2017]**
    (a) Nylon 6 (b) Bakelite
    (c) Nylon 6, 6 (d) Terylene

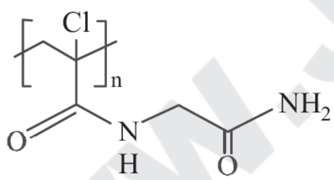
13. Major product of the following reaction is: **[2019]**



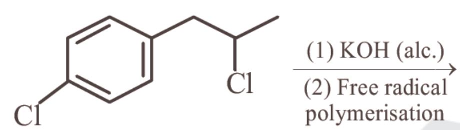
(a) 

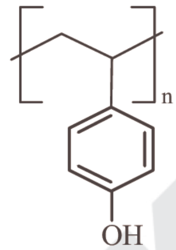
(b) 

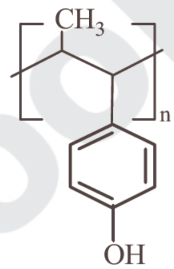
(c) 

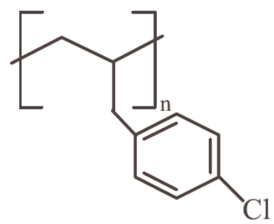
(d) 

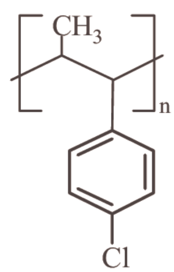
14. The major product of the following reaction is: **[2019]**



(a) 

(b) 

(c) 

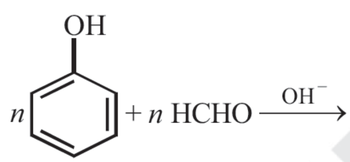
(d) 

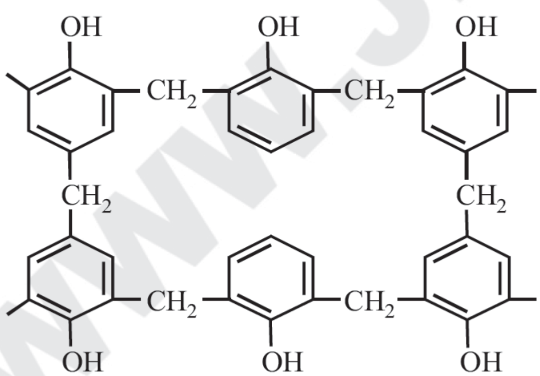
Answer Key

1	2	3	4	5	6	7	8	9	10	11	12	13	14	
(a)	(b)	(c)	(d)	(b)	(b)	(b)	(c)	(a)	(d)	(b)	(a)	(d)	(d)	

Solutions

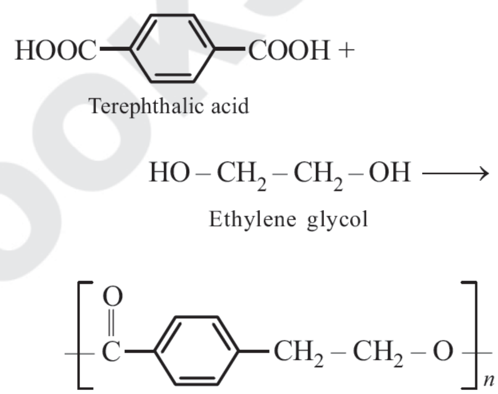
1. **(a)** Polymerisation starts either by condensation or addition reactions between monomers. Condensation polymers are formed by the combination of monomers with the elimination of simple molecules. Whereas addition polymers are formed by the addition of the molecules of the monomer or monomers to form a large molecule without elimination of any thing.
2. **(b)** Nylon is a polyamide polymer.
3. **(c)** Teflon is polymer of $CF_2 = CF_2$.
4. **(d)** Bakelite is formed by the reaction of formaldehyde (HCHO) and phenol .





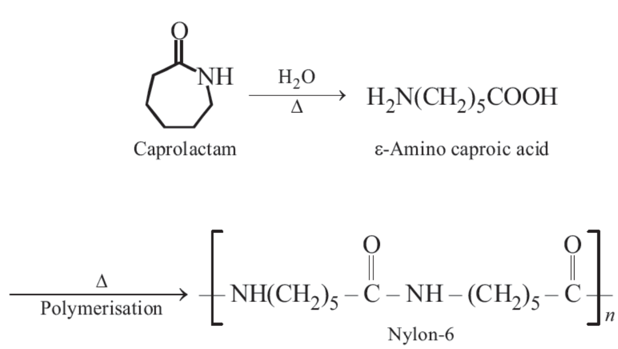
Bakelite

5. **(b)** Buna – N is a copolymer of butadiene ($CH_2 = CH - CH = CH_2$) and acrylonitrile ($CH_2 = CHCN$).
6. **(b)** Nylon 6, 6 has amide linkage capable of forming hydrogen bonding.
7. **(b)**
8. **(c)** Lewis acids are the most common compounds used for initiation of cationic polymerisation. The more popular Lewis acids are $SnCl_4$, $AlCl_3$, BF_3 and $TiCl_4$.
9. **(a)** Except dacron (terylene) all are additive polymers. Terephthalic acid condenses with ethylene glycol to give dacron.

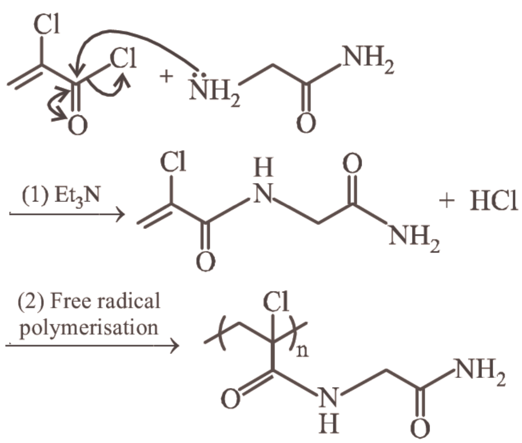


Dacron (Polyester)

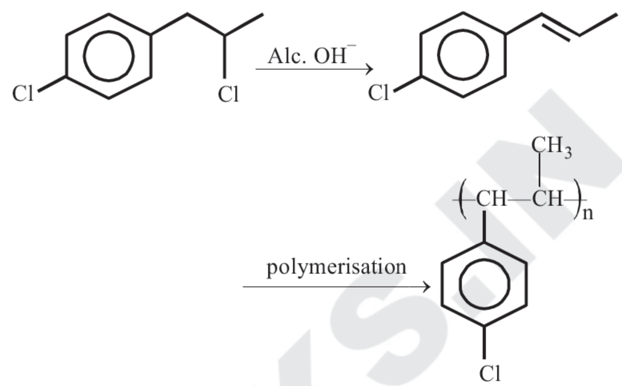
10. **(d)** Glyptal is used in the manufacture of paints and lacquers.
11. **(b)** High density polythene is used in the manufacture of housewares like buckets, dustbins, bottles, pipes etc. Low density polythene is used for insulating electric wires and in the manufacture of flexible pipes, toys, coats, bottles etc.
12. **(a)** Formation of nylon-6 involves hydrolysis of caprolactam, (its monomer) in initial state.



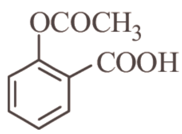
13. (d) Mechanism for the formation of major product is as follows:



14. (d)



30 Chemistry in Everyday Life

1. The compound  is used as

(a) antiseptic (b) antibiotic **[2002]**
(c) analgesic (d) pesticide.

2. Which of the following could act as a propellant for rockets? **[2003]**

(a) Liquid oxygen + liquid argon
(b) Liquid hydrogen + liquid oxygen
(c) Liquid nitrogen + liquid oxygen
(d) Liquid hydrogen + liquid nitrogen

3. Which one of the following types of drugs reduces fever ? **[2005]**

(a) Tranquiliser (b) Antibiotic
(c) Antipyretic (d) Analgesic

4. Aspirin is known as : **[2012]**

(a) Acetyl salicylic acid
(b) Phenyl salicylate
(c) Acetyl salicylate
(d) Methyl salicylic acid

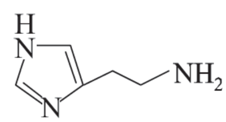
5. Which of the following compounds is not an antacid ? **[2015]**

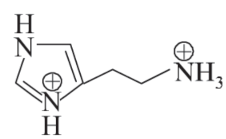
(a) Phenelzine
(b) Ranitidine
(c) Aluminium hydroxide
(d) Cimetidine

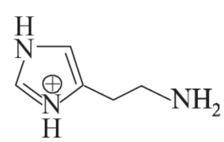
6. Which of the following is an anionic detergent? **[2016]**

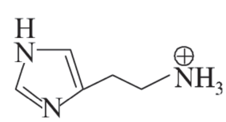
(a) Cetyltrimethyl ammonium bromide.
(b) Glyceryl oleate.
(c) Sodium stearate.
(d) Sodium lauryl sulphate.

7. The predominant form of histamine present in human blood is (pK_a, Histidine – 6.0) **[2018]**

(a) 

(b) 

(c) 

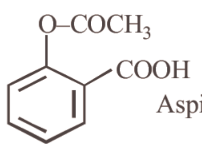
(d) 

8. The number of chiral carbons in chloramphenicol is_______. **[2020]**

Answer Key

1	2	3	4	5	6	7	8							
(c)	(b)	(c)	(a)	(a)	(d)	(d)	(2.00)							

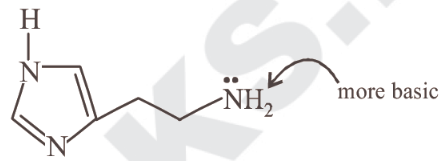
Solutions

1. **(c)** The given compound is aspirin which is antipyretic and analgesic
2. **(b)** Liquid hydrogen and liquid oxygen are used as excellent fuel for rockets. $H_2(\ell)$ has low mass and high enthalpy of combustion, whereas oxygen is a strong supporter of combustion.
3. **(c)** An antipyretic is a drug which is responsible for lowering the temperature of the feverish body to normal but has no effect on normal temperature states.
4. **(a)** 

Aspirin (Acetyl salicylic acid)
5. **(a)** Phenelzine is an antidepressant, while others are antacids.
6. **(d)** Sodium lauryl sulphate ($C_{11}H_{23}CH_2OSO_3^-$ Na^+) is an anionic detergent. Glyceryl oleate is a glyceryl ester of oleic acid. Sodium stearate ($C_{17}H_{35}COO^-Na^+$) is a soap. Cetyltrimethyl ammonium bromide

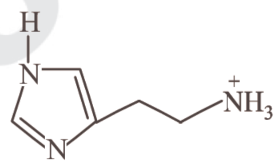
$$\left[CH_3(CH_2)_{15}\overset{+}{N}(CH_3)_3\right]Br^-$$

is a cationic detergent.
7. **(d)** Structure of histamine

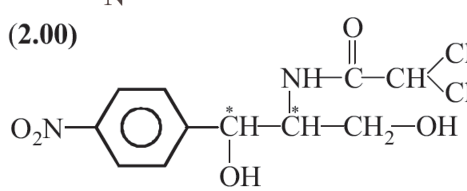


Blood is slightly basic in nature (7.35 pH). At this pH, terminal NH_2 will get protonated due to more basic nature.

∴ Predominant structure of histamine is



8. **(2.00)**



31 Analytical Chemistry

1. When H_2S is passed through Hg_2S we get **[2002]**
   (a) HgS (b) $HgS + Hg_2S$
   (c) $Hg_2S + Hg$ (d) None of these.

2. How do we differentiate between Fe^{3+} and Cr^{3+} in group III? **[2002]**
   (a) by taking excess of NH_4OH solution
   (b) by increasing NH_4^+ ion concentration
   (c) by decreasing OH^- ion concentration
   (d) both (b) and (c)

3. Which one of the following statements is correct ? **[2003]**
   (a) From a mixed precipitate of AgCl and AgI, ammonia solution dissolves only AgCl
   (b) Ferric ions give a deep green precipitate on adding potassium ferrocyanide solution
   (c) On boiling a solution having K^+, Ca^{2+} and HCO_3^- ions we get a precipitate of $K_2Ca(CO_3)_2$
   (d) Manganese salts give a violet borax bead test in the reducing flame

4. The compound formed in the positive test for nitrogen with the Lassaigne solution of an organic compound is **[2004]**
   (a) $Fe_4[Fe(CN)_6]_3$ (b) $Na_3[Fe(CN)_6]$
   (c) $Fe(CN)_3$ (d) $Na_4[Fe(CN)_5NOS]$

5. The ammonia evolved from the treatment of 0.30 g of an organic compound for the estimation of nitrogen was passed in 100 mL of 0.1 M sulphuric acid. The excess of acid required 20 mL of 0.5 M sodium hydroxide solution for complete neutralization. The organic compound is **[2004]**
   (a) urea
   (b) benzamide
   (c) acetamide
   (d) thiourea

6. An organic compound having molecular mass 60 is found to contain $C = 20\%$, $H = 6.67\%$ and $N = 46.67\%$, while rest is oxygen. On heating it gives NH_3 along with a solid residue. The solid residue gives violet colour with alkaline copper sulphate solution. The compound is **[2005]**
   (a) $CH_3CH_2CONH_2$
   (b) $(NH_2)_2CO$
   (c) CH_3CONH_2
   (d) CH_3NCO

7. 29.5 mg of an organic compound containing nitrogen was digested according to Kjeldahl's method and the evolved ammonia was absorbed in 20 mL of 0.1 M HCl solution. The excess of the acid required 15 mL of 0.1 M NaOH solution for complete neutralization. The percentage of nitrogen in the compound is **[2010]**
   (a) 59.0 (b) 47.4
   (c) 23.7 (d) 29.5

8. For the estimation of nitrogen, 1.4 g of an organic compound was digested by Kjeldahl method and the evolved ammonia was absorbed in 60 mL of $\frac{M}{10}$ sulphuric acid. The unreacted acid required 20 mL of $\frac{M}{10}$ sodium hydroxide for complete neutralization. The percentage of nitrogen in the compound is: **[2014]**
   (a) 6% (b) 10% (c) 3% (d) 5%

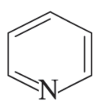
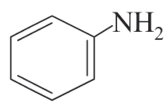
9. In Carius method of estimation of halogens, 250 mg of an organic compound gave 141 mg of AgBr. The percentage of bromine in the compound is : **[2015]**

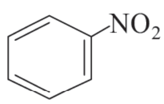
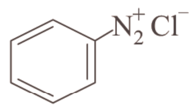
(at. mass Ag = 108; Br = 80)

(a) 48 (b) 60

(c) 24 (d) 36

10. Which of the following compounds will be suitable for Kjeldahl's method for nitrogen estimation? **[2018]**

(a)  (b) 

(c)  (d) 

11. An alkali is titrated against an acid with methyl orange as indicator, which of the following is a correct combination? **[2018]**

	Base	Acid	End point
(a)	Weak	Strong	Colourless to pink
(b)	Strong	Strong	Pinkish red to yellow
(c)	Weak	Strong	Yellow to pinkish red
(d)	Strong	Strong	Pink to colourless

12. The correct match between Item-I and Item-II is: **[2019]**

	Item-I (Drug)		Item-II (Test)
A.	Chloroxylenol	p.	Carbylamine test
B.	Norethindrone	q.	Sodium hydrogen carbonate test
C.	Sulphapyridine	r.	Ferric chloride test
D.	Penicillin	s.	Bayer's test

(a) $A \to r$; $B \to p$; $C \to s$; $D \to q$

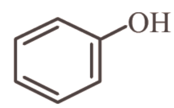
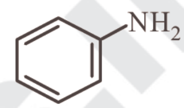
(b) $A \to q$; $B \to s$; $C \to p$; $D \to r$

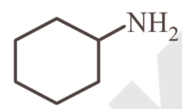
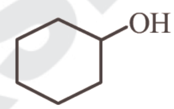
(c) $A \to r$; $B \to s$; $C \to p$; $D \to q$

(d) $A \to q$; $B \to p$; $C \to s$; $D \to r$

13. The organic compound that gives following qualitative analysis is: **[2019]**

	Test	Inference
(a)	Dil. HCl	Insoluble
(b)	NaOH solution	soluble
(c)	Br_2/water	Decolourization

(a)  (b) 

(c)  (d) 

14. A solution of *m*-chloroaniline, *m*-chlorophenol and *m*-chlorobenzoic acid in ethyl acetate was extracted initially with a saturated solution of $NaHCO_3$ to give fraction *A*. The left over organic phase was extracted with dilute NaOH solution to give fraction *B*. The final organic layer was labelled as fraction *C*. Fractions *A*, *B* and *C*, contain respectively: **[2020]**

(a) *m*-chlorobenzoic acid, *m*-chloroaniline and *m*-chlorophenol

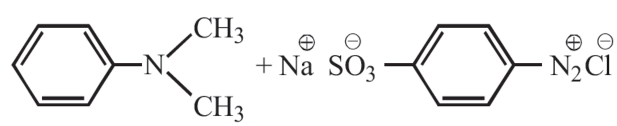
(b) *m*-chlorobenzoic acid, *m*-chlorophenol and *m*-chloroaniline

(c) *m*-chlorophenol, *m*-chlorobenzoic acid and *m*-chloroaniline

(d) *m*-chloroaniline, *m*-chlorobenzoic acid and *m*-chlorophenol

15. Consider the following reaction: **[2020]**

The product '*X*' is used:



$\xrightarrow{OH^-}$ '*X*'

(a) in protein estimation as an alternative to ninhydrin

(b) in acid base titration as an indicator

(c) as food grade colourant

(d) in laboratory test for phenols

Answer Key

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
(c)	(b)	(a)	(a)	(a)	(b)	(c)	(b)	(c)	(b)	(c)	(c)	(a)	(b)	(b)

Solutions

1. **(c)** When H_2S is passed through Hg_2S we get a mixture of mercurous sulphide and mercury $(Hg_2S + Hg)$.

2. **(b)** When we add NH_4Cl, it suppresses the ionisation of NH_4OH and prevents the precipitation of higher group hydroxide in gp(III).

NOTE Further ferric chloride and chromium chloride form different colour precipitates with NH_4OH.

$$FeCl_3 + 3NH_4OH \longrightarrow \underset{\text{Reddish brown}}{Fe(OH)_3\downarrow} + 3NH_4Cl$$

$$CrCl_3 + 3NH_4OH \longrightarrow \underset{\text{Bluish green}}{Cr(OH)_3} + 3NH_4Cl$$

3. **(a)** Between AgCl and AgI, AgI is less soluble, hence ammonia can dissolve ppt. of AgCl only due to formation of complex as given below:

$$AgCl + 2NH_3 \longrightarrow [Ag(NH_3)_2]Cl$$

4. **(a)** Prussian blue $Fe_4[Fe(CN)_6]_3$ is formed in Lassaigne test for nitrogen.

$$Na + \underbrace{C + N}_{\text{(From org. comp.)}} \xrightarrow{\text{Fuse}} NaCN$$

$$FeSO_4 + 2NaOH \longrightarrow \underset{\text{Green ppt.}}{Fe(OH)_2\downarrow} + Na_2SO_4$$

$$Fe(OH)_2 + 2NaCN \longrightarrow Fe(CN)_2 + 2NaOH$$

$$Fe(CN)_2 + 4NaCN \longrightarrow \underset{\text{Sod. ferrocyanide}}{Na_4[Fe(CN)_6]}$$

$$3Na_4[Fe(CN)_6] + 4FeCl_3 \longrightarrow \underset{\substack{\text{Ferricferrocyanide}\\ \text{(Prussian blue)}}}{Fe_4[Fe(CN)_6]_3} + 12NaCl$$

5. **(a)** H_2SO_4 is dibasic.

$$0.1\,M\,H_2SO_4 = 0.2\,N\,H_2SO_4 \qquad [\because M = 2 \times N]$$

M_{eq} of H_2SO_4 taken $= 100 \times 0.2 = 20$

M_{eq} of H_2SO_4 neutralised by NaOH $= 20 \times 0.5 = 10$

M_{eq} of H_2SO_4 neutralised by NH_3 $= 20 - 10 = 10$

$$N_2 = \frac{1.4 \times M_{eq} \text{ of acid neutrialised by } NH_3}{\text{Wt. of organic compound}}$$

$$= \frac{1.4 \times 10}{0.3} = 46.6$$

$$\%\text{ of nitrogen in urea} = \frac{14 \times 2 \times 100}{60} = 46.6$$

[Mol.wt of urea =60]

Similarly % of nitrogen in benzamide

$$= \frac{14 \times 100}{121} = 11.5\% \quad [C_6H_5CONH_2 = 121]$$

$$\text{Acetamide} = \frac{14 \times 1 \times 100}{59} = 23.4\%$$

$[CH_3CONH_2 = 59]$

$$\text{Thiourea} = \frac{14 \times 2 \times 100}{76} = 36.8\%$$

$[NH_2CSNH_2 = 76]$

Hence the compound must be urea.

6. **(b)**

Element	%	Relative no. of attoms	Simplest ratio of attoms
C	20	20/12 = 1.66	1.0
H	6.67	6.67/1 = 6.67	4.16
N	46.67	46.67/14 = 3.33	2.02
O	26.64	26.64/16 = 1.66	1.0

$\therefore$ The empirical formulae is CH_4N_2O

Empirical weight = 60; Mol. wt. = 60;

$$\therefore \quad n = \frac{60}{60} = 1$$

Molecular formula $= CH_4N_2O$

$NH_2 - CO - NH_2$ (Urea)

On heating, urea loses ammonia to give biuret.

$$2NH_2CONH_2 \longrightarrow H_2NCO.NH.CONH_2 + NH_3$$

Biuret with alkaline $CuSO_4$ gives violet colour. Test for –CONH– group.

7. **(c)** Moles of HCl taken $= 20 \times 0.1 \times 10^{-3}$
$= 2 \times 10^{-3}$

Moles of HCl neutralised by NaOH solution
$= 15 \times 0.1 \times 10^{-3} = 1.5 \times 10^{-3}$

Moles of HCl neutralised by ammonia
$= 2 \times 10^{-3} - 1.5 \times 10^{-3}$
$= 0.5 \times 10^{-3}$

Moles of NH_3 absorbed $= 0.5 \times 10^{-3}$

Moles of N in $NH_3 = 0.5 \times 10^{-3}$

Mass of N in organic compound
$= 14 \times 0.5 \times 10^{-3}$ g

% N in organic compound

$$= \frac{14 \times 0.5 \times 10^{-3}}{29.5 \times 10^{-3}} \times 100$$
$= 23.7\%$

8. **(b)** $\%\text{ of N} = \dfrac{1.4 \times \text{meq. of acid}}{\text{mass of organic compound (in g)}}$

$$\text{meq. of } H_2SO_4 = 60 \times \frac{M}{10} \times 2 = 12$$

$$\text{meq. of NaOH} = 20 \times \frac{M}{10} = 2$$

$\therefore$ meq. of acid consumed $= 12 - 2 = 10$

$$\therefore \%\text{ of N} = \frac{1.4 \times 10}{1.4} = 10\%$$

9. **(c)** Mass of substance = 250 mg = 0.250 g

Mass of AgBr = 141 mg = 0.141 g

1 mole of AgBr = 1 g atom of Br

188 g of AgBr = 80 g of Br

$\therefore$ 188 g of AgBr contain bromine = 80 g

0.141 g of AgBr contain bromine

$$= \frac{80}{188} \times 0.141 = 0.06 \text{ g}$$

0.06 g of bromine is present in 0.250 g of organic compound

$\therefore$ % of bromine = 24%

10. **(b)** Kjeldahl's method is not applicable for compounds containing nitrogen in nitro and azo groups and nitrogen in ring, as N of these compounds does not change to ammonium sulphate under these conditions.

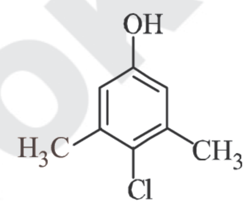
11. **(c)** pH range for methyl orange is

$\xleftarrow[\text{Pinkish red}]{} 3.9 - 4.5 \xrightarrow[\text{Yellow}]{}$

Generally, weak bases have pH greater than 7. When methyl orange is added to a weak basic solution, solution becomes yellow. This solution is then titrated against a strong acid, at the end point pH will be less than 3.1.

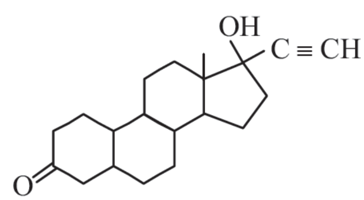
$\therefore$ Solution becomes pinkish red.

12. **(c)** As chloroxylenol contains phenolic group so it gives positive ferric chloride test.



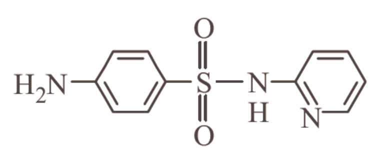
Chloroxylenol

Norethindrone has double bond, thus it will give Bayer's test.



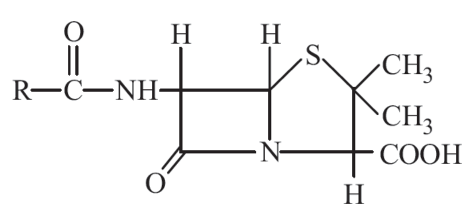
Norethindrone

Sulphapyridine contains $-NH_2$ group so it gives carbylamine test.



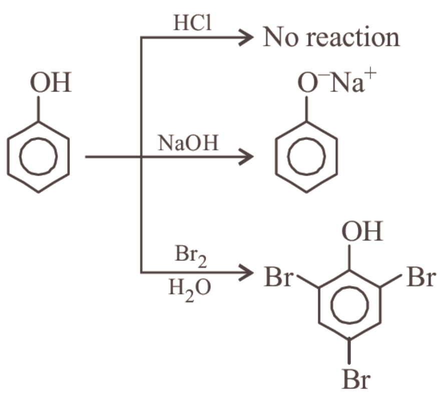
Sulphapyridine

Penicillin contains –COOH group so it will give sodium hydrogen carbonate ($NaHCO_3$) test.

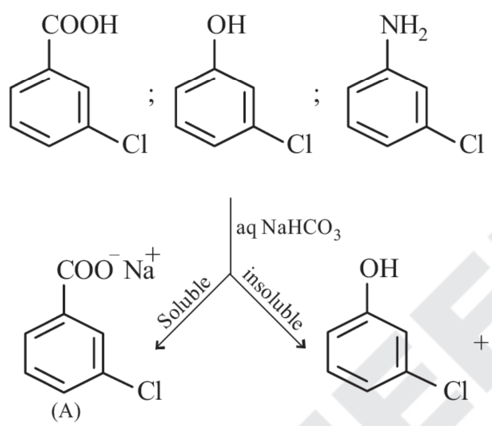


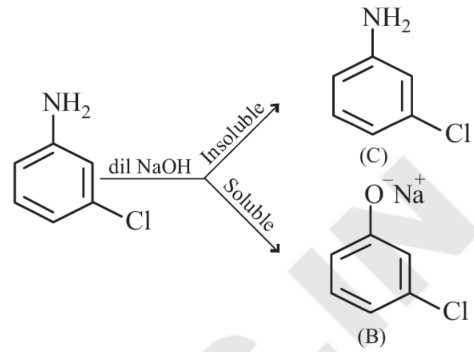
Pencillin

13. (a)

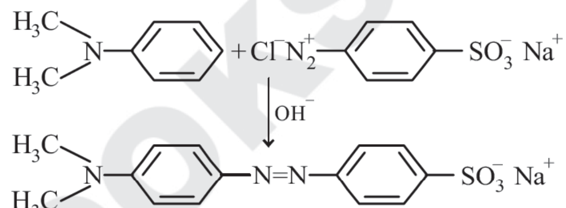


14. (b)





15. (b)



Methyl orange

Methyl orange is used as an indicator in acid base titrations.

MATHEMATICS

1 Sets

1. If A, B and C are three sets such that $A \cap B = A \cap C$ and $A \cup B = A \cup C$, then **[2009]**
   (a) $A = C$ (b) $B = C$
   (c) $A \cap B = \phi$ (d) $A = B$

2. Let $X = \{1,2,3,4,5\}$. The number of different ordered pairs (Y,Z) that can formed such that $Y \subseteq X$, $Z \subseteq X$ and $Y \cap Z$ is empty is : **[2012]**
   (a) 5^2 (b) 3^5
   (c) 2^5 (d) 5^3

3. If $f(x) + 2f\left(\frac{1}{x}\right) = 3x$, $x \neq 0$ and $S = \{x \in R : f(x) = f(-x)\}$; then S: **[2016]**
   (a) contains exactly two elements.
   (b) contains more than two elements.
   (c) is an empty set.
   (d) contains exactly one element.

4. Let $S = \{x \in R : x \geq 0$ and $2|\sqrt{x} - 3| + \sqrt{x}(\sqrt{x} - 6) + 6 = 0\}$. Then S : **[2018]**
   (a) contains exactly one element.
   (b) contains exactly two elements.
   (c) contains exactly four elements.
   (d) is an empty set.

Answer Key

1	2	3	4											
(b)	(b)	(a)	(b)											

Solutions

1. **(b)** $\because B = (B \cap A) \cup B$

$= (A \cap C) \cup B$

$= (A \cup B) \cap (C \cup B)$

$= (A \cup C) \cap (B \cup C)$

$= (A \cap B) \cup C$

$= (A \cap C) \cup C$

$= C$

2. **(b)** Let $X = \{1, 2, 3, 4, 5\}$
   $n(x) = 5$
   Each element of x has 3 options. Either in set Y or set Z or none. ($\because Y \cap Z = \phi$)
   So, number of ordered pairs = 3^5

3. **(a)** $f(x) + 2f\left(\frac{1}{x}\right) = 3x$ ....(1)

$f\left(\frac{1}{x}\right) + 2f(x) = \frac{3}{x}$ ....(2)

Adding (1) and (2)

$$\Rightarrow f(x)+f\left(\frac{1}{x}\right)=x+\frac{1}{x} \quad ...(3)$$

Substracting (1) from (2)

$$\Rightarrow f(x)-f\left(\frac{1}{x}\right)=\frac{3}{x}-3x \quad ...(4)$$

On adding (3) and (4)

$$\Rightarrow f(x)=\frac{2}{x}-x$$

$$f(x)=f(-x)\Rightarrow \frac{2}{x}-x=\frac{-2}{x}+x\Rightarrow x=\frac{2}{x}$$

$x^2=2$ or $x=\sqrt{2},-\sqrt{2}$

4. **(b)** **Case-I:** $x\in[0,9]$

$$2(3-\sqrt{x})+x-6\sqrt{x}+6=0$$

$$\Rightarrow x-8\sqrt{x}+12=0\Rightarrow \sqrt{x}=4,2$$

$$\Rightarrow x=16,4$$

Since $x\in[0,9]$

$\therefore$ $x=4$

Case-II: $x\in[9,\infty]$

$$2(\sqrt{x}-3)+x-6\sqrt{x}+6=0$$

$$\Rightarrow x-4\sqrt{x}=0\Rightarrow x=16,0$$

Since $x\in[9,\infty]$

$\therefore$ $x=16$

Hence, $x=4$ & 16

2 Relations and Functions

1. Domain of definition of the function $f(x)=\dfrac{3}{4-x^2}+\log_{10}(x^3-x)$, is **[2003]**
(a) $(-1,0)\cup(1,2)\cup(2,\infty)$ (b) $(a,2)$
(c) $(-1,0)\cup(a,2)$ (d) $(1,2)\cup(2,\infty)$.

2. If $f:R\to R$ satisfies $f(x+y)=f(x)+f(y)$, for all $x,\ y\in R$ and $f(1)=7$, then $\sum\limits_{r=1}^{n} f(r)$ is **[2003]**
(a) $\dfrac{7n(n+1)}{2}$ (b) $\dfrac{7n}{2}$
(c) $\dfrac{7(n+1)}{2}$ (d) $7n+(n+1)$

3. The graph of the function $y=f(x)$ is symmetrical about the line $x=2$, then **[2004]**
(a) $f(x)=-f(-x)$
(b) $f(2+x)=f(2-x)$
(c) $f(x)=f(-x)$
(d) $f(x+2)=f(x-2)$

4. The domain of the function $f(x)=\dfrac{1}{\sqrt{|x|-x}}$ is **[2011]**
(a) $(0,\infty)$ (b) $(-\infty,0)$
(c) $(-\infty,\infty)-\{0\}$ (d) $(-\infty,\infty)$

Answer Key

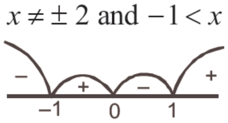
1	2	3	4											
(a)	(a)	(b)	(b)											

Solutions

1. **(a)** $f(x)=\dfrac{3}{4-x^2}+\log_{10}(x^3-x)$

$4-x^2\neq 0$ and $x^3-x>0$;

$x\neq\pm 2$ and $-1<x<0$ or $1<x<\infty$



$\therefore D=(-1,\ 0)\cup(1,\ \infty)-\{2\}$

$D=(-1,\ 0)\cup(1,\ 2)\cup(2,\ \infty)$.

2. **(a)** $f(x+y)=f(x)+f(y)$.

$\because f(1)=7$

$f(2)=f(1+1)=f(1)+f(1)=14$

$f(3)=f(1+2)=f(1)+f(2)=21$

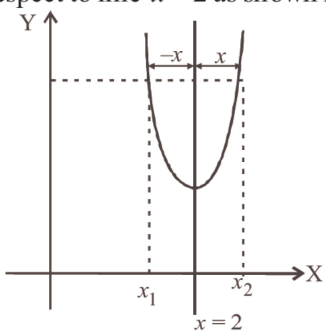
— — — — — — — — — —

— — — — — — — — — —

$$\therefore \sum_{r=1}^{n} f(r) = 7(1+2+3.....+n)$$

$$= \frac{7n(n+1)}{2}$$

3. (b) Given that a graph symmetrical. with respect to line $x = 2$ as shown in the figure.



From the figure

$f(x_1) = f(x_2)$, where $x_1 = 2 - x$

and $x_2 = 2 + x$

$\therefore f(2-x) = f(2+x)$

4. (b) $f(x) = \dfrac{1}{\sqrt{|x| - x}}$, $f(x)$ is define if $|x| - x > 0$

$\Rightarrow |x| > x, \Rightarrow x < 0$

Hence domain of $f(x)$ is $(-\infty, 0)$

3 Trigonometric Functions

1. The number of solution of $\tan x + \sec x = 2\cos x$ in $[0, 2\pi)$ is **[2002]**
   (a) 2 (b) 3
   (c) 0 (d) 1

2. Let α, β be such that $\pi < \alpha - \beta < 3\pi$.
   If $\sin\alpha + \sin\beta = -\frac{21}{65}$ and $\cos\alpha + \cos\beta = -\frac{27}{65}$, then the value of $\cos\frac{\alpha-\beta}{2}$ **[2004]**
   (a) $\frac{-6}{65}$ (b) $\frac{3}{\sqrt{130}}$
   (c) $\frac{6}{65}$ (d) $-\frac{3}{\sqrt{130}}$

3. If $u = \sqrt{a^2\cos^2\theta + b^2\sin^2\theta} + \sqrt{a^2\sin^2\theta + b^2\cos^2\theta}$ then the difference between the maximum and minimum values of u^2 is given by **[2004]**
   (a) $(a-b)^2$ (b) $2\sqrt{a^2+b^2}$
   (c) $(a+b)^2$ (d) $2(a^2+b^2)$

4. The number of values of x in the interval $[0, 3\pi]$ satisfying the equation $2\sin^2 x + 5\sin x - 3 = 0$ is **[2006]**
   (a) 4 (b) 6
   (c) 1 (d) 2

5. If $0 < x < \pi$ and $\cos x + \sin x = \frac{1}{2}$, then $\tan x$ is **[2006]**
   (a) $\frac{(1-\sqrt{7})}{4}$ (b) $\frac{(4-\sqrt{7})}{3}$
   (c) $-\frac{(4+\sqrt{7})}{3}$ (d) $\frac{(1+\sqrt{7})}{4}$

6. Let **A** and **B** denote the statements
   A : $\cos\alpha + \cos\beta + \cos\gamma = 0$
   B : $\sin\alpha + \sin\beta + \sin\gamma = 0$
   If $\cos(\beta-\gamma) + \cos(\gamma-\alpha) + \cos(\alpha-\beta) = -\frac{3}{2}$, then : **[2009]**
   (a) **A** is false and **B** is true
   (b) both **A** and **B** are true
   (c) both **A** and **B** are false
   (d) **A** is true and **B** is false

7. Let $\cos(\alpha+\beta) = \frac{4}{5}$ and $\sin(\alpha-\beta) = \frac{5}{13}$, where $0 \le \alpha, \beta \le \frac{\pi}{4}$. Then $\tan 2\alpha =$ **[2010]**
   (a) $\frac{56}{33}$ (b) $\frac{19}{12}$
   (c) $\frac{20}{7}$ (d) $\frac{25}{16}$

8. If $A = \sin^2 x + \cos^4 x$, then for all real x : **[2011]**
   (a) $\frac{13}{16} \le A \le 1$ (b) $1 \le A \le 2$
   (c) $\frac{3}{4} \le A \le \frac{13}{16}$ (d) $\frac{3}{4} \le A \le 1$

9. The possible values of $\theta \in (0, \pi)$ such that $\sin(\theta) + \sin(4\theta) + \sin(7\theta) = 0$ are **[2011RS]**

(a) $\frac{\pi}{4}, \frac{5\pi}{12}, \frac{\pi}{2}, \frac{2\pi}{3}, \frac{3\pi}{4}, \frac{8\pi}{9}$

(b) $\frac{2\pi}{9}, \frac{\pi}{4}, \frac{\pi}{2}, \frac{2\pi}{3}, \frac{3\pi}{4}, \frac{35\pi}{36}$

(c) $\frac{2\pi}{9}, \frac{\pi}{4}, \frac{\pi}{2}, \frac{2\pi}{3}, \frac{3\pi}{4}, \frac{8\pi}{9}$

(d) $\frac{2\pi}{9}, \frac{\pi}{4}, \frac{4\pi}{9}, \frac{\pi}{2}, \frac{3\pi}{4}, \frac{8\pi}{9}$

10. The equation $e^{\sin x} - e^{-\sin x} - 4 = 0$ has : 1 **[2012]**

(a) infinite number of real roots
(b) no real roots
(c) exactly one real root
(d) exactly four real roots

11. ABCD is a trapezium such that AB and CD are parallel and $BC \perp CD$. If $\angle ADB = \theta$, $BC = p$ and $CD = q$, then AB is equal to : **[2013]**

(a) $\frac{(p^2+q^2)\sin\theta}{p\cos\theta + q\sin\theta}$

(b) $\frac{p^2+q^2\cos\theta}{p\cos\theta + q\sin\theta}$

(c) $\frac{p^2+q^2}{p^2\cos\theta + q^2\sin\theta}$

(d) $\frac{(p^2+q^2)\sin\theta}{(p\cos\theta + q\sin\theta)^2}$

12. The expression $\frac{\tan A}{1-\cot A} + \frac{\cot A}{1-\tan A}$ can be written as : **[2013]**

(a) sinA cosA + 1
(b) secA cosecA + 1
(c) tanA + cotA
(d) secA + cosecA

13. Let $f_k(x) = \frac{1}{k}\left(\sin^k x + \cos^k x\right)$ where $x \in R$ and $k \geq 1$. Then $f_4(x) - f_6(x)$ equals **[2014]**

(a) $\frac{1}{4}$ (b) $\frac{1}{12}$

(c) $\frac{1}{6}$ (d) $\frac{1}{3}$

14. If the angles of elevation of the top of a tower from three collinear points A, B and C, on a line leading to the foot of the tower, are 30°, 45° and 60° respectively, then the ratio, AB : BC, is : **[2015]**

(a) $1:\sqrt{3}$ (b) $2:3$

(c) $\sqrt{3}:1$ (d) $\sqrt{3}:\sqrt{2}$

15. A man is walking towards a vertical pillar in a straight path, at a uniform speed. At a certain point A on the path, he observes that the angle of elevation of the top of the pillar is 30°. After walking for 10 minutes from A in the same direction, at a point B, he observes that the angle of elevation of the top of the pillar is 60°. Then the time taken (in minutes) by him, from B to reach the pillar, is: **[2016]**

(a) 20 (b) 5
(c) 6 (d) 10

16. If $0 \leq x < 2\pi$, then the number of real values of x, which satisfy the equation $\cos x + \cos 2x + \cos 3x + \cos 4x = 0$ is: **[2016]**

(a) 7 (b) 9
(c) 3 (d) 5

17. If $5(\tan^2 x - \cos^2 x) = 2\cos 2x + 9$, then the value of $\cos 4x$ is : **[2017]**

(a) $-\frac{7}{9}$ (b) $-\frac{3}{5}$

(c) $\frac{1}{3}$ (d) $\frac{2}{9}$

18. PQR is a triangular park with PQ = PR = 200 m. A T.V. tower stands at the mid-point of QR. If the angles of elevation of the top of the tower at P, Q and R are respectively 45°, 30° and 30°, then the height of the tower (in m) is : **[2018]**

(a) 50 (b) $100\sqrt{3}$

(c) $50\sqrt{2}$ (d) 100

19. If sum of all the solutions of the equation

$$8\cos x \cdot \left(\cos\left(\frac{\pi}{6}+x\right) \cdot \cos\left(\frac{\pi}{6}-x\right) - \frac{1}{2}\right) = 1$$

in $[0, \pi]$ is $k\pi$, then k is equal to : **[2018]**

(a) $\frac{13}{9}$ (b) $\frac{8}{9}$

(c) $\frac{20}{9}$ (d) $\frac{2}{3}$

20. The value of $\cos^2 10° - \cos 10° \cos 50° + \cos^2 50°$ is : **[2019]**

(a) $\frac{3}{4} + \cos 20°$ (b) 3/4

(c) $\frac{3}{2}(1 + \cos 20°)$ (d) 3/2

21. Let $S = \{\theta \in [-2\pi, 2\pi] : 2\cos^2\theta + 3\sin\theta = 0\}$. Then the sum of the elements of S is: **[2019]**

(a) $\frac{13\pi}{6}$ (b) $\frac{5\pi}{3}$

(c) 2π (d) π

Answer Key

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
(b)	(d)	(a)	(a)	(c)	(b)	(a)	(d)	(d)	(b)	(a)	(b)	(b)	(c)	(b)
16	17	18	19	20	21									
(a)	(a)	(d)	(a)	(b)	(c)									

Solutions

1. **(b)** $\because$ $\tan x + \sec x = 2\cos x$;

$\Rightarrow \sin x + 1 = 2\cos^2 x$

$\Rightarrow \sin x + 1 = 2(1 - \sin^2 x)$;

$\Rightarrow 2\sin^2 x + \sin x - 1 = 0$;

$\Rightarrow (2\sin x - 1)(\sin x + 1) = 0$

$\Rightarrow \sin x = \frac{1}{2}, -1.$;

$\Rightarrow x = 30°, 150°, 270°.$

Number of solution = 3

2. **(d)** $\pi < \alpha - \beta < 3\pi$

$$\Rightarrow \frac{\pi}{2} < \frac{\alpha-\beta}{2} < \frac{3\pi}{2} \Rightarrow \cos\frac{\alpha-\beta}{2} < 0 \quad ....(1)$$

$$\sin\alpha + \sin\beta = -\frac{21}{65}$$

$$\Rightarrow 2\sin\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2} = -\frac{21}{65} \quad ....(2)$$

$$\cos\alpha + \cos\beta = -\frac{27}{65}$$

$$\Rightarrow 2\cos\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2} = -\frac{27}{65} \quad ....(3)$$

Squaring and adding (2) and (3), we get

$$4\cos^2\frac{\alpha-\beta}{2} = \frac{(21)^2 + (27)^2}{(65)^2} = \frac{1170}{65\times 65}$$

$$\therefore \cos^2\frac{\alpha-\beta}{2} = \frac{9}{130} \Rightarrow \cos\frac{\alpha-\beta}{2} = -\frac{3}{\sqrt{130}}$$

[from (1)]

3. **(a)** $$u^2 = a^2 + b^2 + 2\sqrt{\begin{array}{l}(a^4+b^4)\cos^2\theta\sin^2\theta \\ \quad + a^2b^2(\cos^4\theta + \sin^4\theta)\end{array}} \quad \ldots(1)$$

Now, $(a^4 + b^4)\cos^2\theta\sin^2\theta + a^2b^2(\cos^4\theta + \sin^4\theta)$

$= (a^4 + b^4)\cos^2\theta\sin^2\theta + a^2b^2(1 - 2\cos^2\theta\sin^2\theta)$

$= (a^4 + b^4 - 2a^2b^2)\cos^2\theta\sin^2\theta + a^2b^2$

$= (a^2 - b^2)^2.\frac{\sin^2 2\theta}{4} + a^2b^2 \quad \ldots(2)$

$\because 0 \le \sin^2 2\theta \le 1$

$$\Rightarrow 0 \le (a^2 - b^2)^2 \frac{\sin^2 2\theta}{4} \le \frac{(a^2 - b^2)^2}{4}$$

$$\Rightarrow a^2b^2 \le (a^2 - b^2)^2 \frac{\sin^2 2\theta}{4} + a^2b^2$$

$$\le (a^2 - b^2)^2 . \frac{1}{4} + a^2b^2 \qquad ....(3)$$

From (1)

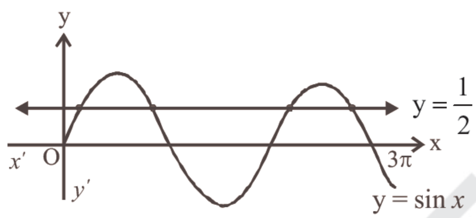
$$a^2 + b^2 + 2\sqrt{a^2b^2} \le u^2 \le a^2 + b^2 + \frac{2}{2}\sqrt{\left(a^2 + b^2\right)^2}$$

$$(a+b)^2 \le u^2 \le 2\left(a^2 + b^2\right)$$

$\therefore$ Max. value – Min. value

$$= 2(a^2 + b^2) - (a + b^2) = (a - b)^2$$

4. **(a)**



$$2\sin^2 x + 5\sin x - 3 = 0$$

$$\Rightarrow \quad (\sin x + 3)(2\sin x - 1) = 0$$

$$\Rightarrow \quad \sin x = \frac{1}{2} \quad \text{and } \sin x \ne -3$$

$\therefore$ In $[0, 3\pi]$, x has 4 values.

5. **(c)** $\cos x + \sin x = \frac{1}{2} \;\Rightarrow\; 1 + \sin 2x = \frac{1}{4}$

$$\Rightarrow \quad \sin 2x = -\frac{3}{4},$$

$$\therefore \pi < 2x < 2\pi$$

$$\Rightarrow \frac{\pi}{2} < x \le \pi \qquad ...(i)$$

$$\frac{2\tan x}{1 + \tan^2 x} = -\frac{3}{4}$$

$$\Rightarrow \quad 3\tan^2 x + 8\tan x + 3 = 0$$

$$\therefore \quad \tan x = \frac{-8 \pm \sqrt{64 - 36}}{6} = -\frac{-4 \pm \sqrt{7}}{3}$$

for $\frac{\pi}{2} < x < \pi$, $\tan x < 0$

$$\therefore \quad \tan x = \frac{-4 - \sqrt{7}}{3}$$

6. **(b)** Given that

$\cos(\beta - \gamma) + \cos(\gamma - \alpha) + \cos(\alpha - \beta)$

$= -\frac{3}{2}$

$\Rightarrow$ $2[\cos(\beta - \gamma) + \cos(\gamma - \alpha) + \cos(\alpha - \beta)] + 3 = 0$

$\Rightarrow$ $2[\cos(\beta - \gamma) + \cos(\gamma - \alpha) + \cos(\alpha - \beta)] + \sin^2\alpha + \cos^2\alpha + \sin^2\beta + \cos^2\beta + \sin^2\gamma + \cos^2\alpha = 0$

$\Rightarrow$ $[\sin^2\alpha + \sin^2\beta + \sin^2\gamma + 2\sin\alpha\,\sin\beta + 2\sin\beta\,\sin\gamma + 2\sin\gamma\,\sin\alpha] + [\cos^2\alpha + \cos^2\beta + \cos^2\gamma + 2\cos\alpha\,\cos\beta + 2\cos\beta\,\cos\gamma + 2\cos\gamma\,\cos\alpha] = 0$

$[\because \cos(A - B) = \cos A . \cos B + \sin A . \sin B]$

$\Rightarrow$ $[\sin\alpha + \sin\beta + \sin\gamma]^2 + (\cos\alpha + \cos\beta + \cos\gamma)^2 = 0$

$\Rightarrow$ $\sin\alpha + \sin\beta + \sin\gamma = 0$ and $\cos\alpha + \cos\beta + \cos\gamma = 0$

$\therefore$ A and B both are true.

7. **(a)** $\cos(\alpha + \beta) = \frac{4}{5} \Rightarrow \tan(\alpha + \beta) = \frac{3}{4}$

$$\sin(\alpha - \beta) = \frac{5}{13} \Rightarrow \tan(\alpha - \beta) = \frac{5}{12}$$

$$\tan 2\alpha = \tan\left[(\alpha + \beta) + (\alpha - \beta)\right]$$

$$= \frac{\tan(\alpha + \beta) + \tan(\alpha - \beta)}{1 - \tan(\alpha + \beta)\tan(\alpha - \beta)} = \frac{\frac{3}{4} + \frac{5}{12}}{1 - \frac{3}{4} . \frac{5}{12}} = \frac{56}{33}$$

8. **(d)** $A = \sin^2 x + \cos^4 x$

$$= \sin^2 x + \cos^2 x(1 - \sin^2 x)$$

$$= \sin^2 x + \cos^2 x - \frac{1}{4}(2\sin x . \cos x)^2$$

$$= 1 - \frac{1}{4}\sin^2(2x)$$

$\because -1 \le \sin 2x \le 1$

$\Rightarrow 0 \le \sin^2(2x) \le 1$

$\Rightarrow 0 \ge -\frac{1}{4}\sin^2(2x) \ge -\frac{1}{4}$

$\Rightarrow 1 \ge 1 - \frac{1}{4}\sin^2(2x) \ge 1 - \frac{1}{4}$

$\Rightarrow 1 \ge A \ge \frac{3}{4}$

9. **(d)** $\sin 4\theta + 2\sin 4\theta \cos 3\theta = 0$

$\because \sin A + \sin B = 2\sin\left(\frac{A+B}{2}\right)\cdot\cos\left(\frac{A-B}{2}\right)$

$\sin 4\theta(1 + 2\cos 3\theta) = 0$

$\sin 4\theta = 0$; $4\theta = n\pi; n \in I$

$\Rightarrow \quad \theta = \frac{\pi}{4}, \frac{\pi}{2}, \frac{3\pi}{4}$

or $\quad \cos 3\theta = -\frac{1}{2}$

or $\quad 3\theta = 2n\pi \pm \frac{2\pi}{3}, n \in I$

$\Rightarrow \quad \theta = \frac{2\pi}{9}, \frac{8\pi}{9}, \frac{4\pi}{9} \qquad [\because \theta, \in (0, \pi)]$

10. **(b)** Given equation is $e^{\sin x} - e^{-\sin x} - 4 = 0$

Put $e^{\sin x} = t$ in the given equation, we get

$t^2 - 4t - 1 = 0$

$\Rightarrow \quad t = \frac{4 \pm \sqrt{16+4}}{2} = \frac{4 \pm \sqrt{20}}{2}$

$= \frac{4 \pm 2\sqrt{5}}{2} = 2 \pm \sqrt{5}$

$\Rightarrow \quad e^{\sin x} = 2 \pm \sqrt{5} \quad \left(\because t = e^{\sin x}\right)$

$\Rightarrow \quad e^{\sin x} = 2 - \sqrt{5}$ and $e^{\sin x} = 2 + \sqrt{5}$

$\Rightarrow \quad e^{\sin x} = 2 - \sqrt{5} < 0$

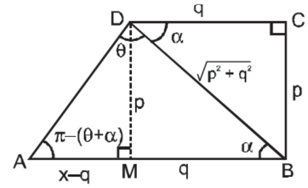
and $\sin x = \ln\left(2 + \sqrt{5}\right) > 1$

So, rejected.

Hence, given equation has no solution.

$\therefore$ The equation has no real roots.

11. **(a)** In ΔABD, from Sine Rule



$$\frac{AB}{\sin\theta} = \frac{\sqrt{p^2+q^2}}{\sin(\pi - (\theta+\alpha))}$$

$$AB = \frac{\sqrt{p^2+q^2}\,\sin\theta}{\sin\theta\cos\alpha + \cos\theta\sin\alpha}$$

$$= \frac{(p^2+q^2)\sin\theta}{q\sin\theta + p\cos\theta}$$

$$\left(\because \cos\alpha = \frac{q}{\sqrt{p^2+q^2}} \text{ and } \sin\alpha = \frac{p}{\sqrt{p^2+q^2}}\right)$$

12. **(b)** Given expression can be written as

$$\frac{\sin A}{\cos A} \times \frac{\sin A}{\sin A - \cos A} + \frac{\cos A}{\sin A} \times \frac{\cos A}{\cos A - \sin A}$$

$$\left(\because \quad \tan A = \frac{\sin A}{\cos A} \text{ and } \cot A = \frac{\cos A}{\sin A}\right)$$

$$= \frac{1}{\sin A - \cos A}\left\{\frac{\sin^3 A - \cos^3 A}{\cos A \sin A}\right\}$$

$\because a^3 - b^3 = (a-b)(a^2 + ab + b^2)$

$$= \frac{\sin^2 A + \sin A\cos A + \cos^2 A}{\sin A\cos A}$$

$= 1 + \sec A \operatorname{cosec} A$

13. **(b)** Let $f_k(x) = \frac{1}{k}(\sin^k x + \cos^k x)$

Consider

$$f_4(x) - f_6(x) = \frac{1}{4}(\sin^4 x + \cos^4 x)$$

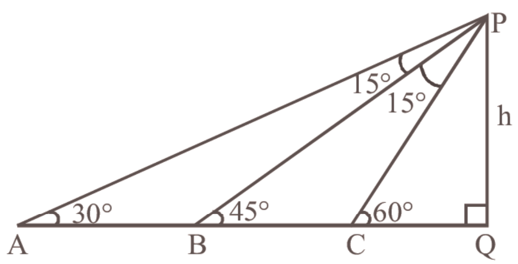
$$-\frac{1}{6}(\sin^6 x + \cos^6 x)$$

$$=\frac{1}{4}[1-2\sin^2 x\cos^2 x]$$

$$-\frac{1}{6}[1-3\sin^2 x\cos^2 x]$$

$$=\frac{1}{4}-\frac{1}{6}=\frac{1}{12}$$

14. (c)



$\because$ PB bisects $\angle APC$, therefore

AB : BC = PA : PC

Also in ΔAPQ, $\sin 30° = \frac{h}{PA} \Rightarrow PA = 2h$

and in ΔCPQ, $\sin 60° = \frac{h}{PC} \Rightarrow PC = \frac{2h}{\sqrt{3}}$

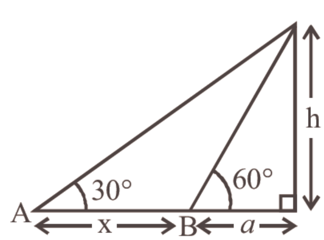
$\therefore$ AB : BC $= 2h : \frac{2h}{\sqrt{3}} = \sqrt{3} : 1$

15. (b) $\tan 30° = \frac{h}{x+a}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{x+a} \Rightarrow \sqrt{3}h = x + a$ ...(1)

$\tan 60° = \frac{h}{a} \Rightarrow \sqrt{3} = \frac{h}{a}$

$\Rightarrow h = \sqrt{3}a$ ...(2)



From (1) and (2)

$3a = x + a \Rightarrow x = 2a$

Here, the speed is uniform

So, time taken to cover x = 2 (time taken to cover a)

$\therefore$ Time taken to cover $a = \frac{10}{2}$ minutes

= 5 minutes

16. (a) $\cos x + \cos 2x + \cos 3x + \cos 4x = 0$

$\Rightarrow 2\cos 2x\cos x + 2\cos 3x\cos x = 0$

$\Rightarrow 2\cos x\left(2\cos\frac{5x}{2}\cos\frac{x}{2}\right) = 0$

$\cos x = 0,\ \cos\frac{5x}{2} = 0,\ \cos\frac{x}{2} = 0$

$x = \pi, \frac{\pi}{2}, \frac{3\pi}{2}, \frac{\pi}{5}, \frac{3\pi}{5}, \frac{7\pi}{5}, \frac{9\pi}{5}$

17. (a) We have

$5\tan^2 x - 5\cos^2 x = 2(2\cos^2 x - 1) + 9$

$\Rightarrow 5\tan^2 x - 5\cos^2 x = 4\cos^2 x - 2 + 9$

$\Rightarrow 5\tan^2 x = 9\cos^2 x + 7$

$\Rightarrow 5(\sec^2 x - 1) = 9\cos^2 x + 7$

Let $\cos^2 x = t$

$\Rightarrow \frac{5}{t} - 9t - 12 = 0$

$\Rightarrow 9t^2 + 12t - 5 = 0$

$\Rightarrow 9t^2 + 15t - 3t - 5 = 0$

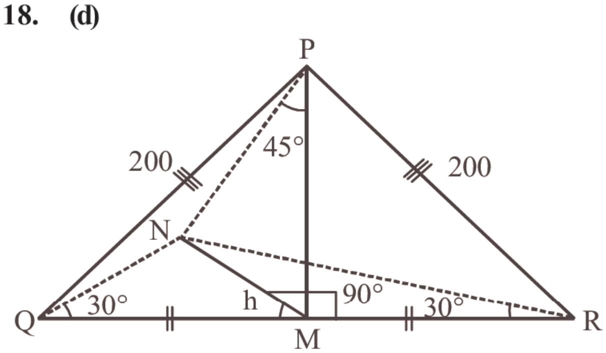
$\Rightarrow (3t-1)(3t+5) = 0$

$\Rightarrow t = \frac{1}{3}$ as $t \neq -\frac{5}{3}$.

$\cos 2x = 2\cos^2 x - 1 = 2\left(\frac{1}{3}\right) - 1 = -\frac{1}{3}$

$\cos 4x = 2\cos^2 2x - 1 = 2\left(-\frac{1}{3}\right)^2 - 1 = -\frac{7}{9}$

18. (d)



Let height of tower MN = h
In ΔQMN we have

$$\tan 30° = \frac{MN}{QM}$$

$\therefore$ $QM = \sqrt{3}h = MR$ ...(1)
Now in ΔMNP
MN = PM ...(2)
In ΔPMQ we have :

$$MP = \sqrt{(200)^2 - (\sqrt{3}h)^2}$$

$\therefore$ From (2), we get :

$$\sqrt{(200)^2 - (\sqrt{3}h)^2} = h \Rightarrow h = 100m$$

19. (a) $\because\ 8\cos x\left(\cos^2\frac{\pi}{6} - \sin^2 x - \frac{1}{2}\right) = 1$

$\Rightarrow\ 8\cos x\left(\frac{3}{4} - \frac{1}{2} - \sin^2 x\right) = 1$

$\Rightarrow\ 8\cos x\left(\frac{1}{4} - (1 - \cos^2 x)\right) = 1$

$\Rightarrow\ 8\cos x\left(\frac{1}{4} - 1 + \cos^2 x\right) = 1$

$\Rightarrow\ 8\cos x\left(\cos^2 x - \frac{3}{4}\right) = 1$

$\Rightarrow\ 8\left(\frac{4\cos^3 x - 3\cos x}{4}\right) = 1$

$\Rightarrow\ 2(4\cos^3 x - 3\cos x) = 1$

$\Rightarrow\ 2\cos 3x = 1 \Rightarrow \cos 3x = \frac{1}{2}$

$\therefore\ 3x = 2n\pi \pm \frac{\pi}{3}, n \in I$

$\Rightarrow\ x = \frac{2n\pi}{3} \pm \frac{\pi}{9}$

In $x \in [0, \pi] : x = \frac{\pi}{9}, \frac{2\pi}{3} + \frac{\pi}{9}, \frac{2\pi}{3} - \frac{\pi}{9}$, only

Sum of all the solutions of the equation

$$= \left(\frac{1}{9} + \frac{2}{3} + \frac{1}{9} + \frac{2}{3} - \frac{1}{9}\right)\pi = \frac{13}{9}\pi$$

20. (b) $\cos^2 10° - \cos 10° \cos 50° + \cos^2 50°$

$$= \left(\frac{1 + \cos 20°}{2}\right) + \left(\frac{1 + \cos 100°}{2}\right) - \frac{1}{2}(2\cos 10° \cos 50°)$$

$$= 1 + \frac{1}{2}(\cos 20° + \cos 100°) - \frac{1}{2}[\cos 60° + \cos 40°]$$

$$= \left(1 - \frac{1}{4}\right) + \frac{1}{2}[\cos 20° + \cos 100° - \cos 40°]$$

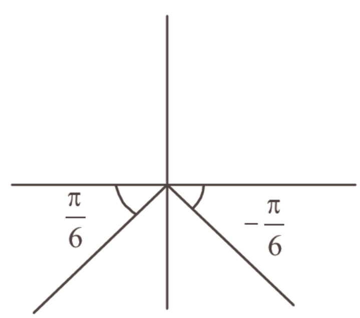
$$= \frac{3}{4} + \frac{1}{2}[2\cos 60° \times \cos 40° - \cos 40°]$$

$$= \frac{3}{4}$$

21. (c) $2\cos^2\theta + 3\sin\theta = 0$

$\Rightarrow (2\sin\theta + 1)(\sin\theta - 2) = 0$

$\Rightarrow \sin\theta = -\frac{1}{2}$ or $\sin\theta = 2 \rightarrow$ Not possibe



The required sum of all solutions in $[-2\pi, 2\pi]$ is

$$= \left(\pi + \frac{\pi}{6}\right) + \left(2\pi - \frac{\pi}{6}\right) + \left(-\frac{\pi}{6}\right) + \left(-\pi + \frac{\pi}{6}\right) = 2\pi$$

4 Principle of Mathematical Induction

1. If $a_n = \sqrt{7+\sqrt{7+\sqrt{7+......}}}$ having n radical signs then by methods of mathematical induction which is true **[2002]**

(a) $a_n > 7, \forall\ n \geq 1$ (b) $a_n < 7, \forall\ n \geq 1$

(c) $a_n < 4, \forall\ n \geq 1$ (d) $a_n < 3, \forall\ n \geq 1$

2. Let $S(K) = 1+3+5...+(2K-1) = 3+K^2$. Then which of the following is true **[2004]**

(a) Principle of mathematical induction can be used to prove the formula

(b) $S(K) \Rightarrow S(K+1)$

(c) $S(K) \not\Rightarrow S(K+1)$

(d) $S(1)$ is correct

Answer Key

1	2														
(b)	(b)														

Solutions

1. **(b)** For $n = 1$, $a_1 = \sqrt{7} < 7$. Let $a_m < 7$.

Then $a_{m+1} = \sqrt{7+a_m}$

$\Rightarrow a^2_{m+1} = 7 + a_m < 7+7 < 14.$

$\Rightarrow a_{m+1} < \sqrt{14} < 7$; So, by the principle of mathematical induction $a_n < 7,\ \forall\ n.$

2. **(b)** $S(K) = 1+3+5+...+(2K-1) = 3+K^2$

$S(1): 1 = 3+1$, which is not true

$\because S(1)$ is not true.

$\therefore$ P.M.I cannot be applied

Let $S(K)$ is true, i.e.

$S(K): 1+3+5....+(2K-1) = 3+K^2$

Adding $2K+1$ on both sides

$\Rightarrow 1+3+5....+(2K-1)+2K+1$

$= 3+K^2+2K+1 = 3+(K+1)^2 = S(K+1)$

$\therefore S(K) \Rightarrow S(K+1)$

5 Complex Numbers and Quadratic Equations

1. z and w are two non zero complex numbers such that $|z|=|w|$ and Arg z + Arg w = π then z equals **[2002]**

(a) $\overline{\omega}$ (b) $-\overline{\omega}$

(c) ω (d) $-\omega$

2. If $|z-4|<|z-2|$, its solution is given by **[2002]**

(a) $\text{Re}(z)>0$ (b) $\text{Re}(z)<0$

(c) $\text{Re}(z)>3$ (d) $\text{Re}(z)>2$

3. The locus of the centre of a circle which touches the circle $|z-z_1|=a$ and $|z-z_2|=b$ externally (z, z_1 & z_2 are complex numbers) will be **[2002]**

(a) an ellipse (b) a hyperbola

(c) a circle (d) none of these

4. If $\alpha \neq \beta$ but $\alpha^2=5\alpha-3$ and $\beta^2=5\beta-3$ then the equation having α/β and β/α as its roots is **[2002]**

(a) $3x^2-19x+3=0$ (b) $3x^2+19x-3=0$

(c) $3x^2-19x-3=0$ (d) $x^2-5x+3=0$

5. Difference between the corresponding roots of $x^2+ax+b=0$ and $x^2+bx+a=0$ is same and $a \neq b$, then **[2002]**

(a) $a+b+4=0$ (b) $a+b-4=0$

(c) $a-b-4=0$ (d) $a-b+4=0$

6. Product of real roots of the equation $t^2x^2+|x|+9=0$ **[2002]**

(a) is always positive

(b) is always negative

(c) does not exist

(d) none of these

7. If p and q are the roots of the equation $x^2+px+q=0$, then **[2002]**

(a) $p=1, q=-2$ (b) $p=0, q=1$

(c) $p=-2, q=0$ (d) $p=-2, q=1$

8. If z and ω are two non-zero complex numbers such that $|z\omega|=1$ and $Arg(z)-Arg(\omega)=\dfrac{\pi}{2}$, then $\bar{z}\omega$ is equal to **[2003]**

(a) -1 (b) 1

(c) $-i$ (d) i

9. Let Z_1 and Z_2 be two roots of the equation $Z^2+aZ+b=0$, Z being complex. Further, assume that the origin, Z_1 and Z_2 form an equilateral triangle. Then **[2003]**

(a) $a^2=4b$ (b) $a^2=b$

(c) $a^2=2b$ (d) $a^2=3b$

10. If $\left(\dfrac{1+i}{1-i}\right)^x=1$ then **[2003]**

(a) $x=2n+1$, where n is any positive integer

(b) $x=4n$, where n is any positive integer

(c) $x=2n$, where n is any positive integer

(d) $x=4n+1$, where n is any positive integer.

11. The value of 'a' for which one root of the quadratic equation $(a^2-5a+3)x^2+(3a-1)x+2=0$ is twice as large as the other is **[2003]**

(a) $-\frac{1}{3}$ (b) $\frac{2}{3}$

(c) $-\frac{2}{3}$ (d) $\frac{1}{3}$

12. The number of real solutions of the equation $x^2 - 3|x| + 2 = 0$ is **[2003]**

(a) 3 (b) 2
(c) 4 (d) 1

13. Let z and w be complex numbers such that $\overline{z} + i\,\overline{w} = 0$ and arg $zw = \pi$. Then arg z equals **[2004]**

(a) $\frac{5\pi}{4}$ (b) $\frac{\pi}{2}$

(c) $\frac{3\pi}{4}$ (d) $\frac{\pi}{4}$

14. If $z = x - i\,y$ and $z^{\frac{1}{3}} = p + iq$, then $\left(\frac{x}{p} + \frac{y}{q}\right) \Big/ (p^2 + q^2)$ is equal to **[2004]**

(a) –2 (b) –1
(c) 2 (d) 1

15. If $|z^2 - 1| = |z|^2 + 1$, then z lies on **[2004]**

(a) an ellipse
(b) the imaginary axis
(c) a circle
(d) the real axis

16. If $(1-p)$ is a root of quadratic equation $x^2 + px + (1-p) = 0$, then its root are **[2004]**

(a) –1, 2 (b) –1, 1
(c) 0, –1 (d) 0, 1

17. If one root of the equation $x^2 + px + 12 = 0$ is 4, while the equation $x^2 + px + q = 0$ has equal roots, then the value of 'q' is **[2004]**

(a) 4 (b) 12

(c) 3 (d) $\frac{49}{4}$

18. The value of a for which the sum of the squares of the roots of the equation $x^2 - (a-2)x - a - 1 = 0$ assume the least value is **[2005]**

(a) 1 (b) 0
(c) 3 (d) 2

19. If the cube roots of unity are 1, ω, ω^2 then the roots of the equation $(x-1)^3 + 8 = 0$, are **[2005]**

(a) $-1, -1 + 2\omega, -1 - 2\omega^2$
(b) $-1, -1, -1$
(c) $-1, 1 - 2\omega, 1 - 2\omega^2$
(d) $-1, 1 + 2\omega, 1 + 2\omega^2$

20. If z_1 and z_2 are two non- zero complex numbers such that $|z_1 + z_2| = |z_1| + |z_2|$, then arg z_1 – arg z_2 is equal to **[2005]**

(a) $\frac{\pi}{2}$ (b) $-\pi$

(c) 0 (d) $\frac{-\pi}{2}$

21. If $\omega = \frac{z}{z - \frac{1}{3}i}$ and $|\omega| = 1$, then z lies on **[2005]**

(a) an ellipse (b) a circle
(c) a straight line (d) a parabola

22. In a triangle PQR, $\angle R = \frac{\pi}{2}$. If $\tan\left(\frac{P}{2}\right)$ and $\tan\left(\frac{Q}{2}\right)$ are the roots of $ax^2 + bx + c = 0$, $a \neq 0$ then **[2005]**

(a) $a = b + c$ (b) $c = a + b$
(c) $b = c$ (d) $b = a + c$

23. If both the roots of the quadratic equation $x^2 - 2kx + k^2 + k - 5 = 0$ are less than 5, then k lies in the interval **[2005]**

(a) (5, 6] (b) $(6, \infty)$
(c) $(-\infty, 4)$ (d) [4, 5]

24. The value of $\sum_{k=1}^{10}\left(\sin\frac{2k\pi}{11} + i\cos\frac{2k\pi}{11}\right)$ is **[2006]**

(a) i (b) 1
(c) -1 (d) $-i$

25. If $z^2+z+1=0$, where z is complex number, then the value of

$$\left(z+\frac{1}{z}\right)^2+\left(z^2+\frac{1}{z^2}\right)^2+\left(z^3+\frac{1}{z^3}\right)^2+$$

$$\ldots\ldots\ldots+\left(z^6+\frac{1}{z^6}\right)^2 \text{ is}$$ **[2006]**

(a) 18 (b) 54
(c) 6 (d) 12

26. If the roots of the quadratic equation $x^2+px+q=0$ are $\tan 30°$ and $\tan 15°$, respectively, then the value of $2+q-p$ is **[2006]**

(a) 2 (b) 3
(c) 0 (d) 1

27. All the values of m for which both roots of the equation $x^2-2mx+m^2-1=0$ are greater than -2 but less than 4, lie in the interval **[2006]**

(a) $-2<m<0$ (b) $m>3$
(c) $-1<m<3$ (d) $1<m<4$

28. If $|z+4|\le 3$, then the maximum value of $|z+1|$ is **[2007]**

(a) 6 (b) 0
(c) 4 (d) 10

29. If the difference between the roots of the equation $x^2+ax+1=0$ is less than $\sqrt{5}$, then the set of possible values of a is **[2007]**

(a) $(3,\infty)$ (b) $(-\infty,-3)$
(c) $(-3,3)$ (d) $(-3,\infty)$

30. The conjugate of a complex number is $\frac{1}{i-1}$ then that complex number is **[2008]**

(a) $\frac{-1}{i-1}$ (b) $\frac{1}{i+1}$
(c) $\frac{-1}{i+1}$ (d) $\frac{1}{i-1}$

31. The quadratic equations $x^2-6x+a=0$ and $x^2-cx+6=0$ have one root in common. The other roots of the first and second equations are integers in the ratio 4 : 3. Then the common root is **[2009]**

(a) 1 (b) 4
(c) 3 (d) 2

32. If the roots of the equation $bx^2+cx+a=0$ be imaginary, then for all real values of x, the expression $3b^2x^2+6bcx+2c^2$ is : **[2009]**

(a) less than $4ab$
(b) greater than $-4ab$
(c) less than $-4ab$
(d) greater than $4ab$

33. If $\left|z-\frac{4}{z}\right|=2$, then the maximum value of $|z|$ is equal to : **[2009]**

(a) $\sqrt{5}+1$ (b) 2
(c) $2+\sqrt{2}$ (d) $\sqrt{3}+1$

34. The number of complex numbers z such that $|z-1|=|z+1|=|z-i|$ equals **[2010]**

(a) 1 (b) 2
(c) ∞ (d) 0

35. If α and β are the roots of the equation $x^2-x+1=0$, then $\alpha^{2009}+\beta^{2009}=$ **[2010]**

(a) -1 (b) 1
(c) 2 (d) -2

36. Let α, β be real and z be a complex number. If $z^2+\alpha z+\beta=0$ has two distinct roots on the line $\text{Re } z=1$, then it is necessary that : **[2011]**

(a) $\beta\in(-1,0)$ (b) $|\beta|=1$
(c) $\beta\in(1,\infty)$ (d) $\beta\in(0,1)$

37. If $\omega(\neq 1)$ is a cube root of unity, and $(1+\omega)^7=A+B\omega$. Then (A, B) equals **[2011]**

(a) $(1,1)$ (b) $(1,0)$
(c) $(-1,1)$ (d) $(0,1)$

38. Sachin and Rahul attempted to solve a quadratic equation. Sachin made a mistake in writing down the constant term and ended up in roots (4,3). Rahul made a mistake in writing down coefficient of x to get roots (3,2). The correct roots of equation are : **[2011 RS]**

(a) 6, 1 (b) 4, 3

(c) – 6, – 1 (d) – 4, – 3

39. Let for $a \neq a_1 \neq 0$,

$f(x) = ax^2 + bx + c,\ g(x) = a_1x^2 + b_1x + c_1$

and $p(x) = f(x) - g(x)$. If $p(x) = 0$ only for $x = -1$ and $p(-2) = 2$, then the value of $p(2)$ is : **[2011 RS]**

(a) 3 (b) 9

(c) 6 (d) 18

40. If $z \neq 1$ and $\frac{z^2}{z-1}$ is real, then the point represented by the complex number z lies : **[2012]**

(a) either on the real axis or on a circle passing through the origin.

(b) on a circle with centre at the origin

(c) either on the real axis or on a circle not passing through the origin.

(d) on the imaginary axis.

41. If the equations $x^2 + 2x + 3 = 0$ and $ax^2 + bx + c = 0$, $a, b, c \in R$, have a common root, then $a : b : c$ is **[2013]**

(a) 1 : 2 : 3 (b) 3 : 2 : 1

(c) 1 : 3 : 2 (d) 3 : 1 : 2

42. If z is a complex number of unit modulus and argument θ, then $\arg\left(\frac{1+z}{1+\bar{z}}\right)$ equals: **[2013]**

(a) $-\theta$ (b) $\frac{\pi}{2} - \theta$

(c) θ (d) $\pi - \theta$

43. If z is a complex number such that $|z| \geq 2$, then the minimum value of $\left|z + \frac{1}{2}\right|$: **[2014]**

(a) is strictly greater than $\frac{5}{2}$

(b) is strictly greater than $\frac{3}{2}$ but less than $\frac{5}{2}$

(c) is equal to $\frac{5}{2}$

(d) lie in the interval $(1, 2)$

44. If $a \in R$ and the equation

$-3(x-[x])^2 + 2(x-[x]) + a^2 = 0$, (where $[x]$ denotes the greatest integer $\leq x$) has no integral solution, then all possible values of a lie in the interval: **[2014]**

(a) $(-2, -1)$

(b) $(-\infty, -2) \cup (2, \infty)$

(c) $(-1, 0) \cup (0, 1)$

(d) $(1, 2)$

45. A complex number z is said to be unimodular if $|z| = 1$. Suppose z_1 and z_2 are complex numbers such that $\frac{z_1 - 2z_2}{2 - z_1\bar{z}_2}$ is unimodular and z_2 is not unimodular. Then the point z_1 lies on a: **[2015]**

(a) circle of radius 2.

(b) circle of radius $\sqrt{2}$.

(c) straight line parallel to x-axis

(d) straight line parallel to y-axis.

46. Let α and β be the roots of equation $x^2 - 6x - 2 = 0$. If $a_n = \alpha^n - \beta^n$, for $n \geq 1$, then the value of $\frac{a_{10} - 2a_8}{2a_9}$ is equal to : **[2015]**

(a) 3 (b) – 3

(c) 6 (d) – 6

47. The sum of all real values of x satisfying the equation $(x^2 - 5x + 5)^{x^2+4x-60} = 1$ is : **[2016]**

(a) 6 (b) 5

(c) 3 (d) – 4

48. A value of θ for which $\frac{2 + 3i\sin\theta}{1 - 2i\sin\theta}$ is purely imaginary, is: **[2016]**

(a) $\sin^{-1}\left(\frac{\sqrt{3}}{4}\right)$ (b) $\sin^{-1}\left(\frac{1}{\sqrt{3}}\right)$

(c) $\frac{\pi}{3}$ (d) $\frac{\pi}{6}$

49. If, for a positive integer n, the quadratic equation, $x(x+1)+(x+1)(x+2)+.....+(x+\overline{n-1})(x+n)=10n$ has two consecutive integral solutions, then n is equal to : **[2017]**

(a) 11 (b) 12
(c) 9 (d) 10

50. If $\alpha, \beta \in C$ are the distinct roots, of the equation $x^2 - x + 1 = 0$, then $\alpha^{101} + \beta^{107}$ is equal to : **[2018]**

(a) 0 (b) 1
(c) 2 (d) – 1

51. Let p, q ∈ R. If $2-\sqrt{3}$ is a root of the quadratic equation, $x^2 + px + q = 0$, then: **[2019]**

(a) $p^2 - 4q + 12 = 0$ (b) $q^2 - 4p - 16 = 0$
(c) $q^2 + 4p + 14 = 0$ (d) $p^2 - 4q - 12 = 0$

52. Let

$$A = \left\{\theta \in \left(-\frac{\pi}{2}, \pi\right) : \frac{3+2i\sin\theta}{1-2i\sin\theta} \text{ is purely imaginary}\right\}$$

Then the sum of the elements in A is: **[2019]**

(a) $\frac{5\pi}{6}$ (b) π
(c) $\frac{3\pi}{4}$ (d) $\frac{2\pi}{3}$

53. Let α and β be two roots of the equation $x^2 + 2x + 2 = 0$, then $\alpha^{15} + \beta^{15}$ is equal to: **[2019]**

(a) –256 (b) 512
(c) –512 (d) 256

54. Let α and β be two real roots of the equation $(k+1)\tan^2 x - \sqrt{2}\,.\,\lambda \tan x = (1-k)$, where $k(\neq -1)$ and λ are real numbers. If $\tan^2(\alpha+\beta) = 50$, then a value of λ is: **[2020]**

(a) $10\sqrt{2}$ (b) 10
(c) 5 (d) $5\sqrt{2}$

Answer Key

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
(b)	(c)	(b)	(a)	(a)	(a)	(a)	(a)	(d)	(b)	(b)	(c)	(c)	(a)	(b)
16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
(c)	(d)	(a)	(c)	(c)	(c)	(b)	(c)	(d)	(d)	(b)	(c)	(a)	(c)	(c)
31	32	33	34	35	36	37	38	39	40	41	42	43	44	45
(d)	(b)	(a)	(a)	(b)	(c)	(a)	(a)	(d)	(a)	(a)	(c)	(d)	(c)	(a)
46	47	48	49	50	51	52	53	54						
(a)	(c)	(b)	(a)	(b)	(d)	(d)	(a)	(b)						

Solutions

1. **(b)** Let $|z| = |\omega| = r$

$\therefore z = re^{i\theta}, \omega = re^{i\phi}$ where $\theta + \phi = \pi$.

$\therefore z = re^{i(\pi-\phi)} = re^{i\pi}\,.\,e^{-i\phi} = -re^{-i\phi} = -\overline{\omega}$.

$[\because e^{i\pi} = -1 \text{ and } \overline{\omega} = re^{-i\phi}]$

2. **(c)** Given that $|z-4| < |z-2|$

Let $z = x + iy$

$\Rightarrow |(x-4)+iy)| < |(x-2)+iy|$

$\Rightarrow (x-4)^2 + y^2 < (x-2)^2 + y^2$

$\Rightarrow x^2 - 8x + 16 < x^2 - 4x + 4 \Rightarrow 12 < 4x$

$\Rightarrow x > 3 \Rightarrow \text{Re}(z) > 3$

3. **(b)** Let the circle be $|z - z_0| = r$. Then according to given conditions $|z_0 - z_1| = r + a$ ...(i)

$|z_0 - z_2| = r + b$ ...(ii)

Subtract (ii) from (i)

we get $|z_0 - z_1| - |z_0 - z_2| = a - b$.

$\therefore$ Locus of centre z_0 is $|z - z_1| - |z - z_2| = a - b$, which represents a hyperbola.

4. **(a)** Given that $\alpha^2 = 5\alpha - 3$ and $\beta^2 = 5\beta - 3$;
$\Rightarrow \alpha$ & β are roots of equation, $x^2 = 5x - 3$
or $x^2 - 5x + 3 = 0$
$\therefore \alpha + \beta = 5$ and $\alpha\beta = 3$

Thus, the equation whose roots are $\frac{\alpha}{\beta}$ and $\frac{\beta}{\alpha}$ is

$$x^2 - x\left(\frac{\alpha}{\beta} + \frac{\beta}{\alpha}\right) + \frac{\alpha\beta}{\alpha\beta} = 0$$

$$\Rightarrow x^2 - x\left(\frac{\alpha^2 + \beta^2}{\alpha\beta}\right) + 1 = 0$$

or $3x^2 - 19x + 3 = 0$

5. **(a)** Let α and β are roots of the equation $x^2 + ax + b = 0$ and γ and δ be the roots of the equation $x^2 + bx + a = 0$ respectively.
$\therefore \alpha + \beta = -a, \alpha\beta = b$ and $\gamma + \delta = -b, \gamma\delta = a$.
Given $|\alpha - \beta| = |\gamma - \delta| \Rightarrow (\alpha - \beta)^2 = (\gamma - \delta)^2$
$\Rightarrow (\alpha + \beta)^2 - 4\alpha\beta = (\gamma + \delta)^2 - 4\gamma\delta$
$\Rightarrow a^2 - 4b = b^2 - 4a$
$\Rightarrow (a^2 - b^2) + 4(a - b) = 0$
$\Rightarrow a + b + 4 = 0 \ (\because a \neq b)$

6. **(a)** Product of real roots

$$= \frac{c}{a} = \frac{9}{t^2} > 0, \ \forall \ t \in R$$

$\therefore$ Product of real roots is always positive.

7. **(a)** $p + q = -p \Rightarrow q = 2p$
and $pq = q \Rightarrow q(p - 1) = 0$
$\Rightarrow q = 0$ or $p = 1$.
If $q = 0$, then $p = 0$.
or $p = 1$, then $q = -2$.

8. **(a)** $|\bar{z}\omega| = |\bar{z}||\omega| = |z||\omega| = |z\omega| = 1 \ [\because |\bar{z}| = |z|]$

$\text{Arg}(\bar{z}\omega) = \arg(\bar{z}) + \arg(\omega)$

$= -\arg(z) + \arg\omega = -\frac{\pi}{2}$

$[\because \arg(\bar{z}) = -\arg(z)]$

$\therefore \bar{z}\omega = -1$

9. **(d)** Given that $Z^2 + aZ + b = 0$;
$Z_1 + Z_2 = -a$ & $Z_1 Z_2 = b$
$0, Z_1, Z_2$ form an equilateral triangle
$\therefore 0^2 + Z_1^2 + Z_2^2 = 0.Z_1 + Z_1.Z_2 + Z_2.0$
(for an equilateral triangle,
$Z_1^2 + Z_2^2 + Z_3^2 = Z_1Z_2 + Z_2Z_3 + Z_3Z_1$)
$\Rightarrow Z_1^2 + Z_2^2 = Z_1Z_2$
$\Rightarrow (Z_1 + Z_2)^2 = 3Z_1Z_2$
$\therefore a^2 = 3b$

10. **(b)** Given that

$$\left(\frac{1+i}{1-i}\right)^x = 1 \Rightarrow \left[\frac{(1+i)^2}{1-i^2}\right]^x = 1$$

$$\left(\frac{1+i^2+2i}{1+1}\right)^x = 1 \Rightarrow (i)^x = 1; \ \therefore x = 4n; \ n \in I^+$$

11. **(b)** Let one roots of given equation be α
$\therefore$ Second roots be 2α then

$$\alpha + 2\alpha = 3\alpha = \frac{1-3a}{a^2 - 5a + 3}$$

$$\Rightarrow \alpha = \frac{1-3a}{3\left(a^2 - 5a + 3\right)} \quad \text{....(i)}$$

and $\alpha.2\alpha = 2\alpha^2 = \frac{2}{a^2 - 5a + 3}$

$$\therefore 2\left[\frac{1}{9}\frac{(1-3a)^2}{(a^2 - 5a + 3)^2}\right] = \frac{2}{a^2 - 5a + 3}$$

[from (i)]

$$\frac{(1-3a)^2}{(a^2 - 5a + 3)} = 9$$

$$\Rightarrow 9a^2 - 6a + 1 = 9a^2 - 45a + 27$$

$$\Rightarrow 39a = 26 \Rightarrow a = \frac{2}{3}$$

12. **(c)** Given that
$x^2 - 3|x| + 2 = 0 \Rightarrow |x|^2 - 3|x| + 2 = 0$
$\Rightarrow (|x| - 2)(|x| - 1) = 0$
$\Rightarrow |x| = 1, 2 \Rightarrow x = \pm 1, \pm 2$
$\therefore$ No. of solution $= 4$

13. **(c)** Given that $\arg zw = \pi$
$\Rightarrow \arg z + \arg w = \pi$ ...(i)
$\bar{z} + i\bar{w} = 0 \Rightarrow \bar{z} = -i\bar{w}$
Replace i by $-i$, we get
$\therefore z = iw \Rightarrow \arg z = \frac{\pi}{2} + \arg w$

$\Rightarrow \arg z = \frac{\pi}{2} + \pi - \arg z$ (from (i))

$\therefore \arg z = \frac{3\pi}{4}$

14. (a) Given that $z^{\frac{1}{3}} = p + iq$

$\Rightarrow z = p^3 + (iq)^3 + 3p(iq)(p + iq)$

$\Rightarrow x - iy = p^3 - 3pq^2 + i(3p^2q - q^3)$

Comparing both side, we get

$\therefore x = p^3 - 3pq^2 \Rightarrow \frac{x}{p} = p^2 - 3q^2$ ...(i)

and $y = q^3 - 3p^2q \Rightarrow \frac{y}{q} = q^2 - 3p^2$ ...(ii)

Adding (i) and (ii), we get

$\therefore \frac{x}{p} + \frac{y}{q} = -2p^2 - 2q^2$

$\therefore \left(\frac{x}{p} + \frac{y}{q}\right) \Big/ (p^2 + q^2) = -2$

15. (b) Given that

$|z^2 - 1| = |z|^2 + 1 \Rightarrow |z^2 - 1|^2 = (z\bar{z} + 1)^2$ $[\because |z|^2 = z\bar{z}]$

$\Rightarrow (z^2 - 1)(\bar{z}^2 - 1) = (z\bar{z} + 1)^2 \ (\because \overline{z_1 - z_2} = \bar{z}_1 - \bar{z}_2)$

$\Rightarrow z^2\bar{z}^2 - z^2 - \bar{z}^2 + 1 = z^2\bar{z}^2 + 2z\bar{z} + 1$

$\Rightarrow z^2 + 2z\bar{z} + \bar{z}^2 = 0$

$\Rightarrow (z + \bar{z})^2 = 0 \Rightarrow z = -\bar{z}$

$\Rightarrow z$ is purely imaginary

16. (c) Let the second root be α.

Then $\alpha + (1 - p) = -p \Rightarrow \alpha = -1$

Also $\alpha.(1 - p) = 1 - p$

$\Rightarrow (\alpha - 1)(1 - p) = 0 \Rightarrow p = 1 \ [\because \alpha = -1]$

$\therefore$ Roots are $\alpha = -1$ and $1 - p = 0$

17. (d) Given that 4 is a root of $x^2 + px + 12 = 0$

$\Rightarrow 16 + 4p + 12 = 0 \Rightarrow p = -7$

Now, the equation $x^2 + px + q = 0$ has equal roots.

$\therefore D = 0$

$\Rightarrow p^2 - 4q = 0 \Rightarrow q = \frac{p^2}{4} = \frac{49}{4}$

18. (a) Given equation is $x^2 - (a - 2)x - a - 1 = 0$

$\Rightarrow \quad \alpha + \beta = a - 2;\ \alpha\beta = -(a + 1)$

$\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$

$= a^2 - 2a + 6 = (a - 1)^2 + 5$

For min. value of $\alpha^2 + \beta^2$, $a - 1 = 0$

$\Rightarrow \quad a = 1.$

19. (c) $\because (x - 1)^3 + 8 = 0 \Rightarrow (x - 1) = (-2)\,(1)^{1/3}$

$\Rightarrow x - 1 = -2$ or -2ω or $-2\omega^2$

or $x = -1$ or $1 - 2\omega$ or $1 - 2\omega^2$.

20. (c) $|z_1 + z_2| = |z_1| + |z_2| \Rightarrow z_1$ and z_2 are collinear and are to the same side of origin; hence $\arg z_1 - \arg z_2 = 0$.

21. (c) Given that $w = \dfrac{z}{z - \frac{1}{3}i}$

$\Rightarrow |w| = \dfrac{|z|}{|z - \frac{1}{3}i|} = 1 \quad \left[\because \left|\frac{z_1}{z_2}\right| = \frac{|z_1|}{|z_2|}\right]$

$\Rightarrow |z| = \left|z - \frac{1}{3}i\right|$

$\Rightarrow$ distance of z from origin and point $\left(0, \frac{1}{3}\right)$ is same hence z lies on bisector of the line joining points (0, 0) and (0, 1/3).

Hence z lies on a straight line.

22. (b) $\tan\left(\frac{P}{2}\right), \tan\left(\frac{Q}{2}\right)$ are the roots of

$ax^2 + bx + c = 0$

$\tan\left(\frac{P}{2}\right) + \tan\left(\frac{Q}{2}\right) = -\frac{b}{a}$

$\tan\left(\frac{P}{2}\right) \cdot \tan\left(\frac{Q}{2}\right) = \frac{c}{a}$

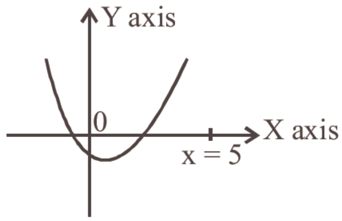
$$\frac{\tan\left(\frac{P}{2}\right) + \tan\left(\frac{Q}{2}\right)}{1 - \tan\left(\frac{P}{2}\right)\tan\left(\frac{Q}{2}\right)} = \tan\left(\frac{P}{2} + \frac{Q}{2}\right) = 1$$

$\left[\because P + Q = \frac{\pi}{2}\right]$

$$\Rightarrow \frac{-\frac{b}{a}}{1-\frac{c}{a}}=1 \Rightarrow -\frac{b}{a}=\frac{a-c}{a}$$

$$\Rightarrow -b=a-c \Rightarrow c=a+b$$

23. (c) Given that both roots of quadratic equation are less than 5



then (i) Discriminant ≥ 0

$4k^2-4(k^2+k-5)\geq 0$

$4k^2-4k^2-4k+20\geq 0$

$4k\leq 20 \Rightarrow k\leq 5$

(ii) $p(5)>0$

$\Rightarrow f(5)>0$; $25-10k+k^2+k-5>0$

$\Rightarrow k^2-9k+20>0$

$\Rightarrow k(k-4)-5(k-4)>0$

$\Rightarrow (k-5)(k-4)>0$

$\Rightarrow k\in(-\infty,4)\cup(5,\infty)$

(iii) $\dfrac{\text{Sum of roots}}{2}<5$

$\Rightarrow -\dfrac{b}{2a}=\dfrac{2k}{2}<5$

$\Rightarrow$ k < 5

The intersection of (i), (ii) & (iii) gives $k\in(-\infty,4)$.

24. (d) $\displaystyle\sum_{k=1}^{10}\left(\sin\frac{2k\pi}{11}+i\cos\frac{2k\pi}{11}\right)$

$$=i\sum_{k=1}^{10}\left(\cos\frac{2k\pi}{11}-i\sin\frac{2k\pi}{11}\right)$$

$$\left[\because e^{i\theta}=\cos\theta+i\sin\theta\right]$$

$$=i\sum_{k=1}^{10}e^{-\frac{2k\pi}{11}i}=i\left\{\sum_{k=0}^{10}e^{-\frac{2k\pi}{11}i}-1\right\}$$

$$=i\left[1+e^{-\frac{2\pi}{11}i}+e^{-\frac{4\pi}{11}i}+\text{....11 terms}\right]-i$$

$$=i\left[\frac{1-\left(e^{-\frac{2\pi}{11}}\right)^{11}}{1-e^{-\frac{2\pi}{11}i}}\right]-i=i\left[\frac{1-e^{-2\pi i}}{1-e^{-\frac{2\pi}{11}i}}\right]-i$$

$$=i\times 0-i \qquad [\because e^{-2\pi i}=1]$$

$$=-i$$

25. (d) $z^2+z+1=0 \Rightarrow$ z = ω or ω^2

So, $z+\dfrac{1}{z}=\omega+\omega^2=-1$

$$\left[\because \frac{1}{z}=\omega^2 \text{ and } 1+\omega+\omega^2=0\right]$$

$z^2+\dfrac{1}{z^2}=\omega^2+\omega=-1, \qquad [\because \omega^3=1]$

$z^3+\dfrac{1}{z^3}=\omega^3+\omega^3=2$

$z^4+\dfrac{1}{z^4}=-1,\ z^5+\dfrac{1}{z^5}=-1$

and $z^6+\dfrac{1}{z^6}=2$

$\therefore$ The given sum = 1+1 + 4 + 1 + 1 + 4 = 12

26. (b) Given that $x^2+px+q=0$

Sum of roots = $\tan 30°+\tan 15°=-p$

Product of roots = $\tan 30°.\tan 15°=q$

$$\tan 45°=\frac{\tan 30°+\tan 15°}{1-\tan 30°.\tan 15°}\Rightarrow\frac{-p}{1-q}=1$$

$\Rightarrow -p=1-q \Rightarrow q-p=1$

$\therefore 2+q-p=3$

27. (c) Given equation is $x^2-2mx+m^2-1=0$

$\Rightarrow (x-m)^2-1=0$

$\Rightarrow (x-m+1)(x-m-1)=0$

$\Rightarrow x=m-1, m+1$

$m-1>-2$ and $m+1<4$

$\Rightarrow m>-1$ and $m<3 \Rightarrow -1<m<3$

28. (a) $|z+1| = |z+4-3|$

$\leq |z+4| + |-3| \leq |3| + |-3|$

$\Rightarrow |z+1| \leq 6 \Rightarrow |z+1|_{max} = 6$

29. (c) Let α and β are roots of the equation $x^2 + ax + 1 = 0$

$\alpha + \beta = -a$ and $\alpha\beta = 1$

Given that $|\alpha - \beta| < \sqrt{5}$

$\Rightarrow \sqrt{(\alpha+\beta)^2 - 4\alpha\beta} < \sqrt{5}$

$\left(\because (\alpha-\beta)^2 = (\alpha+\beta)^2 - 4\alpha\beta\right)$

$\Rightarrow \sqrt{a^2-4} < \sqrt{5} \Rightarrow a^2 - 4 < 5$

$\Rightarrow a^2 - 9 < 0 \Rightarrow a^2 < 9 \Rightarrow -3 < a < 3$

$\Rightarrow a \in (-3, 3)$

30. (c) $\overline{\left(\frac{1}{i-1}\right)} = \frac{1}{\overline{(i-1)}} = \frac{1}{-i-1} = \frac{-1}{i+1}$

31. (d) Let the roots of equation $x^2 - 6x + a = 0$ be α and 4β and that of the equation $x^2 - cx + 6 = 0$ be α and 3β. Then

$\alpha + 4\beta = 6$ ...(i) $4\alpha\beta = a$ ...(ii)

and $\alpha + 3\beta = c$ ...(iii) $3\alpha\beta = 6$ ...(iv)

$\Rightarrow a = 8$ (from (ii) and (iv))

$\therefore$ The equation becomes $x^2 - 6x + 8 = 0$

$\Rightarrow (x-2)(x-4) = 0$

$\Rightarrow$ roots are 2 and 4

$\Rightarrow \alpha = 2, \beta = 1$ $\therefore$ Common root is 2.

32. (b) Given that roots of the equation $bx^2 + cx + a = 0$ are imaginary

$\therefore c^2 - 4ab < 0$ ....(i)

Let $y = 3b^2x^2 + 6bcx + 2c^2$

$\Rightarrow 3b^2x^2 + 6bcx + 2c^2 - y = 0$

As x is real, $D \geq 0$

$\Rightarrow 36b^2c^2 - 12b^2(2c^2 - y) \geq 0$

$\Rightarrow 12b^2(3c^2 - 2c^2 + y) \geq 0$ $[\because b^2 \geq 0]$

$\Rightarrow c^2 + y \geq 0 \Rightarrow y \geq -c^2$

But from eqn. (i), $c^2 < 4ab$ or $-c^2 > -4ab$

$\therefore$ we get $y \geq -c^2 > -4ab$

$\Rightarrow y > -4ab$

33. (a) Given that $\left|z - \frac{4}{z}\right| = 2$

$|z| = \left|z - \frac{4}{z} + \frac{4}{z}\right| \leq \left|z - \frac{4}{z}\right| + \frac{4}{|z|}$

$\Rightarrow |z| \leq 2 + \frac{4}{|z|}$

$\Rightarrow |z|^2 - 2|z| - 4 \leq 0$

$\Rightarrow \left(|z| - \frac{2+\sqrt{20}}{2}\right)\left(|z| - \frac{2-\sqrt{20}}{2}\right) \leq 0$

$\Rightarrow \left(|z| - (1+\sqrt{5})\right)\left(|z| - (1-\sqrt{5})\right) \leq 0$

$-\infty$ + $(1-\sqrt{5})$ − $(1+\sqrt{5})$ + ∞

$\Rightarrow \left(-\sqrt{5}+1\right) \leq |z| \leq \left(\sqrt{5}+1\right)$

$\Rightarrow |z|_{max} = \sqrt{5} + 1$

34. (a) Let $z = x + iy$

$|z-1| = |z+1| \Rightarrow (x-1)^2 + y^2 = (x+1)^2 + y^2$

$\Rightarrow x = 0 \Rightarrow \text{Re } z = 0$

$|z-1| = |z-i| \Rightarrow (x-1)^2 + y^2 = x^2 + (y-1)^2$

$\Rightarrow x = y$

$|z+1| = |z-i| \Rightarrow (x+1)^2 + y^2 = x^2 + (y-1)^2$

$\Rightarrow x = -y$

Only (0, 0) will satisfy all conditions.

$\Rightarrow$ Number of complex number $z = 1$

35. (b) $x^2 - x + 1 = 0 \Rightarrow x = \frac{1 \pm \sqrt{1-4}}{2}$

$x = \frac{1 \pm \sqrt{3}\,i}{2}$

$\alpha = \frac{1}{2} + i\frac{\sqrt{3}}{2} = -\omega^2$

$\beta = \frac{1}{2} - \frac{i\sqrt{3}}{2} = -\omega$

$\alpha^{2009} + \beta^{2009} = (-\omega^2)^{2009} + (-\omega)^{2009}$

$= -\omega^2 - \omega = 1$ $[\because \omega^3 = 1]$

36. (c) Since both the roots of given quadratic equation lie in the line $Re\, z = 1$ i.e., $x = 1$, hence real part of both the roots are 1.

Let both roots be $1 + i\alpha$ and $1 - i\alpha$

Product of the roots, $1 + \alpha^2 = \beta$

$\because \alpha^2 + 1 \geq 1$

$\therefore \beta \geq 1 \Rightarrow \because \beta \in (1, \infty)$

37. (a) $(1+\omega)^7 = A + B\omega$
$(-\omega^2)^7 = A + B\omega$ $(\because \omega^{14} = \omega^{12}.\omega^2 = \omega^2)$
$-\omega^2 = A + B\omega$
$1 + \omega = A + B\omega$
$\Rightarrow A = 1, B = 1.$

38. (a) Let the correct equation be

$ax^2 + bx + c = 0$
Now, Sachin's equation

$ax^2 + bx + c' = 0$
Given that, roots found by Sachin's are 4 and 3

$\Rightarrow -\dfrac{b}{a} = 7$ ....(i)

Rahul's equation, $ax^2 + b'x + c = 0$
Given that roots found by Rahul's are 3 and 2

$\Rightarrow \dfrac{c}{a} = 6$ ...(ii)

From (i) and (ii), roots of the correct equation $x^2 - 7x + 6 = 0$ are 6 and 1.

39. (d) $p(x) = 0$

$\Rightarrow f(x) = g(x)$

$\Rightarrow ax^2 + bx + c = a_1x^2 + b_1x + c_1$

$\Rightarrow (a-a_1)x^2 + (b-b_1)x + (c-c_1) = 0.$

It has only one solution, $x = -1$

$\Rightarrow b - b_1 = a - a_1 + c - c_1$ ...(i)

Sum of roots $\dfrac{-(b-b_1)}{(a-a_1)} = -1-1$

$\Rightarrow \dfrac{b-b_1}{2(a-a_1)} = 1$

$\Rightarrow b - b_1 = 2(a-a_1)$ ...(ii)

Now $p(-2) = 2$
$\Rightarrow f(-2) - g(-2) = 2$
$\Rightarrow 4a - 2b + c - 4a_1 + 2b_1 - c_1 = 2$
$\Rightarrow 4(a-a_1) - 2(b-b_1) + (c-c_1) = 2$ ...(iii)
From equations, (i), (ii) and (iii)

$a - a_1 = c - c_1 = \dfrac{1}{2}(b-b_1) = 2$

Now, $p(2) = f(2) - g(2)$

$= 4(a-a_1) + 2(b-b_1) + (c-c_1)$

$= 8 + 8 + 2 = 18$

40. (a) $\dfrac{z^2}{z-1} = \dfrac{\bar{z}^2}{\bar{z}-1}$ $\left[\because \overline{\left(\dfrac{z_1}{z_2}\right)} = \dfrac{\overline{z_1}}{\overline{z_2}}\right]$

$\Rightarrow z\bar{z}z - z^2 = z.\bar{z}.\bar{z} - \bar{z}^2$

$\Rightarrow |z|^2.z - z^2 = |z|^2.\bar{z} - \bar{z}^2$

$\Rightarrow |z|^2(z-\bar{z}) - (z-\bar{z})(z+\bar{z}) = 0$

$\Rightarrow (z-\bar{z})\left(|z|^2 - (z+\bar{z})\right) = 0$

Either $z - \bar{z} = 0$ or $|z|^2 - (z+\bar{z}) = 0$

Either $z = \bar{z} \Rightarrow$ real axis

or $|z|^2 = z + \bar{z} \Rightarrow z\bar{z} - z - \bar{z} = 0$

represents a circle passing through origin.

41. (a) Given equations are
$x^2 + 2x + 3 = 0$ …(i)
$ax^2 + bx + c = 0$ …(ii)
Roots of equation (i) are imaginary roots in order pair.
According to the question (ii) will also have both roots same as (i). Thus

$\dfrac{a}{1} = \dfrac{b}{2} = \dfrac{c}{3} = \lambda$ (say)
$\Rightarrow a = \lambda, b = 2\lambda, c = 3\lambda$
Hence, required ratio is 1 : 2 : 3

42. (c) Given $|z| = 1$, $\arg z = \theta$

$\Rightarrow \bar{z} = \dfrac{1}{z}$

$\therefore \arg\left(\dfrac{1+z}{1+\bar{z}}\right) = \arg\left(\dfrac{1+z}{1+\dfrac{1}{z}}\right) = \arg(z) = \theta.$

43. (d) We know minimum value of $|Z_1 + Z_2|$ is

$||Z_1| - |Z_2||$. Thus minimum value of $\left|Z + \dfrac{1}{2}\right|$

is $\left||Z| - \dfrac{1}{2}\right| \le \left|Z + \dfrac{1}{2}\right| \le |Z| + \dfrac{1}{2}$

Since, $|Z| \ge 2$ therefore

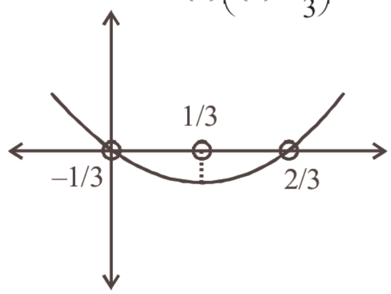
$$2 - \frac{1}{2} < \left|Z + \frac{1}{2}\right| < 2 + \frac{1}{2}$$

$$\Rightarrow \frac{3}{2} < \left|Z + \frac{1}{2}\right| < \frac{5}{2}$$

44. (c) Consider $-3(x-[x])^2 + 2[x-[x]) + a^2 = 0$
$\Rightarrow 3\{x\}^2 - 2\{x\} - a^2 = 0$
$(\because x - [x] = \{x\})$

$$\Rightarrow \quad 3\left(\{x\}^2 - \frac{2}{3}\{x\}\right) = a^2, a \neq 0$$

$$\Rightarrow \quad a^2 = 3\{x\}\left(\{x\} - \frac{2}{3}\right)$$



Now, $\{x\} \in (0,1)$ and $\frac{-2}{3} \leq a^2 < 1$

(by graph)

Since , x is not an integer

$\therefore \quad a \in (-1,1) - \{0\}$

$\Rightarrow a \in (-1,0) \cup (0,1)$

45. **(a)** $\left|\frac{z_1 - 2z_2}{2 - z_1\overline{z}_2}\right| = 1$

$$\Rightarrow \quad |z_1 - 2z_2|^2 = |2 - z_1\overline{z}_2|^2$$

$$\Rightarrow \quad (z_1 - 2z_2)\overline{(z_1 - 2z_2)} = (2 - z_1\overline{z}_2)\overline{(2 - z_1\overline{z}_2)}$$

$$\Rightarrow \quad (z_1 - 2z_2)(\overline{z}_1 - 2\overline{z}_2) = (2 - z_1\overline{z}_2)(2 - \overline{z}_1 z_2)$$

$$\Rightarrow \quad (z_1\overline{z}_1) - 2z_1\overline{z}_2 - 2\overline{z}_1 z_2 + 4z_2\overline{z}_2$$

$$= 4 - 2\overline{z}_1 z_2 - 2z_1\overline{z}_2 + z_1\overline{z}_1 z_2\overline{z}_2$$

$$\Rightarrow \quad |z_1|^2 + 4|z_2|^2 = 4 + |z_1|^2 |z_2|^2$$

$$\Rightarrow \quad |z_1|^2 + 4|z_2|^2 - 4 - |z_1|^2 |z_2|^2 = 0$$

$$\left(|z_1|^2 - 4\right)\left(1 - |z_2|^2\right) = 0$$

$\because \quad |z_2| \neq 1$

$\therefore \quad |z_1|^2 = 4$

$\Rightarrow \quad |z_1| = 2$

$\Rightarrow$ Point z_1 lies on circle of radius 2.

46. **(a)** $\alpha, \beta = \frac{6 \pm \sqrt{36+8}}{2} = 3 \pm \sqrt{11}$

$\alpha = 3 + \sqrt{11}, \beta = 3 - \sqrt{11}$

$\therefore a_n = \left(3+\sqrt{11}\right)^n - \left(3-\sqrt{11}\right)^n$

$$\frac{a_{10} - 2a_8}{2a_9}$$

$$= \frac{\left(3+\sqrt{11}\right)^{10} - \left(3-\sqrt{11}\right)^{10} - 2\left(3+\sqrt{11}\right)^8 + 2\left(3-\sqrt{11}\right)^8}{2\left[\left(3+\sqrt{11}\right)^9 - \left(3-\sqrt{11}\right)^9\right]}$$

$$= \frac{\left(3+\sqrt{11}\right)^8\left[\left(3+\sqrt{11}\right)^2 - 2\right] + \left(3-\sqrt{11}\right)^8\left[2 - \left(3-\sqrt{11}\right)^2\right]}{2\left[\left(3+\sqrt{11}\right)^9 - \left(3-\sqrt{11}\right)^9\right]}$$

$$= \frac{\left(3+\sqrt{11}\right)^8\left(9+11+6\sqrt{11}-2\right) + \left(3-\sqrt{11}\right)^8\left(2-9-11+6\sqrt{11}\right)}{2\left[\left(3+\sqrt{11}\right)^9 - \left(3-\sqrt{11}\right)^9\right]}$$

$$= \frac{6\left(3+\sqrt{11}\right)^9 - 6\left(3-\sqrt{11}\right)^9}{2\left[\left(3+\sqrt{11}\right)^9 - \left(3-\sqrt{11}\right)^9\right]} = \frac{6}{2} = 3$$

47. **(c)** $(x^2 - 5x + 5)^{x^2+4x-60} = 1$

Case I

$x^2 - 5x + 5 = 1$ and $x^2 + 4x - 60$ can be any real number

$\Rightarrow x = 1, 4$

Case II

$x^2 - 5x + 5 = -1$ and $x^2 + 4x - 60$ has to be an even number

$\Rightarrow x = 2, 3$

where 3 is rejected because for x = 3, $x^2 + 4x - 60$ is odd.

Case III

$x^2 - 5x + 5$ can be any real number and $x^2 + 4x - 60 = 0$

$\Rightarrow x = -10, 6$

$\Rightarrow$ Sum of all values of x

$= -10 + 6 + 2 + 1 + 4 = 3$

48. **(b)** Rationalizing the given expression

$$\frac{(2+3i\sin\theta)(1+2i\sin\theta)}{1+4\sin^2\theta}$$

For the given expression to be purely imaginary, real part of the above expression should be equal to zero.

$$\Rightarrow \frac{2-6\sin^2\theta}{1+4\sin^2\theta} = 0 \quad \Rightarrow \sin^2\theta = \frac{1}{3}$$

$$\Rightarrow \sin\theta = \pm\frac{1}{\sqrt{3}} \Rightarrow \theta = \sin^{-1}\left(\frac{1}{\sqrt{3}}\right)$$

49. **(a)** We have

$$\sum_{r=1}^{n}(x+r-1)(x+r)=10n$$

$$\sum_{r=1}^{n}(x^2+xr+(r-1)x+r^2-r)=10n$$

$$\Rightarrow \sum_{r=1}^{n}(x^2+(2r-1)x+r(r-1))=10n$$

$\Rightarrow nx^2+\{1+3+5+....+(2n-1)\}x+\{1.2+2.3$
$+....+(n-1)n\}=10n$

$$\Rightarrow nx^2+n^2x+\frac{(n-1)n(n+1)}{3}=10n$$

$$\Rightarrow x^2+nx+\frac{n^2-31}{3}=0$$

Let α and $\alpha+1$ be its two solutions
($\because$ it has two consequtive integral solutions)

$\Rightarrow \alpha+(\alpha+1)=-n$

$$\Rightarrow \alpha=\frac{-n-1}{2} \quad ...(i)$$

$$\text{Also } \alpha(\alpha+1)=\frac{n^2-31}{3} \quad ...(ii)$$

Putting value of (i) in (ii), we get

$$-\left(\frac{n+1}{2}\right)\left(\frac{1-n}{2}\right)=\frac{n^2-31}{3}$$

$\Rightarrow n^2=121 \Rightarrow n=11$

50. **(b)** α, β are roots of $x^2-x+1=0$

$\therefore$ $\alpha=-\omega$ and $\beta=-\omega^2$
where ω is cube root of unity

$\therefore$ $\alpha^{101}+\beta^{107}=(-\omega)^{101}+(-\omega^2)^{107}$
$=-[\omega^2+\omega]=-[-1]=1$

51. **(d)** Since $2-\sqrt{3}$ is a root of the quadratic equation
$x^2+px+q=0$

$\therefore 2+\sqrt{3}$ is the other root

$\Rightarrow$ Sum of roots = 4, Product of roots = 1

$\Rightarrow p=-4, q=1$

$\Rightarrow p^2-4q-12=16-4-12=0$

52. **(d)** Suppose $z=\dfrac{3+2i\sin\theta}{1-2i\sin\theta}$

Since, z is purely imaginary, then $z+\bar{z}=0$

$$\Rightarrow \frac{3+2i\sin\theta}{1-2i\sin\theta}+\frac{3-2i\sin\theta}{1+2i\sin\theta}=0$$

$$\Rightarrow \frac{(3+2i\sin\theta)(1+2i\sin\theta)+(3-2i\sin\theta)(1-2i\sin\theta)}{1+4\sin^2\theta}$$
$=0$

$$\Rightarrow \sin^2\theta=\frac{3}{4} \Rightarrow \sin\theta=\frac{\sqrt{3}}{2}$$

$$\Rightarrow \theta=-\frac{\pi}{3},\frac{\pi}{3},\frac{2\pi}{3}$$

Now, the sum of elements in A

$$=-\frac{\pi}{3}+\frac{\pi}{3}+\frac{2\pi}{3}=\frac{2\pi}{3}$$

53. **(a)** Consider the equation

$x^2+2x+2=0$

$$x=\frac{-2\pm\sqrt{4-8}}{2}=-1\pm i$$

Let $\alpha=-1+i, \beta=-1-i$

$\alpha^{15}+\beta^{15}=(-1+i)^{15}+(-1-i)^{15}$

$$=\left(\sqrt{2}e^{i\frac{3\pi}{4}}\right)^{15}+\left(\sqrt{2}e^{-i\frac{3\pi}{4}}\right)^{15}$$

$$=\left(\sqrt{2}\right)^{15}\left[e^{\frac{i45\pi}{4}}+e^{\frac{-i45\pi}{4}}\right]$$

$$=\left(\sqrt{2}\right)^{15}.2\cos\frac{45\pi}{4}=\left(\sqrt{2}\right)^{15}.2\cos\frac{3\pi}{4}$$

$$=\frac{-2}{\sqrt{2}}\left(\sqrt{2}\right)^{15}=-2\left(\sqrt{2}\right)^{14}=-256$$

54. **(b)** $(k+1)\tan^2 x-\sqrt{2}\lambda\tan x+(k-1)=0$

$$\tan\alpha+\tan\beta=\frac{\sqrt{2}\lambda}{k+1} \quad \text{[Sum of roots]}$$

$$\tan\alpha\cdot\tan\beta=\frac{k-1}{k+1} \quad \text{[Product of roots]}$$

$$\therefore \quad \tan(\alpha+\beta)=\frac{\frac{\sqrt{2}\lambda}{k+1}}{1-\frac{k-1}{k+1}}=\frac{\sqrt{2}\lambda}{2}=\frac{\lambda}{\sqrt{2}}$$

$$\tan^2(\alpha+\beta)=\frac{\lambda^2}{2}=50$$

$\lambda=10.$

6 Linear Inequality

1. If a, b, c are distinct +ve real numbers and $a^2+b^2+c^2=1$ then $ab+bc+ca$ is **[2002]**
   (a) less than 1 (b) equal to 1
   (c) greater than 1 (d) any real no.

2. If x is real, the maximum value of $\frac{3x^2+9x+17}{3x^2+9x+7}$ is **[2006]**
   (a) $\frac{1}{4}$ (b) 41
   (c) 1 (d) $\frac{17}{7}$

3. **Statement-1 :** For every natural number $n \geq 2$,
$$\frac{1}{\sqrt{1}}+\frac{1}{\sqrt{2}}+\ldots\ldots+\frac{1}{\sqrt{n}}>\sqrt{n}.$$
Statement-2 : For every natural number $n \geq 2$,
$\sqrt{n(n+1)}<n+1.$ **[2008]**
   (a) Statement -1 is false, Statement-2 is true
   (b) Statement -1 is true, Statement-2 is true; Statement -2 is a correct explanation for Statement-1
   (c) Statement -1 is true, Statement-2 is true; Statement -2 is not a correct explanation for Statement-1
   (d) Statement -1 is true, Statement-2 is false

Answer Key

1	2	3										
(a)	(b)	(b)										

Solutions

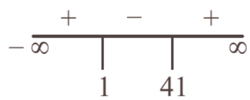
1. **(a)** $\because (a-b)^2+(b-c)^2+(c-a)^2>0$
$\Rightarrow 2(a^2+b^2+c^2-ab-bc-ca)>0$
$[\because a^2+b^2+c^2=1]$
$\Rightarrow 2>2(ab+bc+ca) \Rightarrow ab+bc+ca<1$

2. **(b)** $y=\frac{3x^2+9x+17}{3x^2+9x+7}$
$3x^2(y-1)+9x(y-1)+7y-17=0$
$D \geq 0 \because x$ is real

$$81(y-1)^2 - 4 \times 3(y-1)(7y-17) \geq 0$$

$$\Rightarrow \ (y-1)(y-41) \leq 0$$

$$\Rightarrow \ 1 \leq y \leq 41$$



$\therefore$ Max value of y is 41

3. (b) Given that statement 2 is

$$\sqrt{n(n+1)} < n+1, n \geq 2$$

$\Rightarrow \sqrt{n} < \sqrt{n+1}, n \geq 2$ which is true

$$\Rightarrow \sqrt{2} < \sqrt{3} < \sqrt{4} < \sqrt{5} < -------\sqrt{n} \quad ...(i)$$

Now, $\sqrt{2} < \sqrt{n} \Rightarrow \dfrac{1}{\sqrt{2}} > \dfrac{1}{\sqrt{n}}$ (from (i))

$\sqrt{3} < \sqrt{n} \Rightarrow \dfrac{1}{\sqrt{3}} > \dfrac{1}{\sqrt{n}}$; (from (i))

$\sqrt{n} \leq \sqrt{n} \Rightarrow \dfrac{1}{\sqrt{n}} \geq \dfrac{1}{\sqrt{n}}$ (from (i))

Also $\dfrac{1}{\sqrt{1}} > \dfrac{1}{\sqrt{n}}$ $\therefore$ Adding all, we get

$$\frac{1}{\sqrt{1}} + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} + ... + \frac{1}{n} > \frac{n}{\sqrt{n}} = \sqrt{n}$$

Hence both the statements are correct and statement 2 is a correct explanation of statement-1.

7 Permutations and Combinations

1. Total number of four digit odd numbers that can be formed using 0, 1, 2, 3, 5, 7 (using repetition allowed) are **[2002]**
(a) 216 (b) 375
(c) 400 (d) 720

2. Number greater than 1000 but less than 4000 is formed using the digits 0, 1, 2, 3, 4 (repetition allowed). Their number is **[2002]**
(a) 125 (b) 105
(c) 375 (d) 625

3. Five digit number divisible by 3 is formed using 0, 1, 2, 3, 4, 6 and 7 without repetition. Total number of such numbers are **[2002]**
(a) 312 (b) 3125
(c) 120 (d) 216

4. The sum of integers from 1 to 100 that are divisible by 2 or 5 is **[2002]**
(a) 3000 (b) 3050
(c) 3600 (d) 3250

5. If ${}^nC_r$ denotes the number of combination of n things taken r at a time, then the expression ${}^nC_{r+1} + {}^nC_{r-1} + 2 \times {}^nC_r$ equals **[2003]**
(a) ${}^{n+1}C_{r+1}$ (b) ${}^{n+2}C_r$
(c) ${}^{n+2}C_{r+1}$ (d) ${}^{n+1}C_r$.

6. A student is to answer 10 out of 13 questions in an examination such that he must choose at least 4 from the first five questions. The number of choices available to him is **[2003]**
(a) 346 (b) 140
(c) 196 (d) 280

7. The number of ways in which 6 men and 5 women can dine at a round table if no two women are to sit together is given by **[2003]**
(a) $6! \times 5!$ (b) 6×5
(c) 30 (d) 5×4

8. How many ways are there to arrange the letters in the word GARDEN with vowels in alphabetical order **[2004]**
(a) 480 (b) 240
(c) 360 (d) 120

9. The number of ways of distributing 8 identical balls in 3 distinct boxes so that none of the boxes is empty is **[2004]**
(a) ${}^8C_3$ (b) 21
(c) 3^8 (d) 5

10. If the letters of the word SACHIN are arranged in all possible ways and these words are written out as in dictionary, then the word SACHIN appears at serial number **[2005]**
(a) 601 (b) 600
(c) 603 (d) 602

11. At an election, a voter may vote for any number of candidates, not greater than the number to be elected. There are 10 candidates and 4 are of be selected, if a voter votes for at least one candidate, then the number of ways in which he can vote is **[2006]**
(a) 5040 (b) 6210
(c) 385 (d) 1110

12. The set $S = \{1, 2, 3, ..., 12\}$ is to be partitioned into three sets A, B, C of equal size.
Thus $A \cup B \cup C = S$,
$A \cap B = B \cap C = A \cap C = \phi$. The number of ways to partition S is **[2007]**
(a) $\frac{12!}{(4!)^3}$ (b) $\frac{12!}{(4!)^4}$
(c) $\frac{12!}{3!(4!)^3}$ (d) $\frac{12!}{3!(4!)^4}$

13. How many different words can be formed by jumbling the letters in the word MISSISSIPPI in which no two S are adjacent? **[2008]**

(a) $8.{}^6C_4.{}^7C_4$ (b) $6.7.{}^8C_4$

(c) $6.8.{}^7C_4$. (d) $7.{}^6C_4.{}^8C_4$

14. From 6 different novels and 3 different dictionaries,4 novels and 1 dictionary are to be selected and arranged in a row on a shelf so that the dictionary is always in the middle. Then the number of such arrangement is: **[2009]**

(a) at least 500 but less than 750

(b) at least 750 but less than 1000

(c) at least 1000

(d) less than 500

15. There are two urns. Urn A has 3 distinct red balls and urn B has 9 distinct blue balls. From each urn two balls are taken out at random and then transferred to the other. The number of ways in which this can be done is **[2010]**

(a) 36 (b) 66

(c) 108 (d) 3

16. **Statement-1:** The number of ways of distributing 10 identical balls in 4 distinct boxes such that no box is empty is ${}^9C_3$.

Statement-2: The number of ways of choosing any 3 places from 9 different places is ${}^9C_3$. **[2011]**

(a) Statement-1 is true, Statement-2 is true; Statement-2 is not a correct explanation for Statement-1.

(b) Statement-1 is true, Statement-2 is false.

(c) Statement-1 is false, Statement-2 is true.

(d) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1.

17. **Statement - 1 :** For each natural number n, $(n+1)^7 - n^7 - 1$ is divisible by 7.

Statement - 2 : For each natural number $n, n^7 - n$ is divisible by 7. **[2011 RS]**

(a) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1.

(b) Statement-1 is true, Statement-2 is true; Statement-2 is NOT a correct explanation for Statement-1

(c) Statement-1 is true, Statement-2 is false

(d) Statement-1 is false, Statement-2 is true

18. There are 10 points in a plane, out of these 6 are collinear. If N is the number of triangles formed by joining these points. Then : **[2011RS]**

(a) $N \le 100$ (b) $100 < N \le 140$

(c) $140 < N \le 190$ (d) $N > 190$

19. Assuming the balls to be identical except for difference in colours, the number of ways in which one or more balls can be selected from 10 white, 9 green and 7 black balls is : **[2012]**

(a) 880 (b) 629

(c) 630 (d) 879

20. Let A and B be two sets containing four and two elements respectively. Then the number of subsets of the set A × B, each having at least three elements is : **[2013, 2015]**

(a) 275 (b) 510

(c) 219 (d) 256

21. The number of integers greater than 6,000 that can be formed, using the digits 3, 5, 6, 7 and 8, without repetition, is : **[2015]**

(a) 120 (b) 72

(c) 216 (d) 192

22. If all the words (with or without meaning) having five letters, formed using the letters of the word SMALL and arranged as in a dictionary; then the position of the word SMALL is : **[2016]**

(a) 52^{nd} (b) 58^{th}

(c) 46^{th} (d) 59^{th}

23. A man X has 7 friends, 4 of them are ladies and 3 are men. His wife Y also has 7 friends, 3 of them are ladies and 4 are men. Assume X and Y have no common friends. Then the total number of ways in which X and Y together can throw a party inviting 3 ladies and 3 men, so that 3 friends of each of X and Y are in this party, is : **[2017]**

(a) 484 (b) 485

(c) 468 (d) 469

24. From 6 different novels and 3 different dictionaries, 4 novels and 1 dictionary are to be selected and arranged in a row on a shelf so that the dictionary is always in the middle. The number of such arrangements is : **[2018]**

(a) less than 500

(b) at least 500 but less than 750

(c) at least 750 but less than 1000

(d) at least 1000

25. A committee of 11 members is to be formed from 8 males and 5 females. If m is the number of ways the committee is formed with at least 6 males and n is the number of ways the committee is formed with at least 3 females, then: **[2019]**

(a) m + n = 68 (b) m = n = 78

(c) n = m – 8 (d) m = n = 68

26. Consider a class of 5 girls and 7 boys. The number of different teams consisting of 2 girls and 3 boys that can be formed from this class, if there are two specific boys A and B, who refuse to be the members of the same team, is: **[2019]**

(a) 500 (b) 200

(c) 300 (d) 350

27. Total number of 6-digit numbers in which only and all the five digits 1, 3, 5, 7 and 9 appear, is: **[2020]**

(a) $\frac{1}{2}(6!)$ (b) 6!

(c) 5^6 (d) $\frac{5}{2}(6!)$

Answer Key														
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
(d)	(c)	(d)	(b)	(c)	(c)	(a)	(c)	(b)	(a)	(c)	(a)	(d)	(c)	(c)
16	17	18	19	20	21	22	23	24	25	26	27			
(a)	(a)	(a)	(d)	(c)	(d)	(b)	(b)	(d)	(b)	(c)	(d)			

Solutions

1. **(d)** Total number of numbers formed using 0, 1, 2, 3, 5, 7
$= 5 \times 6 \times 6 \times 4 = 36 \times 20 = 720.$

2. **(c)** Total number of numbers
$= 3 \times 5 \times 5 \times 5 - 1 = 374$

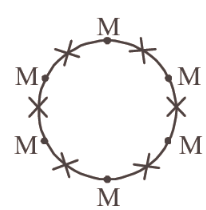
3. **(d)** We know that a number is divisible by 3 only when the sum of the digits is divisible by 3. The given digits are 0, 1, 2, 3, 4, 5.
Here the possible number of combinations of 5 digits out of 6 are ${}^5C_4 = 5$, which are as follows–
$1 + 2 + 3 + 4 + 5 = 15 = 3 \times 5$ (divisible by 3)
$0 + 2 + 3 + 4 + 5 = 14$ (not divisible by 3)
$0 + 1 + 3 + 4 + 5 = 13$ (not divisible by 3)
$0 + 1 + 2 + 4 + 5 = 12 = 3 \times 4$ (divisible by 3)
$0 + 1 + 2 + 3 + 5 = 11$ (not divisible by 3)
$0 + 1 + 2 + 3 + 4 = 10$ (not divisible by 3)
Thus the number should contain the digits 1, 2, 3, 4, 5 or the digits 0, 1, 2, 4, 5.
Taking 1, 2, 3, 4, 5, the 5 digit numbers are $= 5! = 120$
Taking 0, 1, 2, 4, 5, the 5 digit numbers are $= 5! - 4! = 96$
$\therefore$ Total number of numbers $= 120 + 96 = 216$

4. **(b)** Required sum
$$= (2 + 4 + 6 + ... + 100) + (5 + 10 + 15 + ... + 100) - (10 + 20 + ... + 100)$$
$$= 2(1 + 2 + 3 ... + 50) + 5(1 + 2 + 3 + ... + 50) - 10(1 + 2 + 3 + ... + 10)$$
$$= 2550 + 1050 - 530 = 3050.$$

5. **(c)** ${}^nC_{r+1} + {}^nC_{r-1} + 2\,{}^nC_r$
$= {}^nC_{r-1} + {}^nC_r + {}^nC_r + {}^nC_{r+1}$
$\left[\because {}^nC_r + {}^nC_{r-1} = {}^{n+1}C_r\right]$
$= {}^{n+1}C_r + {}^{n+1}C_{r+1} = {}^{n+2}C_{r+1}$

6. **(c)** According to given question two cases are possible.
(i) Selecting 4 out of first five question and 6 out of remaining question
$= {}^5C_4 \times {}^8C_6 = 140$ ways
(ii) Selecting 5 out of first five question and 5 out of remaining 8 questions
$= {}^5C_5 \times {}^8C_5 = 56$ ways
Therefore, total number of choices $= 140 + 56 = 196.$

7. **(a)** No. of ways in which 6 men can be arranged at a round table $= (6 - 1)! = 5!$



Now women can be arranged in ${}^6P_5$ $= 6!$ ways.
Total Number of ways $= 6! \times 5!$

8. **(c)** Total number of arrangements of letters in the word GARDEN = 6 ! = 720 there are two vowels A and E, in half of the arrangements A preceeds E and other half A follows E.
So, numbers of word with vowels in alphabetical order in $\frac{1}{2} \times 720 = 360$

9. **(b)** We know that the number of ways of distributing n identical items among r persons, when each one of them receives at least one item is $^{n-1}C_{r-1}$

$\therefore$ The required number of ways

$= {}^{8-1}C_{3-1} = {}^{7}C_2 = \frac{7!}{2!5!} \quad = \frac{7 \times 6}{2 \times 1} = 21$

10. **(a)** Alphabetical order is
A, C, H, I, N, S
No. of words starting with A = 5! = 120
No. of words starting with C = 5! = 120
No. of words starting with H = 5! = 120
No. of words starting with I = 5! = 120
No. of words starting with N = 5! = 120
SACHIN – 1
$\therefore$ Sachin appears at serial no. 601

11. **(c)** The number of ways can vote

$= {}^{10}C_1 + {}^{10}C_2 + {}^{10}C_3 + {}^{10}C_4$

$= 10 + 45 + 120 + 210 = 385$

12. **(a)** Set S = {1, 2, 3, ... 12}

$A \cup B \cup C = S,\ A \cap B = B \cap C = A \cap C = \phi$

$\therefore$ Each sets contain 4 elements.
$\therefore$ The number of ways to partition

$= {}^{12}C_4 \times {}^{8}C_4 \times {}^{4}C_4$

$= \frac{12!}{4!8!} \times \frac{8!}{4!4!} \times \frac{4!}{4!0!} = \frac{12!}{(4!)^3}$

13. **(d)** First let us arrange M, I, I, I, I, P, P

Which can be done in $\frac{7!}{4!2!}$ ways

* M * I * I * I * I * P * P *
Now 4 S can be kept at any of the * places in $^{8}C_4$ ways so that no two S are adjacent.
Total required ways

$= \frac{7!}{4!2!}\,{}^{8}C_4 = \frac{7!}{4!2!}\,{}^{8}C_4 = 7 \times {}^{6}C_4 \times {}^{8}C_4$

14. **(c)** 4 novels, out of 6 novels and 1 dictionary out of 3 can be selected in $^{6}C_4 \times {}^{3}C_1$ ways
Then 4 novels with one dictionary in the middle can be arranged in 4! ways.
$\therefore$ Total ways of arrangement

$= {}^{6}C_4 \times {}^{3}C_1 \times 4! = 1080$

15. **(c)** Two balls are taken from each urn

Total number of ways = $^{3}C_2 \times {}^{9}C_2$

$= 3 \times \frac{9 \times 8}{2} = 3 \times 36 = 108$

16. **(a)** The number of ways of distributing 10 identical balls in 4 distinct boxes

$= {}^{10-1}C_{4-1} = {}^{9}C_3$

17. **(a)** **Statement 2 :**

$P(n): n^7 - n$ is divisible by 7

Put $n = 1$, $1 - 1 = 0$ is divisible by 7, which is true for $n = 1$.
Let $n = k$, $P(k) : k^7 - k$ is divisible by 7, true for $n = k$.
Put $n = k + 1$

$\therefore \quad P(k+1): (k+1)^7 - (k+1)$ is div. by 7

$P(k+1) : k^7 + {}^{7}C_1 k^6 + {}^{7}C_2 k^2 + ... + {}^{7}C_6 k + 1 - k - 1$, is div. by 7.

$P(k+1) : (k^7 - k) + ({}^{7}C_1 k^6 + {}^{7}C_2 k^5 + ... + {}^{7}C_6 k)$ is div. by 7.

Since 7 is coprime with 1, 2, 3, 4, 5, 6.

So $^{7}C_1, {}^{7}C_2, ... {}^{7}C_6$ are all divisible by 7

$\therefore \quad P(k+1)$ is divisible by 7, it is also true for $n = k + 1$

Hence $P(n) : n^7 - n$ is divisible by 7

Statement 1 : $n^7 - n$ is divisible by 7

$\Rightarrow \quad (n+1)^7 - (n+1)$ is divisible by 7

$\Rightarrow \quad (n+1)^7 - n^7 - 1 + (n^7 - n)$

is divisible by 7

$\Rightarrow \quad (n+1)^7 - n^7 - 1$ is divisible by 7

Hence both Statements 1 and 2 are correct and Statement 2 is the correct explanation of Statement -1.

18. **(a)** Number of required triangles $= {}^{10}C_3 - {}^{6}C_3$

$$= \frac{10\times 9\times 8}{6} - \frac{6\times 5\times 4}{6} = 120 - 20 = 100$$

19. **(d)** Given that number of white balls = 10
Number of green balls = 9
and Number of black balls = 7
$\therefore$ Required probability
$= (10+1)(9+1)(7+1) - 1$
$= 11.10.8 - 1 = 879$
[$\because$ The total number of ways of selecting one or more items from p identical items of one kind, q identical items of second kind; r identical items of third kind is
$(p+1)(q+1)(r+1) - 1$]

20. **(c)** Given
$n(A) = 4, n(B) = 2, n(A\times B) = 8$
Required number of subsets
$= {}^8C_3 + {}^8C_4 + ... + {}^8C_8 = 2^8 - {}^8C_0 - {}^8C_1 - {}^8C_2$
$= 256 - 1 - 8 - 28 = 219$

21. **(d)** Four digits number can be arranged in $3 \times 4!$ ways.
Five digits number can be arranged in 5! ways.
Number of integers $= 3 \times 4! + 5! = 192$.

22. **(b)** ALLMS
No. of words starting with

A : $\underline{A}$ _ _ _ _ $\frac{4!}{2!} = 12$

L : $\underline{L}$ _ _ _ _ $4! = 24$

M : $\underline{M}$ _ _ _ _ $\frac{4!}{2!} = 12$

S : $\underline{S}\,\underline{A}$ _ _ _ _ $\frac{3!}{2!} = 3$

: $\underline{S}\,\underline{L}$ _ _ _ $3! = 6$

SMALL $\rightarrow$ 58th word

23. **(b)**
X → 4 ladies, 3 men
Y → 3 ladies, 4 men

Possible cases for X are
(1) 3 ladies, 0 man
(2) 2 ladies, 1 man
(3) 1 lady, 2 men
(4) 0 ladies, 3 men
Possible cases for Y are
(1) 0 ladies, 3 men
(2) 1 lady, 2 men
(3) 2 ladies, 1 man
(4) 3 ladies, 0 man
No. of ways $= {}^4C_3 \,.\, {}^4C_3 + ({}^4C_2 \,.\, {}^3C_1)^2 + ({}^4C_1 \,.\, {}^3C_2)^2 + ({}^3C_3)^2$
$= 16 + 324 + 144 + 1 = 485$

24. **(d)** $\therefore$ Required number of ways
$= {}^6C_4 \times {}^3C_1 \times 4!$
$= 15 \times 3 \times 24 = 1080$

25. **(b)** Since, m = number of ways the committee is formed with at least 6 males
$= {}^8C_6 \,.\, {}^5C_5 + {}^8C_7 \,.\, {}^5C_4 + {}^8C_8 \,.\, {}^5C_3 = 78$
and n = number of ways the committee is formed with at least 3 females
$= {}^5C_3 \,.\, {}^8C_8 + {}^5C_4 \,.\, {}^8C_7 + {}^5C_5 \,.\, {}^8C_6 = 78$
Hence, m = n = 78

26. **(c)** Since, the number of ways to select 2 girls is ${}^5C_2$.
Now, 3 boys can be selected in 3 ways.
(1) Selection of A and selection of any 2 other boys (except B) in ${}^5C_2$ ways
(2) Selection of B and selection of any 2 two other boys (except A) in ${}^5C_2$ ways
(3) Selection of 3 boys (except A and B) in ${}^5C_3$ ways
Hence, required number of different teams
$= {}^5C_2\,({}^5C_2 + {}^5C_2 + {}^5C_3) = 300$

27. **(d)** Five digits numbers be 1, 3, 5, 7, 9
For selection of one digit, we have ${}^5C_1$ choice.

And six digits can be arrange in $\frac{6!}{2!}$ ways.

Hence, total such numbers $= \frac{5.6!}{2!} = \frac{5}{2}.6!$

Binomial Theorem

8

1. The coefficients of x^p and x^q in the expansion of $(1+x)^{p+q}$ are **[2002]**
(a) equal
(b) equal with opposite signs
(c) reciprocals of each other
(d) none of these

2. If the sum of the coefficients in the expansion of $(a+b)^n$ is 4096, then the greatest coefficient in the expansion is **[2002]**
(a) 1594 (b) 792
(c) 924 (d) 2924

3. The positive integer just greater than $(1+0.0001)^{10000}$ is **[2002]**
(a) 4 (b) 5
(c) 2 (d) 3

4. r and n are positive integers $r > 1$, $n > 2$ and coefficient of $(r+2)^{th}$ term and $3r^{th}$ term in the expansion of $(1+x)^{2n}$ are equal, then n equals **[2002]**
(a) $3r$ (b) $3r+1$
(c) $2r$ (d) $2r+1$

5. If x is positive, the first negative term in the expansion of $(1+x)^{27/5}$ is **[2003]**
(a) 6th term (b) 7th term
(c) 5th term (d) 8th term

6. The number of integral terms in the expansion of $(\sqrt{3}+\sqrt[8]{5})^{256}$ is **[2003]**
(a) 35 (b) 32
(c) 33 (d) 34

7. The coefficient of the middle term in the binomial expansion in powers of x of $(1+\alpha x)^4$ and of $(1-\alpha x)^4$ is the same if α equals **[2004]**
(a) $\frac{3}{5}$ (b) $\frac{10}{3}$
(c) $\frac{-3}{10}$ (d) $\frac{-5}{3}$

8. The coefficient of x^n in expansion of $(1+x)(1-x)^n$ is **[2004]**
(a) $(-1)^{n-1}n$ (b) $(-1)^n(1-n)$
(c) $(-1)^{n-1}(n-1)^2$ (d) $(n-1)$

9. The value of ${}^{50}C_4 + \sum_{r=1}^{6} {}^{56-r}C_3$ is **[2005]**
(a) ${}^{55}C_4$ (b) ${}^{55}C_3$
(c) ${}^{56}C_3$ (d) ${}^{56}C_4$

10. If the coefficient of x^7 in $\left[ax^2+\left(\frac{1}{bx}\right)\right]^{11}$ equals the coefficient of x^{-7} in $\left[ax-\left(\frac{1}{bx^2}\right)\right]^{11}$, then a and b satisfy the relation **[2005]**
(a) $a-b=1$ (b) $a+b=1$
(c) $\frac{a}{b}=1$ (d) $ab=1$

11. If x is so small that x^3 and higher powers of x may be neglected, then $\dfrac{(1+x)^{\frac{3}{2}}-\left(1+\frac{1}{2}x\right)^3}{(1-x)^{\frac{1}{2}}}$ may be approximated as **[2005]**
(a) $1-\frac{3}{8}x^2$ (b) $3x+\frac{3}{8}x^2$
(c) $-\frac{3}{8}x^2$ (d) $\frac{x}{2}-\frac{3}{8}x^2$

12. For natural numbers m, n if $(1-y)^m(1+y)^n$ $= 1+a_1y+a_2y^2+.......$ and $a_1 = a_2 = 10$, then (m,n) is **[2006]**

(a) $(20,45)$ (b) $(35,20)$
(c) $(45,35)$ (d) $(35,45)$

13. In the binomial expansion of $(a-b)^n, n \geq 5$, the sum of 5th and 6th terms is zero, then a/b equals **[2007]**

(a) $\frac{n-5}{6}$ (b) $\frac{n-4}{5}$
(c) $\frac{5}{n-4}$ (d) $\frac{6}{n-5}$.

14. The sum of the series **[2007]**

$^{20}C_0 - {}^{20}C_1 + {}^{20}C_2 - {}^{20}C_3 + ... - ..... + {}^{20}C_{10}$ is

(a) 0 (b) $^{20}C_{10}$
(c) $-{}^{20}C_{10}$ (d) $\frac{1}{2}\,{}^{20}C_{10}$

15. **Statement -1 :** $\sum_{r=0}^{n}(r+1)\ {}^nC_r = (n+2)2^{n-1}$.

Statement-2:
$\sum_{r=0}^{n}(r+1)\ {}^nC_r x^r = (1+x)^n + nx(1+x)^{n-1}$. **[2008]**

(a) Statement -1 is false, Statement-2 is true
(b) Statement -1 is true, Statement-2 is true; Statement -2 is a correct explanation for Statement-1
(c) Statement -1 is true, Statement-2 is true; Statement -2 is not a correct explanation for Statement-1
(d) Statement -1 is true, Statement-2 is false

16. The remainder left out when $8^{2n} - (62)^{2n+1}$ is divided by 9 is: **[2009]**

(a) 2 (b) 7
(c) 8 (d) 0

17. Let $S_1 = \sum_{j=1}^{10} j(j-1)^{10}C_j$, $S_2 = \sum_{j=1}^{10} j^{10}C_j$ and $S_3 = \sum_{j=1}^{10} j^{2\ 10}C_j$. **[2010]**

Statement -1 : $S_3 = 55 \times 2^9$.

Statement - 2: $S_1 = 90 \times 2^8$ and $S_2 = 10 \times 2^8$.

(a) Statement -1 is true, Statement -2 is true ; Statement -2 is **not** a correct explanation or Statement -1.
(b) Statement -1 is true, Statement -2 is false.
(c) Statement -1 is false, Statement -2 is true .
(d) Statement - 1 is true, Statement 2 is true ; Statement -2 is a correct explanation for Statement -1.

18. The coefficient of x^7 in the expansion of $(1-x-x^2+x^3)^6$ is **[2011]**

(a) –132 (b) –144
(c) 132 (d) 144

19. If n is a positive integer , then $\left(\sqrt{3}+1\right)^{2n} - \left(\sqrt{3}-1\right)^{2n}$ is : **[2012]**

(a) an irrational number
(b) an odd positive integer
(c) an even positive integer
(d) a rational number other than positive integers

20. The term independent of x in expansion of $\left(\frac{x+1}{x^{2/3}-x^{1/3}+1} - \frac{x-1}{x-x^{1/2}}\right)^{10}$ is **[2013]**

(a) 4 (b) 120
(c) 210 (d) 310

21. Let T_n be the number of all possible triangles formed by joining vertices of an n-sided regular polygon. If $T_{n+1} - T_n = 10$, then the value of n is : **[2013]**

(a) 7 (b) 5
(c) 10 (d) 8

22. If $X = \{4^n - 3n - 1 : n \in N\}$ and $Y = \{9(n-1) : n \in N\}$, where N is the set of natural numbers, then $X \cup Y$ is equal to: **[2014]**

(a) X (b) Y
(c) N (d) $Y - X$

23. The sum of coefficients of integral power of x in the binomial expansion $\left(1-2\sqrt{x}\right)^{50}$ is : **[2015]**

(a) $\frac{1}{2}\left(3^{50}-1\right)$ (b) $\frac{1}{2}\left(2^{50}+1\right)$
(c) $\frac{1}{2}\left(3^{50}+1\right)$ (d) $\frac{1}{2}\left(3^{50}\right)$

24. If the number of terms in the expansion of $\left(1-\frac{2}{x}+\frac{4}{x^2}\right)^n$, $x \neq 0$, is 28, then the sum of the coefficients of all the terms in this expansion, is : **[2016]**

(a) 243 (b) 729
(c) 64 (d) 2187

25. The value of $({}^{21}C_1 - {}^{10}C_1) + ({}^{21}C_2 - {}^{10}C_2) + ({}^{21}C_3 - {}^{10}C_3) + ({}^{21}C_4 - {}^{10}C_4) + ... + ({}^{21}C_{10} - {}^{10}C_{10})$ is: **[2017]**

(a) $2^{20} - 2^{10}$ (b) $2^{21} - 2^{11}$
(c) $2^{21} - 2^{10}$ (d) $2^{20} - 2^{9}$

26. The sum of the co-efficients of all odd degree terms in the expansion of

$(x+\sqrt{x^3-1})^5 + (x-\sqrt{x^3-1})^5, (x>1)$ is : **[2018]**

(a) 0 (b) 1
(c) 2 (d) – 1

27. If the fourth term in the Binomial expansion of $\left(\frac{2}{x}+x^{\log_8 x}\right)^6$ $(x>0)$ is 20×8^7, then a value of x is: **[2019]**

(a) 8^3 (b) 8^2
(c) 8 (d) 8^{-2}

28. If the fractional part of the number $\frac{2^{403}}{15}$ is $\frac{k}{15}$, then k is equal to: **[2019]**

(a) 6 (b) 8
(c) 4 (d) 14

29. If the sum of the coefficients of all even powers of x in the product

$(1+x+x^2+...+x^{2n})(1-x+x^2-x^3+...+x^{2n})$ is 61,

then n is equal to ———. **[2020]**

Answer Key

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
(a)	(c)	(d)	(c)	(d)	(c)	(c)	(b)	(d)	(d)	(c)	(d)	(b)	(d)	(b)
16	17	18	19	20	21	22	23	24	25	26	27	28	29	
(a)	(b)	(b)	(a)	(c)	(b)	(b)	(c)	(b)	(a)	(c)	(b)	(b)	(30)	

Solutions

1. **(a)** We know that $t_{p+1} = {}^{p+q}C_p\, x^p$ and

$t_{q+1} = {}^{p+q}C_q\, x^q$

$\because\ {}^{p+q}C_p = {}^{p+q}C_q$. [Remember ${}^nC_r = {}^nC_{n-r}$]

2. **(c)** Take $a = 1$ and $b = 1$ in $(a+b)^n$.

$2^n = 4096 = 2^{12} \Rightarrow n = 12;$

The greatest coeff = coeff of middle term.

So middle term = t_7.

$\Rightarrow t_7 = t_{6+1} = {}^{12}C_6 a^6 b^6$

$\Rightarrow$ Coeff of $t_7 = {}^{12}C_6 = \frac{12!}{6!6!} = 924.$

3. **(d)** $(1+0.0001)^{10000} = \left(1+\frac{1}{n}\right)^n, n = 10000$

$$= 1 + n.\frac{1}{n} + \frac{n(n-1)}{2!}\frac{1}{n^2} + \frac{n(n-1)(n-2)}{3!}\frac{1}{n^3} + ... + \frac{1}{n^n}$$

$$= 1 + 1 + \frac{1}{2!}\left(1-\frac{1}{n}\right) + \frac{1}{3!}\left(1-\frac{1}{n}\right)\left(1-\frac{2}{n}\right) + ... + \frac{1}{n^n}$$

$$< 1 + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + ... + \frac{1}{(9999)!}$$

$$= 1 + \frac{1}{1!} + \frac{1}{2!} + ...\infty = e < 3$$

4. **(c)** $t_{r+2} = {}^{2n}C_{r+1}\ x^{r+1}; t_{3r} = {}^{2n}C_{3r-1}\ x^{3r-1}$

Given that, ${}^{2n}C_{r+1} = {}^{2n}C_{3r-1}$;

$\Rightarrow\ r+1+3r-1=2n$

$\Rightarrow\ 2n=4r \Rightarrow\ n=2r$

5. **(d)** $T_{r+1} = \dfrac{n(n-1)(n-2).........(n-r+1)}{r!}(x)^r$

For first negative term,

$n-r+1<0 \Rightarrow r>n+1$

$\Rightarrow r > \dfrac{32}{5}\ \therefore r = 7 \cdot \left(\because n = \dfrac{27}{5}\right)$

Therefore, first negative term is T_8.

6. **(c)** $T_{r+1} = {}^{256}C_r(\sqrt{3})^{256-r}(\sqrt[8]{5})^r$

$= {}^{256}C_r(3)^{\frac{256-r}{2}}(5)^{r/8}$

Terms will be integral if $\dfrac{256-r}{2}$ & $\dfrac{r}{8}$ both are +ve integer. It is possible if r is an integral multiple of 8 and $0 \le r \le 256$

$\therefore\ r=33$

7. **(c)** The middle term in the expansion of

$(1+\alpha x)^4 = T_3 = {}^4C_2(\alpha x)^2 = 6\alpha^2x^2$

The middle term in the expansion of

$(1-\alpha x)^6 = T_4 = {}^6C_3(-\alpha x)^3 = -20\alpha^3x^3$

According to the question

$6\alpha^2 = -20\alpha^3 \Rightarrow \alpha = -\dfrac{3}{10}$

8. **(b)** Coeff. of x^n in $(1+x)(1-x)^n$

= coeff of x^n in

$(1+x)(1-{}^nC_1x+{}^nC_2x^2-....+(-1)^{n\,n}C_nx^n)$

$=(-1)^{n\,n}C_n+(-1)^{n-1\,n}C_{n-1}$

$=(-1)^n+(-1)^{n-1}.n$

$=(-1)^n(1-n)$

9. **(d)** ${}^{50}C_4+\sum_{r=1}^{6}{}^{56-r}C_3$

$= {}^{50}C_4+\left[\begin{array}{r}{}^{55}C_3+{}^{54}C_3+{}^{53}C_3+{}^{52}C_3\\+{}^{51}C_3+{}^{50}C_3\end{array}\right]$

$\left[\because {}^nC_r+{}^nC_{r-1}={}^{n+1}C_r\right]$

$=({}^{50}C_4+{}^{50}C_3)$

$+{}^{51}C_3+{}^{52}C_3+{}^{53}C_3+{}^{54}C_3+{}^{55}C_3$

$=({}^{51}C_4+{}^{51}C_3)$

$+{}^{52}C_3+{}^{53}C_3+{}^{54}C_3+{}^{55}C_3$

Proceeding in the same way, we get

$={}^{55}C_4+{}^{55}C_3={}^{56}C_4$.

10. **(d)** T_{r+1} in the expansion

$\left[ax^2+\dfrac{1}{bx}\right]^{11} = {}^{11}C_r(ax^2)^{11-r}\left(\dfrac{1}{bx}\right)^r$

$= {}^{11}C_r(a)^{11-r}(b)^{-r}(x)^{22-2r-r}$

For the Coefficient of x^7, we have

$22-3r=7 \Rightarrow r=5$

$\therefore$ Coefficient of x^7

$={}^{11}C_5(a)^6\ (b)^{-5}$ ...(1)

Again T_{r+1} in the expansion

$\left[ax-\dfrac{1}{bx^2}\right]^{11} = {}^{11}C_r(ax^2)^{11-r}\left(-\dfrac{1}{bx^2}\right)^r$

$= {}^{11}C_r(a)^{11-r}(-1)^r\times(b)^{-r}(x)^{-2r+11-r}$

For the Coefficient of x^{-7}, we have

Now $11-3r=-7 \Rightarrow 3r=18 \Rightarrow r=6$

$\therefore$ Coefficient of x^{-7}

$={}^{11}C_6\,a^5\times1\times(b)^{-6}$ ...(2)

$\because$ Coefficient of x^7 = Coefficient of x^{-7}

From (1) and (2),

$\therefore\ {}^{11}C_5\,(a)^6\,(b)^{-5}={}^{11}C_6\,a^5\times(b)^{-6}$

$\Rightarrow ab=1.$

11. **(c)** $\because$ x^3 and higher powers of x may be neglected

$$\therefore \frac{(1+x)^{\frac{3}{2}}-\left(1+\frac{x}{2}\right)^3}{\left(1-x^{\frac{1}{2}}\right)}$$

$$=(1-x)^{\frac{-1}{2}}\left[\left(1+\frac{3}{2}x+\frac{\frac{3}{2}\cdot\frac{1}{2}}{2!}x^2\right)-\left(1+\frac{3x}{2}+\frac{3.2}{2!}\frac{x^2}{4}\right)\right]$$

$$=\left[1+\frac{x}{2}+\frac{\frac{1}{2}\cdot\frac{3}{2}}{2!}x^2\right]\left[\frac{-3}{8}x^2\right]=\frac{-3}{8}x^2$$

12. **(d)** $(1-y)^m(1+y)^n$

$$=[1-{}^mC_1y+{}^mC_2y^2-......]$$

$$[1+{}^nC_1y+{}^nC_2y^2+.....]$$

$$=1+(n-m)y+\left\{\frac{m(m-1)}{2}+\frac{n(n-1)}{2}-mn\right\}y^2+.....$$

$\therefore\ a_1=n-m=10$ ...(i)

and $a_2=\dfrac{m^2+n^2-m-n-2mn}{2}=10$

$(m-n)^2-(m+n)=20$

$\Rightarrow\ m+n=80$ ...(ii) [from (i)]

Solving (i) and (ii), we get

$\therefore\ m=35, n=45$

13. **(b)** $T_{r+1}=(-1)^r.\ {}^nC_r\ (a)^{n-r}.\ (b)^r$ is an expansion of $(a-b)^n$

$\therefore$ 5th term $=t_5=t_{4+1}$

$=(-1)^4.{}^nC_4\ (a)^{n-4}.(b)^4={}^nC_4\ .\ a^{n-4}\ .\ b^4$

6th term $=t_6=t_{5+1}=(-1)^5\,{}^nC_5\,(a)^{n-5}\,(b)^5$

Given $t_5+t_6=0$

$\therefore\ {}^nC_4\ .\ a^{n-4}\ .\ b^4+(-{}^nC_5\ .\ a^{n-5}\ .\ b^5)=0$

$$\Rightarrow \frac{n!}{4!(n-4)!}.\frac{a^n}{a^4}.b^4-\frac{n!}{5!(n-5)!}.\frac{a^nb^5}{a^5}=0$$

$$\Rightarrow \frac{n!.a^nb^4}{4!(n-5)!.a^4}\left[\frac{1}{(n-4)}-\frac{b}{5.a}\right]=0$$

$[\because a\neq 0, b\neq 0]$

$$\Rightarrow \frac{1}{n-4}-\frac{b}{5a}=0 \Rightarrow \frac{a}{b}=\frac{n-4}{5}$$

14. **(d)** We know that, $(1+x)^{20}={}^{20}C_0+{}^{20}C_1x+{}^{20}C_2x^2+......\ {}^{20}C_{10}x^{10}+.....\ {}^{20}C_{20}x^{20}$

Put $x=-1$, $(0)={}^{20}C_0-{}^{20}C_1+{}^{20}C_2-{}^{20}C_3+......+{}^{20}C_{10}-{}^{20}C_{11}....+{}^{20}C_{20}$

$\Rightarrow\ 0=2[{}^{20}C_0-{}^{20}C_1+{}^{20}C_2-{}^{20}C_3+.....-{}^{20}C_9]+{}^{20}C_{10}$

$\Rightarrow\ {}^{20}C_{10}=2[{}^{20}C_0-{}^{20}C_1+{}^{20}C_2-{}^{20}C_3+......-{}^{20}C_9+{}^{20}C_{10}]$

$\Rightarrow\ {}^{20}C_0-{}^{20}C_1+{}^{20}C_2-{}^{20}C_3+....+{}^{20}C_{10}=\frac{1}{2}\ {}^{20}C_{10}$

15. **(b)** **From statement 2:**

$$\sum_{r=0}^{n}(r+1)\,{}^nC_rx^r=\sum_{r=0}^{n}r.{}^nC_rx^r+\sum_{r=0}^{n}{}^nC_rx^r$$

$$=\sum_{r=1}^{n}r\cdot\frac{n}{r}\,{}^{n-1}C_{r-1}x^r+(1+x)^n$$

$$=nx\sum_{r=1}^{n}{}^{n-1}C_{r-1}x^{r-1}+(1+x)^n$$

$=nx\,(1+x)^{n-1}+(1+x)^n=$ RHS

$\therefore$ Statement 2 is correct.

Putting $x=1$, we get

$$\sum_{r=0}^{n}(r+1)^nC_r=n\cdot 2^{n-1}+2^n=(n+2)\cdot 2^{n-1}.$$

$\therefore$ Statement 1 is also true and statement 2 is a correct explanation for statement 1.

16. **(a)** $(8)^{2n}-(62)^{2n+1}$

$= (64)^n-(62)^{2n+1}$

$= (63+1)^n-(63-1)^{2n+1}$

$$=\left[{}^nC_0(63)^n+{}^nC_1(63)^{n-1}+{}^nC_2(63)^{n-2}+\ldots\ldots+{}^nC_{n-1}(63)+{}^nC_n\right]$$

$$-\left[{}^{2n+1}C_0(63)^{2n+1}-{}^{2n+1}C_1(63)^{2n}+{}^{2n+1}C_2(63)^{2n-1}-\ldots\ldots+(-1)^{2n+1}\,{}^{2n+1}C_{2n+1}\right]$$

$$=63\times\left[{}^nC_0(63)^{n-1}+{}^nC_1(63)^{n-2}+{}^nC_2(63)^{n-3}+\ldots\ldots+{}^nC_{n-1}\right]+1-63\times$$

$$\left[{}^{2n+1}C_0(63)^{2n}-{}^{2n+1}C_1(63)^{2n-1}+\ldots\ldots+{}^{2n+1}C_{2n}\right]+1$$

$= 63 \times$ some integral value $+ 2$

Hence, when divided by 9 leaves 2 as the remainder.

17. **(b)** $S_2=\sum_{j=1}^{10} j\,{}^{10}C_j=\sum_{j=1}^{10}10\,{}^9C_{j-1}$

$$\left[\because {}^nC_r=\frac{n}{r}\,{}^{n-1}C_{r-1}\right]$$

$$=10\left[{}^9C_0+{}^9C_1+{}^9C_2+\ldots.+{}^9C_9\right]=10.2^9$$

18. **(b)** $(1-x-x^2+x^3)^6=[(1-x)-x^2(1-x)]^6$

$=(1-x)^6(1-x^2)^6$

$=(1-6x+15x^2-20x^3+15x^4-6x^5+x^6)$

$\times(1-6x^2+15x^4-20x^6+15x^8-6x^{10}+x^{12})$

Coefficient of $x^7=(-6)(-20)+(-20)(15)+(-6)(-6)=-144$

19. **(a)** Consider $\left(\sqrt{3}+1\right)^{2n}-\left(\sqrt{3}-1\right)^{2n}$

$$=2\left[{}^{2n}C_1\left(\sqrt{3}\right)^{2n-1}+{}^{2n}C_3\left(\sqrt{3}\right)^{2n-3}+{}^{2n}C_5\left(\sqrt{3}\right)^{2n-5}+\ldots.\right]$$

$\because (a+b)^n-(a-b)^n$

$=2[{}^nC_1a^{n-1}b+{}^nC_3a^{n-3}b^3\ldots]$

= which is an irrational number.

20. **(c)** Given expression can be written as

$$\left[\frac{\left(x^{1/3}\right)^3+1^3}{x^{2/3}-x^{1/3}+1}-\frac{\left(\sqrt{x}\right)^2-1^2}{\sqrt{x}(\sqrt{x}-1)}\right]^{10}$$

$$=\left((x^{1/3}+1)-\left(\frac{\sqrt{x}+1}{\sqrt{x}}\right)\right)^{10}$$

$$=\left(x^{1/3}+1-1-\frac{1}{\sqrt{x}}\right)^{10}$$

$=(x^{1/3}-x^{-1/2})^{10}$

General term $= T_{r+1}$

$={}^{10}C_r\,(x^{1/3})^{10-r}(-x^{-1/2})^r$

$={}^{10}C_r\,x^{\frac{10-r}{3}}\cdot(-1)^r\cdot x^{-\frac{r}{2}}$

$={}^{10}C_r(-1)^r\cdot x^{\frac{10-r}{3}-\frac{r}{2}}$

Term will be independent of x when

$\frac{10-r}{3}-\frac{r}{2}=0$

$\Rightarrow\ r=4$

So, required term $= T_5={}^{10}C_4=210$

21. **(b)** We know that,

$T_n={}^nC_3\Rightarrow T_{n+1}={}^{n+1}C_3$

ATQ, $T_{n+1}-T_n={}^{n+1}C_3-{}^nC_3=10$

$\Rightarrow\ {}^nC_2=10$

$\Rightarrow\ n=5.$

22. **(b)** $4^n-3n-1=(1+3)^n-3n-1$

$=[{}^nC_0+{}^nC_1.3+{}^nC_2.3^2+\ldots\ldots+{}^nC_n3^n]-3n-1$

$=9\,[{}^nC_2+{}^nC_3.3+\ldots.+{}^nC_n.3^{n-2}]$

$\therefore\ 4^n-3n-1$ is a multiple of 9 for all n.

$\therefore\ X=\{x : x$ is a multiple of $9\}$

Also, $Y=\{9(n-1) : n\in\mathbf{N}\}$

$=\{$All multiples of $9\}$

Clearly $X\subset Y.\ \therefore\ X\cup Y=Y$

23. (c) We know that $(a+b)^n+(a-b)^n$

$$=2[{}^nC_0a^nb^0+{}^nC_2a^{n-2}b^2+{}^nC_4a^{n-4}b^4...]$$

$$\left(1-2\sqrt{x}\right)^{50}+\left(1+2\sqrt{x}\right)^{50}$$

$$2\left[{}^{50}C_0+{}^{50}C_2(2\sqrt{x})^2+{}^{50}C_4(2\sqrt{x})^4...\right]$$

$$=2\left[{}^{50}C_0+{}^{50}C_2 2^2x+{}^{50}C_4 2^4x^2+...\right]$$

Putting x = 1, we get,

$${}^{50}C_0+{}^{50}C_2 2^2+{}^{50}C_4 2^4...$$

$$=\frac{3^{50}+1}{2}$$

24. (b) Total number of terms $= {}^{n+2}C_2 = 28$

$(n+2)(n+1) = 56;\ n = 6$

$\therefore$ Put $x = 1$ in expansion $\left(1-\frac{2}{x}+\frac{4}{x^2}\right)^6$,

we get sum of coefficient $= (1-2+4)^6$

$= 3^6 = 729.$

25. (a) We have $({}^{21}C_1+{}^{21}C_2\ ......+{}^{21}C_{10})$

$-({}^{10}C_1+{}^{10}C_2\ .....\ {}^{10}C_{10})$

$$=\frac{1}{2}[({}^{21}C_1+....+{}^{21}C_{10})+({}^{21}C_{11}+....{}^{21}C_{20})]$$

$-(2^{10}-1)$

$(\because\ {}^{10}C_1+{}^{10}C_2+....+{}^{10}C_{10}=2^{10}-1)$

$$=\frac{1}{2}[2^{21}-2]-(2^{10}-1)$$

$= (2^{20}-1)-(2^{10}-1) = 2^{20}-2^{10}$

26. (c) Since we know that,

$(x+a)^5+(x-a)^5$

$= 2[{}^5C_0x^5+{}^5C_2x^3\cdot a^2+{}^5C_4x\cdot a^4]$

$$\therefore\quad \left(x+\sqrt{x^3-1}\right)^5+\left(x-\sqrt{x^3-1}\right)^5$$

$= 2[{}^5C_0x^5+{}^5C_2x^3(x^3-1)+{}^5C_4x(x^3-1)^2]$

$\Rightarrow\ 2[x^5+10x^6-10x^3+5x^7-10x^4+5x]$

$\therefore$ Sum of coefficients of odd degree terms = 2.

27. (b) $\because\ T_4 = 20\times 8^7$

$$\Rightarrow {}^6C_3\left(\frac{2}{x}\right)^3\times(x^{\log_8 x})^3 = 20\times 8^7$$

$$\Rightarrow 8\times 20\times\left(\frac{x^{\log_8 x}}{x}\right)^3 = 20\times 8^7$$

$$\Rightarrow \frac{x^{\log_8 x}}{x} = 64$$

Now, take $\log_8$ on both sides, we get

$(\log_8 x)^2-(\log_8 x) = 2$

$\Rightarrow \log_8 x = -1$ or $\log_8 x = 2$

$$\Rightarrow x = \frac{1}{8}\quad\text{or}\quad x = 8^2$$

28. (b) $2^{403} = 2^{400}\cdot 2^3$

$= 2^{4\times 100}\cdot 2^3 = (2^4)^{100}\cdot 8$

$= 8(2^4)^{100} = 8(16)^{100}$

$= 8(1+15)^{100} = 8+15\mu$

When 2^{403} is divided by 15, then remainder is 8.

Hence, fractional part of the number is

$$\frac{8}{15}$$

Therefore value of k is 8

29. (30) Let $(1-x+x^2\ .....\ x^{2n})(1+x+x^2\ .....\ x^{2n})$

$= a_0+a_1x+a_2x^2+.....$

put $x = 1$

$1(2n+1) = a_0+a_1+a_2+.....\ a_{2n}$ ...(i)

put $x = -1$

$(2n+1)\times 1 = a_0-a_1+a_2+......\ a_{2n}$ ...(ii)

Adding (i) and (ii), we get,

$4n+2 = 2(a_0+a_2+.....) = 2\times 61$

$\Rightarrow\ 2n+1 = 61\ \Rightarrow\ n = 30.$

9 Sequences and Series

1. If 1, $\log_9(3^{1-x}+2)$, $\log_3(4.3^x-1)$ are in A.P. then x equals **[2002]**
   (a) $\log_3 4$ (b) $1-\log_3 4$
   (c) $1-\log_4 3$ (d) $\log_4 3$
2. The value of $2^{1/4}.\ 4^{1/8}.\ 8^{1/16}\ ...\ \infty$ is **[2002]**
   (a) 1 (b) 2
   (c) 3/2 (d) 4
3. Fifth term of a GP is 2, then the product of its 9 terms is **[2002]**
   (a) 256 (b) 512
   (c) 1024 (d) none of these
4. Sum of infinite number of terms of GP is 20 and sum of their square is 100. The common ratio of GP is **[2002]**
   (a) 5 (b) 3/5
   (c) 8/5 (d) 1/5
5. $1^3-2^3+3^3-4^3+...+9^3=$ **[2002]**
   (a) 425 (b) –425
   (c) 475 (d) –475
6. The sum of the series **[2003]**
   $\frac{1}{1.2}-\frac{1}{2.3}+\frac{1}{3.4}$........... up to ∞ is equal to
   (a) $\log_e\left(\frac{4}{e}\right)$ (b) $2\log_e 2$
   (c) $\log_e 2-1$ (d) $\log_e 2$
7. If the sum of the roots of the quadratic equation $ax^2+bx+c=0$ is equal to the sum of the squares of their reciprocals, then $\frac{a}{c},\frac{b}{a}$ and $\frac{c}{b}$ are in **[2003]**
   (a) Arithmetic - Geometric Progression
   (b) Arithmetic Progression
   (c) Geometric Progression
   (d) Harmonic Progression.
8. If $S_n=\sum_{r=0}^{n}\frac{1}{{}^nC_r}$ and $t_n=\sum_{r=0}^{n}\frac{r}{{}^nC_r}$, then $\frac{t_n}{S_n}$ is equal to **[2004]**
   (a) $\frac{2n-1}{2}$ (b) $\frac{1}{2}n-1$
   (c) $n-1$ (d) $\frac{1}{2}n$
9. Let T_r be the rth term of an A.P. whose first term is a and common difference is d. If for some positive integers $m, n,\ m \neq n,\ T_m=\frac{1}{n}$ and $T_n=\frac{1}{m}$, then $a-d$ equals **[2004]**
   (a) $\frac{1}{m}+\frac{1}{n}$ (b) 1
   (c) $\frac{1}{mn}$ (d) 0
10. The sum of the first n terms of the series $1^2+2.2^2+3^2+2.4^2+5^2+2.6^2+...$ is $\frac{n(n+1)^2}{2}$ when n is even. When n is odd the sum is **[2004]**

(a) $\left[\frac{n(n+1)}{2}\right]^2$ (b) $\frac{n^2(n+1)}{2}$

(c) $\frac{n(n+1)^2}{4}$ (d) $\frac{3n(n+1)}{2}$

11. The sum of series $\frac{1}{2!}+\frac{1}{4!}+\frac{1}{6!}+.....$ is **[2004]**

(a) $\frac{(e^2-2)}{e}$ (b) $\frac{(e-1)^2}{2e}$

(c) $\frac{(e^2-1)}{2e}$ (d) $\frac{(e^2-1)}{2}$

12. Let two numbers have arithmetic mean 9 and geometric mean 4. Then these numbers are the roots of the quadratic equation **[2004]**

(a) $x^2-18x-16=0$

(b) $x^2-18x+16=0$

(c) $x^2+18x-16=0$

(d) $x^2+18x+16=0$

13. If the coefficients of rth, $(r+1)$th, and $(r+2)$th terms in the the binomial expansion of $(1+y)^m$ are in A.P., then m and r satisfy the equation **[2005]**

(a) $m^2-m(4r-1)+4r^2-2=0$

(b) $m^2-m(4r+1)+4r^2+2=0$

(c) $m^2-m(4r+1)+4r^2-2=0$

(d) $m^2-m(4r-1)+4r^2+2=0$

14. If $x=\sum_{n=0}^{\infty}a^n$, $y=\sum_{n=0}^{\infty}b^n$, $z=\sum_{n=0}^{\infty}c^n$ where a, b, c are in A.P and $|a|<1, |b|<1, |c|<1$ then x, y, z are in **[2005]**

(a) G.P.

(b) A.P.

(c) Arithmetic - Geometric Progression

(d) H.P.

15. The sum of the series **[2005]**

$1+\frac{1}{4.2!}+\frac{1}{16.4!}+\frac{1}{64.6!}+$ ....upto ∞ is

(a) $\frac{e-1}{\sqrt{e}}$ (b) $\frac{e+1}{\sqrt{e}}$

(c) $\frac{e-1}{2\sqrt{e}}$ (d) $\frac{e+1}{2\sqrt{e}}$

16. Let a_1, a_2, a_3........... be terms on A.P. If

$\frac{a_1+a_2+..........a_p}{a_1+a_2+..........+a_q}=\frac{p^2}{q^2}$, $p \neq q$, then $\frac{a_6}{a_{21}}$ equals **[2006]**

(a) $\frac{41}{11}$ (b) $\frac{7}{2}$

(c) $\frac{2}{7}$ (d) $\frac{11}{41}$

17. If a_1, a_2, ..., a_n are in H.P., then the expression $a_1a_2+a_2a_3+.........+a_{n-1}a_n$ is equal to **[2006]**

(a) $n(a_1-a_n)$ (b) $(n-1)(a_1-a_n)$

(c) na_1a_n (d) $(n-1)a_1a_n$

18. The sum of series $\frac{1}{2!}-\frac{1}{3!}+\frac{1}{4!}-$ .... upto infinity is **[2007]**

(a) $e^{-\frac{1}{2}}$ (b) $e^{+\frac{1}{2}}$

(c) e^{-2} (d) e^{-1}

19. In a geometric progression consisting of positive terms, each term equals the sum of the next two terms. Then the common ratio of its progression is equals **[2007]**

(a) $\sqrt{5}$ (b) $\frac{1}{2}\left(\sqrt{5}-1\right)$

(c) $\frac{1}{2}\left(1-\sqrt{5}\right)$ (d) $\frac{1}{2}\sqrt{5}$

20. The first two terms of a geometric progression add up to 12. the sum of the third and the fourth terms is 48. If the terms of the geometric progression are alternately positive and negative, then the first term is **[2008]**

(a) –4 (b) –12
(c) 12 (d) 4

21. The sum to infinite term of the series

$1+\frac{2}{3}+\frac{6}{3^2}+\frac{10}{3^3}+\frac{14}{3^4}+...$ is **[2009]**

(a) 3 (b) 4
(c) 6 (d) 2

22. A person is to count 4500 currency notes. Let a_n denote the number of notes he counts in the nth minute. If $a_1 = a_2 = ... = a_{10} = 150$ and $a_{10}, a_{11}, ...$ are in an AP with common difference –2, then the time taken by him to count all notes is **[2010]**

(a) 34 minutes (b) 125 minutes
(c) 135 minutes (d) 24 minutes

23. A man saves ₹ 200 in each of the first three months of his service. In each of the subsequent months his saving increases by ₹ 40 more than the saving of immediately previous month. His total saving from the start of service will be ₹ 11040 after **[2011]**

(a) 19 months (b) 20 months
(c) 21 months (d) 18 months

24. Let a_n be the n^{th} term of an A.P. If $\sum_{r=1}^{100} a_{2r} = \alpha$ and $\sum_{r=1}^{100} a_{2r-1} = \beta,$ then the common difference of the A.P. is **[2011]**

(a) $\alpha-\beta$ (b) $\frac{\alpha-\beta}{100}$
(c) $\beta-\alpha$ (d) $\frac{\alpha-\beta}{200}$

25. **Statement-1:** The sum of the series $1+(1+2+4)+(4+6+9)+(9+12+16)+....+(361+380+400)$ is 8000.

Statement-2: $\sum_{k=1}^{n}\left(k^3-(k-1)^3\right)=n^3,$ for any natural number n. **[2012]**

(a) Statement-1 is false, Statement-2 is true.
(b) Statement-1 is true, statement-2 is true; statement-2 is a correct explanation for Statement-1.
(c) Statement-1 is true, statement-2 is true; statement-2 is **not** a correct explanation for Statement-1.
(d) Statement-1 is true, statement-2 is false.

26. If 100 times the 100th term of an AP with non zero common difference equals the 50 times its 50th term, then the 150th term of this AP is : **[2012]**

(a) – 150
(b) 150 times its 50th term
(c) 150
(d) Zero

27. The sum of first 20 terms of the sequence 0.7, 0.77, 0.777,....., is **[2013]**

(a) $\frac{7}{81}(179-10^{-20})$
(b) $\frac{7}{9}(99-10^{-20})$
(c) $\frac{7}{81}(179+10^{-20})$
(d) $\frac{7}{9}(99+10^{-20})$

28. Let α and β be the roots of equation $px^2 + qx + r = 0, p \neq 0$. If p, q, r are in A.P and $\frac{1}{\alpha}+\frac{1}{\beta}=4$, then the value of $|\alpha-\beta|$ is: **[2014]**

(a) $\frac{\sqrt{34}}{9}$ (b) $\frac{2\sqrt{13}}{9}$
(c) $\frac{\sqrt{61}}{9}$ (d) $\frac{2\sqrt{17}}{9}$

29. If $(10)^9+2(11)^1(10^8)+3(11)^2(10)^7+.....+10(11)^9=k(10)^9$, then k is equal to: **[2014]**

(a) 100 (b) 110
(c) $\frac{121}{10}$ (d) $\frac{441}{100}$

30. Three positive numbers form an increasing G. P. If the middle term in this G.P. is doubled, the new numbers are in A.P. then the common ratio of the G.P. is: **[2014]**

(a) $2-\sqrt{3}$ (b) $2+\sqrt{3}$

(c) $\sqrt{2}+\sqrt{3}$ (d) $3+\sqrt{2}$

31. The sum of first 9 terms of the series.

$$\frac{1^3}{1}+\frac{1^3+2^3}{1+3}+\frac{1^3+2^3+3^3}{1+3+5}+....$$ **[2015]**

(a) 142 (b) 192

(c) 71 (d) 96

32. If m is the A.M. of two distinct real numbers l and n(l, n > 1) and G_1, G_2 and G_3 are three geometric means between l and n, then $G_1^4+2G_2^4+G_3^4$ equals. **[2015]**

(a) $4\,lmn^2$ (b) $4\,l^2m^2n^2$

(c) $4\,l^2\,mn$ (d) $4\,lm^2n$

33. If the 2^{nd}, 5^{th} and 9^{th} terms of a non-constant A.P. are in G.P., then the common ratio of this G.P. is: **[2016]**

(a) 1 (b) $\frac{7}{4}$

(c) $\frac{8}{5}$ (d) $\frac{4}{3}$

34. If the sum of the first ten terms of the series

$$\left(1\frac{3}{5}\right)^2+\left(2\frac{2}{5}\right)^2+\left(3\frac{1}{5}\right)^2+4^2+\left(4\frac{4}{5}\right)^2+.......,$$

is $\frac{16}{5}$m, then m is equal to : **[2016]**

(a) 100 (b) 99

(c) 102 (d) 101

35. For any three positive real numbers a, b and c, $9(25a^2+b^2)+25(c^2-3ac)=15b(3a+c)$. Then : **[2017]**

(a) a, b and c are in G.P.

(b) b, c and a are in G.P.

(c) b, c and a are in A.P.

(d) a, b and c are in A.P.

36. Let a, b, c ∈ R. If $f(x)=ax^2+bx+c$ is such that $a+b+c=3$ and $f(x+y)=f(x)+f(y)+xy,\ \forall\ x,y\in R$, then $\sum_{n=1}^{10} f(n)$ is equal to : **[2017]**

(a) 2553 (b) 330

(c) 165 (d) 190

37. Let $a_1, a_2, a_3, ..., a_{49}$ be in A.P. such that $\sum_{k=0}^{12} a_{4k+1}=416$ and $a_9+a_{43}=66$. If $a_1^2+a_2^2+...+a_{17}^2=140m$, then m is equal to : **[2018]**

(a) 68 (b) 34

(c) 33 (d) 66

38. Let A be the sum of the first 20 terms and B be the sum of the first 40 terms of the series

$$1^2+2\cdot2^2+3^2+2.4^2+5^2+2.6^2+...$$

If $B-2A=100\lambda$, then λ is equal to : **[2018]**

(a) 248 (b) 464

(c) 496 (d) 232

39. Let the sum of the first n terms of a non-constant A.P., a_1, a_2, a_3, ………….. be $50n+\frac{n(n-7)}{2}A$, where A is a constant. If d is the common difference of this A.P., then the ordered pair (d, a_{50}) is equal to: **[2019]**

(a) $(50, 50+46A)$ (b) $(50, 50+45A)$

(c) $(A, 50+45A)$ (d) $(A, 50+46A)$

40. Let $\sum_{k=1}^{10} f(a+k)=16(2^{10}-1)$, where the function f satisfies f(x + y) = f(x) f(y) for all natural numbers x, y and f(1) = 2. Then the natural number 'a' is: **[2019]**

(a) 2 (b) 16 (c) 4 (d) 3

41. If a, b and c be three distinct real numbers in G.P. and a + b + c = xb, then x cannot be: **[2019]**

(a) – 2 (b) – 3

(c) 4 (d) 2

42. Five numbers are in A.P., whose sum is 25 and product is 2520. If one of these five numbers is $-\frac{1}{2}$, then the greatest number amongst them is: **[2020]**

(a) 27 (b) 7

(c) $\frac{21}{2}$ (d) 16

43. The greatest positive integer k, for which $49^k + 1$ is a factor of the sum $49^{125} + 49^{124} + ... + 49^2 + 49 + 1$, is: **[2020]**

(a) 32 (b) 63

(c) 60 (d) 65

Answer Key

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
(b)	(b)	(b)	(b)	(a)	(a)	(d)	(d)	(d)	(b)	(b)	(b)	(c)	(d)	(d)
16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
(d)	(d)	(d)	(b)	(b)	(a)	(a)	(c)	(b)	(b)	(d)	(c)	(b)	(a)	(b)
31	32	33	34	35	36	37	38	39	40	41	42	43		
(d)	(d)	(d)	(d)	(c)	(b)	(b)	(a)	(d)	(d)	(d)	(d)	(b)		

Solutions

1. **(b)** $1, \log_9(3^{1-x}+2), \log_3(4.3^x-1)$ are in A.P.

$\because$ a, b, c are in A.P then $b = a + c$

$\Rightarrow 2\log_9(3^{1-x}+2) = 1 + \log_3(4.3^x - 1)$

$\because \log_{b^q} a^p = \frac{p}{q}\log_b a$

$\Rightarrow \log_3(3^{1-x}+2) = \log_3 3 + \log_3(4.3^x - 1)$

$\Rightarrow \log_3(3^{1-x}+2) = \log_3[3(4.3^x - 1)]$

$\Rightarrow 3^{1-x} + 2 = 3(4.3^x - 1)$

$\Rightarrow 3.3^{-x} + 2 = 12.3^x - 3.$

Put $3^x = t$

$\Rightarrow \frac{3}{t} + 2 = 12t - 3 \Rightarrow 12t^2 - 5t - 3 = 0;$

Hence $t = -\frac{1}{3}, \frac{3}{4}$

$\Rightarrow 3^x = \frac{3}{4}$ (as $3^x \neq -ve$)

$\Rightarrow x = \log_3\left(\frac{3}{4}\right)$ or $x = \log_3 3 - \log_3 4$

$\Rightarrow x = 1 - \log_3 4$

2. **(b)** Let $P = 2^{1/4}.2^{2/8}.2^{3/16}.........\infty$

$= 2^{1/4+2/8+3/16+.........\infty}$

Now, let $S = \frac{1}{4} + \frac{2}{8} + \frac{3}{16} + ...\infty$ ......(1)

$\frac{1}{2}S = \frac{1}{8} + \frac{2}{16} + ...\infty$ ......(2)

Subtracting (2) from (1)

$\Rightarrow \frac{1}{2}S = \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + ...\infty$

or $\frac{1}{2}S = \frac{a}{1-r} = \frac{1/4}{1-1/2} = \frac{1}{2} \Rightarrow S = 1$

$\therefore P = 2^S = 2$

3. **(b)** $\because a_4 = 2 \Rightarrow ar^4 = 2$

Now, $a \times ar \times ar^2 \times ar^3 \times ar^4$

$\times ar^5 \times ar^6 \times ar^7 \times ar^8$

$= a^9 r^{36} = (ar^4)^9 = 2^9 = 512$

4. **(b)** Let a = first term of G.P. and r = common ratio of G.P.; Then G.P. is a, ar, ar^2

Given $S_\infty = 20 \Rightarrow \frac{a}{1-r} = 20$

$\Rightarrow a = 20(1-r)$ ... (i)

Also $a^2 + a^2r^2 + a^2r^4 + ...$ to $\infty = 100$

$$\Rightarrow \frac{a^2}{1-r^2} = 100 \Rightarrow \frac{[20(1-r)]^2}{1-r^2} = 100$$ [from (i)]

$$\Rightarrow \frac{400(1-r)^2}{(1-r)(1+r)} = 100 \Rightarrow 4(1-r) = 1+r$$

$\Rightarrow 1 + r = 4 - 4r \Rightarrow 5r = 3 \Rightarrow r = 3/5.$

5. **(a)** $1^3 - 2^3 + 3^3 - 4^3 + ... + 9^3$

$= 1^3 + 2^3 + 3^3 + ... + 9^3 - 2(2^3 + 4^3 + 6^3 + 8^3)$

$$\left[\because \Sigma n^3 = \left(\frac{n(n+1)}{2}\right)^2\right]$$

$$= \left[\frac{9 \times 10}{2}\right]^2 - 2.2^3\left[1^3 + 2^3 + 3^3 + 4^3\right]$$

$$= (45)^2 - 16.\left[\frac{4 \times 5}{2}\right]^2 = 2025 - 1600 = 425$$

6. **(a)** Let $S = \frac{1}{1.2} - \frac{1}{2.3} + \frac{1}{3.4} ....\infty$

$$T_n = \frac{1}{n(n+1)} = \left(\frac{1}{n} - \frac{1}{n+1}\right)$$

$$\therefore S = \left(\frac{1}{1} - \frac{1}{2}\right) - \left(\frac{1}{2} - \frac{1}{3}\right) + \left(\frac{1}{3} - \frac{1}{4}\right) - \left(\frac{1}{4} - \frac{1}{5}\right).....$$

$$= 1 - 2\left[\frac{1}{2} - \frac{1}{3} + \frac{1}{4} - \frac{1}{5}...............\infty\right]$$

$$\left[\because \log(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + ...\infty\right]$$

$$= 1 - 2[-\log(1+1) + 1] = 2\log 2 - 1 = \log\left(\frac{4}{e}\right).$$

7. **(d)** $ax^2 + bx + c = 0,\ \alpha + \beta = \frac{-b}{a},\ \alpha\beta = \frac{c}{a}$

ATQ, $\alpha + \beta = \frac{1}{\alpha^2} + \frac{1}{\beta^2}$

$$\alpha + \beta = \frac{\alpha^2 + \beta^2}{\alpha^2\beta^2} \Rightarrow -\frac{b}{a} = \frac{\frac{b^2}{a^2} - \frac{2c}{a}}{\frac{c^2}{a^2}}$$

On simplification $2a^2c = ab^2 + bc^2$

$\Rightarrow \frac{2a}{b} = \frac{c}{a} + \frac{b}{c}$ [Divide both side by abc]

$\Rightarrow \frac{c}{a}, \frac{a}{b}, \frac{b}{c}$ are in A.P.

$\therefore \frac{a}{c}, \frac{b}{a}, \& \frac{c}{b}$ are in H.P.

8. **(d)** $S_n = \frac{1}{^nC_0} + \frac{1}{^nC_1} + \frac{1}{^nC_2} + ... + \frac{1}{^nC_n}$

$$t_n = \frac{0}{^nC_0} + \frac{1}{^nC_1} + \frac{2}{^nC_2} + ... + \frac{n}{^nC_n} \quad ...(i)$$

$$t_n = \frac{n}{^nC_n} + \frac{n-1}{^nC_{n-1}} + \frac{n-2}{^nC_{n-2}} + ... + \frac{0}{^nC_0} \quad ...(ii)$$

Adding (i) and (ii), we get,

$$2t_n = (n)\left[\frac{1}{^nC_0} + \frac{1}{^nC_1} + ...\frac{1}{^nC_n}\right] = nS_n$$

$\because {^nC_r} = {^nC_{n-r}}$

$\therefore \frac{t_n}{S_n} = \frac{n}{2}$

9. **(d)** $T_m = a + (m-1)\,d = \frac{1}{n}$ .....(i)

$T_n = a + (n-1)d = \frac{1}{m}$ .....(ii)

Subtracting (ii) from (i), we get

$(m-n)d = \frac{1}{n} - \frac{1}{m} \Rightarrow d = \frac{1}{mn}$

From (i) $a = \frac{1}{mn} \Rightarrow a - d = 0$

10. (b) If n is odd, the required sum is

$$1^2+2.2^2+3^2+2.4^2+......+2.(n-1)^2+n^2$$

$$=\frac{(n-1)(n-1+1)^2}{2}+n^2$$

$\because$ If n is even then
$1^2+2.2^2+3^2+2.4^2....+2(n-1)^2$
$=n(n+1)^2/2$

$$=\left(\frac{n-1}{2}+1\right)n^2=\frac{n^2(n+1)}{2}$$

11. (b) We know that

$$e^x=1+\frac{x}{1!}+\frac{x^2}{2!}+\frac{x^3}{3!}.....\infty$$

$$\therefore\ e=1+\frac{1}{1!}+\frac{1}{2!}+\frac{1}{3!}+.......$$

and $e^{-1}=1-\frac{1}{1!}+\frac{1}{2!}-\frac{1}{3!}+.......$

$$\therefore e+e^{-1}=2\left[1+\frac{1}{2!}+\frac{1}{4!}+....\right]$$

$$\therefore\frac{1}{2!}+\frac{1}{4!}+\frac{1}{6!}+......=\frac{e+e^{-1}}{2}-1$$

$$=\frac{e^2+1-2e}{2e}=\frac{(e-1)^2}{2e}$$

12. (b) Let two numbers be a and b then $\frac{a+b}{2}=9$

$\Rightarrow a+b=18$ and $\sqrt{ab}=4\Rightarrow ab=16$
$\therefore$ Equation with roots a and b is

$$x^2-(a+b)x+ab=0$$

$$\Rightarrow x^2-18x+16=0$$

13. (c) Coeficient of r^{th}, $(r+1)^{th}$ and $(r+2)^{th}$ terms is ${}^mC_{r-1}$, ${}^mC_r$ and ${}^mC_{r+1}$ resp.

Given that ${}^mC_{r-1}$, ${}^mC_r$, ${}^mC_{r+1}$ are in A.P.

$$2\,{}^mC_r={}^mC_{r-1}+{}^mC_{r+1}$$

$$\Rightarrow\ 2=\frac{{}^mC_{r-1}}{{}^mC_r}+\frac{{}^mC_{r+1}}{{}^mC_r}$$

$$=\frac{r}{m-r+1}+\frac{m-r}{r+1}$$

$$\Rightarrow\ m^2-m(4r+1)+4r^2-2=0.$$

14. (d) $x=\sum_{n=0}^{\infty}a^n=\frac{1}{1-a}\ \Rightarrow a=1-\frac{1}{x}$

$$y=\sum_{n=0}^{\infty}b^n=\frac{1}{1-b}\ \Rightarrow b=1-\frac{1}{y}$$

$$z=\sum_{n=0}^{\infty}c^n=\frac{1}{1-c}\ \Rightarrow c=1-\frac{1}{z}$$

a, b, c are in A.P. $\Rightarrow$ $2b=a+c$

$$2\left(1-\frac{1}{y}\right)=1-\frac{1}{x}+1-\frac{1}{z}$$

$$\frac{2}{y}=\frac{1}{x}+\frac{1}{z}\ \Rightarrow x, y, z \text{ are in H.P.}$$

15. (d) We know that

$$\frac{e^x+e^{-x}}{2}=1+\frac{x^2}{2!}+\frac{x^4}{4!}+\frac{x^6}{6!}.............$$

Putting $x=\frac{1}{2}$, we get

$$1+\frac{1}{4.2!}+\frac{1}{16.4!}+\frac{1}{64.6!}+......\infty=\frac{e^{\frac{1}{2}}+e^{\frac{-1}{2}}}{2}$$

$$=\frac{\sqrt{e}+\frac{1}{\sqrt{e}}}{2}=\frac{e+1}{2\sqrt{e}}$$

16. (d) Given that

$$\frac{S_p}{S_q}=\frac{\frac{p}{2}[2a_1+(p-1)d]}{\frac{q}{2}[2a_1+(q-1)d]}=\frac{p^2}{q^2}$$

$$\Rightarrow\ \frac{2a_1+(p-1)d}{2a_1+(q-1)d}=\frac{p}{q}$$

Put $p = 11$ and $q = 41$

$$\frac{a_1 + 5d}{a_1 + 20d} = \frac{11}{41} \Rightarrow \frac{a_6}{a_{21}} = \frac{11}{41}$$

17. **(d)** $\because a_1, a_2, a_3 \ldots\ldots a_n$ are in H.P.

$\therefore \frac{1}{a_1}, \frac{1}{a_2}, \frac{1}{a_3} \ldots\ldots \frac{1}{a_n}$ are in A.P.

$$\therefore \frac{1}{a_2} - \frac{1}{a_1} = \frac{1}{a_3} - \frac{1}{a_2} = \ldots\ldots\ldots = \frac{1}{a_n} - \frac{1}{a_{n-1}} = d$$

(say)

Then $a_1 a_2 = \frac{a_1 - a_2}{d}$, $a_2 a_3 = \frac{a_2 - a_3}{d}$,

$\ldots\ldots\ldots\ldots, a_{n-1}a_n = \frac{a_{n-1} - a_n}{d}$

Adding all equations, we get

$\therefore \quad a_1 a_2 + a_2 a_3 + \ldots\ldots\ldots + a_{n-1}a_n$

$$= \frac{a_1 - a_2}{d} + \frac{a_2 - a_3}{d} + \ldots. + \frac{a_{n-1} - a_n}{d}$$

$$= \frac{1}{d}[a_1 - a_2 + a_2 - a_3 + \ldots. + a_{n-1} - a_n]$$

$$= \frac{a_1 - a_n}{d}$$

Also, $\frac{1}{a_n} = \frac{1}{a_1} + (n-1)d$

$$\Rightarrow \frac{a_1 - a_n}{a_1 a_n} = (n-1)d$$

$$\Rightarrow \frac{a_1 - a_n}{d} = (n-1)a_1 a_n$$

Which is the required result.

18. **(d)** We know that $e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots\ldots\ldots\infty$

Put $x = -1$

$$\therefore \quad e^{-1} = 1 - 1 + \frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} \ldots\ldots\ldots\infty$$

$$\therefore \quad e^{-1} = \frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} - \frac{1}{5!} \ldots\ldots\ldots\infty$$

19. **(b)** Let the series $a, ar, ar^2, \ldots..$ are in geometric progression.

Given that, $a = ar + ar^2$

$\Rightarrow \quad 1 = r + r^2 \quad \Rightarrow \quad r^2 + r - 1 = 0$

$$\Rightarrow \quad r = \frac{-1 \pm \sqrt{1 - 4 \times -1}}{2} = \frac{-1 \pm \sqrt{5}}{2}$$

$\Rightarrow \quad r = \frac{\sqrt{5} - 1}{2}$ [$\because$ terms of G.P. are positive

$\therefore$ r should be positive]

20. **(b)** ATQ,

$a + ar = 12 \qquad \ldots(1)$

$ar^2 + ar^3 = 48 \qquad \ldots(2)$

$$\Rightarrow \frac{ar^2(1+r)}{a(1+r)} = \frac{48}{12} \Rightarrow r^2 = 4, \Rightarrow r = -2$$

($\because$ terms are alternately + ve and –ve)

$\Rightarrow \quad a = -12$

21. **(a)** Let $S = 1 + \frac{2}{3} + \frac{6}{3^2} + \frac{10}{3^3} + \frac{14}{3^4} + \ldots\ldots.\infty$ ....(1)

Multiplying both sides by $\frac{1}{3}$, we get

$$\frac{1}{3}S = \frac{1}{3} + \frac{2}{3^2} + \frac{6}{3^3} + \frac{10}{3^4} + \ldots\ldots..\infty \qquad \ldots.(2)$$

Subtracting eqn. (2) from eqn. (1), we get

$$\frac{2}{3}S = 1 + \frac{1}{3} + \frac{4}{3^2} + \frac{4}{3^3} + \frac{4}{3^4} + \ldots\ldots..\infty$$

$$\Rightarrow \frac{2}{3}S = \frac{4}{3} + \frac{4}{3^2} + \frac{4}{3^3} + \frac{4}{3^4} + \ldots\ldots..\infty$$

$$\Rightarrow \frac{2}{3}S = \frac{\frac{4}{3}}{1 - \frac{1}{3}} = \frac{4}{3} \times \frac{3}{2} \Rightarrow S = 3$$

22. **(a)** Till 10th minute number of counted notes = 1500

Remaining notes = 4500 – 1500 = 3000

$$3000 = \frac{n}{2}[2 \times 148 + (n-1)(-2)] = n[148 - n + 1]$$

$$n^2 - 149n + 3000 = 0$$

$\Rightarrow\ n = 125, 24$

But $n = 125$ is not possible

$\therefore$ Total time $= 24 + 10 = 34$ minutes.

23. (c) Let number of months $= n$

$\therefore 200 \times 3 + (240 + 280 + 320 + \ldots + (n-3)^{\text{th}} \text{ term})$

$= 11040$

$$\Rightarrow \frac{n-3}{2}[2 \times 240 + (n-4) \times 40] = 11040 - 600$$

$\Rightarrow (n-3)[240 + 20n - 80] = 10440$

$\Rightarrow (n-3)(20n + 160) = 10440$

$\Rightarrow (n-3)(n+8) = 522$

$\Rightarrow n^2 + 5n - 546 = 0$

$\Rightarrow (n+26)(n-21) = 0$

$\therefore n = 21$

24. (b) Let A.P. be $a, a+d, a+2d, \ldots\ldots\ldots$

$a_2 + a_4 + \ldots\ldots\ldots\ldots + a_{200} = \alpha$

$$\Rightarrow \frac{100}{2}\left[2(a+d) + (100-1)2d\right] = \alpha \quad \ldots\text{(i)}$$

and $a_1 + a_3 + a_5 + \ldots\ldots\ldots + a_{199} = \beta$

$$\Rightarrow \frac{100}{2}[2a + (100-1)\,2d] = \beta \quad \ldots\text{(ii)}$$

Subtracting (ii) from (i), we get

$$d = \frac{\alpha - \beta}{100}$$

25. (b) nth term of the given series

$$= T_n = (n-1)^2 + (n-1)n + n^2$$

$$= \frac{\left((n-1)^3 - n^3\right)}{(n-1) - n} = n^3 - (n-1)^3$$

$$\Rightarrow S_n = \sum_{k=1}^{n}\left[k^3 - (k-1)^3\right] \Rightarrow 8000 = n^3$$

$\Rightarrow\ n = 20$ which is a natural number.

Hence, both the given statements are true. and statement 2 is correct explanation for statement 1.

26. (d) Let 'a' is the first term and 'd' is the common difference of an $A.P.$

Now, According to the question

$100a_{100} = 50a_{50}$

$100\,(a + 99d) = 50\,(a + 49d)$

$\Rightarrow\ 2a + 198\,d = a + 49d \ \Rightarrow\ a + 149\,d = 0$

Hence, $T_{150} = a + 149\,d = 0$

27. (c) Let $S = \frac{7}{10} + \frac{77}{100} + \frac{777}{10^3} + \ldots\ldots +$ up to 20 terms

$$= 7\left[\frac{1}{10} + \frac{11}{100} + \frac{111}{10^3} + \ldots\ldots + \text{up to 20 terms}\right]$$

Multiply and divide by 9

$$= \frac{7}{9}\left[\frac{9}{10} + \frac{99}{100} + \frac{999}{1000} + \ldots\ldots \text{up to 20 terms}\right]$$

$$= \frac{7}{9}\left[\left(1 - \frac{1}{10}\right) + \left(1 - \frac{1}{10^2}\right) + \left(1 - \frac{1}{10^3}\right) + \ldots\ldots \text{up to 20 terms}\right]$$

$$= \frac{7}{9}\left[20 - \frac{\frac{1}{10}\left(1 - \left(\frac{1}{10}\right)^{20}\right)}{1 - \frac{1}{10}}\right]$$

$$= \frac{7}{9}\left[\frac{179}{9} + \frac{1}{9}\left(\frac{1}{10}\right)^{20}\right]$$

$$= \frac{7}{81}\,[179 + (10)^{-20}]$$

28. (b) Let p, q, r are in AP

$\Rightarrow 2q = p + r \quad \ldots\text{(i)}$

Given $\frac{1}{\alpha} + \frac{1}{\beta} = 4$

$$\Rightarrow \frac{\alpha + \beta}{\alpha\beta} = 4$$

We have $\alpha + \beta = -q/p$ and $\alpha\beta = \frac{r}{p}$

$$\Rightarrow \frac{-\frac{q}{p}}{\frac{r}{p}} = 4 \Rightarrow q = -4r \quad \ldots\text{(ii)}$$

From (i), we have

$2(-4r) = p + r$

$p = -9r$ ....(iii)

Now, $|\alpha - \beta| = \sqrt{(\alpha+\beta)^2 - 4\alpha\beta}$

$= \sqrt{\left(\frac{-q}{p}\right)^2 - \frac{4r}{p}} = \frac{\sqrt{q^2 - 4pr}}{|p|}$

From (ii) and (iii)

$= \frac{\sqrt{16r^2 + 36r^2}}{|-9r|} = \frac{2\sqrt{13}}{9}$

29. (a) Given that $10^9 + 2.(11)(10)^8 + 3(11)^2(10)^7 + ... + 10(11)^9 = k(10)^9$

Let $x = 10^9 + 2.(11)(10)^8 + 3(11)^2(10)^7 + ... + 10(11)^9$ ...(i)

Multiplied by $\frac{11}{10}$ on both the sides

$\frac{11}{10}x = 11.10^8 + 2.(11)^2.(10)^7 + ... + 9(11)^9 + 11^{10}$ ...(ii)

Subtract (ii) from (i), we get

$x\left(1 - \frac{11}{10}\right) = 10^9 + 11(10)^8 + 11^2 \times (10)^7 + ... + 11^9 - 11^{10}$

$\Rightarrow -\frac{x}{10} = 10^9\left[\frac{\left(\frac{11}{10}\right)^{10} - 1}{\frac{11}{10} - 1}\right] - 11^{10}$

$\Rightarrow -\frac{x}{10} = (11^{10} - 10^{10}) - 11^{10} = -10^{10}$

$\Rightarrow x = 10^{11} = k.10^9$ Given

$\Rightarrow k = 100$

30. (b) Let a, ar, ar^2 are in G.P.

According to the question

$a, 2ar, ar^2$ are in A.P.

$\Rightarrow 2 \times 2ar = a + ar^2$

$\Rightarrow 4r = 1 + r^2 \Rightarrow r^2 - 4r + 1 = 0$

$r = \frac{4 \pm \sqrt{16-4}}{2} = 2 \pm \sqrt{3}$

Since $r > 1$

$\therefore \quad r = 2 - \sqrt{3}$ is rejected

Hence, $r = 2 + \sqrt{3}$

31. (d) n^{th} term of series

$= \frac{\left[\frac{n(n+1)}{2}\right]^2}{n^2} = \frac{1}{4}(n+1)^2$

Sum of n term $= \Sigma \frac{1}{4}(n+1)^2$

$= \frac{1}{4}\left[\Sigma n^2 + 2\Sigma n + n\right]$

$= \frac{1}{4}\left[\frac{n(n+1)(2n+1)}{6} + \frac{2n(n+1)}{2} + n\right]$

Sum of 9 terms

$= \frac{1}{4}\left[\frac{9 \times 10 \times 19}{6} + \frac{18 \times 10}{2} + 9\right]$

$= \frac{384}{4} = 96$

32. (d) $m = \frac{l+n}{2}$ and common ratio of G.P.

$= r = \left(\frac{n}{l}\right)^{\frac{1}{4}}$

$\therefore \quad G_1 = l^{3/4}n^{1/4}, G_2 = l^{1/2}n^{1/2}, G_3 = l^{1/4}n^{3/4}$

$G_1^4 + 2G_2^4 + G_3^4 = l^3n + 2l^2n^2 + ln^3$

$= ln(l+n)^2$

$= ln \times (2m)^2$

$= 4lm^2n$

33. (d) Let the GP be a, ar and ar^2 then $a = A + d$; $ar = A + 4d$; $ar^2 = A + 8d$

$\Rightarrow \frac{ar^2 - ar}{ar - a} = \frac{(A+8d)-(A+4d)}{(A+4d)-(A+d)}$

$r = \frac{4}{3}$

34. **(d)** $\left(\frac{8}{5}\right)^2 + \left(\frac{12}{5}\right)^2 + \left(\frac{16}{5}\right)^2 + \left(\frac{20}{5}\right)^2 \ldots + \left(\frac{44}{5}\right)^2$

$S = \frac{16}{25}\left(2^2 + 3^2 + 4^2 + \ldots + 11^2\right)$

$= \frac{16}{25}\left(\frac{11(11+1)(22+1)}{6} - 1\right)$

$= \frac{16}{25} \times 505 = \frac{16}{5} \times 101$

$\Rightarrow \frac{16}{5}m = \frac{16}{5} \times 101$

$\Rightarrow m = 101.$

35. **(c)** We have

$9(25a^2 + b^2) + 25(c^2 - 3ac) = 15b(3a + c)$

$\Rightarrow 225a^2 + 9b^2 + 25c^2 - 75ac = 45ab + 15bc$

$\Rightarrow (15a)^2 + (3b)^2 + (5c)^2 - 75ac - 45ab - 15bc = 0$

$\frac{1}{2}[(15a - 3b)^2 + (3b - 5c)^2 + (5c - 15a)^2] = 0$

It is possible when $15a - 3b = 0$, $3b - 5c = 0$ and $5c - 15a = 0$

$\Rightarrow 15a = 3b \Rightarrow b = 5a$

$\Rightarrow b = \frac{5c}{3}, a = \frac{c}{3}$

$\Rightarrow a + b = \frac{c}{3} + \frac{5c}{3} = \frac{6c}{3}$

$\Rightarrow a + b = 2c$

$\Rightarrow$ b, c, a are in A.P.

36. **(b)** $f(x) = ax^2 + bx + c$

$f(1) = a + b + c = 3 \Rightarrow f(1) = 3$

Now $f(x + y) = f(x) + f(y) + xy$ ...(i)

Put $x = y = 1$ in eqn (i)

$f(2) = f(1) + f(1) + 1$

$= 2f(1) + 1$

$f(2) = 7$

$\Rightarrow f(3) = 12$

$S_n = 3 + 7 + 12 + \ldots\ldots\ldots + t_n$

$S_n = \quad 3 + 7 + 12 + \ldots\ldots\ldots + t_{n-1} + t_n$

$- \quad - \quad - \quad - \quad - \quad -$

$0 = 3 + 4 + 5 \ldots\ldots\ldots$ to n term $- t_n$

$t_n = 3 + 4 + 5 + \ldots$ upto n terms

$t_n = \frac{(n^2 + 5n)}{2}$

$S_n = \sum t_n = \sum \frac{(n^2 + 5n)}{2}$

$S_n = \frac{1}{2}\left[\frac{n(n+1)(2n+1)}{6} + \frac{5n(n+1)}{2}\right]$

$= \frac{n(n+1)(n+8)}{6}$

$S_{10} = \frac{10 \times 11 \times 18}{6} = 330$

37. **(b)** $\because \sum_{k=0}^{12} a_{4k+1} = 416$

$\Rightarrow \frac{13}{2}[2a_1 + 48d] = 416$

$\Rightarrow a_1 + 24d = 32$ ...(i)

Now, $a_9 + a_{43} = 66 \Rightarrow 2a_1 + 50d = 66$ ...(ii)

From eqs. (i) & (ii) we get; $d = 1$ and $a_1 = 8$

Also, $\sum_{r=1}^{17} a_r^2 = \sum_{r=1}^{17} [8 + (r-1)1]^2 = 140\text{ m}$

$\Rightarrow \sum_{r=1}^{17} (r+7)^2 = 140\text{ m}$

$\Rightarrow \sum_{r=1}^{17} (r^2 + 14r + 49) = 140\text{ m}$

$\Rightarrow \left(\frac{17 \times 18 \times 35}{6}\right) + 14\left(\frac{17 \times 18}{2}\right) + (49 \times 17) = 140\text{ m}$

$\Rightarrow m = 34$

38. **(a)** Here, $B - 2A$

$= \sum_{n=1}^{40} a_n - 2\sum_{n=1}^{20} a_n = \sum_{n=21}^{40} a_n - \sum_{n=1}^{20} a_n$

$B - 2A = (21^2 + 2.22^2 + 23^2 + 2.24^2 + \ldots.. + 40^2)$
$- (1^2 + 2.2^2 + 3^2 + 2.4^2 \ldots.. + 20^2)$

$= 20[22 + 2.24 + 26 + 2.28 + \ldots.. + 60]$

$= 20\left[\underbrace{(22 + 24 + 26 \ldots. 60)}_{20\text{ terms}} + \underbrace{(24 + 28 + \ldots. + 60)}_{10\text{ terms}}\right]$

$$20\left[\frac{20}{2}(22+60)+\frac{10}{2}(24+60)\right]$$
$$=10[20.82+10.84]$$
$$=100[164+84]=100.248$$

39. (d) $\because S_n=\left(50-\frac{7A}{2}\right)n+n^2\times\frac{A}{2}$

$\Rightarrow$ a = S_1 = 50 – 3A

$\therefore d=a_2-a_1=(S_2-S_1)-S_1$

$\Rightarrow d=\frac{A}{2}\times 2=A$

Now, $a_{50}=a_1+49\times d$

$=(50-3A)+49A=50+46A$

So, $(d, a_{50})=(A, 50+46A)$

40. (d) $\because f(x+y)=f(x).f(y)$

$\Rightarrow$ Let $f(x)=t^x$

$\because f(1)=2 \therefore t=2$

$\Rightarrow f(x)=2^x$

Since, $\sum_{k=1}^{10} f(a+k)=16(2^{10}-1)$

Then, $\sum_{k=1}^{10} 2^{a+k}=16(2^{10}-1)$

$\Rightarrow 2^a\sum_{k=1}^{10}2^k=16(2^{10}-1)$

$\Rightarrow 2^a\times\frac{((2^{10})-1)\times 2}{(2-1)}=16\times(2^{10}-1)$

$\Rightarrow 2.2^a=16\Rightarrow a=3$

41. (d) $\because a, b, c,$ are in G.P.

$\Rightarrow \quad b^2=ac$

Since, $a+b+c=xb$

$\Rightarrow \quad a+c=(x-1)b$

Take square on both sides, we get

$a^2+c^2+2ac=(x-1)^2b^2$

$\Rightarrow \quad a^2+c^2=(x-1)^2ac-2ac \quad [\because b^2=ac]$

$\Rightarrow \quad a^2+c^2=ac[(x-1)^2-2]$

$\Rightarrow \quad a^2+c^2=ac[x^2-2x-1]$

$\because a^2+c^2$ are positive and $b^2=ac$ which is also positive. Then, x^2-2x-1 would be positive but for $x=2$, x^2-2x-1 is negative. Hence, x cannot be taken as 2.

42. (d) Let 5 terms of A.P. be

$a-2d, a-d, a, a+d, a+2d.$

Sum = 25 $\Rightarrow$ $5a=25$ $\Rightarrow$ $a=5$

Product = 2520

$(5-2d)(5-d)5(5+d)(5+2d)=2520$

$\Rightarrow \quad (25-4d^2)(25-d^2)=504$

$\Rightarrow \quad 625-100d^2-25d^2+4d^4=504$

$\Rightarrow \quad 4d^4-125d^2+625-504=0$

$\Rightarrow \quad 4d^4-125d^2+121=0$

$\Rightarrow \quad 4d^4-121d^2-4d^2+121=0$

$\Rightarrow \quad (d^2-1)(4d^2-121)=0$

$\Rightarrow \quad d=\pm 1, d=\pm\frac{11}{2}$

$d=\pm 1$ and $d=-\frac{11}{2}$, does not give $\frac{-1}{2}$ as a term

$\therefore \quad d=\frac{11}{2}$

$\therefore$ Largest term $=5+2d=5+11=16$

43. (b) $\frac{(49)^{126}-1}{48}=\frac{((49)^{63}+1)(49^{63}-1)}{48}$

$\left[\because S_n=\frac{a(r^n-1)}{r-1}\right]$

$\therefore$ K = 63

10 Straight Lines & Pair of Straight Lines

1. A triangle with vertices (4, 0), (–1, –1), (3, 5) is **[2002]**
   (a) isosceles and right angled
   (b) isosceles but not right angled
   (c) right angled but not isosceles
   (d) neither right angled nor isosceles

2. Locus of mid point of the portion between the axes of $x \cos\alpha + y\sin\alpha = p$ whre p is constant is **[2002]**
   (a) $x^2+y^2=\frac{4}{p^2}$ (b) $x^2+y^2=4p^2$
   (c) $\frac{1}{x^2}+\frac{1}{y^2}=\frac{2}{p^2}$ (d) $\frac{1}{x^2}+\frac{1}{y^2}=\frac{4}{p^2}$

3. If the pair of lines $ax^2+2hxy+by^2+2gx+2fy+c=0$ intersect on the y-axis then **[2002]**
   (a) $2fgh=bg^2+ch^2$ (b) $bg^2 \neq ch^2$
   (c) $abc=2fgh$ (d) none of these

4. The pair of lines represented by
   $3ax^2+5xy+(a^2-2)y^2=0$
   are perpendicular to each other for **[2002]**
   (a) two values of a (b) $\forall\, a$
   (c) for one value of a (d) for no values of a

5. A square of side a lies above the x-axis and has one vertex at the origin. The side passing through the origin makes an angle $\alpha\left(0<\alpha<\frac{\pi}{4}\right)$ with the positive direction of x-axis. The equation of its diagonal not passing through the origin is **[2003]**
   (a) $y(\cos\alpha+\sin\alpha)+x(\cos\alpha-\sin\alpha)=a$
   (b) $y(\cos\alpha-\sin\alpha)-x(\sin\alpha-\cos\alpha)=a$
   (c) $y(\cos\alpha+\sin\alpha)+x(\sin\alpha-\cos\alpha)=a$
   (d) $y(\cos\alpha+\sin\alpha)+x(\sin\alpha+\cos\alpha)=a$.

6. If the pair of straight lines $x^2-2pxy-y^2=0$ and $x^2-2qxy-y^2=0$ be such that each pair bisects the angle between the other pair, then **[2003]**
   (a) $pq=-1$ (b) $p=q$
   (c) $p=-q$ (d) $pq=1$

7. Locus of centroid of the triangle whose vertices are $(a\cos t, a\sin t), (b\sin t, -b\cos t)$ and $(1,0)$, where t is a parameter, is **[2003]**
   (a) $(3x+1)^2+(3y)^2=a^2-b^2$
   (b) $(3x-1)^2+(3y)^2=a^2-b^2$
   (c) $(3x-1)^2+(3y)^2=a^2+b^2$
   (d) $(3x+1)^2+(3y)^2=a^2+b^2$

8. If x_1, x_2, x_3 and y_1, y_2, y_3 are both in G.P. with the same common ratio, then the points $(x_1, y_1), (x_2, y_2)$ and (x_3, y_3) **[2003]**
   (a) are vertices of a triangle
   (b) lie on a straight line
   (c) lie on an ellipse
   (d) lie on a circle.

9. If the equation of the locus of a point equidistant from the point (a_1, b_1) and (a_2, b_2) is $(a_1 - b_2)x + (a_1 - b_2)y + c = 0$, then the value of `c` is **[2003]**

(a) $\sqrt{a_1^2 + b_1^2 - a_2^2 - b_2^2}$

(b) $\frac{1}{2}a_2^2 + b_2^2 - a_1^2 - b_1^2$

(c) $a_1^2 - a_2^2 + b_1^2 - b_2^2$

(d) $\frac{1}{2}(a_1^2 + a_2^2 + b_1^2 + b_2^2)$.

10. Let $A(2, -3)$ and $B(-2, 3)$ be vertices of a triangle ABC. If the centroid of this triangle moves on the line $2x + 3y = 1$, then the locus of the vertex C is the line **[2004]**

(a) $3x - 2y = 3$ (b) $2x - 3y = 7$

(c) $3x + 2y = 5$ (d) $2x + 3y = 9$

11. The equation of the straight line passing through the point (4, 3) and making intercepts on the co-ordinate axes whose sum is –1 is **[2004]**

(a) $\frac{x}{2} - \frac{y}{3} = 1$ and $\frac{x}{-2} + \frac{y}{1} = 1$

(b) $\frac{x}{2} - \frac{y}{3} = -1$ and $\frac{x}{-2} + \frac{y}{1} = -1$

(c) $\frac{x}{2} + \frac{y}{3} = 1$ and $\frac{x}{2} + \frac{y}{1} = 1$

(d) $\frac{x}{2} + \frac{y}{3} = -1$ and $\frac{x}{-2} + \frac{y}{1} = -1$

12. If the sum of the slopes of the lines given by $x^2 - 2cxy - 7y^2 = 0$ is four times their product. c has the value **[2004]**

(a) –2 (b) –1

(c) 2 (d) 1

13. If one of the lines given by $6x^2 - xy + 4cy^2 = 0$ is $3x + 4y = 0$, then c equals **[2004]**

(a) –3 (b) 1

(c) 3 (d) 1

14. The line parallel to the x- axis and passing through the intersection of the lines $ax + 2by + 3b = 0$ and $bx - 2ay - 3a = 0$, where $(a, b) \neq (0, 0)$ is **[2005]**

(a) below the x - axis at a distance of $\frac{3}{2}$ from it

(b) below the x - axis at a distance of $\frac{2}{3}$ from it

(c) above the x - axis at a distance of $\frac{3}{2}$ from it

(d) above the x - axis at a distance of $\frac{2}{3}$ from it

15. If a vertex of a triangle is (1, 1) and the mid points of two sides through this vertex are (–1, 2) and (3, 2) then the centroid of the triangle is **[2005]**

(a) $\left(-1, \frac{7}{3}\right)$ (b) $\left(\frac{-1}{3}, \frac{7}{3}\right)$

(c) $\left(1, \frac{7}{3}\right)$ (d) $\left(\frac{1}{3}, \frac{7}{3}\right)$

16. A straight line through the point A (3, 4) is such that its intercept between the axes is bisected at A. Its equation is **[2006]**

(a) $x + y = 7$ (b) $3x - 4y + 7 = 0$

(c) $4x + 3y = 24$ (d) $3x + 4y = 25$

17. If (a, a^2) falls inside the angle made by the lines $y = \frac{x}{2}$, $x > 0$ and $y = 3x$, $x > 0$, then a belong to **[2006]**

(a) $\left(0, \frac{1}{2}\right)$ (b) $(3, \infty)$

(c) $\left(\frac{1}{2}, 3\right)$ (d) $\left(-3, -\frac{1}{2}\right)$

18. Let A (1, k), B (1, 1) and C (2, 1) be the vertices of a right angled triangle with AC as its hypotenuse. If the area of the triangle is 1 square unit, then the set of values which 'k' can take is given by **[2007]**

(a) {–1, 3} (b) {–3, –2}

(c) {1, 3} (d) {0, 2}

19. Let P = (–1, 0), Q = (0, 0) and R = $(3, 3\sqrt{3})$ be three point. The equation of the bisector of the angle PQR is **[2007]**

(a) $\frac{\sqrt{3}}{2}x+y=0$ (b) $x+\sqrt{3y}=0$

(c) $\sqrt{3}x+y=0$ (d) $x+\frac{\sqrt{3}}{2}y=0$

20. If one of the lines of $my^2+(1-m^2)xy-mx^2=0$ is a bisector of the angle between the lines xy = 0, then m is **[2007]**

(a) 1 (b) 2

(c) –1/2 (d) –2

21. The perpendicular bisector of the line segment joining P (1, 4) and Q(k, 3) has y-intercept –4. Then a possible value of k is **[2008]**

(a) 1 (b) 2

(c) –2 (d) – 4

22. The shortest distance between the line y – x = 1 and the curve $x = y^2$ is : **[2009]**

(a) $\frac{2\sqrt{3}}{8}$ (b) $\frac{3\sqrt{2}}{5}$

(c) $\frac{\sqrt{3}}{4}$ (d) $\frac{3\sqrt{2}}{8}$

23. The lines $p(p^2+1)x-y+q=0$ and $(p^2+1)^2x+(p^2+1)y+2q=0$ are perpendicular to a common line for : **[2009]**

(a) exactly one values of p

(b) exactly two values of p

(c) more than two values of p

(d) no value of p

24. Three distinct points A, B and C are given in the 2-dimensional coordinates plane such that the ratio of the distance of any one of them from the point (1, 0) to the distance from the point (–1, 0) is equal to $\frac{1}{3}$. Then the circumcentre of the triangle ABC is at the point: **[2009]**

(a) $\left(\frac{5}{4},0\right)$ (b) $\left(\frac{5}{2},0\right)$

(c) $\left(\frac{5}{3},0\right)$ (d) (0, 0)

25. The lines $L_1 : y-x=0$ and $L_2 : 2x+y=0$ intersect the line $L_3 : y+2=0$ at P and Q respectively. The bisector of the acute angle between L_1 and L_2 intersects L_3 at R. **[2011]**

Statement-1: The ratio $PR : RQ$ equals $2\sqrt{2}:\sqrt{5}$

Statement-2: In any triangle, bisector of an angle divides the triangle into two similar triangles.

(a) Statement-1 is true, Statement-2 is true; Statement-2 is **not** a correct explanation for Statement-1.

(b) Statement-1 is true, Statement-2 is false.

(c) Statement-1 is false, Statement-2 is true.

(d) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1.

26. The lines $x+y=|a|$ and $ax-y=1$ intersect each other in the first quadrant. Then the set of all possible values of a in the interval : **[2011RS]**

(a) $(0,\infty)$ (b) $[1,\infty)$

(c) $(-1,\infty)$ (d) $(-1,1)$

27. If A (2, – 3) and B (– 2, 1) are two vertices of a triangle and third vertex moves on the line $2x+3y=9$, then the locus of the centroid of the triangle is : **[2011RS]**

(a) $x-y=1$ (b) $2x+3y=1$

(c) $2x+3y=3$ (d) $2x-3y=1$

28. If the line $2x+y=k$ passes through the point which divides the line segment joining the points (1,1) and (2,4) in the ratio 3 :2, then k equals : **[2012]**

(a) $\frac{29}{5}$ (b) 5

(c) 6 (d) $\frac{11}{5}$

29. A line is drawn through the point (1,2) to meet the coordinate axes at P and Q such that it forms a triangle OPQ, where O is the origin. If the area of the triangle OPQ is least, then the slope of the line PQ is : **[2012]**

(a) $-\frac{1}{4}$ (b) -4

(c) -2 (d) $-\frac{1}{2}$

30. A ray of light along $x + \sqrt{3}y = \sqrt{3}$ gets reflected upon reaching x-axis, the equation of the reflected ray is **[2013]**

(a) $y = x + \sqrt{3}$ (b) $\sqrt{3}y = x - \sqrt{3}$

(c) $y = \sqrt{3}x - \sqrt{3}$ (d) $\sqrt{3}y = x - 1$

31. The x-coordinate of the incentre of the triangle that has the coordinates of mid points of its sides as (0, 1) (1, 1) and (1, 0) is : **[2013]**

(a) $2+\sqrt{2}$ (b) $2-\sqrt{2}$

(c) $1+\sqrt{2}$ (d) $1-\sqrt{2}$

32. Let PS be the median of the triangle vertices $P(2, 2)$, $Q(6, -1)$ and $R(7, 3)$. The equation of the line passing through (1, –1) and parallel to PS is: **[2014]**

(a) $4x+7y+3=0$ (b) $2x-9y-11=0$

(c) $4x-7y-11=0$ (d) $2x+9y+7=0$

33. Let a, b, c and d be non-zero numbers. If the point of intersection of the lines $4ax+2ay+c=0$ and $5bx+2by+d=0$ lies in the fourth quadrant and is equidistant from the two axes then **[2014]**

(a) $3bc-2ad=0$ (b) $3bc+2ad=0$

(c) $2bc-3ad=0$ (d) $2bc+3ad=0$

34. Two sides of a rhombus are along the lines, $x-y+1=0$ and $7x-y-5=0$. If its diagonals intersect at (–1, –2), then which one of the following is a vertex of this rhombus? **[2016]**

(a) $\left(\frac{1}{3},\frac{-8}{3}\right)$ (b) $\left(\frac{-10}{3},\frac{-7}{3}\right)$

(c) $(-3,-9)$ (d) $(-3,-8)$

35. A straight the through a fixed point (2, 3) intersects the coordinate axes at distinct points P and Q. If O is the origin and the rectangle OPRQ is completed, then the locus of R is :

(1) $2x+3y=xy$ (2) $3x+2y=xy$

(3) $3x+2y=6xy$ (4) $3x+2y=6$

36. Let the orthocentre and centroid of a triangle be A(–3, 5) and B(3, 3) respectively. If C is the circumcentre of this triangle, then the radius of the circle having line segment AC as diameter, is : **[2018]**

(a) $2\sqrt{10}$ (b) $3\sqrt{\frac{5}{2}}$

(c) $\frac{3\sqrt{5}}{2}$ (d) $\sqrt{10}$

37. Slope of a line passing through P(2, 3) and intersecting the line x + y = 7 at a distance of 4 units from P, is: **[2019]**

(a) $\frac{1-\sqrt{5}}{1+\sqrt{5}}$ (b) $\frac{1-\sqrt{7}}{1+\sqrt{7}}$

(c) $\frac{\sqrt{7}-1}{\sqrt{7}+1}$ (d) $\frac{\sqrt{5}-1}{\sqrt{5}+1}$

38. Consider the set of all lines $px + qy + r = 0$ such that $3p + 2q + 4r = 0$. Which one of the following statements is true? **[2019]**

(a) The lines are concurrent at the point $\left(\frac{3}{4},\frac{1}{2}\right)$.

(b) Each line passes through the origin.

(c) The lines are all parallel .

(d) The lines are not concurrent.

39. Let $A(1, 0)$, $B(6, 2)$ and C $\left(\frac{3}{2},6\right)$ be the vertices of a triangle ABC. If P is a point inside the triangle ABC such that the triangles APC, APB and BPC have equal areas, then the length of the line segment PQ, where Q is the point $\left(-\frac{7}{6},-\frac{1}{3}\right)$, is ______. **[2020]**

Answer Key

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
(a)	(d)	(a)	(a)	(a)	(a)	(c)	(b)	(b)	(d)	(a)	(c)	(a)	(a)	(c)
16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
(c)	(c)	(a)	(c)	(a)	(d)	(d)	(a)	(a)	(b)	(b)	(b)	(c)	(c)	(b)
31	32	33	34	35	36	37	38	39						
(b)	(d)	(a)	(a)	(b)	(b)	(b)	(a)	(5)						

Solutions

1. **(a)** $AB = \sqrt{(4+1)^2 + (0+1)^2} = \sqrt{26}$;

$BC = \sqrt{(3+1)^2 + (5+1)^2} = \sqrt{52}$

$CA = \sqrt{(4-3)^2 + (0-5)^2} = \sqrt{26}$;

$\because AB = CA$

$\therefore$ Isosceles triangle

$\because (\sqrt{26})^2 + (\sqrt{26})^2 = 52$

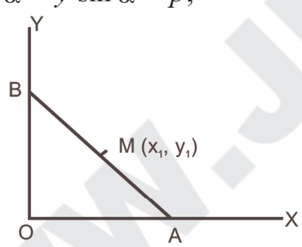
$BC^2 = AB^2 + AC^2$

$\therefore$ right angled triangle,

So, the given triangle is isosceles right angled.

2. **(d)** Equation of AB is

$x \cos\alpha + y \sin\alpha = p$;



$\Rightarrow \dfrac{x\cos\alpha}{p} + \dfrac{y\sin\alpha}{p} = 1$;

$\Rightarrow \dfrac{x}{p/\cos\alpha} + \dfrac{y}{p/\sin\alpha} = 1$

So, co-ordinates of A and B are

$\left(\dfrac{p}{\cos\alpha}, 0\right)$ and $\left(0, \dfrac{p}{\sin\alpha}\right)$;

So, coordinates of midpoint of AB are

$M(x_1, y_1) = \left(\dfrac{p}{2\cos\alpha}, \dfrac{p}{2\sin\alpha}\right)$

$x_1 = \dfrac{p}{2\cos\alpha}$ & $y_1 = \dfrac{p}{2\sin\alpha}$;

$\Rightarrow \cos\alpha = p/2x_1$ and $\sin\alpha = p/2y_1$;

$\because \cos^2\alpha + \sin^2\alpha = 1$

Locus of (x_1, y_1) is $\dfrac{1}{x^2} + \dfrac{1}{y^2} = \dfrac{4}{p^2}$.

3. **(a)** Put $x = 0$ in the given equation

$\Rightarrow by^2 + 2fy + c = 0$.

For unique point of intersection, $f^2 - bc = 0$

$\Rightarrow af^2 - abc = 0$.

We know that for pair of straight line

$abc + 2fgh - af^2 - bg^2 - ch^2 = 0$

$\Rightarrow 2fgh - bg^2 - ch^2 = 0$

4. **(a)** We know that pair of straighty lines

$ax^2 + 2hxy + by^2 = 0$ are perpendicular when

$a + b = 0$

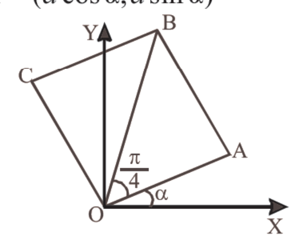
$3a + a^2 - 2 = 0 \Rightarrow a^2 + 3a - 2 = 0.$;

$\Rightarrow a = \dfrac{-3 \pm \sqrt{9+8}}{2} = \dfrac{-3 \pm \sqrt{17}}{2}$

5. **(a)** Co-ordinates of $A = (a\cos\alpha, a\sin\alpha)$

Equation of OB,

$y = \tan\left(\dfrac{\pi}{4} + \alpha\right)x$



CA $\perp^r$ to OB

$\therefore$ Slope of CA $= -\cot\left(\dfrac{\pi}{4} + \alpha\right)$

Equation of CA

$y - a\sin\alpha = -\cot\left(\dfrac{\pi}{4} + \alpha\right)(x - a\cos\alpha)$

$\Rightarrow (y - a\sin\alpha)\left(\tan\left(\dfrac{\pi}{4} + \alpha\right)\right) = (a\cos\alpha - x)$

$\Rightarrow (y - a\sin\alpha)\left(\dfrac{\tan\frac{\pi}{4} + \tan\alpha}{1 - \tan\frac{\pi}{4}\tan\alpha}\right) = (a\cos\alpha - x)$

$\Rightarrow (y - a\sin\alpha)(1 + \tan\alpha)$
$= (a\cos\alpha - x)(1 - \tan\alpha)$
$\Rightarrow (y - a\sin\alpha)(\cos\alpha + \sin\alpha)$
$= (a\cos\alpha - x)(\cos\alpha - \sin\alpha)$
$\Rightarrow y(\cos + \sin\alpha) - a\sin\alpha\cos\alpha - a\sin^2\alpha$
$= a\cos^2\alpha - a\cos\alpha\sin\alpha - x(\cos\alpha - \sin\alpha)$
$\Rightarrow y(\cos\alpha + \sin\alpha) + x(\cos\alpha - \sin\alpha) = a$
$y(\sin\alpha + \cos\alpha) + x(\cos\alpha - \sin\alpha) = a.$

6. **(a)** Equation of bisectors of second pair of straight lines is,
$qx^2 + 2xy - qy^2 = 0$ ....(i)
It must be identical to the first pair
$x^2 - 2pxy - y^2 = 0$ ....(ii)
from (i) and (ii)
$\frac{q}{1} = \frac{2}{-2p} = \frac{-q}{-1} \Rightarrow pq = -1.$

7. **(c)** We know that centroid
$(x, y) = \left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}\right)$
$x = \frac{a\cos t + b\sin t + 1}{3}$
$\Rightarrow a\cos t + b\sin t = 3x - 1$
$y = \frac{a\sin t - b\cos t}{3}$
$\Rightarrow a\sin t - b\cos t = 3y$
Squaring and adding,
$(3x - 1)^2 + (3y)^2 = a^2 + b^2$

8. **(b)** Taking co-ordinates as
$A\left(\frac{x}{r}, \frac{y}{r}\right); B(x, y)$ and $C(xr, yr)$.
Then slope of line joining
$A\left(\frac{x}{r}, \frac{y}{r}\right), B(x, y) = \frac{y\left(1 - \frac{1}{r}\right)}{x\left(1 - \frac{1}{r}\right)} = \frac{y}{x}$
and slope of line joining $B(x, y)$ and $C(xr, yr)$
$= \frac{y(r - 1)}{x(r - 1)} = \frac{y}{x}$
$\therefore m_1 = m_2$
$\because$ Slope of AB and BC are same and one point B common.
$\Rightarrow$ Points lie on the straight line.

9. **(b)** $(x - a_1)^2 + (y - b_1)^2$
$= (x - a_2)^2 + (y - b_2)^2$
$(a_1 - a_2)x + (b_1 - b_2)y$
$+ \frac{1}{2}(a_2{}^2 + b_2{}^2 - a_1{}^2 - b_1{}^2) = 0$
Comparing with given eqn. we get
$c = \frac{1}{2}(a_2{}^2 + b_2{}^2 - a_1{}^2 - b_1{}^2)$

10. **(d)** Let the vertex C be (h, k), then the centroid of
ΔABC is $\left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}\right)$
$= \left(\frac{2 - 2 + h}{3}, \frac{-3 + 1 + k}{3}\right)$
$= \left(\frac{h}{3}, \frac{-2 + k}{3}\right)$. It lies on $2x + 3y = 1$
$\Rightarrow \frac{2h}{3} - 2 + k = 1 \Rightarrow 2h + 3k = 9$
$\Rightarrow$ Locus of C is $2x + 3y = 9$

11. **(a)** Let the required line be $\frac{x}{a} + \frac{y}{b} = 1$ ....(i)
then $a + b = -1 \Rightarrow b = -a - 1$ ....(ii)
(i) passes through $(4, 3)$, $\Rightarrow \frac{4}{a} + \frac{3}{b} = 1$
$\Rightarrow 4b + 3a = ab$ ....(iii)
Putting value of b from (ii) in (iii), we get
$a^2 - 4 = 0 \Rightarrow a = \pm 2 \Rightarrow b = -3$ or 1
$\therefore$ Equations of straight lines are
$\frac{x}{2} + \frac{y}{-3} = 1$ or $\frac{x}{-2} + \frac{y}{1} = 1$

12. **(c)** Let the lines be $y = m_1x$ and $y = m_2x$ then
$m_1 + m_2 = -\frac{2c}{7}$ and $m_1m_2 = -\frac{1}{7}$
Given that $m_1 + m_2 = 4\, m_1m_2$
$\Rightarrow -\frac{2c}{7} = -\frac{4}{7} \Rightarrow c = 2$

13. **(a)** $3x + 4y = 0$ is one of the line of the pair equations. of lines
$6x^2 - xy + 4cy^2 = 0$, Put $y = -\frac{3}{4}x$,
we get, $6x^2 + \frac{3}{4}x^2 + 4c\left(-\frac{3}{4}x\right)^2 = 0$
$\Rightarrow 6 + \frac{3}{4} + \frac{9c}{4} = 0 \Rightarrow c = -3$

14. (a) The eqn. of line passing through the intersection of lines $ax+2by+3b=0$ and $bx-2ay-3a=0$ is

$ax+2by+3b+\lambda(bx-2ay-3a)=0$

$\Rightarrow (a+b\lambda)x+(2b-2a\lambda)y+3b-3\lambda a=0$

Required line is parallel to x-axis.

$\therefore\ a+b\lambda=0 \Rightarrow \lambda=-a/b$

$\Rightarrow ax+2by+3b-\dfrac{a}{b}(bx-2ay-3a)=0$

$\Rightarrow ax+2by+3b-ax+\dfrac{2a^2}{b}y+\dfrac{3a^2}{b}=0$

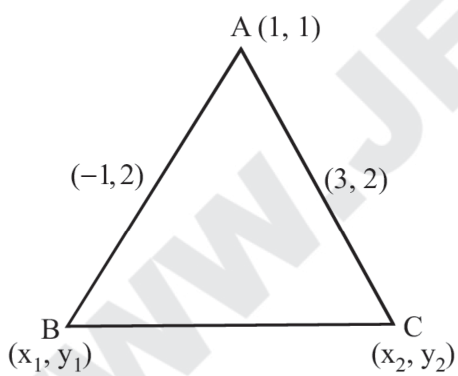
$y\left(2b+\dfrac{2a^2}{b}\right)+3b+\dfrac{3a^2}{b}=0$

$y\left(\dfrac{2b^2+2a^2}{b}\right)=-\left(\dfrac{3b^2+3a^2}{b}\right)$

$y=\dfrac{-3(a^2+b^2)}{2(b^2+a^2)}=\dfrac{-3}{2}$

So it is 3/2 units below x-axis.

15. (c) Vertex of triangle is (1, 1) and midpoint of sides through this vertex is (– 1, 2) and (3, 2)



$\dfrac{1+x_1}{2}=-1,\ \dfrac{1+y_1}{2}=2$

$\Rightarrow$ B (–3, 3)

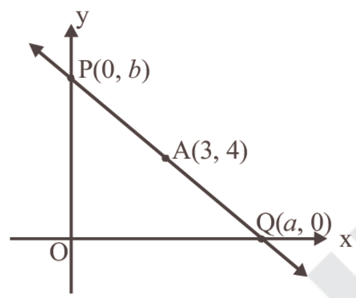
$\dfrac{1+x_2}{2}=3,\dfrac{1+y_2}{2}=2$

$\Rightarrow$ C (–5, 3)

$\therefore$ Centroid is $\dfrac{1-3+5}{3},\dfrac{1+3+3}{3}$

$\Rightarrow\left(1,\dfrac{7}{3}\right)$

16. (c)



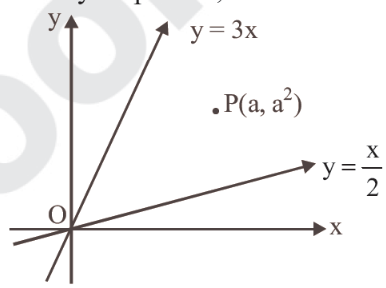
$\because$ A is the mid point of PQ,

$\therefore\ \dfrac{a+0}{2}=3,\ \dfrac{0+b}{2}=4 \Rightarrow a=6, b=8$

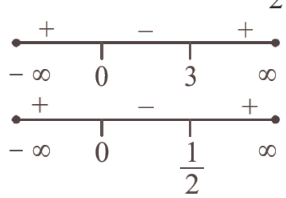
$\therefore$ Equation of line is $\dfrac{x}{6}+\dfrac{y}{8}=1$

or $4x+3y=24$

17. (c) Clearly for point P,

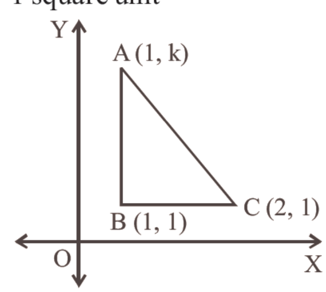


$a^2-3a<0$ and $a^2-\dfrac{a}{2}>0$



$\Rightarrow\ \dfrac{1}{2}<a<3$

18. (a) **Given :** A(l, k), B(1, 1) and C(2, 1) are vertices of a right angled triangle and area of ΔABC = 1 square unit

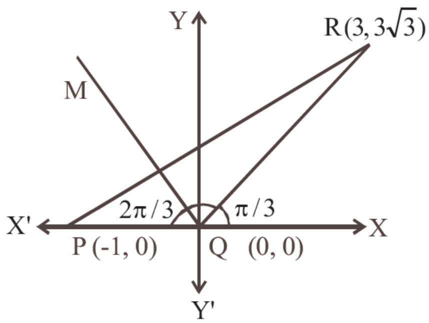


We know that, area of right angled triangle

$=\dfrac{1}{2}\times BC\times AB\ =\ 1=\dfrac{1}{2}(1)\,|(k-1)|$

$\Rightarrow\ \pm(k-1)=2\ \Rightarrow\ k=-1, 3$

19. (c) Given : The coordinates of points P, Q, R are $(-1, 0)$, $(0, 0)$, $(3, 3\sqrt{3})$ respectively.



Slope of QR $= \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{3\sqrt{3}}{3}$

$\Rightarrow \quad \tan\theta = \sqrt{3} \Rightarrow \theta = \dfrac{\pi}{3}$

$\Rightarrow \angle RQX = \dfrac{\pi}{3}$

$\therefore \quad \angle RQP = \pi - \dfrac{\pi}{3} = \dfrac{2\pi}{3}$

Let QM bisects the $\angle PQR$,

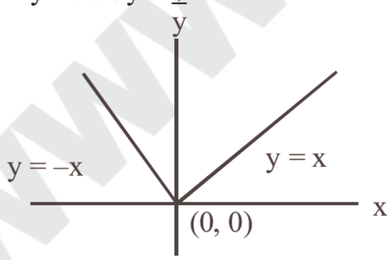
$\therefore \quad \angle MQR = \dfrac{\pi}{3} \Rightarrow \angle MQX = \dfrac{2\pi}{3}$

$\therefore$ Slope of the line QM $= \tan\dfrac{2\pi}{3} = -\sqrt{3}$

$\therefore$ Equation of line QM is $(y - 0) = -\sqrt{3}\,(x - 0)$

$\Rightarrow \quad y = -\sqrt{3}\,x \Rightarrow \sqrt{3}\,x + y = 0$

20. (a) From figure equation of bisectors of lines, $xy = 0$ are $y = \pm x$



$\therefore$ Put $y = \pm x$ in the given equation $my^2 + (1 - m^2)xy - mx^2 = 0$

$\therefore \quad mx^2 \pm (1 - m^2)x^2 - mx^2 = 0$

$\Rightarrow \quad 1 - m^2 = 0 \Rightarrow m = \pm 1$

21. (d) Slope of $PQ = \dfrac{3-4}{k-1} = \dfrac{-1}{k-1}$

$\therefore$ Slope of perpendicular bisector of PQ $= (k - 1)$

Also, mid point of PQ $\left(\dfrac{k+1}{2}, \dfrac{7}{2}\right)$.

$\therefore$ Equation of perpendicular bisector of PQ is

$$y - \frac{7}{2} = (k-1)\left(x - \frac{k+1}{2}\right)$$

$\Rightarrow \quad 2y - 7 = 2(k-1)x - (k^2 - 1)$

$\Rightarrow \quad 2(k-1)x - 2y + (8 - k^2) = 0$

Given that y-intercept

$= \dfrac{8 - k^2}{2} = -4$

$\Rightarrow \quad 8 - k^2 = -8$ or $k^2 = 16 \Rightarrow k = \pm 4$

22. (d) Let (a^2, a) be the point of shortest distance on $x = y^2$

Then distance between (a^2, a) and line $x - y + 1 = 0$ is given by

$$D = \left|\frac{ax_1 + by_1 + c}{\sqrt{a^2 + b^2}}\right| = \frac{|a^2 - a + 1|}{\sqrt{2}}$$

$$= \frac{1}{\sqrt{2}}\left|(a - \frac{1}{2})^2 + \frac{3}{4}\right|$$

It is min when $a = \dfrac{1}{2}$ and

$D_{min} = \dfrac{3}{4\sqrt{2}} = \dfrac{3\sqrt{2}}{8}$

23. (a) Given that the lines $p(p^2 + 1)x - y + q = 0$ and $(p^2 + 1)^2 x + (p^2 + 1)y + 2q = 0$ are perpendicular to a common line then these lines must be parallel to each other,

$\therefore \quad m_1 = m_2 \quad \Rightarrow -\dfrac{p(p^2+1)}{-1} = -\dfrac{(p^2+1)^2}{p^2+1}$

$\Rightarrow (p^2 + 1)^2\ (p + 1) = 0$

$\Rightarrow p = -1$

$\therefore$ p can have exactly one value.

24. (a) Given that $P(1, 0), Q(-1, 0)$

and $\dfrac{AP}{AQ} = \dfrac{BP}{BQ} = \dfrac{CP}{CQ} = \dfrac{1}{3}$

$\Rightarrow 3AP = AQ \Rightarrow 9\,AP^2 = AQ^2$

Let $A = (x, y)$ then

$9(x - 1)^2 + 9y^2 = (x + 1)^2 + y^2$

$\Rightarrow 9x^2 - 18x + 9 + 9y^2 = x^2 + 2x + 1 + y^2$

$\Rightarrow\ 8x^2 - 20x + 8y^2 + 8 = 0$

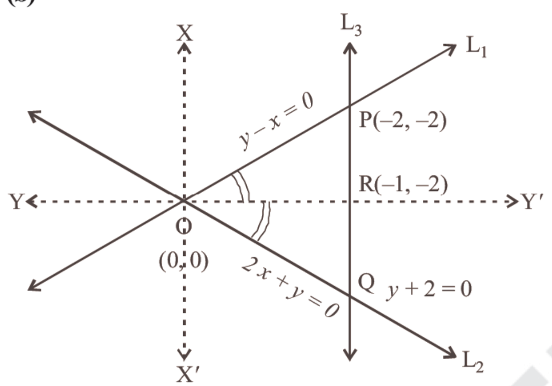
$\Rightarrow\ x^2 + y^2 - \frac{5}{2}x + 1 = 0$ ....(i)

$\therefore$ A lies on the circle given by eq (i). As B and C also follow the same condition, they must lie on the same circle.

$\therefore$ Centre of circumcircle of ΔABC

= Centre of circle given by (1) = $\left(\frac{5}{4}, 0\right)$

25. (b)



$L_1 : y - x = 0$
$L_2 : 2x + y = 0$
$L_3 : y + 2 = 0$

On solving the equation of line L_1 and L_2 we get their point of intersection (0, 0) i.e., origin O.

On solving the equation of line L_1 and L_3, we get $P = (-2, -2)$.

Similarly, solving equation of line L_2 and L_3, we get $Q = (-1, -2)$

We know that bisector of an angle of a triangle, divide the opposite side the triangle in the ratio of the sides including the angle [Angle Bisector Theorem of a Triangle]

$$\therefore \frac{PR}{RQ} = \frac{OP}{OQ} = \frac{\sqrt{(-2)^2 + (-2)^2}}{\sqrt{(-1)^2 + (-2)^2}} = \frac{2\sqrt{2}}{\sqrt{5}}$$

$\therefore$ Statement 1 is true but $\angle OPR \neq \angle OQR$

So ΔOPR and ΔOQR not similar

$\therefore$ Statement 2 is false.

26. (b) Given that $x + y = |a|$

and $ax - y = 1$

Case I : If $a > 0$

$x + y = a$ .... (1)

$ax - y = 1$ .... (2)

On adding equations (1) and (2), we get

$x(1 + a) = 1 + a \Rightarrow x = 1$

$y = a - 1$

Since given that intersection point lies in first quadrant

So, $a - 1 \geq 0$

$\Rightarrow\ a \geq 1$

$\Rightarrow\ a \in [1, \infty)$

Case II : If $a < 0$

$x + y = -a$ .... (3)

$ax - y = 1$ .... (4)

On adding equations (3) and (4), we get

$x(1 + a) = 1 - a$

$x = \frac{1-a}{1+a} > 0 \Rightarrow \frac{a-1}{a+1} < 0$

Since $a - 1 < 0$

$\therefore\ a + 1 > 0$

$\Rightarrow\ a > -1$ .... (5)

–1

$y = -a - \frac{1-a}{1+a} > 0\ = \frac{-a - a^2 - 1 + a}{1+a} > 0$

$\Rightarrow\ -\left(\frac{a^2+1}{a+1}\right) > 0 \Rightarrow \frac{a^2+1}{a+1} < 0$

Since $a^2 + 1 > 0$

$\therefore\ a + 1 < 0$

$\Rightarrow\ a < -1$ .... (6)

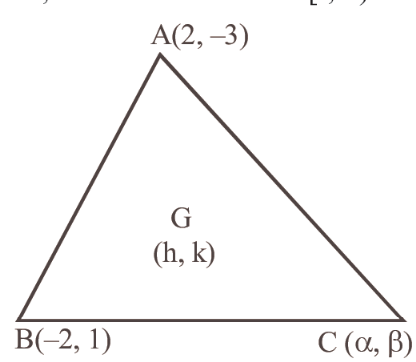
–1

From (5) and (6), $a \in \phi$

Hence, Case-II is not possible.

So, correct answer is $a \in [1, \infty)$

27. (b)



Centroid $(h,k) = \left(\frac{2-2+\alpha}{3}, \frac{-3+1+\beta}{3}\right)$

$\therefore \quad \alpha = 3h$

$\beta - 2 = 3k$

$\beta = 3k + 2$

Third vertex (α, β) lies on the line

$2x + 3y = 9$

$2\alpha + 3\beta = 9$

$2(3h) + 3(3k+2) = 9$

$2h + 3k = 1$

$2x + 3y = 1$

28. **(c)** Let the points be A (1,1) and B (2,4).
Let point C divides line AB in the ratio $3:2$.
So, by section formula we have

$$C = \left(\frac{3\times2+2\times1}{3+2}, \frac{3\times4+2\times1}{3+2}\right)$$

$$= \left(\frac{8}{5}, \frac{14}{5}\right)$$

Since Line $2x + y = k$ passes through $C\left(\frac{8}{5}, \frac{14}{5}\right)$

$$\Rightarrow \quad \frac{2\times8}{5} + \frac{14}{5} = k \quad \Rightarrow \quad k = 6$$

29. **(c)** Equation of a line passing through (x_1, y_1) having slope m is given by
$y - y_1 = m(x - x_1)$
The equation of line PQ which is passing through (1, 2) is $(y-2) = m(x-1)$
where m is the slope of the line PQ.
Now, point $P(x, 0)$ lies on x-axis

$\therefore \quad y - 2 = m(x-1) \Rightarrow 0 - 2 = m(x-1)$

$\Rightarrow \quad -2 = m(x-1) \Rightarrow \quad x - 1 = \frac{-2}{m}$

$\Rightarrow \quad x = \frac{-2}{m} + 1$

Also, $OP = x$

$= \frac{-2}{m} + 1$

Similarly, point $Q(0, y)$ lies on y-axis.

$\therefore \quad y - 2 = m(x-1)$

$\Rightarrow \quad y - 2 = m(-1)$

$\Rightarrow \quad y = 2 - m$ and $OQ = y = 2 - m$

Area of $\Delta POQ = \frac{1}{2}(OP)(OQ)$

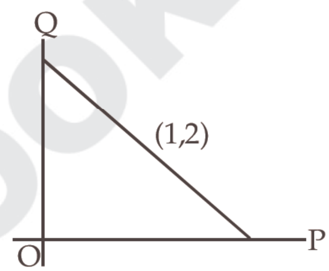
$= \frac{1}{2}\left(1 - \frac{2}{m}\right)(2-m)$

$\left(\because \text{Area of } \Delta = \frac{1}{2} \times \text{base} \times \text{height}\right)$

$= \frac{1}{2}\left[2 - m - \frac{4}{m} + 2\right]$

$= \frac{1}{2}\left[4 - \left(m + \frac{4}{m}\right)\right]$

$= 2 - \frac{m}{2} - \frac{2}{m}$



Let Area $= f(m) = 2 - \frac{m}{2} - \frac{2}{m}$

Now, $f'(m) = \frac{-1}{2} + \frac{2}{m^2}$

Put $f'(m) = 0$

$\Rightarrow m^2 = 4 \Rightarrow m = \pm 2$

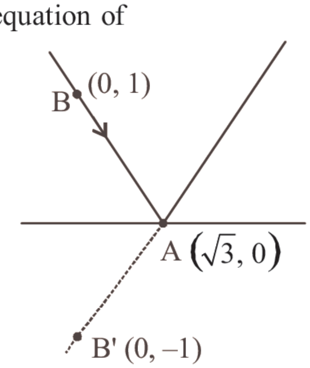
Now, $f''(m) = \frac{-4}{m^3}$

$f''(m)\big|_{m=2} = -\frac{1}{2} < 0$

$f''(m)\big|_{m=-2} = \frac{1}{2} > 0$

Area will be least at $m = -2$
Hence, slope of PQ is -2.

30. **(b)** Suppose B(0, 1) be any point on given line and co-ordinate of A is $(\sqrt{3}, 0)$. So, equation of



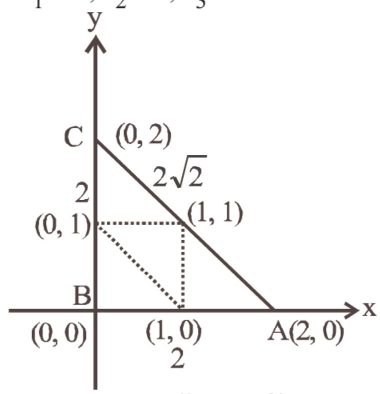
Reflected ray is $\frac{-1-0}{0-\sqrt{3}} = \frac{y-0}{x-\sqrt{3}}$

$\Rightarrow \quad \sqrt{3}y = x - \sqrt{3}$

31. (b) From the figure, we have

$a = 2, b = 2\sqrt{2}, \ c = 2$

$x_1 = 0, x_2 = 0, x_3 = 2$



Now, x-co-ordinate of incentre is given as

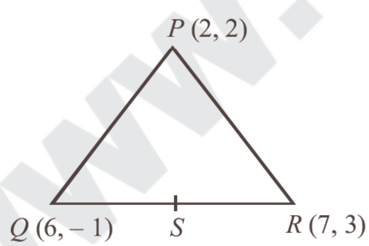
$\frac{ax_1 + bx_2 + cx_3}{a+b+c}$

$\Rightarrow$ x-coordinate of incentre

$= \frac{2 \times 0 + 2\sqrt{2}.0 + 2.2}{2 + 2 + 2\sqrt{2}}$

$= \frac{2}{2+\sqrt{2}} = 2 - \sqrt{2}$

32. (d) Let P, Q, R, be the vertices of ΔPQR



Since PS is the median

S is mid-point of QR

So, $S = \left(\frac{7+6}{2}, \frac{3-1}{2}\right) = \left(\frac{13}{2}, 1\right)$

Now, slope of $PS = \frac{2-1}{2-\frac{13}{2}} = -\frac{2}{9}$

Since, required line is parallel to PS therefore slope of required line = slope of PS

Now, eqn. of line passing through $(1, -1)$ and having slope $-\frac{2}{9}$ is

$y - (-1) = -\frac{2}{9}(x-1)$

$9y + 9 = -2x + 2 \Rightarrow 2x + 9y + 7 = 0$

33. (a) Given lines are

$4ax + 2ay + c = 0$

$5bx + 2by + d = 0$

The point of intersection will be

$\frac{x}{2ad - 2bc} = \frac{-y}{4ad - 5bc} = \frac{1}{8ab - 10ab}$

$\Rightarrow \quad x = \frac{2(ad - bc)}{-2ab} = \frac{bc - ad}{ab}$

$\Rightarrow \quad y = \frac{5bc - 4ad}{-2ab} = \frac{4ad - 5bc}{2ab}$

$\because$ Point of intersection is in fourth quadrant so x is positive and y is negative.

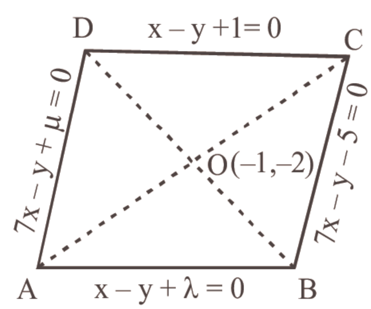
Also distance from axes is same

So $x = -y$

($\because$ distance from x-axis is $-y$ as y is negative)

$\frac{bc - ad}{ab} = \frac{5bc - 4ad}{2ab} \Rightarrow 3bc - 2ad = 0$

34. (a)



Let other two sides of rhombus are

$x - y + \lambda = 0$

and $7x - y + \mu = 0$

then O is equidistant from AB and DC and from AD and BC

$\therefore \ |-1 + 2 + 1| = |-1 + 2 + \lambda| \Rightarrow \lambda = -3$

and $|-7 + 2 - 5| = |-7 + 2 + \mu| \Rightarrow \mu = 15$

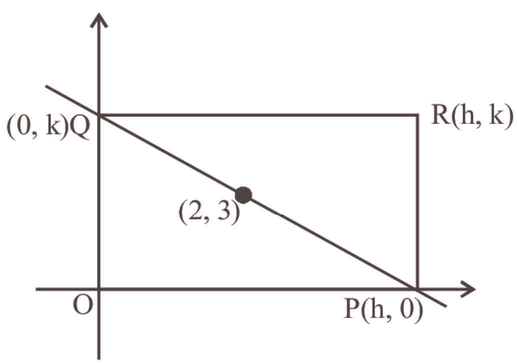
$\therefore$ Other two sides are $x - y - 3 = 0$ and $7x - y + 15 = 0$

On solving the eq^{n}s of sides pairwise, we get
the vertices as

$$\left(\frac{1}{3},\frac{-8}{3}\right),(1,2),\left(\frac{-7}{3},\frac{-4}{3}\right),(-3,-6)$$

35. (b) Equation of PQ is

$$\frac{x}{h}+\frac{y}{k}=1 \quad ...(1)$$



Since, (1) passes through the fixed point (2, 3) Then, $\frac{2}{h}+\frac{3}{k}=1$

Then, the locus of R is $\frac{2}{x}+\frac{3}{y}=1$

or $3x+2y=xy$.

36. (b) Since Orthocentre of the triangle is A(–3, 5) and centriod of the triangle is B(3, 3), then

AB $=\sqrt{40}=2\sqrt{10}$

Centroid divides orthocentre and circumcentre of the triangle in ratio 2 : 1

$\therefore$ AB : BC = 2 : 1

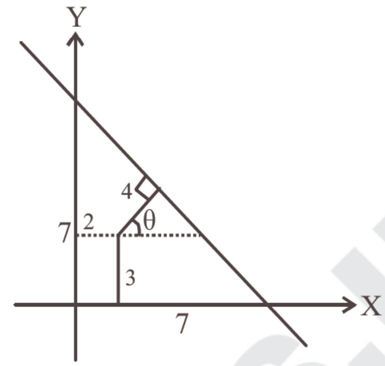
Now, $AB=\frac{2}{3}AC$

$\Rightarrow \quad AC=\frac{3}{2}AB=\frac{3}{2}(2\sqrt{10}) \quad \Rightarrow AC=3\sqrt{10}$

$\therefore$ Radius of circle with AC as diametre

$$=\frac{AC}{2}=\frac{3}{2}\sqrt{10}=3\sqrt{\frac{5}{2}}$$

37. (b)



Since point at 4 units from P (2, 3) will be A (4 cosθ + 2, 4 sin (θ + 3) and this point will satisfy the equation of line x + y = 7

$$\Rightarrow \cos\theta+\sin\theta=\frac{1}{2}$$

On squaring

$$\Rightarrow \sin 2\theta=-\frac{3}{4}\Rightarrow\frac{2\tan\theta}{1+\tan^2\theta}=-\frac{3}{4}$$

$$\Rightarrow 3\tan^2\theta+8\tan\theta+3=0$$

$$\Rightarrow \tan\theta=\frac{-8\pm 2\sqrt{7}}{6}$$

(θ is acute so, ignoring –ve sign)

$$\Rightarrow \tan\theta=\frac{-8+2\sqrt{7}}{6}\times\frac{1+\sqrt{7}}{1+\sqrt{7}}=\frac{1-\sqrt{7}}{1+\sqrt{7}}$$

38. (a) The given equations of the set of all lines

$px+qy+r=0$ .....(1)

and given condition is :

$3p+2q+4r=0$

$$\Rightarrow \frac{3}{4}p+\frac{2}{4}q+r=0 \quad ...(2)$$

From (1) & (2) we get :

$$\therefore x=\frac{3}{4}, y=\frac{1}{2}$$

Hence the set of lines are concurrent and passing through the fixed point $\left(\frac{3}{4},\frac{1}{2}\right)$

39. (5) P will be centroid of ΔABC

$$P\left(\frac{17}{6},\frac{8}{3}\right)\Rightarrow PQ=\sqrt{(4)^2+(3)^2}=5$$

11 Conic Sections

1. If the chord $y = mx + 1$ of the circle $x^2+y^2=1$ subtends an angle of measure 45° at the major segment of the circle then value of m is **[2002]**

(a) $2 \pm \sqrt{2}$ (b) $-2 \pm \sqrt{2}$

(c) $-1 \pm \sqrt{2}$ (d) none of these

2. The centres of a set of circles, each of radius 3, lie on the circle $x^2+y^2=25$. The locus of any point in the set is **[2002]**

(a) $4 \le x^2+y^2 \le 64$ (b) $x^2+y^2 \le 25$

(c) $x^2+y^2 \ge 25$ (d) $3 \le x^2+y^2 \le 9$

3. The centre of the circle passing through (0, 0) and (1, 0) and touching the circle $x^2+y^2=9$ is **[2002]**

(a) $\left(\frac{1}{2},\frac{1}{2}\right)$ (b) $\left(\frac{1}{2},-\sqrt{2}\right)$

(c) $\left(\frac{3}{2},\frac{1}{2}\right)$ (d) $\left(\frac{1}{2},\frac{3}{2}\right)$

4. The equation of a circle with origin as a centre and passing through equilateral triangle whose median is of length $3a$ is **[2002]**

(a) $x^2+y^2=9a^2$ (b) $x^2+y^2=16a^2$

(c) $x^2+y^2=4a^2$ (d) $x^2+y^2=a^2$

5. Two common tangents to the circle $x^2+y^2=2a^2$ and parabola $y^2=8ax$ are **[2002]**

(a) $x=\pm(y+2a)$ (b) $y=\pm(x+2a)$

(c) $x=\pm(y+a)$ (d) $y=\pm(x+a)$

6. If the two circles $(x-1)^2+(y-3)^2=r^2$ and $x^2+y^2-8x+2y+8=0$ intersect in two distinct point, then **[2003]**

(a) $r > 2$ (b) $2 < r < 8$

(c) $r < 2$ (d) $r = 2$.

7. The lines $2x-3y=5$ and $3x-4y=7$ are diameters of a circle having area as 154 sq.units.Then the equation of the circle is **[2003]**

(a) $x^2+y^2-2x+2y=62$

(b) $x^2+y^2+2x-2y=62$

(c) $x^2+y^2+2x-2y=47$

(d) $x^2+y^2-2x+2y=47$.

8. The normal at the point $(bt_1^2, 2bt_1)$ on a parabola meets the parabola again in the point $(bt_2^2, 2bt_2)$, then **[2003]**

(a) $t_2=t_1+\frac{2}{t_1}$ (b) $t_2=-t_1-\frac{2}{t_1}$

(c) $t_2=-t_1+\frac{2}{t_1}$ (d) $t_2=t_1-\frac{2}{t_1}$

9. The foci of the ellipse $\frac{x^2}{16}+\frac{y^2}{b^2}=1$ and the hyperbola $\frac{x^2}{144}-\frac{y^2}{81}=\frac{1}{25}$ coincide. Then the value of b^2 is **[2003]**

(a) 9 (b) 1

(c) 5 (d) 7

10. If a circle passes through the point (a, b) and cuts the circle $x^2 + y^2 = 4$ orthogonally, then the locus of its centre is **[2004]**

(a) $2ax - 2by - (a^2 + b^2 + 4) = 0$

(b) $2ax + 2by - (a^2 + b^2 + 4) = 0$

(c) $2ax - 2by + (a^2 + b^2 + 4) = 0$

(d) $2ax + 2by + (a^2 + b^2 + 4) = 0$

11. A variable circle passes through the fixed point $A(p, q)$ and touches x-axis. The locus of the other end of the diameter through A is **[2004]**

(a) $(y - q)^2 = 4px$ (b) $(x - q)^2 = 4py$

(c) $(y - p)^2 = 4qx$ (d) $(x - p)^2 = 4qy$

12. If the lines $2x + 3y + 1 = 0$ and $3x - y - 4 = 0$ lie along diameter of a circle of circumference 10π, then the equation of the circle is **[2004]**

(a) $x^2 + y^2 + 2x - 2y - 23 = 0$

(b) $x^2 + y^2 - 2x - 2y - 23 = 0$

(c) $x^2 + y^2 + 2x + 2y - 23 = 0$

(d) $x^2 + y^2 - 2x + 2y - 23 = 0$

13. Intercept on the line $y = x$ by the circle $x^2 + y^2 - 2x = 0$ is AB. Equation of the circle on AB as a diameter is **[2004]**

(a) $x^2 + y^2 + x - y = 0$

(b) $x^2 + y^2 - x + y = 0$

(c) $x^2 + y^2 + x + y = 0$

(d) $x^2 + y^2 - x - y = 0$

14. If $a \neq 0$ and the line $2bx + 3cy + 4d = 0$ passes through the points of intersection of the parabolas $y^2 = 4ax$ and $x^2 = 4ay$, then **[2004]**

(a) $d^2 + (3b - 2c)^2 = 0$

(b) $d^2 + (3b + 2c)^2 = 0$

(c) $d^2 + (2b - 3c)^2 = 0$

(d) $d^2 + (2b + 3c)^2 = 0$

15. The eccentricity of an ellipse, with its centre at the origin, is $\frac{1}{2}$. If one of the directrices is $x = 4$, then the equation of the ellipse is: **[2004]**

(a) $4x^2 + 3y^2 = 1$ (b) $3x^2 + 4y^2 = 12$

(c) $4x^2 + 3y^2 = 12$ (d) $3x^2 + 4y^2 = 1$

16. If the circles $x^2 + y^2 + 2ax + cy + a = 0$ and $x^2 + y^2 - 3ax + dy - 1 = 0$ intersect in two distinct points P and Q then the line $5x + by - a = 0$ passes through P and Q for **[2005]**

(a) exactly one value of a

(b) no value of a

(c) infinitely many values of a

(d) exactly two values of a

17. A circle touches the x- axis and also touches the circle with centre at (0,3) and radius 2. The locus of the centre of the circle is **[2005]**

(a) an ellipse (b) a circle

(c) a hyperbola (d) a parabola

18. If a circle passes through the point (a, b) and cuts the circle $x^2 + y^2 = p^2$ orthogonally, then the equation of the locus of its centre is **[2005]**

(a) $x^2 + y^2 - 3ax - 4by + (a^2 + b^2 - p^2) = 0$

(b) $2ax + 2by - (a^2 - b^2 + p^2) = 0$

(c) $x^2 + y^2 - 2ax - 3by + (a^2 - b^2 - p^2) = 0$

(d) $2ax + 2by - (a^2 + b^2 + p^2) = 0$

19. If the pair of lines $ax^2 + 2(a + b)xy + by^2 = 0$ lie along diameters of a circle and divide the circle into four sectors such that the area of one of the sectors is thrice the area of another sector then **[2005]**

(a) $3a^2 - 10ab + 3b^2 = 0$

(b) $3a^2 - 2ab + 3b^2 = 0$

(c) $3a^2 + 10ab + 3b^2 = 0$

(d) $3a^2 + 2ab + 3b^2 = 0$

20. Let P be the point (1, 0) and Q a point on the locus $y^2 = 8x$. The locus of mid point of PQ is **[2005]**

(a) $y^2 - 4x + 2 = 0$ (b) $y^2 + 4x + 2 = 0$

(c) $x^2 + 4y + 2 = 0$ (d) $x^2 - 4y + 2 = 0$

21. The locus of a point $P(\alpha, \beta)$ moving under the condition that the line $y = \alpha x + \beta$ is a tangent to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ is **[2005]**

(a) an ellipse (b) a circle

(c) a parabola (d) a hyperbola

22. An ellipse has OB as semi minor axis, F and F' its focii and the angle FBF' is a right angle. Then the eccentricity of the ellipse is **[2005]**

(a) $\frac{1}{\sqrt{2}}$ (b) $\frac{1}{2}$

(c) $\frac{1}{4}$ (d) $\frac{1}{\sqrt{3}}$

23. If the lines $3x - 4y - 7 = 0$ and $2x - 3y - 5 = 0$ are two diameters of a circle of area 49π square units, the equation of the circle is **[2006]**

(a) $x^2 + y^2 + 2x - 2y - 47 = 0$

(b) $x^2 + y^2 + 2x - 2y - 62 = 0$

(c) $x^2 + y^2 - 2x + 2y - 62 = 0$

(d) $x^2 + y^2 - 2x + 2y - 47 = 0$

24. Let C be the circle with centre (0, 0) and radius 3 units. The equation of the locus of the mid points of the chords of the circle C that subtend an angle of $\frac{2\pi}{3}$ at its center is **[2006]**

(a) $x^2 + y^2 = \frac{3}{2}$ (b) $x^2 + y^2 = 1$

(c) $x^2 + y^2 = \frac{27}{4}$ (d) $x^2 + y^2 = \frac{9}{4}$

25. The locus of the vertices of the family of parabolas $y = \frac{a^3x^2}{3} + \frac{a^2x}{2} - 2a$ is **[2006]**

(a) $xy = \frac{105}{64}$ (b) $xy = \frac{3}{4}$

(c) $xy = \frac{35}{16}$ (d) $xy = \frac{64}{105}$

26. In an ellipse, the distance between its foci is 6 and minor axis is 8. Then its eccentricity is **[2006]**

(a) $\frac{3}{5}$ (b) $\frac{1}{2}$

(c) $\frac{4}{5}$ (d) $\frac{1}{\sqrt{5}}$

27. Consider a family of circles which are passing through the point (– 1, 1) and are tangent to x-axis. If (h, k) are the coordinate of the centre of the circles, then the set of values of k is given by the interval **[2007]**

(a) $-\frac{1}{2} \le k \le \frac{1}{2}$ (b) $k \le \frac{1}{2}$

(c) $0 \le k \le \frac{1}{2}$ (d) $k \ge \frac{1}{2}$

28. For the Hyperbola $\frac{x^2}{\cos^2\alpha} - \frac{y^2}{\sin^2\alpha} = 1$, which of the following remains constant when α varies = ? **[2007]**

(a) abscissae of vertices

(b) abscissae of foci

(c) eccentricity

(d) directrix.

29. The equation of a tangent to the parabola $y^2 = 8x$ is $y = x + 2$. The point on this line from which the other tangent to the parabola is perpendicular to the given tangent is **[2007]**

(a) (2,4) (b) (–2,0)

(c) (–1, 1) (d) (0, 2)

30. The point diametrically opposite to the point $P(1, 0)$ on the circle $x^2 + y^2 + 2x + 4y - 3 = 0$ is **[2008]**

(a) (3, – 4) (b) (–3, 4)

(c) (–3, –4) (d) (3, 4)

31. A focus of an ellipse is at the origin. The directrix is the line $x = 4$ and the eccentricity is $\frac{1}{2}$. Then the length of the semi-major axis is **[2008]**

(a) $\frac{8}{3}$ (b) $\frac{2}{3}$ (c) $\frac{4}{3}$ (d) $\frac{5}{3}$

32. A parabola has the origin as its focus and the line $x = 2$ as the directrix. Then the vertex of the parabola is at **[2008]**

(a) $(0, 2)$ (b) $(1, 0)$

(c) $(0, 1)$ (d) $(2, 0)$

33. If P and Q are the points of intersection of the circles $x^2 + y^2 + 3x + 7y + 2p - 5 = 0$ and $x^2 + y^2 + 2x + 2y - p^2 = 0$ then there is a circle passing through P, Q and (1, 1) for: **[2009]**

(a) all except one value of p

(b) all except two values of p

(c) exactly one value of p

(d) all values of p

34. The ellipse $x^2 + 4y^2 = 4$ is inscribed in a rectangle aligned with the coordinate axes, which in turn is inscribed in another ellipse that passes through the point (4, 0). Then the equation of the ellipse is : **[2009]**

(a) $x^2 + 12y^2 = 16$ (b) $4x^2 + 48y^2 = 48$

(c) $4x^2 + 64y^2 = 48$ (d) $x^2 + 16y^2 = 16$

35. The circle $x^2 + y^2 = 4x + 8y + 5$ intersects the line $3x - 4y = m$ at two distinct points if **[2010]**

(a) $-35 < m < 15$ (b) $15 < m < 65$

(c) $35 < m < 85$ (d) $-85 < m < -35$

36. If two tangents drawn from a point P to the parabola $y^2 = 4x$ are at right angles, then the locus of P is **[2010]**

(a) $2x + 1 = 0$ (b) $x = -1$

(c) $2x - 1 = 0$ (d) $x = 1$

37. The two circles $x^2 + y^2 = ax$ and $x^2 + y^2 = c^2$ $(c > 0)$ touch each other if **[2011]**

(a) $|a| = c$ (b) $a = 2c$

(c) $|a| = 2c$ (d) $2|a| = c$

38. The shortest distance between line $y - x = 1$ and curve $x = y^2$ is **[2011]**

(a) $\frac{3\sqrt{2}}{8}$ (b) $\frac{8}{3\sqrt{2}}$

(c) $\frac{4}{\sqrt{3}}$ (d) $\frac{\sqrt{3}}{4}$

39. Equation of the ellipse whose axes are the axes of coordinates and which passes through the point (–3, 1) and has eccentricity $\sqrt{\frac{2}{5}}$ is **[2011]**

(a) $5x^2 + 3y^2 - 48 = 0$ (b) $3x^2 + 5y^2 - 15 = 0$

(c) $5x^2 + 3y^2 - 32 = 0$ (d) $3x^2 + 5y^2 - 32 = 0$

40. The equation of the circle passing through the point (1, 0) and (0, 1) and having the smallest radius is **[2011 RS]**

(a) $x^2 + y^2 - 2x - 2y + 1 = 0$

(b) $x^2 + y^2 - x - y = 0$

(c) $x^2 + y^2 + 2x + 2y - 7 = 0$

(d) $x^2 + y^2 + x + y - 2 = 0$

41. The equation of the hyperbola whose foci are (– 2, 0) and (2, 0) and eccentricity is 2 is given by : **[2011RS]**

(a) $x^2 - 3y^2 = 3$ (b) $3x^2 - y^2 = 3$

(c) $-x^2 + 3y^2 = 3$ (d) $-3x^2 + y^2 = 3$

42. The length of the diameter of the circle which touches the x-axis at the point (1,0) and passes through the point (2,3) is: **[2012]**

(a) $\frac{10}{3}$ (b) $\frac{3}{5}$

(c) $\frac{6}{5}$ (d) $\frac{5}{3}$

43. **Statement-1 :** An equation of a common tangent to the parabola $y^2 = 16\sqrt{3}x$ and the ellipse $2x^2 + y^2 = 4$ is $y = 2x + 2\sqrt{3}$

Statement-2 : If the line $y = mx + \frac{4\sqrt{3}}{m}, (m \neq 0)$ is a common tangent to the parabola $y^2 = 16\sqrt{3}x$ and the ellipse $2x^2 + y^2 = 4$, then m satisfies $m^4 + 2m^2 = 24$ **[2012]**

(a) Statement-1 is false, Statement-2 is true.

(b) Statement-1 is true, statement-2 is true; statement-2 is a correct explanation for Statement-1.

(c) Statement-1 is true, statement-2 is true; statement-2 is **not** a correct explanation for Statement-1.

(d) Statement-1 is true, statement-2 is false.

44. An ellipse is drawn by taking a diameter of the circle $(x-1)^2+y^2=1$ as its semi-minor axis and a diameter of the circle $x^2+(y-2)^2=4$ is semi-major axis. If the centre of the ellipse is at the origin and its axes are the coordinate axes, then the equation of the ellipse is : **[2012]**

(a) $4x^2+y^2=4$ (b) $x^2+4y^2=8$
(c) $4x^2+y^2=8$ (d) $x^2+4y^2=16$

45. The chord PQ of the parabola $y^2=x$, where one end P of the chord is at point $(4,-2)$, is perpendicular to the axis of the parabola. Then the slope of the normal at Q is **[2012]**

(a) -4 (b) $-\frac{1}{4}$
(c) 4 (d) $\frac{1}{4}$

46. The circle passing through $(1,-2)$ and touching the axis of x at $(3,0)$ also passes through the point **[2013]**

(a) $(-5,2)$ (b) $(2,-5)$
(c) $(5,-2)$ (d) $(-2,5)$

47. The equation of the circle passing through the foci of the ellipse $\frac{x^2}{16}+\frac{y^2}{9}=1$, and having centre at $(0,3)$ is **[2013]**

(a) $x^2+y^2-6y-7=0$
(b) $x^2+y^2-6y+7=0$
(c) $x^2+y^2-6y-5=0$
(d) $x^2+y^2-6y+5=0$

48. **Given :** A circle, $2x^2+2y^2=5$ and a parabola, $y^2=4\sqrt{5}x$.

Statement-1 : An equation of a common tangent to these curves is $y=x+\sqrt{5}$.

Statement-2 : If the line, $y=mx+\frac{\sqrt{5}}{m}\ (m\neq 0)$ is their common tangent, then m satisfies $m^4-3m^2+2=0$. **[2013]**

(a) Statement-1 is true; Statement-2 is true; Statement-2 is a correct explanation for Statement-1.
(b) Statement-1 is true; Statement-2 is true; Statement-2 is not a correct explanation for Statement-1.
(c) Statement-1 is true; Statement-2 is false.
(d) Statement-1 is false; Statement-2 is true.

49. The locus of the foot of perpendicular drawn from the centre of the ellipse $x^2+3y^2=6$ on any tangent to it is **[2014]**

(a) $\left(x^2+y^2\right)^2=6x^2+2y^2$
(b) $\left(x^2+y^2\right)^2=6x^2-2y^2$
(c) $\left(x^2-y^2\right)^2=6x^2+2y^2$
(d) $\left(x^2-y^2\right)^2=6x^2-2y^2$

50. Let C be the circle with centre at $(1,1)$ and radius $=1$. If T is the circle centred at $(0,y)$, passing through origin and touching the circle C externally, then the radius of T is equal to **[2014]**

(a) $\frac{1}{2}$ (b) $\frac{1}{4}$
(c) $\frac{\sqrt{3}}{\sqrt{2}}$ (d) $\frac{\sqrt{3}}{2}$

51. The slope of the line touching both the parabolas $y^2=4x$ and $x^2=-32y$ is **[2014]**

(a) $\frac{1}{8}$ (b) $\frac{2}{3}$
(c) $\frac{1}{2}$ (d) $\frac{3}{2}$

52. Let O be the vertex and Q be any point on the parabola, $x^2=8y$. If the point P divides the line segment OQ internally in the ratio 1 : 3, then locus of P is : **[2015]**

(a) $y^2=2x$ (b) $x^2=2y$
(c) $x^2=y$ (d) $y^2=x$

53. The number of common tangents to the circles $x^2+y^2-4x-6x-12=0$ and $x^2+y^2+6x+18y+26=0$, is : **[2015]**

(a) 3 (b) 4
(c) 1 (d) 2

54. The area (in sq. units) of the quadrilateral formed by the tangents at the end points of the latus rectum to the ellipse $\frac{x^2}{9}+\frac{y^2}{5}=1$, is : **[2015]**

(a) $\frac{27}{2}$ (b) 27
(c) $\frac{27}{4}$ (d) 18

55. Locus of the image of the point (2, 3) in the line $(2x - 3y + 4) + k(x - 2y + 3) = 0$, $k \in \mathbf{R}$, is a : **[2015]**

(a) circle of radius $\sqrt{2}$.

(b) circle of radius $\sqrt{3}$.

(c) straight line parallel to x-axis

(d) straight line parallel to y-axis

56. The centres of those circles which touch the circle, $x^2 + y^2 - 8x - 8y - 4 = 0$, externally and also touch the x-axis, lie on: **[2016]**

(a) a hyperbola

(b) a parabola

(c) a circle

(d) an ellipse which is not a circle

57. The eccentricity of the hyperbola whose length of the latus rectum is equal to 8 and the length of its conjugate axis is equal to half of the distance between its foci, is : **[2016]**

(a) $\frac{2}{\sqrt{3}}$ (b) $\sqrt{3}$

(c) $\frac{4}{3}$ (d) $\frac{4}{\sqrt{3}}$

58. If one of the diameters of the circle, given by the equation, $x^2 + y^2 - 4x + 6y - 12 = 0$, is a chord of a circle S, whose centre is at (–3, 2), then the radius of S is: **[2016]**

(a) 5 (b) 10

(c) $5\sqrt{2}$ (d) $5\sqrt{3}$

59. Let P be the point on the parabola, $y^2 = 8x$ which is at a minimum distance from the centre C of the circle, $x^2 + (y + 6)^2 = 1$. Then the equation of the circle, passing through C and having its centre at P is: **[2016]**

(a) $x^2 + y^2 - \frac{x}{4} + 2y - 24 = 0$

(b) $x^2 + y^2 - 4x + 9y + 18 = 0$

(c) $x^2 + y^2 - 4x + 8y + 12 = 0$

(d) $x^2 + y^2 - x + 4y - 12 = 0$

60. A hyperbola passes through the point $P(\sqrt{2}, \sqrt{3})$ and has foci at $(\pm 2, 0)$. Then the tangent to this hyperbola at P also passes through the point : **[2017]**

(a) $(-\sqrt{2}, -\sqrt{3})$ (b) $(3\sqrt{2}, 2\sqrt{3})$

(c) $(2\sqrt{2}, 3\sqrt{3})$ (d) $(\sqrt{3}, \sqrt{2})$

61. The radius of a circle, having minimum area, which touches the curve $y = 4 - x^2$ and the lines, $y = |x|$ is : **[2017]**

(a) $4(\sqrt{2}+1)$ (b) $2(\sqrt{2}+1)$

(c) $2(\sqrt{2}-1)$ (d) $4(\sqrt{2}-1)$

62. Tangents are drawn to the hyperbola $4x^2 - y^2 = 36$ at the points P and Q. If these tangents intersect at the point T(0, 3) then the area (in sq. units) of ΔPTQ is : **[2018]**

(a) $54\sqrt{3}$ (b) $60\sqrt{3}$

(c) $36\sqrt{5}$ (d) $45\sqrt{5}$

63. Tangent and normal are drawn at P(16, 16) on the parabola $y^2 = 16x$, which intersect the axis of the parabola at A and B, respectively. If C is the centre of the circle through the points P, A and B and $\angle CPB = \theta$, then a value of $\tan\theta$ is : **[2018]**

(a) 2 (b) 3

(c) $\frac{4}{3}$ (d) $\frac{1}{2}$

64. Two sets A and B are as under :

$A = \{(a, b) \in R \times R : |a - 5| < 1 \text{ and } |b - 5| < 1\}$;

$B = \{(a, b) \in R \times R : 4(a-6)^2 + 9(b-5)^2 \le 36\}$.

Then : **[2018]**

(a) $A \subset B$

(b) $A \cap B = \phi$ (an empty set)

(c) neither $A \subset B$ nor $B \subset A$

(d) $B \subset A$

65. If the tangent at (1, 7) to the curve $x^2 = y - 6$ touches the circle $x^2 + y^2 + 16x + 12y + c = 0$ then the value of c is : **[2018]**

(a) 185 (b) 85

(c) 95 (d) 195

66. If the line $y = mx + 7\sqrt{3}$ is normal to the hyperbola $\frac{x^2}{24} - \frac{y^2}{18} = 1$, then a value of m is : **[2019]**

(a) $\frac{\sqrt{5}}{2}$ (b) $\frac{\sqrt{15}}{2}$

(c) $\frac{2}{\sqrt{5}}$ (d) $\frac{3}{\sqrt{5}}$

67. All the points in the set **[2019]**

$S = \left\{\frac{\alpha + i}{\alpha - i} : \alpha \in R\right\} (i = \sqrt{-1})$ lie on a:

(a) straight line whose slope is 1.
(b) circle whose radius is 1.
(c) circle whose radius is $\sqrt{2}$.
(d) straight line whose slope is –1.

68. If a tangent to the circle $x^2 + y^2 = 1$ intersects the coordinate axes at distinct points P and Q, then the locus of the mid-point of PQ is: **[2019]**

(a) $x^2 + y^2 - 4x^2y^2 = 0$
(b) $x^2 + y^2 - 2xy = 0$
(c) $x^2 + y^2 - 16x^2y^2 = 0$
(d) $x^2 + y^2 - 2x^2y^2 = 0$

69. If one end of a focal chord of the parabola, $y^2 = 16x$ is at (1, 4), then the length of this focal chord is: **[2019]**

(a) 25 (b) 22
(c) 24 (d) 20

70. Axis of a parabola lies along x-axis. If its vertex and focus are at distance 2 and 4 respectively from the origin, on the positive *x*-axis then which of the following points does not lie on it? **[2019]**

(a) $(5, 2\sqrt{6})$ (b) (8, 6)
(c) $(6, 4\sqrt{2})$ (d) (4, –4)

71. Equation of a common tangent to the circle, $x^2 + y^2 - 6x = 0$ and the parabola, $y^2 = 4x$, is : **[2019]**

(a) $2\sqrt{3}y = 12x + 1$ (b) $\sqrt{3}y = x + 3$
(c) $2\sqrt{3}y = -x - 12$ (d) $\sqrt{3}y = 3x + 1$

72. Three circles of radii *a*, *b*, *c* ($a < b < c$) touch each other externally. If they have *x*-axis as a common tangent, then: **[2019]**

(a) $\frac{1}{\sqrt{a}} = \frac{1}{\sqrt{b}} + \frac{1}{\sqrt{c}}$
(b) $\frac{1}{\sqrt{b}} = \frac{1}{\sqrt{a}} + \frac{1}{\sqrt{c}}$
(c) *a*, b, c are in A.P
(d) $\sqrt{a}, \sqrt{b}, \sqrt{c}$ are in A.P.

73. If the distance between the foci of an ellipse is 6 and the distance between its directrices is 12, then the length of its latus rectum is: **[2020]**

(a) $\sqrt{3}$ (b) $3\sqrt{2}$
(c) $\frac{3}{\sqrt{2}}$ (d) $2\sqrt{3}$

74. If $y = mx + 4$ is a tangent to both the parabolas, $y^2 = 4x$ and $x^2 = 2by$, then *b* is equal to: **[2020]**

(a) –32 (b) –64
(c) –128 (d) 128

75. If $\text{Re}\left(\frac{z-1}{2z+i}\right) = 1$, where $z = x + iy$, then the point (x, y) lies on *a*: **[2020]**

(a) circle whose centre is at $\left(-\frac{1}{2}, -\frac{3}{2}\right)$.
(b) straight line whose slope is $-\frac{2}{3}$.
(c) straight line whose slope is $\frac{3}{2}$.
(d) circle whose diameter is $\frac{\sqrt{5}}{2}$.

Answer Key

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
(c)	(a)	(b)	(c)	(b)	(b)	(d)	(b)	(d)	(b)	(d)	(d)	(d)	(d)	(b)
16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
(b)	(d)	(d)	(d)	(a)	(d)	(a)	(d)	(d)	(a)	(a)	(d)	(b)	(b)	(c)
31	32	33	34	35	36	37	38	39	40	41	42	43	44	45
(a)	(b)	(a)	(a)	(a)	(b)	(a)	(a)	(d)	(b)	(b)	(a)	(b)	(d)	(a)
46	47	48	49	50	51	52	53	54	55	56	57	58	59	60
(c)	(a)	(b)	(a)	(b)	(c)	(b)	(a)	(b)	(a)	(b)	(a)	(d)	(c)	(c)
61	62	63	64	65	66	67	68	69	70	71	72	73	74	75
(None)	(d)	(a)	(a)	(c)	(c)	(b)	(a)	(a)	(b)	(b)	(a)	(b)	(c)	(d)

Solutions

1. **(c)** Given equation of circle $x^2 + y^2 = 1 = (1)^2$
$\Rightarrow x^2 + y^2 = (y - mx)^2$
$\Rightarrow x^2 = m^2x^2 - 2\,mxy;$
$\Rightarrow x^2\ (1 - m^2) + 2mxy = 0$. Which represents the pair of lines between which the angle is 45^o.

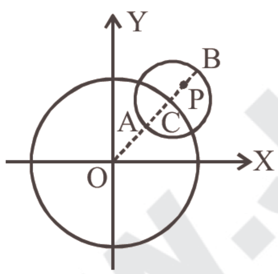
$$\therefore \tan 45 = \pm\frac{2\sqrt{m^2 - 0}}{1 - m^2} = \frac{\pm 2m}{1 - m^2};$$

$\Rightarrow 1 - m^2 = \pm\ 2m \Rightarrow m^2 \pm 2m - 1 = 0$

$$\Rightarrow m = \frac{-2 \pm \sqrt{4 + 4}}{2}$$

$$= \frac{-2 \pm 2\sqrt{2}}{2} = -1 \pm \sqrt{2}.$$

2. **(a)** $\because$ The centre C of circle of radius 3 lies on circle of radius 5. Let $P(x, y)$ in the smaller circle.



we should have
$OA \le OP \le OB$

$$\Rightarrow (5 - 3) \le \sqrt{x^2 + y^2} \le 5 + 3$$

$$\Rightarrow 4 \le x^2 + y^2 \le 64$$

3. **(b)** Let the required circle be
$x^2 + y^2 + 2gx + 2fy + c = 0$
Since it passes through (0, 0) and (1, 0)
On putting these values, we get

$$\Rightarrow c = 0 \text{ and } g = -\frac{1}{2}$$

Points (0, 0) and (1, 0) lie inside the circle $x^2 + y^2 = 9$, so two circles touch internally
$\Rightarrow c_1c_2 = r_1 - r_2$

$$\therefore \sqrt{g^2 + f^2} = 3 - \sqrt{g^2 + f^2} \Rightarrow \sqrt{g^2 + f^2} = \frac{3}{2}$$

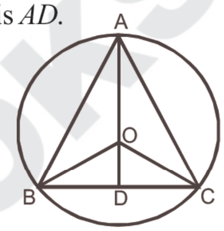
Squaring both side, we get

$$\Rightarrow f^2 = \frac{9}{4} - \frac{1}{4} = 2 \qquad \therefore f = \pm\sqrt{2}.$$

Hence, the centres of required circle are

$$\left(\frac{1}{2}, \sqrt{2}\right) \text{ or } \left(\frac{1}{2}, -\sqrt{2}\right)$$

4. **(c)** Let ABC be an equilateral triangle, whose median is AD.



In equilateral triangle median is also altitude
So, $AD \perp BC$
Given $AD = 3a$.
Let $AB = BC = AC = x$.
In ΔABD, $AB^2 = AD^2 + BD^2$;
$\Rightarrow x^2 = 9a^2 + (x^2/4)$

$$\frac{3}{4}x^2 = 9a^2 \Rightarrow x^2 = 12a^2.$$

In $\Delta\ OBD$, $OB^2 = OD^2 + BD^2$

$$\Rightarrow r^2 = (3a - r)^2 + \frac{x^2}{4}$$

$\Rightarrow r^2 = 9a^2 - 6ar + r^2 + 3a^2$
$\Rightarrow 6ar = 12a^2$
$\Rightarrow r = 2a$
So equation of circle is $x^2 + y^2 = 4a^2$

5. **(b)** The equation of any tangent to the parabola $y^2 = 8ax$ is

$$y = mx + \frac{2a}{m} \qquad ...(i)$$

If (i) is also a tangent to the circle, $x^2 + y^2$ $= 2a^2$ then, $\sqrt{2}a = \pm\dfrac{2a}{m\sqrt{m^2 + 1}}$

$\Rightarrow m^2(1 + m^2) = 2 \Rightarrow (m^2 + 2)(m^2 - 1) = 0$
$\Rightarrow m = \pm 1$.
Putting the value of m in eqn (i), we get
$y = \pm(x + 2a)$.

6. (b) $\because$ Given two circles intersect at two points

$\therefore |r_1 - r_2| < C_1C_2$

$\Rightarrow r - 3 < 5 \Rightarrow 0 < r < 8$ ....(1)

and $r_1 + r_2 > C_1C_2$, $r + 3 > 5 \Rightarrow r > 2$ ....(2)

From (1) and (2), $2 < r < 8$.

7. (d) Area of circle $= \pi r^2 = 154 \Rightarrow r = 7$

For centre, solving equation

$2x - 3y = 5$ & $3x - 4y = 7$

we get, $x = 1, y = -1$

$\therefore$ centre $= (1, -1)$

Equation of circle, $(x-1)^2 + (y+1)^2 = 7^2$

$x^2 + y^2 - 2x + 2y = 47$

8. (b) Equation of the normal to a parabola $y^2 = 4bx$ at point $\left(bt_1^2, 2bt_1\right)$ is

$y = -t_1 x + 2bt_1 + bt_1^3$

Given that, it also passes through $\left(bt_2^2, 2bt_2\right)$ then

$2bt_2 = -t_1 bt_2^2 + 2bt_1 + bt_1^3$

$\Rightarrow 2t_2 - 2t_1 = -t_1\left(t_2^2 - t_1^2\right)$

$\Rightarrow 2(t_2 - t_1) = -t_1(t_2 + t_1)(t_2 - t_1)$

$\Rightarrow 2 = -t_1(t_2 + t_1) \Rightarrow t_2 + t_1 = -\dfrac{2}{t_1}$

$\Rightarrow t_2 = -t_1 - \dfrac{2}{t_1}$

9. (d) $\dfrac{x^2}{144} - \dfrac{y^2}{81} = \dfrac{1}{25}$

$a = \sqrt{\dfrac{144}{25}} = \dfrac{12}{5}, b = \sqrt{\dfrac{81}{25}} = \dfrac{9}{5},$

$e = \sqrt{1 + \dfrac{81}{144}} = \dfrac{15}{12} = \dfrac{5}{4}$

$\therefore$ Foci $= (\pm ae, 0) = (\pm 3, 0)$

$\therefore$ foci of ellipse = foci of hyperbola

$\therefore$ for ellipse $ae = 3$ but $a = 4$,

$\therefore$ $e = \dfrac{3}{4}$

Then, $b^2 = a^2(1 - e^2)$

$\Rightarrow b^2 = 16\left(1 - \dfrac{9}{16}\right) = 7$

10. (b) Let the equation of circle is

$x^2 + y^2 + 2gx + 2fy + c = 0$ ......(1)

It passes through (a, b)

$\therefore a^2 + b^2 + 2ga + 2fb + c = 0$ ......(2)

Circle (1) cuts $x^2 + y^2 = 4$ orthogonally

Two circles intersect orthogonally if

$2g_1g_2 + 2f_1f_2 = c_1 + c_2$

$\therefore 2(g \times 0 + f \times 0) = c - 4 \Rightarrow c = 4$

$\therefore$ from (2) $a^2 + b^2 + 2ga + 2fb + 4 = 0$

$\therefore$ Locus of centre $(-g, -f)$ is

$a^2 + b^2 - 2ax - 2by + 4 = 0$

or $2ax + 2by = a^2 + b^2 + 4$

11. (d) Let the variable circle be

$x^2 + y^2 + 2gx + 2fy + c = 0$ ....(1)

Since it passes through (p, q)

$\therefore p^2 + q^2 + 2gp + 2fq + c = 0$ ....(2)

Circle (1) touches x-axis,

$\therefore g^2 - c = 0 \Rightarrow c = g^2$. From (2)

$p^2 + q^2 + 2gp + 2fq + g^2 = 0$ ....(3)

Let the other end of diameter through (p, q) be (h, k), then

$\dfrac{h+p}{2} = -g$ and $\dfrac{k+q}{2} = -f$

Putting value of g and f in (3), we get

$$p^2 + q^2 + 2p\left(-\frac{h+p}{2}\right) + 2q\left(-\frac{k+q}{2}\right) + \left(\frac{h+p}{2}\right)^2 = 0$$

$\Rightarrow h^2 + p^2 - 2hp - 4kq = 0$

$\therefore$ locus of (h, k) is

$x^2 + p^2 - 2xp - 4yq = 0$

$\Rightarrow (x - p)^2 = 4qy$

12. **(d)** Two diameters are along

$2x+3y+1=0$ and $3x-y-4=0$

On solving we get centre $(1, -1)$

Circumference of circle $= 2\pi r = 10\pi$

$\therefore r = 5.$

Required circle is, $(x-1)^2+(y+1)^2 = 5^2$

$\Rightarrow x^2+y^2-2x+2y-23=0$

13. **(d)** Solving $y = x$ and the circle

$x^2+y^2-2x=0$, we get

$x=0, y=0$ and $x=1, y=1$

$\therefore$ Extremities of diameter of the required circle are A (0, 0) and B (1, 1). Hence, the equation of circle is

$(x-0)(x-1)+(y-0)(y-1)=0$

$\Rightarrow x^2+y^2-x-y=0$

14. **(d)** Solving equations of parabolas

$y^2=4ax$ and $x^2=4ay$, we get (0, 0) and $(4a, 4a)$

Putting in the given equation of line

$2bx+3cy+4d=0$, we get

$d = 0$ and $2b+3c=0$

$\Rightarrow d^2+(2b+3c)^2=0$

15. **(b)** Given that $e=\frac{1}{2}$. Directrix, $x=\frac{a}{e}=4$

$\therefore a = 4\times\frac{1}{2}=2$

$\therefore b = 2\sqrt{1-\frac{1}{4}}=\sqrt{3}$

Equation of ellipse is

$\frac{x^2}{4}+\frac{y^2}{3}=1 \Rightarrow 3x^2+4y^2=12$

16. **(b)** Given that

$s_1 = x^2+y^2+2ax+cy+a=0$

$s_2 = x^2+y^2-3ax+dy-1=0$

Equation of common chord PQ of circles s_1 and s_2 is given by $s_1-s_2=0$

$\Rightarrow 5ax+(c-d)y+a+1=0$

Given that $5x+by-a=0$ passes through P and Q

$\therefore$ The two equations should represent the same line

$\Rightarrow \frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2}$

$\Rightarrow \frac{a}{1}=\frac{c-d}{b}=\frac{a+1}{-a} \Rightarrow a+1=-a^2$

$a^2+a+1=0$ $[\because D=-3]$

$\therefore$ No real value of a.

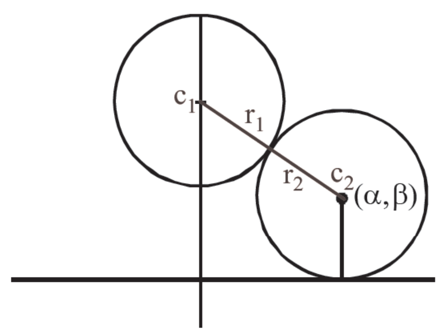
17. **(d)** Equation of circle with centre (0, 3) and radius 2 is $x^2+(y-3)^2=4$

Let locus of the centre of the variable circle is (α, β)

$\because$ It touches x - axis.

$\therefore$ It's equation is

$(x-\alpha)^2+(y-\beta)^2=\beta^2$



Circle touch externally $\Rightarrow c_1c_2 = r_1+r_2$

$\therefore \sqrt{\alpha^2+(\beta-3)^2} = 2+\beta$

$\alpha^2+(\beta-3)^2=\beta^2+4+4\beta$

$\alpha^2+\beta^2-6\beta+9=\beta^2+4+4\beta$

$\Rightarrow \alpha^2=10(\beta-1/2)$

$\therefore$ Locus is $x^2=10\left(y-\frac{1}{2}\right)$

Which is equation of parabola.

18. **(d)** Let the centre variable circle be (α, β)

$\because$ It cuts the circle $x^2+y^2=p^2$ orthogonally

$\therefore$ Using $2g_1g_2 + 2f_1f_2 = c_1 + c_2$, we get

$2(-\alpha)\times 0 + 2(-\beta)\times 0 = c_1 - p^2$

$\Rightarrow c_1 = p^2$

Let equation of circle is

$x^2 + y^2 - 2\alpha x - 2\beta y + p^2 = 0$

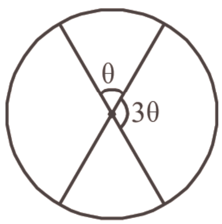
$\because$ It passes through (a, b)

$\Rightarrow a^2 + b^2 - 2\alpha a - 2\beta b + p^2 = 0$

$\therefore$ Locus of (α, β) is

$\therefore 2ax + 2by - (a^2 + b^2 + p^2) = 0$.

19. (d)



Given that area of one sector
= 3 × area of another sector
$\Rightarrow$ Angle at centre by one sector
$= 3\times$ angle at centre by another sector
Let one angle be θ then other $= 3\theta$

Clearly $\theta + 3\theta = 180 \Rightarrow \theta = 45^\circ$ (Linear pair)

$\therefore$ Angle between the diameters represented by pair of equation

$ax^2 + 2(a+b)xy + by^2 = 0$ is 45°

$\therefore$ Using $\tan\theta = \dfrac{2\sqrt{h^2 - ab}}{a+b}$

we get, $\tan 45^\circ = \dfrac{2\sqrt{(a+b)^2 - ab}}{a+b}$

$\Rightarrow 1 = \dfrac{2\sqrt{a^2 + b^2 + ab}}{a+b}$

$\Rightarrow (a+b)^2 = 4\left(a^2 + b^2 + ab\right)$

$\Rightarrow a^2 + b^2 + 2ab = 4a^2 + 4b^2 + 4ab$

$\Rightarrow 3a^2 + 3b^2 + 2ab = 0$

20. (a) Given $P = (1, 0)$, let $Q = (h, k)$
Since Q lies on $y^2 = 8x$

$\therefore K^2 = 8h$ ...(i)

Let (α, β) be the midpoint of PQ

$\therefore \alpha = \dfrac{h+1}{2}, \quad \beta = \dfrac{k+0}{2}$

$2\alpha - 1 = h \qquad 2\beta = k.$

Putting value of h and k in (i)

$(2\beta)^2 = 8(2\alpha - 1) \Rightarrow \beta^2 = 4\alpha - 2$

$\Rightarrow y^2 - 4x + 2 = 0$.

21. (d) We know that tangent to the hyperbola

$\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 1$ is

$y = mx \pm \sqrt{a^2m^2 - b^2}$

Given that $y = \alpha x + \beta$ is the tangent of hyperbola.

$\Rightarrow \quad m = \alpha$ and $a^2m^2 - b^2 = \beta^2$

$\therefore \quad a^2\alpha^2 - b^2 = \beta^2$

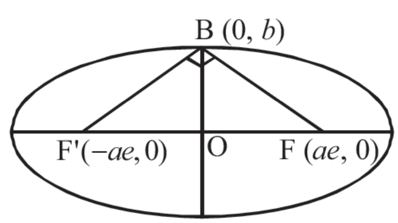
Locus is $a^2x^2 - y^2 = b^2$ which is hyperbola.

22. (a) Given that $\angle FBF' = 90°$

$\Rightarrow FB^2 + F'B^2 = FF'^2$

$\therefore \left(\sqrt{a^2e^2 + b^2}\right)^2 + \left(\sqrt{a^2e^2 + b^2}\right)^2 = (2ae)^2$

$\Rightarrow 2(a^2e^2 + b^2) = 4a^2e^2 \Rightarrow e^2 = \dfrac{b^2}{a^2}$ ...(i)



We know that $e^2 = 1 - b^2/a^2 = 1 - e^2$ [from (i)]

$\Rightarrow 2e^2 = 1, \; e = \dfrac{1}{\sqrt{2}}$.

23. (d) On solving we get point of intersection of $3x-4y-7=0$ and $2x-3y-5=0$ is $(1,-1)$ which is the centre of the circle

Area of circle $= \pi r^2 = 49\pi$

$\therefore$ radius $= 7$

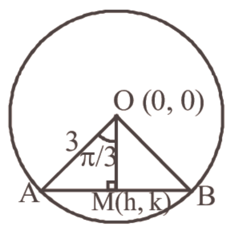
$\therefore$ Equation is $(x-1)^2+(y+1)^2 = 49$

$\Rightarrow \; x^2+y^2-2x+2y-47=0$

24. (d) Given that centre of circle be (0, 0) and radius is 3 unit

Let $M(h, k)$ be the mid point of chord AB

where $\angle AOB = \dfrac{2\pi}{3}$



$\therefore \; \angle AOM = \dfrac{\pi}{3}$. Also $OM = 3\cos\dfrac{\pi}{3} = \dfrac{3}{2}$

$\Rightarrow \sqrt{h^2+k^2} = \dfrac{3}{2} \Rightarrow h^2+k^2 = \dfrac{9}{4}$

$\therefore$ Locus of (h, k) is $x^2+y^2 = \dfrac{9}{4}$

25. (a) Given that family of parabolas is

$$y = \frac{a^3x^2}{3}+\frac{a^2x}{2}-2a$$

$$\Rightarrow y = \frac{a^3}{3}\left(x^2+\frac{3}{2a}x+\frac{9}{16a^2}\right)-\frac{3a}{16}-2a$$

$$\Rightarrow y+\frac{35a}{16} = \frac{a^3}{3}\left(x+\frac{3}{4a}\right)^2$$

$\therefore$ Vertex of parabola is $\left(\dfrac{-3}{4a}, \dfrac{-35a}{16}\right)$

To find locus of this vertex,

$x = \dfrac{-3}{4a}$ and $y = \dfrac{-35a}{16}$

$\Rightarrow a = \dfrac{-3}{4x}$ and $a = -\dfrac{16y}{35}$

$\Rightarrow \dfrac{-3}{4x} = \dfrac{-16y}{35} \Rightarrow 64xy = 105$

$\Rightarrow xy = \dfrac{105}{64}$ which is the required equation of locus.

26. (a) Given that distance between foci is $2ae = 6 \;\Rightarrow\; ae = 3$ and length of minor axis is $2b = 8 \Rightarrow \; b = 4$

we know that $b^2 = a^2(1-e^2)$

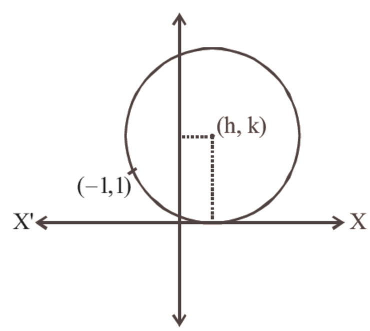
$\Rightarrow 16 = a^2-a^2e^2 \Rightarrow a^2 = 16+9 = 25$

$\Rightarrow a = 5$

$\therefore \; e = \dfrac{3}{a} = \dfrac{3}{5}$

27. (d) Equation of circle whose centre is (h, k) and touch the x-axis

i.e $(x-h)^2+(y-k)^2 = k^2$



(radius of circle $= k$ because circle is tangent to x-axis)

$\because$ Equation of circle passing through $(-1, 1)$

$\therefore \; (-1-h)^2+(1-k)^2 = k^2$

$\Rightarrow \; 1+h^2+2h+1+k^2-2k = k^2$

$\Rightarrow \; h^2+2h-2k+2 = 0$

$D \geq 0$

$\therefore \; (2)^2-4\times 1.(-2k+2) \geq 0$

$\Rightarrow \; 4-4(-2k+2) \geq 0 \Rightarrow 1+2k-2 \geq 0$

$\Rightarrow \; k \geq \dfrac{1}{2}$

28. (b) Given, equation of hyperbola is

$$\frac{x^2}{\cos^2\alpha}-\frac{y^2}{\sin^2\alpha} = 1$$

Compare with equation of hyperbola

$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$, we get $a^2=\cos^2\alpha$ and

$b^2=\sin^2\alpha$

We know that, $b^2=a^2(e^2-1)$

$\Rightarrow\ \sin^2\alpha=\cos^2\alpha(e^2-1)$

$\Rightarrow\ \sin^2\alpha+\cos^2\alpha=\cos^2\alpha.e^2$

$\Rightarrow\ e^2=\sec^2\alpha$

$\Rightarrow\ e=\sec\alpha$

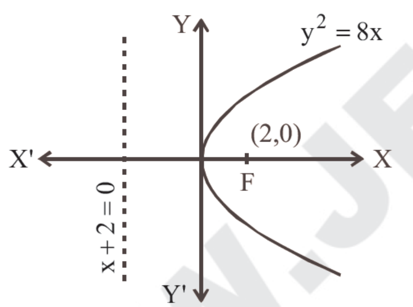
$\therefore\ ae=\cos\alpha\ .\ \frac{1}{\cos\alpha}=1$

Co-ordinates of foci are $(\pm ae, 0)$

i.e. $(\pm 1, 0)$

Hence, abscissae of foci remain constant when α varies.

29. **(b)** Given that parabola $y^2=8x$



We know that the locus of point of intersection of two perpendicular tangents to a parabola is its directrix.

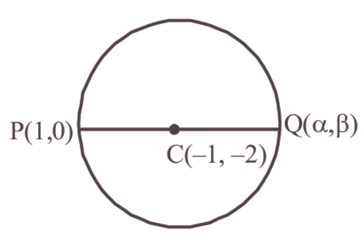
Point must be on the directrix of parabola

$\because$ Equation of directrix $x+2=0$

$\Rightarrow\ x=-2$

Hence the point is $(-2, 0)$

30. **(c)** The given circle is $x^2+y^2+2x+4y-3=0$



Centre $(-g,-f)=(-1,-2)$

Let $Q(h, k)$ be the point diametrically opposite to the point $P(1, 0)$,

then $\frac{1+h}{2}=-1$ and $\frac{0+k}{2}=-2$

$\Rightarrow h=-3, k=-4$

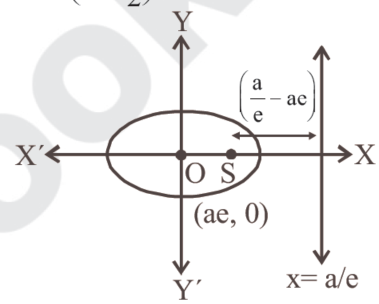
So, Q is $(-3, -4)$

31. **(a)** Perpendicular distance of directrix

$x=\pm\frac{a}{e}$ from focus $(\pm ae, 0)$

$=\frac{a}{e}-ae=4$

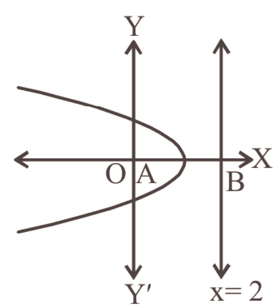
$\Rightarrow a\left(2-\frac{1}{2}\right)=4$



$\Rightarrow a=\frac{8}{3}$

$\therefore$ Semi major axis $=8/3$

32. **(b)** We know that vertex of a parabola is the mid point of focus and the point



where directrix meets the axis of the parabola.

Given that focus is $O(0, 0)$ and directrix meets the axis at $B(2, 0)$

$\therefore$ Vertex of the parabola is $\left(\frac{0+2}{2}, 0\right)=(1,0)$

33. **(a)** The given circles are

$S_1\equiv x^2+y^2+3x+7y+2p-5=0$....(1)

$S_2\equiv x^2+y^2+2x+2y-p^2=0$ ....(2)

$\therefore$ Equation of common chord PQ is
$S_1 - S_2 = 0$ [From (i) and (ii)]

$\Rightarrow L \equiv x + 5y + p^2 + 2p - 5 = 0$

$\Rightarrow$ Equation of circle passing through P and Q is $S_1 + \lambda L = 0$

$\Rightarrow (x^2 + y^2 + 3x + 7y + 2p - 5) + \lambda (x + 5y + p^2 + 2p - 5) = 0$

Given that it passes through (1, 1), therefore
$(7 + 2p) + \lambda (2p + p^2 + 1) = 0$

$\Rightarrow \lambda = -\dfrac{2p+7}{(p+1)^2}$

which does not exist for $p = -1$

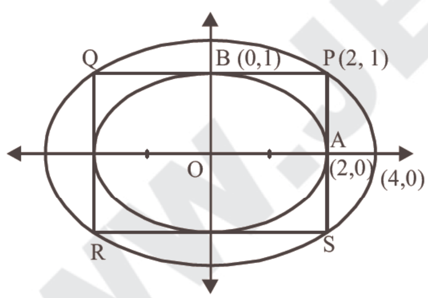
34. (a) The given equation of ellipse is

$$\frac{x^2}{4} + \frac{y^2}{1} = 1$$

So, $A = (2, 0)$ and $B = (0, 1)$

If $PQRS$ is the rectangle in which it is inscribed, then $P = (2, 1)$.

Let $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$ be the ellipse circumscribing the rectangle $PQRS$.



Then it passed through P (2,1)

$\therefore \dfrac{4}{a^2} + \dfrac{1}{b^2} = 1$ ....(i)

Also, given that, it passes through (4, 0)

$\therefore \dfrac{16}{a^2} + 0 = 1 \Rightarrow a^2 = 16$

$\Rightarrow b^2 = 4/3$ [putting $a^2 = 16$ in eq^n (i)]

$\therefore$ The required equation of ellipse is

$$\frac{x^2}{16} + \frac{y^2}{4/3} = 1$$

or $x^2 + 12y^2 = 16$

35. (a) Given equation of circle is
$x^2 + y^2 - 4x - 8y - 5 = 0$

Centre = (2, 4), Radius = $\sqrt{4+16+5} = 5$

Given circle is intersecting the line $3x - 4y = m$, at two distinct points.

$\Rightarrow$ length of perpendicular from centre to the line < radius

$\Rightarrow \dfrac{|6-16-m|}{5} < 5 \quad \Rightarrow |10+m| < 25$

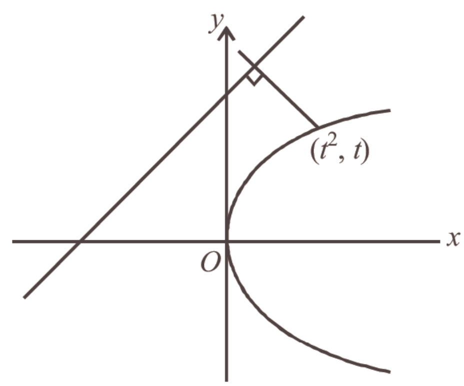
$\Rightarrow -25 < m + 10 < 25 \Rightarrow -35 < m < 15$

36. (b) We know that the locus of perpendicular tangents is directrix i.e., $x = -a; x = -1$

37. (a) If the two circles touch each other and centre (0, 0) of $x^2 + y^2 = c^2$ is lies on circle $x^2 + y^2 = ax$ then they must touch each other internally.

So, $\dfrac{|a|}{2} = c - \dfrac{|a|}{2} \Rightarrow |a| = c$

38. (a)



Let (t^2, t) be point on parabola from that line have shortest distance.

$\therefore$ Distance $= \left|\dfrac{t^2 - t + 1}{\sqrt{2}}\right|$

$= \dfrac{1}{\sqrt{2}}\left[\left(t - \dfrac{1}{2}\right)^2 + \dfrac{3}{4}\right]$

Distance is minimum when $t - \dfrac{1}{2} = 0$

$\therefore$ Shortest distance $= \dfrac{1}{\sqrt{2}}\left[0 + \dfrac{3}{4}\right]$

$= \dfrac{3\sqrt{2}}{8}$

39. **(d)** Let the equation of ellipse be $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$

Given it passes through (–3, 1) so

$\frac{9}{a^2}+\frac{1}{b^2}=1$ ..(i)

Also, we know that

$b^2 = a^2(1-e^2) = a^2(1-2/5)$

$\Rightarrow\ 5b^2 = 3a^2$ ...(ii)

Solving (i) and (ii) we get $a^2 = \frac{32}{3}, b^2 = \frac{32}{5}$

So, the equation of the ellipse is

$$3x^2 + 5y^2 = 32$$

40. **(b)** Given circle whose diametric end points are (1,0) and (0,1) will be of smallest radius. Equation of this smallest circle is

$(x-1)(x-0)+(y-0)(y-1)=0$

$\Rightarrow\ x^2+y^2-x-y=0$

41. **(b)** Given that $ae=2$ and $e=2$

$\therefore\quad a=1$

We know, $b^2 = a^2\left(e^2-1\right)$

$b^2 = 1(4-1)$

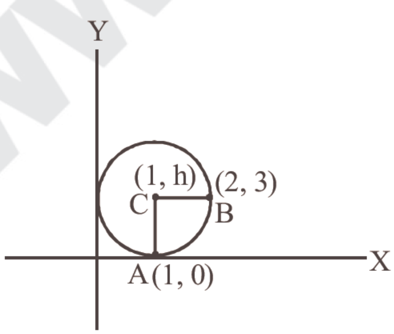
$b^2 = 3$

$\therefore$ Equation of hyperbola, $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$

$\Rightarrow\ \frac{x^2}{1}-\frac{y^2}{3}=1$

$3x^2-y^2=3$

42. **(a)** Since, circle touches , the x-axis at (1, 0). So, let centre of the circle be (1, h)



Given that circle passes through the point B (2,3)

$\therefore\quad CA = CB$ (radius)

$\Rightarrow\quad CA^2 = CB^2$

$\Rightarrow\ (1-1)^2+(h-0)^2=(1-2)^2+(h-3)^2$

$\Rightarrow\ h^2 = 1+h^2+9-6h$

$\Rightarrow\ h=\frac{10}{6}=\frac{5}{3}$

$\therefore$ Length of the diameter $=\frac{10}{3}$

43. **(b)** Given that equation of ellipse is $2x^2+y^2=4$

$\Rightarrow\ \frac{2x^2}{4}+\frac{y^2}{4}=1\ \Rightarrow \frac{x^2}{2}+\frac{y^2}{4}=1$

$\therefore$ Equation of tangent to the ellipse $\frac{x^2}{2}+\frac{y^2}{4}=1$ is

$y = mx \pm \sqrt{2m^2+4}$ ...(i)

($\because$ Equation of tangent to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is $y=mx+c$

where $c=\pm\sqrt{a^2m^2+b^2}$)

Now, Equation of tangent to the parabola

$y^2 = 16\sqrt{3}x$ is $y = mx+\frac{4\sqrt{3}}{m}$ ...(ii)

($\because$ equation of tangent to the parabola $y^2=4ax$ is $y=mx+\frac{a}{m}$)

From (i) and (ii), we get

$\frac{4\sqrt{3}}{m}=\pm\sqrt{2m^2+4}$

Squaring on both the sides, we get

$16(3)=(2m^2+4)m^2$

$\Rightarrow\ 48 = m^2(2m^2+4)$

$\Rightarrow\ 2m^4+4m^2-48=0$

$\Rightarrow\ m^4+2m^2-24=0$

$\Rightarrow\ (m^2+6)(m^2-4)=0$

$\Rightarrow\ m^2=4\ (\because m^2\neq -6)\ \Rightarrow\quad m=\pm 2$

Putting value of m in equ. (i), we get

Equation of common tangents are

$y=\pm 2x\pm 2\sqrt{3}$

Thus, statement-1 and 2 both are true. Statement-2 is correct explanation for statement-1.

44. **(d)** Equation of circle is $(x-1)^2+y^2=1$

$\Rightarrow$ radius = 1 and diameter = 2

$\therefore$ ATQ length of semi-minor axis b is 2.

Equation of circle is $x^2+(y-2)^2=4=(2)^2$

$\Rightarrow$ radius = 2 and diameter = 4

$\therefore$ ATQ length of semi major a axis is 4

We know, equation of ellipse is given by

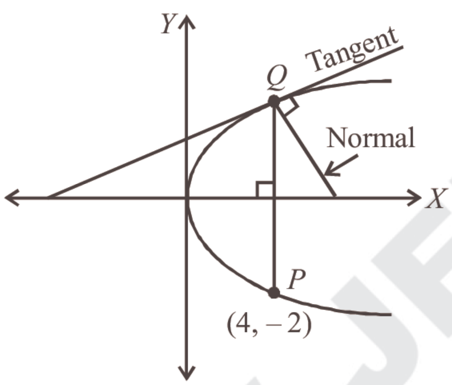
$$\frac{x^2}{(a)^2}+\frac{y^2}{(b)^2}=1$$

$$\Rightarrow \frac{x^2}{(4)^2}+\frac{y^2}{(2)^2}=1 \Rightarrow \frac{x^2}{16}+\frac{y^2}{4}=1$$

$$\Rightarrow x^2+4y^2=16$$

45. **(a)** Given that point P is $(4,-2)$ and $PQ \perp x$-axis

So, from figure $Q=(4, 2)$



Equation of tangent at (4, 2) is

$$yy_1=\frac{1}{2}(x+x_1)$$

$$\Rightarrow 2y=\frac{1}{2}(x+2) \Rightarrow 4y=x+2$$

$$\Rightarrow y=\frac{x}{4}+\frac{1}{2}$$

So, slope of tangent $=\frac{1}{4}$

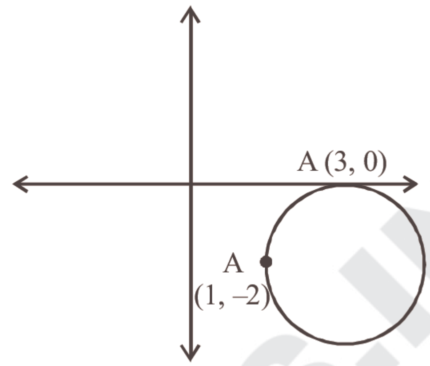
$\because$ product of slope of tangent and normal = –1

$\therefore$ Slope of normal = – 4

46. **(c)** Since circle touches x-axis at (3, 0)

$\therefore$ The equation of circle be

$(x-3)^2+(y-0)^2+\lambda y=0$



As it passes through (1, –2)

$\therefore$ Put $x=1, y=-2$

$\Rightarrow (1-3)^2+(-2)^2+\lambda(-2)=0$

$\Rightarrow \lambda=4$

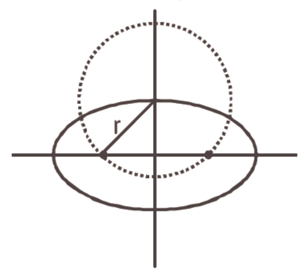
$\therefore$ equation of circle is

$(x-3)^2+y^2-8=0$

Now, from the options (5, –2) satisfies equation of circle.

47. **(a)** From the given equation of ellipse, we have

$a=4, b=3, e=\sqrt{1-\frac{9}{16}}$



$\Rightarrow e=\frac{\sqrt{7}}{4}$

Now, radius of this circle $=a^2=16$

$\Rightarrow$ Focii $=(\pm\sqrt{7}, 0)$

Now equation of circle is

$(x-0)^2+(y-3)^2=16$

$x^2+y^2-6y-7=0$

48. **(b)** Let common tangent be

$$y=mx+\frac{\sqrt{5}}{m}$$

Since, perpendicular distance from centre of the circle to the common tangent is equal to radius of the circle, therefore

$$\frac{\frac{\sqrt{5}}{m}}{\sqrt{1+m^2}}=\sqrt{\frac{5}{2}}$$

On squaring both the side, we get

$m^2(1+m^2)=2$

$\Rightarrow m^4+m^2-2=0$

$\Rightarrow (m^2+2)(m^2-1)=0$

$\Rightarrow m=\pm 1 \qquad (\because m^2 \neq -2)$

$y=\pm\left(x+\sqrt{5}\right)$, both statements are correct as $m = \pm 1$ satisfies the given equation of statement-2.

49. (a) Given equation of ellipse can be written as

$$\frac{x^2}{6}+\frac{y^2}{2}=1$$

$\Rightarrow a^2=6, b^2=2$

Now, equation of any variable tangent is

$$y=mx\pm\sqrt{a^2m^2+b^2} \qquad ...(i)$$

where m is slope of the tangent

So, equation of perpendicular line drawn from centre to tangent is

$$y=\frac{-x}{m} \qquad ...(ii)$$

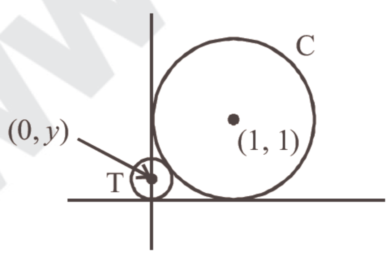
Eliminating m, we get

$$(x^4+y^4+2x^2y^2)=a^2x^2+b^2y^2$$

$$\Rightarrow (x^2+y^2)^2=a^2x^2+b^2y^2$$

$$\Rightarrow \boxed{(x^2+y^2)^2=6x^2+2y^2}$$

50. (b)



Equation of circle

$$C \equiv (x-1)^2+(y-1)^2=1$$

Radius of T = $|y|$

T touches C externally therefore,

Distance between the centres = sum of their radii

$$\Rightarrow \sqrt{(0-1)^2+(y-1)^2}=1+|y|$$

$\Rightarrow (0-1)^2+(y-1)^2=(1+|y|)^2$

$\Rightarrow 1+y^2+1-2y=1+y^2+2|y|$

$2|y|=1-2y$

If $y>0$ then $2y=1-2y \Rightarrow y=\frac{1}{4}$

If $y<0$ then $-2y=1-2y \Rightarrow 0=1$ (not possible)

$\therefore \quad y=\frac{1}{4}$

51. (c) Given parabolas are

$y^2=4x \qquad ...(1)$

$x^2=-32y \qquad ...(2)$

Let m be slope of common tangent

Equation of tangent of parabola (1)

$$y=mx+\frac{1}{m} \qquad ...(i)$$

Equation of tangent of parabola (2)

$y=mx+8m^2 \qquad ...(ii)$

(i) and (ii) are identical

$$\Rightarrow \frac{1}{m}=8m^2 \Rightarrow m^3=\frac{1}{8} \Rightarrow \boxed{m=\frac{1}{2}}$$

ALTERNATIVE METHOD:

Let tangent to $y^2=4x$ be $y=mx+\frac{1}{m}$

Since this is also tangent to $x^2=-32y$

$$\therefore \quad x^2=-32\left(mx+\frac{1}{m}\right)$$

$$\Rightarrow x^2+32mx+\frac{32}{m}=0$$

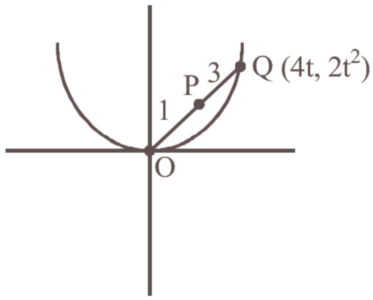
Now, D = 0

$$(32)^2-4\left(\frac{32}{m}\right)=0$$

$$\Rightarrow m^3=\frac{4}{32} \quad \Rightarrow m=\frac{1}{2}$$

52. **(b)** Let P(h, k) divides OQ in the ratio 1 : 3

Let any point Q on $x^2 = 8y$ is $(4t, 2t^2)$.



Then by section formula

$\Rightarrow \quad k = \frac{t^2}{2}$ and $h = t$

$\Rightarrow \quad 2k = h^2$

Required locus of P is $x^2 = 2y$

53. **(a)** $x^2 + y^2 - 4x - 6y - 12 = 0$ ...(i)

Centre, $C_1 = (2, 3)$

Radius, $r_1 = 5$ units

$x^2 + y^2 + 6x + 18y + 26 = 0$ ...(ii)

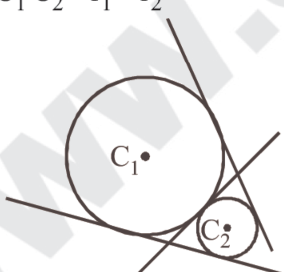
Centre, $C_2 = (-3, -9)$

Radius, $r_2 = 8$ units

$C_1C_2 = \sqrt{(2+3)^2 + (3+9)^2} = 13$ units

$r_1 + r_2 = 5 + 8 = 13$

$\therefore C_1 C_2 = r_1 + r_2$



Therefore there are three common tangents.

54. **(b)** The end point of latus rectum of ellipse

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ in first quadrant is $\left(ae, \frac{b^2}{a}\right)$

and the tangent at this point intersects

x-axis at $\left(\frac{a}{e}, 0\right)$ and y-axis at (0, a).

The given ellipse is $\frac{x^2}{9} + \frac{y^2}{5} = 1$

Then $a^2 = 9, b^2 = 5$

$\Rightarrow \quad e = \sqrt{1 - \frac{5}{9}} = \frac{2}{3}$

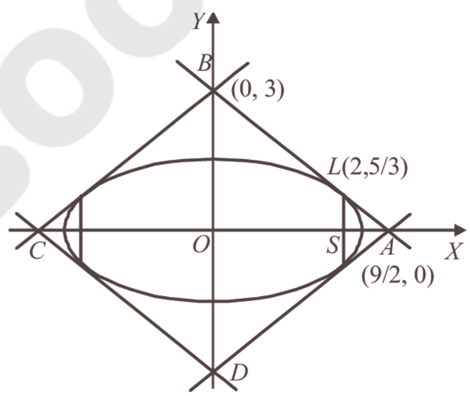
$\therefore$ End point of latus rectum in first quadrant is

$L(2, 5/3)$

Equation of tangent at L is $\frac{2x}{9} + \frac{y}{3} = 1$

[$\because$ It meets x-axis at $A\,(9/2, 0)$ and y-axis at $B\,(0, 3)$]

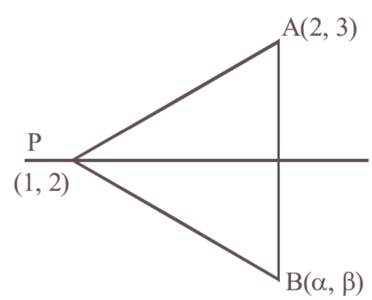
$\therefore \quad$ Area of $\Delta OAB = \frac{1}{2} \times \frac{9}{2} \times 3 = \frac{27}{4}$



By symmetry area of quadrilateral

$= 4 \times (\text{Area } \Delta OAB) = 4 \times \frac{27}{4} = 27$ sq. units.

55. **(a)** Intersection point of $2x - 3y + 4 = 0$ and $x - 2y + 3 = 0$ is (1, 2)



Let image of $A(2, 3)$ is $B(\alpha, \beta)$.

Since, P is the fixed point for given family of lines

So, $PB = PA$

$(\alpha-1)^2+(\beta-2)^2=(2-1)^2+(3-2)^2$

$(\alpha-1)^2+(\beta-2)^2=1+1=2$

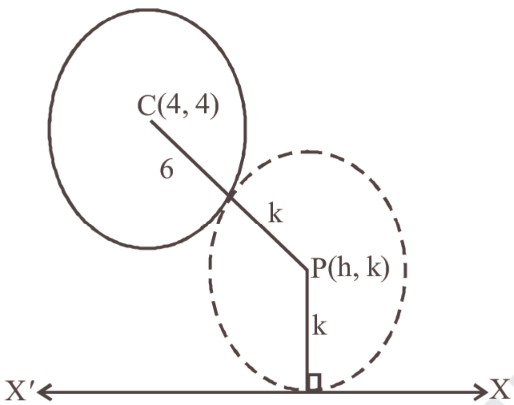
$(x-1)^2+(y-2)^2=(\sqrt{2})^2$

Compare with

$(x-a)^2+(y-b)^2=r^2$

Therefore, given locus is a circle with centre (1, 2) and radius $\sqrt{2}$.

56. (b)



For the given circle,

centre : (4, 4); radius = 6

Let centre of variable circle is (h, k) and it touches x-axis, so radius = k

$\therefore\ 6+k=\sqrt{(h-4)^2+(k-4)^2}$

$(h-4)^2=20k+20$

$\therefore$ locus of (h, k) is

$(x-4)^2=20(y+1)$,

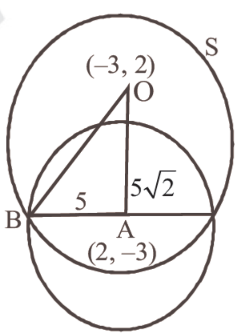
which is a parabola.

57. (a) $\frac{2b^2}{a}=8$ and $2b=\frac{1}{2}(2ae)$

$\Rightarrow 4b^2=a^2e^2 \Rightarrow 4a^2(e^2-1)=a^2e^2$

$\Rightarrow 3e^2=4 \quad \Rightarrow e=\frac{2}{\sqrt{3}}$

58. (d)



Given, centre of S is O (–3, 2) and centre of given circle is A(2, –3) and radius is 5.

$\Rightarrow OA = 5\sqrt{2}$

Also AB = 5 ($\because$ AB = radius of the given circle)

$\Rightarrow$ Using pythagoras theorem in ΔOAB

$r=5\sqrt{3}$

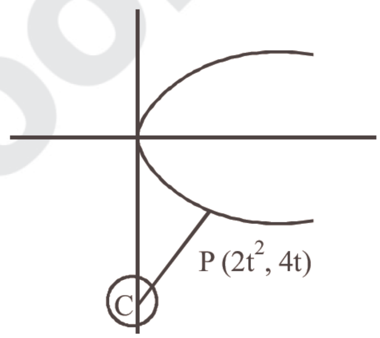
59. (c) Minimum distance $\Rightarrow$ perpendicular distance

Eq^n of normal at $p(2t^2, 4t)$

$y=-tx+4t+2t^3$

It passes through C(0, –6)

$\Rightarrow t^3+2t+3=0 \Rightarrow t=-1$



Centre of new circle $= P(2t^2, 4t)$

$= P(2, -4)$

Radius $= PC = \sqrt{(2-0)^2+(-4+6)^2}$

$= 2\sqrt{2}$

$\therefore$ Equation of circle is :

$(x-2)^2+(y+4)=\left(2\sqrt{2}\right)^2$

$\Rightarrow\ x^2+y^2-4x+8y+12=0$

60. (c) Equation of hyperbola is $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$

foci is $(\pm 2, 0) \Rightarrow ae=\pm 2 \Rightarrow a^2e^2=4$

Since $b^2=a^2(e^2-1)$

$b^2=a^2e^2-a^2 \ \therefore\ a^2+b^2=4$ ...(i)

Hyperbola passes through $(\sqrt{2}, \sqrt{3})$

$\therefore \frac{2}{a^2}-\frac{3}{b^2}=1$ ...(ii)

$$\frac{2}{4-b^2}-\frac{3}{b^2}=1 \qquad \text{[from (i)]}$$

$\Rightarrow b^4+b^2-12=0$

$\Rightarrow (b^2-3)(b^2+4)=0$

$\Rightarrow \; b^2=3$

$b^2=-4$ (Not possible)

For $b^2=3$

$\Rightarrow a^2=1 \;\therefore\; \frac{x^2}{1}-\frac{y^2}{3}=1$

Equation of tangent is $\frac{\sqrt{2}x}{1}-\frac{\sqrt{3}y}{3}=1$

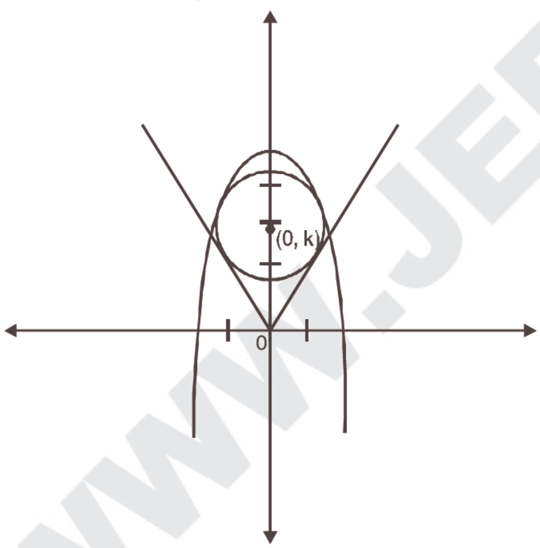
Clearly $(2\sqrt{2},3\sqrt{3})$ satisfies it.

61. **(None)**

(Let the equation of circle be

$x^2+(y-k)^2=r^2$

It touches $x-y=0$



$\Rightarrow \left|\frac{0-k}{\sqrt{2}}\right|=r$

$\Rightarrow \; k=r\sqrt{2}$

$\therefore$ Equation of circle becomes

$$x^2+(y-k)^2=\frac{k^2}{2} \qquad \text{...(i)}$$

It touches $y=4-x^2$ as well

$\therefore$ Solving the two equations

$\Rightarrow \; 4-y+(y-k)^2=\frac{k^2}{2}$

$\Rightarrow \; 1y^2-y(2k+1)+\frac{k^2}{2}+4=0$

It will give equal roots $\therefore D=0$

$\Rightarrow \; (2k+1)^2=4\left(\frac{k^2}{2}+4\right)$

$\Rightarrow \; 2k^2+4k-15=0$

$\Rightarrow \; k=\frac{-2+\sqrt{34}}{2}$

$\therefore \; r=\frac{k}{\sqrt{2}}=\frac{-2+\sqrt{34}}{2\sqrt{2}}$

Which is not matching with any of the option given here.

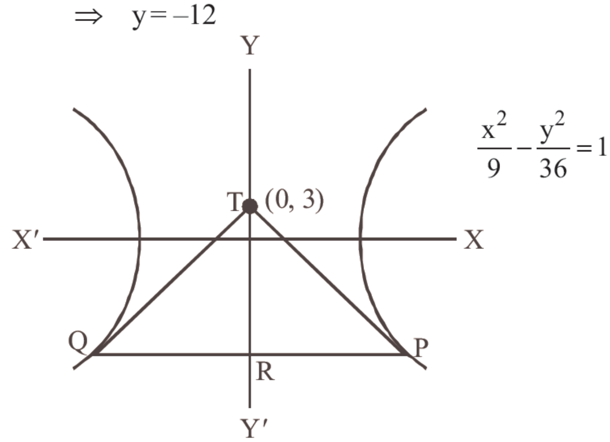
62. **(d)** Here equation of hyperbola is

$$\frac{x^2}{9}-\frac{y^2}{36}=1$$

Now, PQ is the chord of contant

$\therefore$ Equation of PQ is : $\frac{x(0)}{9}-\frac{y(3)}{36}=1$

$\Rightarrow \; y=-12$



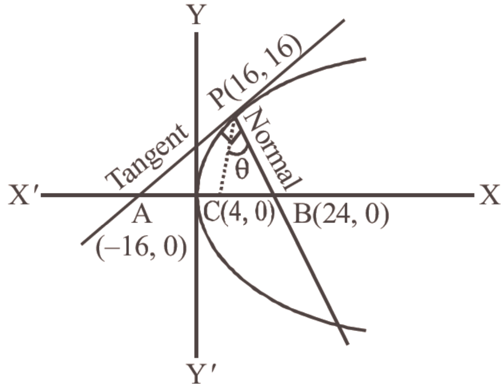
$\therefore$ Area of $\Delta PQT=\frac{1}{2}\times TR\times PQ$

$\because \; P\equiv(3\sqrt{5},-12) \;\therefore\; TR=3+12=15,$

$\therefore$ Area of ΔPQT

$$=\frac{1}{2}\times 15\times 6\sqrt{5}=45\sqrt{5} \text{ sq. units}$$

63. **(a)** Equation of tnagent at P(16, 16) is given as:
$x-2y+16=0$



Slope of PC $(m_1)=\frac{4}{3}$
Slope of PB $(m_2)=-2$

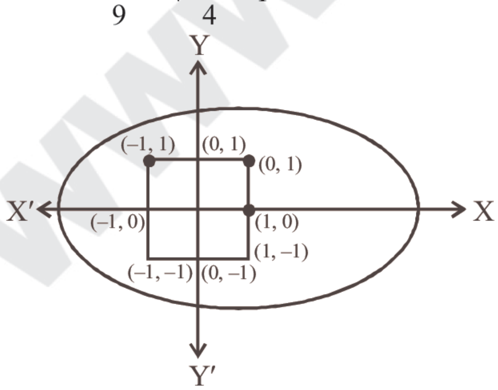
Hence, $\tan\theta=\left|\frac{m_1-m_2}{1+m_1\cdot m_2}\right|=\left|\frac{\frac{4}{3}+2}{1-\frac{4}{3}\cdot 2}\right|$

$\Rightarrow$ $\tan\theta=2$

64. **(a)** $A=\{(a,b)\in R\times R: |a-5|<1, |b-5|<1\}$
Let $a-5=x$, $b-5=y$
Set A contains all points inside $|x|<1, |y|<1$

$B=\{(a,b)\in R\times R: 4(a-6)^2+9(b-5)^2\le 36\}$
Set B contains all points inside or on

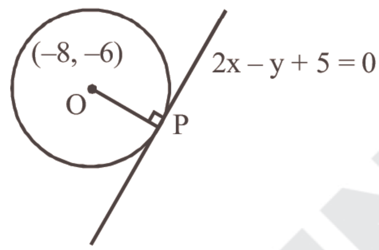
$$\frac{(x-1)^2}{9}+\frac{y^2}{4}=1$$



$\therefore$ $(\pm1,\pm1)$ lies inside the ellipse.

Hence, $A\subset B$.

65. **(c)** Equation of tangent at (1, 7) to $x^2=y-6$ is $2x-y+5=0$.



Now, perpendicular from centre O(–8, –6) to $2x-y+5=0$ should be equal to radius of the circle

$$\therefore\quad \left|\frac{-16+6+5}{\sqrt{5}}\right|=\sqrt{64+36-C}$$

$\Rightarrow \sqrt{5}=\sqrt{100-C}$

$\Rightarrow c=95$

66. **(c)** Since, $lx+my+n=0$ is a normal to $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$,

then $\frac{a^2}{l^2}-\frac{b^2}{m^2}=\frac{(a^2+b^2)^2}{n^2}$

but it is given that $mx-y+7\sqrt{3}$ is normal

to hyperbola $\frac{x^2}{24}-\frac{y^2}{18}=1$

then $\frac{24}{m^2}-\frac{18}{(-1)^2}=\frac{(24+18)^2}{(7\sqrt{3})^2}\Rightarrow m=\frac{2}{\sqrt{5}}$

67. **(b)** Let $z\in S$ then $z=\frac{\alpha+i}{\alpha-i}$

Since, z is a complex number and let $z=x+iy$

Then, $x+iy=\frac{(\alpha+i)^2}{\alpha^2+1}$ (by rationalisation)

$$\Rightarrow x+iy=\frac{(\alpha^2-1)}{\alpha^2+1}+\frac{i(2\alpha)}{\alpha^2+1}$$

Then compare both sides

$$x=\frac{\alpha^2-1}{\alpha^2+1} \quad ...(1)$$

$$y=\frac{2\alpha}{\alpha^2+1} \quad ...(2)$$

Now squaring and adding equations (1) and (2)

$$\Rightarrow x^2+y^2=\frac{(\alpha^2-1)^2}{(\alpha^2+1)^2}+\frac{4\alpha^2}{(\alpha^2+1)^2}=1$$

68. (a) Let any tangent to circle $x^2+y^2=1$ is

$x\cos\theta+y\sin\theta=1$

Since, P and Q are the point of intersection on the co-ordinate axes.

Then $P\equiv\left(\frac{1}{\cos\theta},0\right)$ & $Q\equiv\left(0,\frac{1}{\sin\theta}\right)$

$\therefore$ mid-point of PQ be

$$M\equiv\left(\frac{1}{2\cos\theta},\frac{1}{2\sin\theta}\right)\equiv(h,k)$$

$$\Rightarrow \cos\theta=\frac{1}{2h} \quad ...(1)$$

$$\sin\theta=\frac{1}{2k} \quad ...(2)$$

Now squaring and adding equations (1) and (2)

$$\frac{1}{h^2}+\frac{1}{k^2}=4$$

$\Rightarrow h^2+k^2=4h^2k^2$

$\therefore$ locus of M is : $x^2+y^2-4x^2y^2=0$

69. (a) $\because$ $y^2=16x \Rightarrow a=4$

One end of focal of the parabola is at (1, 4)

$\because$ y-coordinate of focal chord is $2at$

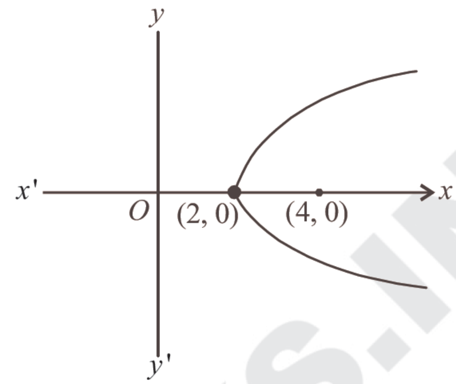
$\therefore 2\ at=4$

$$\Rightarrow t=\frac{1}{2}$$

Hence, the required length of focal chord

$$=a\left(t+\frac{1}{t}\right)^2=4\times\left(2+\frac{1}{2}\right)^2=25$$

70. (b) Since, vertex and focus of given parabola is (2, 0) and (4, 0) respectively



Then, equation of parabola is

$(y-0)^2=4\times 2(x-2)$

$\Rightarrow y^2=8x-16$

Hence, the point (8, 6) does not lie on given parabola.

71. (b) Since, the equation of tangent to parabola $y^2=4x$ is

$$y=mx+\frac{1}{m} \quad ...(1)$$

The line (1) is also the tangent to circle

$x^2+y^2-6x=0$

Then centre of circle = (3, 0)

radius of circle = 3

The perpendicular distance from centre to tangent is equal to the radius of circle

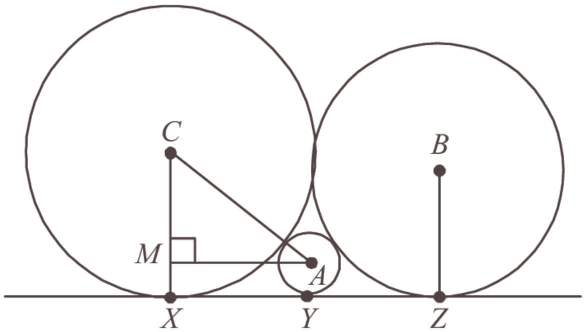
$$\therefore \frac{\left|3m+\frac{1}{m}\right|}{\sqrt{1+m^2}}=3\Rightarrow\left(3m+\frac{1}{m}\right)^2=9(1+m^2)$$

$$\Rightarrow m=\pm\frac{1}{\sqrt{3}}$$

Then, from equation (1): $y=\pm\frac{1}{\sqrt{3}}x\pm\sqrt{3}$

Hence, $\sqrt{3}y=x+3$ is one of the required common tangent.

72. (a)



$AM^2 = AC^2 - MC^2$

$= (a+c)^2 - (c-a)^2 = 4ac$

$\Rightarrow \quad AM^2 = XY^2 = 4ac$

$\Rightarrow \quad XY = 2\sqrt{ac}$

Similarly, $YZ = 2\sqrt{ba}$ and $XZ = 2\sqrt{bc}$

Then, $XZ = XY + YZ$

$\Rightarrow \quad 2\sqrt{bc} = 2\sqrt{ac} + 2\sqrt{ba}$

$\Rightarrow \quad \dfrac{1}{\sqrt{a}} = \dfrac{1}{\sqrt{b}} + \dfrac{1}{\sqrt{c}}$

73. (b) $2ae = 6$ and $\dfrac{2a}{e} = 12$

$\Rightarrow \quad ae = 3 \qquad$ ...(i)

and $\dfrac{a}{e} = 6 \Rightarrow e = \dfrac{a}{6} \quad$ ...(ii)

$\Rightarrow \quad a^2 = 18 \quad$ [From (i) and (ii)]

$\Rightarrow \quad b^2 = a^2 - a^2e^2 = 18 - 9 = 9$

$\therefore \quad$ Latus rectum $= \dfrac{2b^2}{a} = \dfrac{2 \times 9}{3\sqrt{2}} = 3\sqrt{2}$

74. (c) $y = mx + 4 \qquad$ ...(i)

Tangent of $y^2 = 4x$ is

$\Rightarrow \quad y = mx + \dfrac{1}{m} \qquad$ ...(ii)

[$\because$ Equation of tangent of $y^2 = 4\,ax$ is $y = mx + \dfrac{a}{m}$]

From (i) and (ii)

$4 = \dfrac{1}{m} \Rightarrow m = \dfrac{1}{4}$

So, line $y = \dfrac{1}{4}x + 4$ is also tangent to parabola

$x^2 = 2by$, so solve both equations.

$x^2 = 2b\left(\dfrac{x+16}{4}\right)$

$\Rightarrow \quad 2x^2 - bx - 16b = 0$

$\Rightarrow \quad D = 0 \qquad$ [For tangent]

$\Rightarrow \quad b^2 - 4 \times 2 \times (-16b) = 0$

$\Rightarrow \quad b^2 + 32 \times 4b = 0$

$b = -128$, $b = 0$ (not possible)

75. (d) $\because \quad z = x + iy$

$\left(\dfrac{z-1}{2z+i}\right) = \dfrac{(x-1)+iy}{2(x+iy)+i}$

$= \dfrac{(x-1)+iy}{2x+(2y+1)i} \times \dfrac{2x-(2y+1)i}{2x-(2y+1)i}$

$\text{Re}\left(\dfrac{z+1}{2z+i}\right) = \dfrac{2x(x-1)+y(2y+1)}{(2x)^2+(2y+1)^2} = 1$

$\Rightarrow \quad \left(x+\dfrac{1}{2}\right)^2 + \left(y+\dfrac{3}{4}\right)^2 = \left(\dfrac{\sqrt{5}}{4}\right)^2.$

12 Limits and Derivatives

1. $\lim_{x\to 0} \frac{\sqrt{1-\cos 2x}}{\sqrt{2}x}$ is **[2002]**
(a) 1 (b) –1
(c) zero (d) does not exist

2. $\lim_{x\to\infty}\left(\frac{x^2+5x+3}{x^2+x+2}\right)^x$ **[2002]**
(a) e^4 (b) e^2
(c) e^3 (d) 1

3. Let $f(2)=4$ and $f'(2)=4$. Then $\lim_{x\to 2}\frac{xf(2)-2f(x)}{x-2}$ is given by **[2002]**
(a) 2 (b) –2
(c) –4 (d) 3

4. $\lim_{n\to\infty}\frac{1^p+2^p+3^p+.....+n^p}{n^{p+1}}$ is **[2002]**
(a) $\frac{1}{p+1}$ (b) $\frac{1}{1-p}$
(c) $\frac{1}{p}-\frac{1}{p-1}$ (d) $\frac{1}{p+2}$

5. $\lim_{x\to 0}\frac{\log x^n-[x]}{[x]}, n\in N$, ($[x]$ denotes greatest integer less than or equal to x) **[2002]**
(a) has value -1 (b) has value 0
(c) has value 1 (d) does not exist

6. If $f(1)=1, f'(1)=2$, then $\lim_{x\to f}\frac{\sqrt{f(x)}-1}{\sqrt{x}-1}$ is **[2002]**
(a) 2 (b) 4
(c) 1 (d) 1/2

7. If $\lim_{x\to 0}\frac{\log(3+x)-\log(3-x)}{x}=k$, the value of k is **[2003]**
(a) $-\frac{2}{3}$ (b) 0
(c) $-\frac{1}{3}$ (d) $\frac{2}{3}$

8. $\lim_{x\to\frac{\pi}{2}}\frac{\left[1-\tan\left(\frac{x}{2}\right)\right][1-\sin x]}{\left[1+\tan\left(\frac{x}{2}\right)\right][\pi-2x]^3}$ is **[2003]**
(a) ∞ (b) $\frac{1}{8}$
(c) 0 (d) $\frac{1}{32}$

9. If $\lim_{x\to\infty}\left(1+\frac{a}{x}+\frac{b}{x^2}\right)^{2x}=e^2$, then the values of a and b, are **[2004]**
(a) $a=1$ and $b=2$ (b) $a=1, b\in R$
(c) $a\in R, b=2$ (d) $a\in R, b\in R$

10. Let α and β be the distinct roots of $ax^2+bx+c=0$, then $\lim_{x\to\alpha}\frac{1-\cos(ax^2+bx+c)}{(x-\alpha)^2}$ is equal to **[2005]**

(a) $\frac{a^2}{2}(\alpha-\beta)^2$ (b) 0

(c) $\frac{-a^2}{2}(\alpha-\beta)^2$ (d) $\frac{1}{2}(\alpha-\beta)^2$

11. Let $f: R\to R$ be a positive increasing function with $\lim_{x\to\infty}\frac{f(3x)}{f(x)}=1$, then, $\lim_{x\to\infty}\frac{f(2x)}{f(x)}=$ **[2010]**

(a) $\frac{2}{3}$ (b) $\frac{3}{2}$

(c) 3 (d) 1

12. $\lim_{x\to 2}\left(\frac{\sqrt{1-\cos\{2(x-2)\}}}{x-2}\right)$ **[2011]**

(a) equals $\sqrt{2}$ (b) equals $-\sqrt{2}$

(c) equals $\frac{1}{\sqrt{2}}$ (d) does not exist

13. Let $f: R\to[0,\infty)$ be such that $\lim_{x\to 5} f(x)$ exists and $\lim_{x\to 5}\frac{(f(x))^2-9}{\sqrt{|x-5|}}=0$ **[2011RS]**

Then $\lim_{x\to 5} f(x)$ equals :

(a) 0 (b) 1
(c) 2 (d) 3

14. $\lim_{x\to 0}\frac{(1-\cos 2x)(3+\cos x)}{x\tan 4x}$ is equal to **[2013]**

(a) $-\frac{1}{4}$ (b) $\frac{1}{2}$

(c) 1 (d) 2

15. $\lim_{x\to 0}\frac{\sin\left(\pi\cos^2 x\right)}{x^2}$ is equal to: **[2014]**

(a) $-\pi$ (b) π

(c) $\frac{\pi}{2}$ (d) 1

16. $\lim_{x\to\frac{\pi}{2}}\frac{\cot x-\cos x}{(\pi-2x)^3}$ equals : **[2017]**

(a) $\frac{1}{4}$ (b) $\frac{1}{24}$

(c) $\frac{1}{16}$ (d) $\frac{1}{8}$

17. For each $t\in R$, let $[t]$ be the greatest integer less than or equal to t. Then **[2018]**

$\lim_{x\to 0^+} x\left(\left[\frac{1}{x}\right]+\left[\frac{2}{x}\right]+\ldots+\left[\frac{15}{x}\right]\right)$

(a) is equal to 15.
(b) is equal to 120.
(c) does not exist (in R).
(d) is equal to 0.

18. $\lim_{y\to 0}\frac{\sqrt{1+\sqrt{1+y^4}}-\sqrt{2}}{y^4}$ **[2019]**

(a) exists and equals $\frac{1}{4\sqrt{2}}$

(b) exists and equals $\frac{1}{2\sqrt{2}\left(\sqrt{2}+1\right)}$

(c) exists and equals $\frac{1}{2\sqrt{2}}$

(d) does not exist

19. $\lim_{x\to 2}\frac{3^x+3^{3-x}-12}{3^{-x/2}-3^{1-x}}$ is equal to ______. **[2020]**

Answer Key

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
(d)	(a)	(c)	(a)	(d)	(a)	(d)	(d)	(b)	(a)	(d)	(d)	(d)	(d)	(b)
16	17	18	19											
(c)	(b)	(a)	36											

Solutions

1. (d) $\lim_{x\to 0}\frac{\sqrt{1-\cos 2x}}{\sqrt{2}x}$;

$\lim_{x\to 0}\frac{\sqrt{2\sin^2 x}}{\sqrt{2}x} \Rightarrow \lim_{x\to 0}\frac{|\sin x|}{x}$

$\left[\because \lim_{\theta\to 0}\frac{\sin\theta}{\theta}=1\right]$

The limit of above does not exist as LHS = –1 ≠ RHL = 1

2. (a) $\lim_{x\to\infty}\left(\frac{x^2+5x+3}{x^2+x+2}\right)^x$

$= \lim_{x\to\infty}\left(1+\frac{4x+1}{x^2+x+2}\right)^x$

$= \lim_{x\to\infty}\left[\left(1+\frac{4x+1}{x^2+x+2}\right)^{\frac{x^2+x+2}{4x+1}}\right]^{\frac{(4x+1)x}{x^2+x+2}}$

$= e^{\lim_{x\to\infty}\frac{4x^2+x}{x^2+x+2}}\left[\because \lim_{x\to\infty}(1+\lambda x)^{\frac{1}{x}}=e^{\lambda}\right]$

$= e^{\lim_{x\to\infty}\frac{4+\frac{1}{x}}{1+\frac{1}{x}+\frac{2}{x^2}}} = e^4 \qquad \left[\because \frac{1}{\infty}=0\right]$

3. (c) Given that $f(2)=4$ and $f(2)=4$

We have, $\lim_{x\to 2}\frac{xf(2)-2f(x)}{x-2}$, $\left(\frac{0}{0}\right)$

Applying L-Hospital's rule, we get

$= \lim_{x\to 2} f(2)-2f'(x) = f(2)-2f'(2)$

$= 4-2\times 4 = -4.$

4. (a) $\lim_{n\to\infty}\frac{1^p+2^p+....+n^p}{n^{p+1}}$

$= \lim_{n\to\infty}\sum_{r=1}^{n}\frac{r^p}{n^p\cdot n} = \int_0^1 x^p\,dx = \left[\frac{x^{p+1}}{p+1}\right]_0^1 = \frac{1}{p+1}$

5. (d) Since, $\lim_{x\to 0^-}[x]=-1\neq \lim_{x\to 0^+}[x]=0.$ So $\lim_{x\to 0}[x]$ does not exist, hence the required limit does not exist.

6. (a) Given that $f(1)=1$ and $f'(1)=2$

$\lim_{x\to 1}\frac{\sqrt{f(x)}-1}{\sqrt{x}-1}$ $\left(\frac{0}{0}\right)$ form

Applying L-Hospital's rule, we get

$= \lim_{x\to 1}\frac{\frac{1}{2\sqrt{f(x)}}f'(x)}{1/2\sqrt{x}} = \frac{f'(1)}{\sqrt{f(1)}} = \frac{2}{1} = 2.$

7. (d) $\lim_{x\to 0}\frac{\log(3+x)-\log(3-x)}{x}=k$

Applying L'Hospital rule, we get

$\Rightarrow \lim_{x\to 0}\frac{\frac{1}{3+x}-\frac{-1}{3-x}}{1}=k \quad \therefore \frac{2}{3}=k$

8. (d) $\lim_{x\to\frac{\pi}{2}}\frac{\tan\left(\frac{\pi}{4}-\frac{x}{2}\right).(1-\sin x)}{(\pi-2x)^3}$

Let $x=\frac{\pi}{2}+y; y\to 0$

$= \lim_{y\to 0}\frac{\tan\left(-\frac{y}{2}\right).(1-\cos y)}{(-2y)^3}$

$= \lim_{y\to 0}\frac{-\tan\frac{y}{2}2\sin^2\frac{y}{2}}{(-8).\frac{y^3}{8}.8}$

$\left[\because 1-\cos\theta = 2\sin^2\frac{\theta}{2}\right]$

$= \lim_{y\to 0}\frac{1}{32}\frac{\tan\frac{y}{2}}{\left(\frac{y}{2}\right)}.\left[\frac{\sin y/2}{y/2}\right]^2 = \frac{1}{32}$

$\left[\because \lim_{\theta\to 0}\frac{\sin\theta}{\theta} = \lim_{\theta\to 0}\frac{\tan\theta}{\theta}=1\right]$

9. (b) We know that $\lim_{x\to\infty}(1+x)^{\frac{1}{x}}=e$

Given that $\lim_{x\to\infty}\left(1+\frac{a}{x}+\frac{b}{x^2}\right)^{2x}=e^2$

$$\Rightarrow \lim_{x\to\infty}\left[\left(1+\frac{a}{x}+\frac{b}{x^2}\right)^{\left(\frac{1}{\frac{a}{x}+\frac{b}{x^2}}\right)}\right]^{2x\left(\frac{a}{x}+\frac{b}{x^2}\right)}=e^2$$

$$\Rightarrow e^{\lim_{x\to\infty}2\left[a+\frac{b}{x}\right]}=e^2\Rightarrow e^{2a}=e^2$$

$\Rightarrow a=1$ and $b\in R$

10. (a) Given that
$ax^2+bx+c=a(x-\alpha)(x-\beta)$

$$\lim_{x\to\alpha}\frac{1-\cos a(x-\alpha)(x-\beta)}{(x-\alpha)^2}$$

$$=\lim_{x\to\alpha}\frac{2\sin^2\left(a\frac{(x-\alpha)(x-\beta)}{2}\right)}{(x-\alpha)^2}$$

$$=\lim_{x\to\alpha}\frac{2}{(x-\alpha)^2}\times\frac{\sin^2\left(a\frac{(x-\alpha)(x-\beta)}{2}\right)}{\frac{a^2(x-\alpha)^2(x-\beta)^2}{4}}\times\frac{a^2(x-\alpha)^2(x-\beta)^2}{4}$$

$$=\frac{a^2(\alpha-\beta)^2}{2}.$$

11. (d) Given that f(x) is a positive increasing function.

$\therefore\ 0<f(x)<f(2x)<f(3x)$

Divided by $f(x)$

$$\Rightarrow 0<1<\frac{f(2x)}{f(x)}<\frac{f(3x)}{f(x)}$$

$$\Rightarrow \lim_{x\to\infty}1\le\lim_{x\to\infty}\frac{f(2x)}{f(x)}\le\lim_{x\to\infty}\frac{f(3x)}{f(x)}$$

By Sandwich Theorem.

$$\Rightarrow \lim_{x\to\infty}\frac{f(2x)}{f(x)}=1$$

12. (d) $\lim_{x\to2}\frac{\sqrt{1-\cos\{2(x-2)\}}}{x-2}$

$$\left[\because 1-\cos\theta=2\sin^2\frac{v}{2}\right]$$

$$=\lim_{x\to2}\frac{\sqrt{2}|\sin(x-2)|}{x-2}$$

$$\underset{(\text{at }x=2)}{\text{L.H.L}}=-\lim_{x\to2}\frac{\sqrt{2}\sin(x-2)}{(x-2)}=-\sqrt{2}$$

$$\underset{(\text{at }x=2)}{\text{R.H.L}}=\lim_{x\to2}\frac{\sqrt{2}\sin(x-2)}{(x-2)}=\sqrt{2}$$

Thus $\underset{(\text{at }x=2)}{\text{L.H.L}}\neq\underset{(\text{at }x=2)}{\text{R.H.L}}$

Hence, $\lim_{x\to2}\frac{\sqrt{1-\cos\{2(x-2)\}}}{x-2}$ does not exist.

13. (d) Given that $\lim_{x\to5}\frac{(f(x))^2-9}{\sqrt{|x-5|}}=0$

$$\Rightarrow\lim_{x\to5}[(f(x))^2-9]=0$$

$$\Rightarrow\left[\lim_{x\to5}f(x)\right]^2=9\Rightarrow\lim_{x\to5}f(x)=3$$

14. (d) Multiply and divide by x in the given expression, we get

$$\lim_{x\to0}\frac{(1-\cos 2x)}{x^2}\frac{(3+\cos x)}{1}\cdot\frac{x}{\tan 4x}$$

$$\left[\because 1-\cos 2x=2\sin^2\frac{x}{2}\right]$$

$$=\lim_{x\to0}\frac{2\sin^2 x}{x^2}\cdot\frac{3+\cos x}{1}\cdot\frac{x}{\tan 4x}$$

$$=2\lim_{x\to0}\frac{\sin^2 x}{x^2}\cdot\lim_{x\to0}(3+\cos x)\cdot\lim_{x\to0}\frac{4x}{\tan 4x}\times\frac{1}{4}$$

$$=2.4.\frac{1}{4}=2$$

15. (b) $\lim_{x\to0}\frac{\sin(\pi\cos^2 x)}{x^2}$

$$=\lim_{x\to0}\frac{\sin\left[\pi(1-\sin^2 x)\right]}{x^2}$$

$$= \lim_{x\to 0} \sin\frac{(\pi - \pi\sin^2 x)}{x^2}$$

$[\because \sin(\pi - \theta) = \sin\theta]$

$$= \lim_{x\to 0} \sin\frac{(\pi\sin^2 x)}{\pi\sin^2 x} \times \frac{\pi\sin^2 x}{x^2}$$

$$= \lim_{x\to 0} 1\times\pi\left(\frac{\sin x}{x}\right)^2 = \pi$$

16. (c) $\lim_{x\to\frac{\pi}{2}} \frac{\cot x(1-\sin x)}{-8\left(x-\frac{\pi}{2}\right)^3}$

$$= \lim_{x\to\frac{\pi}{2}} \frac{\cot x(1-\sin x)}{8\left(\frac{\pi}{2}-x\right)^3}$$

Put $\frac{\pi}{2} - x = t \Rightarrow$ as $x \to \frac{\pi}{2} \Rightarrow t \to 0$

$$= \lim_{t\to 0} \frac{\cot\left(\frac{\pi}{2}-t\right)\left(1-\sin\left(\frac{\pi}{2}-t\right)\right)}{8t^3}$$

$$= \lim_{t\to 0} \frac{\tan t(1-\cos t)}{8t^3}$$

$$= \lim_{t\to 0} \frac{\tan t}{8t}.\frac{1-\cos t}{t^2}$$

$$= \frac{1}{8}.1.\frac{1}{2} = \frac{1}{16}$$

17. (b) Since, $\lim_{x\to 0^+} x\left(\left[\frac{1}{x}\right]+\left[\frac{2}{x}\right]+\ldots+\left[\frac{15}{x}\right]\right)$

$$= \lim_{x\to 0^+} x\left(\frac{1+2+3+\ldots+15}{x}\right) - \left(\left\{\frac{1}{x}\right\}+\left\{\frac{2}{x}\right\}+\ldots+\left\{\frac{15}{x}\right\}\right)$$

$\because\ 0 \le \left\{\frac{r}{x}\right\} < 1 \quad \Rightarrow \quad 0 \le x\left\{\frac{r}{x}\right\} < x$

$$\therefore \lim_{x\to 0^+} x\left(\frac{1+2+3+\ldots+15}{x}\right) = \frac{15\times 16}{2} = 120$$

18. (a) $L = \lim_{y\to 0} \frac{\sqrt{1+\sqrt{1+y^4}} - \sqrt{2}}{y^4}$

$$= \lim_{y\to 0} \frac{\left(\sqrt{1+\sqrt{1+y^4}}-\sqrt{2}\right)\left(\sqrt{1+\sqrt{1+y^4}}+\sqrt{2}\right)}{y^4\left(\sqrt{1+\sqrt{1+y^4}}+\sqrt{2}\right)}$$

$$= \lim_{y\to 0} \frac{1+\sqrt{1+y^4}-2}{y^4\left(\sqrt{1+\sqrt{1+y^4}}+\sqrt{2}\right)}$$

$$= \lim_{y\to 0} \frac{\left(\sqrt{1+y^4}-1\right)\left(\sqrt{1+y^4}+1\right)}{y^4\left(\sqrt{1+\sqrt{1+y^4}}+\sqrt{2}\right)\left(\sqrt{1+y^4}+1\right)}$$

$$= \lim_{y\to 0} \frac{1+y^4-1}{y^4\left(\sqrt{1+\sqrt{1+y^4}}+\sqrt{2}\right)\left(\sqrt{1+y^4}+1\right)}$$

$$= \frac{1}{2\sqrt{2}\times 2} = \frac{1}{4\sqrt{2}}$$

19. (36) Let $3^x = t^2$

$$\lim_{t\to 3} \frac{t^2+\frac{27}{t^2}-12}{\frac{1}{t}-\frac{3}{t^2}}$$

$$= \lim_{t\to 3} \frac{t^4-12t^2+27}{t-3}$$

$$= \lim_{t\to 3} \frac{(t^2-3)(t+3)(t-3)}{t-3}$$

$$= (3^2-3)(3+3) = 36.$$

13 Mathematical Reasoning

DIRECTIONS: Given below question contains two statements: Statement-1(Assertion) and Statement-2 (Reason). This question also has four alternative choices, only one of which is the correct answer. You have to select the correct choice.

1. Let p be the statement "x is an irrational number", q be the statement "y is a transcendental number", and r be the statement "x is a rational number iff y is a transcendental number". **[2008]**

Statement-1 : r is equivalent to either q or p

Statement-2 : r is equivalent to $\sim(p \leftrightarrow \sim q)$.

(a) Statement -1 is false, Statement-2 is true

(b) Statement -1 is true, Statement-2 is true; Statement -2 is a correct explanation for Statement-1

(c) Statement -1 is true, Statement-2 is true; Statement -2 is not a correct explanation for Statement-1

(d) Statement -1 is true, Statement-2 is false

2. The statement $p \to (q \to p)$ is equivalent to **[2008]**

(a) $p \to (p \to q)$ (b) $p \to (p \vee q)$

(c) $p \to (p \wedge q)$ (d) $p \to (p \leftrightarrow q)$

DIRECTIONS: Given below question contains two statements: Statement-1(Assertion) and Statement -2 (Reason). This question also has four alternative choices, only one of which is the correct answer. You have to select the correct choice.

3. **Statement-1 :** $\sim(p \leftrightarrow \sim q)$ is equivalent to $p \leftrightarrow q$.

Statement-2 : $\sim(p \leftrightarrow \sim q)$ is a tantology **[2009]**

(a) Statement-1 is true, Statement-2 is true; Statement-2 is not a correct explanation for Statement-1.

(b) Statement-1 is true, Statement-2 is false.

(c) Statement-1 is false, Statement-2 is true.

(d) Statement-1 is true, Statement-2 is true, Statement-2 is a correct explanation for statement -1

4. Let S be a non-empty subset of R. Consider the following statement :

P : There is a rational number $x \in$ S such that $x > 0$.

Which of the following statements is the negation of the statement P ? **[2010]**

(a) There is no rational number $x \in$ S such that $x \leq 0$.

(b) Every rational number $x \in$ S satisfies $x \leq 0$.

(c) $x \in$ S and $x \leq 0 \Rightarrow x$ is not rational.

(d) There is a rational number $x \in$ S such that $x \leq 0$.

5. Consider the following statements **[2011]**

***P* :** Suman is brilliant

***Q* :** Suman is rich

***R* :** Suman is honest

The negation of the statement "Suman is brilliant and dishonest if and only if Suman is rich" can be expressed as

(a) $\sim(Q \leftrightarrow (P \wedge \sim R))$

(b) $\sim Q \leftrightarrow \sim P \wedge R$

(c) $\sim(P \wedge \sim R) \leftrightarrow Q$

(d) $\sim P \wedge (Q \leftrightarrow \sim R)$

6. The only statement among the following that is a tautology is **[2011RS]**

(a) $A \wedge (A \vee B)$

(b) $A \vee (A \wedge B)$

(c) $[A \wedge (A \to B)] \to B$

(d) $B \to [A \wedge (A \to B)]$

7. The negation of the statement "If I become a teacher, then I will open a school", is : **[2012]**

(a) I will become a teacher and I will not open a school.

(b) Either I will not become a teacher or I will not open a school.

(c) Neither I will become a teacher nor I will open a school.

(d) I will not become a teacher or I will open a school.

8. Consider

Statement-1 : $(p \wedge \sim q) \wedge (\sim p \wedge q)$ is a fallacy.

Statement-2 : $(p \to q) \leftrightarrow (\sim q \to \sim p)$ is a tautology. **[2013]**

(a) Statement-1 is true; Statement-2 is true; Statement-2 is a correct explanation for Statement-1.

(b) Statement-1 is true; Statement-2 is true; Statement-2 is not a correct explanation for Statement-1.

(c) Statement-1 is true; Statement-2 is false.

(d) Statement-1 is false; Statement-2 is true.

9. The statement $\sim (p \leftrightarrow \sim q)$ is: **[2014]**

(a) a tautology

(b) a fallacy

(c) equavalent to $p \leftrightarrow q$

(d) equivalent to $\sim p \leftrightarrow q$

10. The negation of $\sim s \vee (\sim r \wedge s)$ is equivalent to : **[2015]**

(a) $s \vee (r \vee \sim s)$
(b) $s \wedge r$
(c) $s \wedge \sim r$
(d) $s \wedge (r \wedge \sim s)$

11. The Boolean Expression $(p \wedge \sim q) \vee q \vee (\sim p \wedge q)$ is equivalent to: **[2016]**

(a) $p \vee q$
(b) $p \vee \sim q$
(c) $\sim p \wedge q$
(d) $p \wedge q$

12. The following statement **[2017]**
$(p \to q) \to [(\sim p \to q) \to q]$ is :

(a) a fallacy

(b) a tautology

(c) equivalent to $\sim p \to q$

(d) equivalent to $p \to \sim q$

13. The Boolean expression $\sim (p \vee q) \vee (\sim p \wedge q)$ is equivalent to : **[2018]**

(a) p
(b) q
(c) ~q
(d) ~p

14. For any two statements p and q, the negation of the expression $p \vee (\sim p \wedge q)$ is: **[2019]**

(a) $\sim p \wedge \sim q$
(b) $p \wedge q$
(c) $p \leftrightarrow q$
(d) $\sim p \vee \sim q$

15. If the Boolean expression **[2019]**
$(p \oplus q) \wedge (\sim p \odot q)$ is equivalent to $p \wedge q$, where $\oplus, \odot \in \{\wedge, \vee\}$ then the ordered pair $(\oplus, \odot)$ is:

(a) $(\vee, \wedge)$
(b) $(\vee, \vee)$
(c) $(\wedge, \vee)$
(d) $(\wedge, \wedge)$

16. The logical statement $(p \Rightarrow q) \wedge (q \Rightarrow \sim p)$ is equivalent to: **[2020]**

(a) p
(b) q
(c) $\sim P$
(d) $\sim q$

Answer Key														
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
(None)	(b)	(b)	(b)	(a)	(c)	(a)	(b)	(c)	(b)	(a)	(b)	(d)	(a)	(c)
16														
(c)														

Solutions

1. **(None)**
Given that
$p:\ x$ is an irrational number
$q: y$ is a transcendental number
$r: x$ is a rational number iff y is a transcendental number.
clearly $r: \sim p \leftrightarrow q$
Truth table to check the equivalence of 'r' and 'q or p'; 'r' and $\sim(p \leftrightarrow \sim q)$

p	q	~p	~q	(i) ~p ↔q	(ii) q or p	p↔~q	(iii) ~(p↔~q)
T	T	F	F	F	T	F	T
T	F	F	T	T	T	T	F
F	T	T	F	T	T	T	F
F	F	T	T	F	F	F	T

From columns (i), (ii) and (iii), we observe, that none of the these statements are equivalent to each other.
∴ Statement 1as well as statement 2 both are false.
∴ None of the options is correct.

2. **(b)** The truth table for the given statements, as follows :

p	q	p∨q	q→p	p →(q→p)	p →(p∨q)
T	T	T	T	T	T
T	F	T	T	T	T
F	T	T	F	T	T
F	F	F	T	T	T

From table we observe that
$p \rightarrow (q \rightarrow p)$ is equivalent to $p \rightarrow (p \vee q)$

3. **(b)** The truth table for the logical statements, involved in statement 1, is as follows :

p	q	$\sim q$	$p \leftrightarrow \sim q$	(i) $\sim(p \leftrightarrow \sim q)$	(ii) $p \leftrightarrow q$
T	T	F	F	T	T
T	F	T	T	F	F
F	T	F	T	F	F
F	F	T	F	T	T

We observe the columns (i) and (ii) are identical, therefore
$\sim(p \leftrightarrow \sim q)$ is equivalent to $p \leftrightarrow q$
But $\sim(p \leftrightarrow \sim q)$ is not a tautology as all entries in its column are not T.
∴ Statement-1 is true but statement-2 is false.

4. **(b)** Given that P : there is a rational number $x \in S$ such that $x > 0$.
~P : Every rational number $x \in S$ satisfies $x \le 0$.

5. **(a)** Given that Suman is brilliant and dishonest i.e. $(P \wedge \sim R)$ if and only if Suman is rich is expressed as
$Q \leftrightarrow (P \wedge \sim R)$
Negation of it will be $\sim(Q \leftrightarrow (P \wedge \sim R))$

6. **(c)** Truth table of all options is as follows.

A	B	A ∨ B	A ∧ B	A ∧ (A ∨ B)	A ∨ (A ∧ B)	A → B	A ∧ (A → B)	[A ∧ (A → B) →B]	[B → [A ∧ (A →B)]
T	F	T	F	T	T	F	F	T	T
F	T	T	F	F	F	T	F	T	F
T	T	T	T	T	T	T	T	T	T
F	F	F	F	F	F	T	F	T	T

∴ It is tautology.

7. **(a)** Let p : I become a teacher.
q : I will open a school
Negation of $p \rightarrow q$ is $\sim(p \rightarrow q) = p \wedge \sim q$
i.e. I will become a teacher and I will not open a school.

8. **(b)** **Statement-2 :** $(p \rightarrow q) \leftrightarrow (\sim q \rightarrow \sim p)$
$\equiv (p \rightarrow q) \leftrightarrow (p \rightarrow q)$
which is always true.
So, statement 2 is true

Statement-1: $(p \wedge \sim q) \wedge (\sim p \wedge q)$
$= p \wedge \sim q \wedge \sim p \wedge q$
$= p \wedge \sim p \wedge \sim q \wedge q$
$= f \wedge f = f$
So statement-1 is true

9. **(c)**

p	q	$\sim q$	$p \leftrightarrow \sim q$	$\sim(p \leftrightarrow \sim q)$ (i)	$p \leftrightarrow q$ (ii)
F	F	T	F	T	T
F	T	F	T	F	F
T	F	T	T	F	F
T	T	F	F	T	T

From column (i) and (ii) are equivalent.
Clearly equivalent to $p \leftrightarrow q$

10. **(b)** $\sim[\sim s \vee (\sim r \wedge s)]$
$= s \wedge \sim(\sim r \wedge s)$
$= s \wedge (r \vee \sim s)$
$= (s \wedge r) \vee (s \wedge \sim s)$
$= (s \wedge r) \vee f$
$= s \wedge r$

11. **(a)** $(p \wedge \sim q) \vee q \vee (\sim p \wedge q)$
$\Rightarrow \{(p \vee q) \wedge (\sim q \vee q)\} \vee (\sim p \wedge q)$
$\Rightarrow \{(p \vee q) \wedge T\} \vee (\sim p \wedge q)$
$\Rightarrow (p \vee q) \vee (\sim p \wedge q)$
$\Rightarrow \{(p \vee q) \vee \sim p\} \wedge (p \vee q \vee q)$
$\Rightarrow T \wedge (p \vee q)$
$\Rightarrow p \vee q$

12. **(b)** We have

p	q	$\sim p$	$p \to q$	$\sim p \to q$	$(\sim p \to q) \to q$	$(p \to q) \to ((\sim p \to q) \to q)$
T	F	F	F	T	F	T
T	T	F	T	T	T	T
F	F	T	T	F	T	T
F	T	T	T	T	T	T

$\therefore$ It is tautology.

13. **(d)** $\sim(p \vee q) \vee (\sim p \wedge q)$
$(\sim p \wedge \sim q) \vee (\sim p \wedge q)$
$\Rightarrow \sim p \wedge (\sim q \vee q)$
$\Rightarrow \sim p \wedge t \equiv \sim p$

14. **(a)** $\sim(p \vee (\sim p \wedge q))$
$= \sim((p \vee \sim p) \wedge (p \vee q)) = \sim(t \wedge (p \vee q))$
$= \sim(p \vee q) = \sim p \wedge \sim q$

15. **(c)** Check each option
(a) $(p \vee q) \wedge (\sim p \wedge q) = (\sim p \wedge q)$
(b) $(p \vee q) \wedge (\sim p \vee q) = (p \wedge \sim p) \vee q = F \vee q = q$
(c) $(p \wedge q) \wedge (\sim p \vee q) = (p \wedge q \wedge \sim p) \vee (p \wedge q) \wedge q$
$= F \vee (p \wedge q) = p \wedge q$
(d) $(p \wedge q) \wedge (\sim p \wedge q) = (p \wedge \sim p) \wedge q = F \wedge q = F$

16. **(c)**

p	q	$p \Rightarrow q$	$\sim p$	$q \Rightarrow \sim p$	$(p \Rightarrow q) \quad (p \Rightarrow \sim q)$
T	T	T	F	F	F
T	F	F	F	T	F
F	T	T	T	T	T
F	F	T	T	T	T

Clearly $(p \Rightarrow q) \wedge (q \Rightarrow \sim p)$ is equivalent to $\sim p$

14 Statistics

1. In a class of 100 students there are 70 boys whose average marks in a subject are 75. If the average marks of the complete class is 72, then what is the average of the girls? **[2002]**
(a) 73 (b) 65
(c) 68 (d) 74

2. The median of a set of 9 distinct observations is 20.5. If each of the largest 4 observations of the set is increased by 2, then the median of the new set **[2003]**
(a) remains the same as that of the original set
(b) is increased by 2
(c) is decreased by 2
(d) is two times the original median.

3. In an experiment with 15 observations on x, the following results were available: **[2003]**
$\Sigma x^2 = 2830,\ \Sigma x = 170$
One observation that was 20 was found to be wrong and was replaced by the correct value 30. The corrected variance is **[2003]**
(a) 8.33 (b) 78.00
(c) 188.66 (d) 177.33

4. Consider the following statements :
(A) Mode can be computed from histogram.
(B) Median is not independent of change of scale.
(C) Variance is independent of change of origin and scale.
Which of these is / are correct ? **[2004]**
(a) (A), (B) and (C) (b) Only (B)
(c) Only (A) and (B) (d) Only (A)

5. In a series of 2 n observations, half of them equal a and remaining half equal $-a$. If the standard deviation of the observations is 2, then $|a|$ equals. **[2004]**
(a) $\frac{\sqrt{2}}{n}$ (b) $\sqrt{2}$ (c) 2 (d) $\frac{1}{n}$

6. If in a frequency distribution, the mean and median are 21 and 22 respectively, then its mode is approximately **[2005]**
(a) 22.0 (b) 20.5
(c) 25.5 (d) 24.0

7. Let x_1, x_2, ... x_n be n observations such that $\sum x_i^2 = 400$ and $\sum x_i = 80$. Then the possible value of n among the following is **[2005]**
(a) 15 (b) 18
(c) 9 (d) 12

8. Suppose a population A has 100 observations 101, 102, ..., 200 and another population B has 100 obsevrations 151, 152, ... 250. If V_A and V_B represent the variances of the two populations, respectively then $\frac{V_A}{V_B}$ is **[2006]**
(a) 1 (b) $\frac{9}{4}$
(c) $\frac{4}{9}$ (d) $\frac{2}{3}$

9. The average marks of boys in class is 52 and that of girls is 42. The average marks of boys and girls combined is 50. The percentage of boys in the class is **[2007]**
(a) 80 (b) 60
(c) 40 (d) 20.

10. The mean of the numbers a, b, 8, 5, 10 is 6 and the variance is 6.80. Then which one of the following gives possible values of a and b? **[2008]**
(a) $a = 0, b = 7$ (b) $a = 5, b = 2$
(c) $a = 1, b = 6$ (d) $a = 3, b = 4$

DIRECTIONS: This question contains two statements: statement-1 (Assertion) and statement-2 (Reason). This question also has four alternative choices, only one of which is the correct answer. You have to select the correct choice.

11. **Statement-1 :** The variance of first n even natural numbers is $\frac{n^2-1}{4}$.

Statement-2 : The sum of first n natural numbers is $\frac{n(n+1)}{2}$ and the sum of squares of first n natural numbers is $\frac{n(n+1)(2n+1)}{6}$. **[2009]**

(a) Statement-1 is true, Statement-2 is true Statement-2 is not a correct explanation for Statement-1.
(b) Statement-1 is true, Statement-2 is false.
(c) Statement-1 is false, Statement-2 is true.
(d) Statement-1 is true, Statement-2 is true. Statement-2 is a correct explanation for Statement-1.

12. If the mean deviation of the numbers 1, 1 + d, 1 + 2d, ... 1 + 100d from their mean is 255, then d is equal to : **[2009]**

(a) 20.0 (b) 10.1
(c) 20.2 (d) 10.0

13. For two data sets, each of size 5, the variances are given to be 4 and 5 and the corresponding means are given to be 2 and 4, respectively. The variance of the combined data set is **[2010]**

(a) $\frac{11}{2}$ (b) 6
(c) $\frac{13}{2}$ (d) $\frac{5}{2}$

14. If the mean deviation about the median of the numbers $a, 2a, ..., 50a$ is 50, then $|a|$ equals **[2011]**

(a) 3 (b) 4
(c) 5 (d) 2

15. A scientist is weighing each of 30 fishes. Their mean weight worked out is 30 gm and a standarion deviation of 2 gm. Later, it was found that the measuring scale was misaligned and always under reported every fish weight by 2 gm. The correct mean and standard deviation (in gm) of fishes are respectively : **[2011RS]**

(a) 32, 2 (b) 32, 4
(c) 28,2 (d) 28, 4

16. Let $x_1, x_2, ..., x_n$ be n observations, and let $\bar{x}$ be their arithmetic mean and σ^2 be the variance.

Statement-1 : Variance of $2x_1, 2x_2, ..., 2x_n$ is $4\sigma^2$.

Statement-2 : Arithmetic mean $2x_1, 2x_2, ..., 2x_n$ is $4\bar{x}$. **[2012]**

(a) Statement-1 is false, Statement-2 is true.
(b) Statement-1 is true, statement-2 is true; statement-2 is a correct explanation for Statement-1.
(c) Statement-1 is true, statement-2 is true; statement-2 is ***not*** a correct explanation for Statement-1.
(d) Statement-1 is true, statement-2 is false.

17. All the students of a class performed poorly in Mathematics. The teacher decided to give grace marks of 10 to each of the students. Which of the following statistical measures will not change even after the grace marks were given ? **[2013]**

(a) mean (b) median
(c) mode (d) variance

18. The variance of first 50 even natural numbers is **[2014]**

(a) 437 (b) $\frac{437}{4}$
(c) $\frac{833}{4}$ (d) 833

19. The mean of the data set comprising of 16 observations is 16. If one of the observation valued 16 is deleted and three new observations valued 3, 4 and 5 are added to the data, then the mean of the resultant data, is: **[2015]**

(a) 15.8 (b) 14.0
(c) 16.8 (d) 16.0

20. If the standard deviation of the numbers 2, 3, a and 11 is 3.5, then which of the following is true? **[2016]**

(a) $3a^2 - 34a + 91 = 0$
(b) $3a^2 - 23a + 44 = 0$
(c) $3a^2 - 26a + 55 = 0$
(d) $3a^2 - 32a + 84 = 0$

21. If $\sum_{i=1}^{9}(x_i - 5) = 9$ and $\sum_{i=1}^{9}(x_i - 5)^2 = 45$, then the standard deviation of the 9 items $x_1, x_2, ..., x_9$ is : **[2018]**

(a) 4 (b) 2
(c) 3 (d) 9

22. If the standard deviation of the numbers –1, 0, 1, k is $\sqrt{5}$ where k > 0, then k is equal to: **[2019]**

(a) $2\sqrt{6}$ (b) $2\sqrt{\frac{10}{3}}$

(c) $4\sqrt{\frac{5}{3}}$ (d) $\sqrt{6}$

23. 5 students of a class have an average height 150 cm and variance 18 cm^2. A new student, whose height is 156 cm, joined them. The variance (in cm^2) of the height of these six students is: **[2019]**

(a) 16 (b) 22

(c) 20 (d) 18

24. If the variance of the first n natural numbers is 10 and the variance of the first m even natural numbers is 16, then $m+n$ is equal to ______. **[2020]**

Answer Key

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
(b)	(a)	(b)	(c)	(c)	(d)	(b)	(a)	(a)	(d)	(c)	(b)	(a)	(b)	(a)
16	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**						
(d)	(d)	(d)	(b)	(d)	(b)	(a)	(c)	(18)						

Solutions

1. **(b)** Total student = 100

Total marks of 70 boys = 75 × 70 = 5250

$\Rightarrow$ Total marks of girls = 7200 – 5250 = 1950

Number of girls = 100 – 70 = 30

Average of girls $= \frac{1950}{30} = 65$

2. **(a)** $n = 9$ then median term $= \left(\frac{9+1}{2}\right)^{th} = 5^{th}$ term. That means four observation followed by it. If last four observations are increased by 2. The median is 5th observation which is remaining unchanged.

$\therefore$ There will be no change in median.

3. **(b)** $\Sigma x = 170, \Sigma x^2 = 2830$

New, $\Sigma x' = 170 + (30-20) = 180$

New, $\Sigma x'^2 = 2830 + (900 - 400)$

$= 2830 + 500 = 3330$

Now, Variance $= \frac{1}{n}\Sigma x'^2 - \left(\frac{1}{n}\Sigma x'\right)^2$

$= \frac{1}{15} \times 3330 - \left(\frac{1}{15} \times 180\right)^2 = 222 - 144 = 78.$

4. **(c)** Only first statement (A) and second statements (B) are correct.

5. **(c)** Clearly sum of observations = 0,

$\therefore$ mean $A = 0$

Standard deviation $\sigma = \sqrt{\frac{\sum (x-A)^2}{2n}}$

$2 = \sqrt{\frac{(a-0)^2 + (a-0)^2 + ...(0-a)^2 + ...}{2n}}$ $[\because \sigma = 2]$

$= \sqrt{\frac{a^2.2n}{2n}} = |a|$

Hence, $|a| = 2$

6. **(d)** We know that Mode = 3 Median – 2Mean

$3 \times 22 - 2 \times 21 = 66 - 42 = 24$

7. **(b)** We know that for positive real numbers $x_1, x_2, ..., x_n$,

A.M. of k^{th} powers of $x_i \geq k^{th}$ the power of A.M. of x_i

$\Rightarrow \frac{\sum x_1^2}{n} \geq \left(\frac{\sum x_1}{n}\right)^2 \Rightarrow \frac{400}{n} \geq \left(\frac{80}{n}\right)^2$

$\Rightarrow n \geq 16$. So only possible value for $n = 18$

8. **(a)** $\sigma_x^2 = \frac{\sum d_i^2}{n}$ (Here d_i = deviations are taken from the mean). Since population A and population B both have 100 consecutive integers, therefore both have same standard deviation and hence the variance is also same. $\therefore \frac{V_A}{V_B} = 1$

9. **(a)** Let the number of boys be x and girls be y.

$\Rightarrow 52x + 42y = 50(x + y)$

$\Rightarrow 52x - 50x = 50y - 42y$

$\Rightarrow \; 2x = 8y \Rightarrow \; \frac{x}{y} = \frac{4}{1} \Rightarrow \frac{x}{x+y} = \frac{4}{5}$

$\therefore$ Required % of boys $= \frac{x}{x+y} \times 100$

$= \frac{4}{5} \times 100 = 80\%$

10. **(d)** Mean of $a, b, 8, 5, 10$ is 6

$\Rightarrow \; \frac{a+b+8+5+10}{5} = 6$

$\Rightarrow \; a + b = 6$ ...(i)

Variance of $a, b, 8, 5, 10$ is 6.80

$\Rightarrow \frac{(a-6)^2 + (b-6)^2 + (8-6)^2 + (5-6)^2 + (10-6)^2}{5}$

$= 6.80$

$\Rightarrow \; a^2 - 12a + 36 + (1-a)^2 + 21 = 34$ [using eq. (i)]

$\Rightarrow \; 2a^2 - 14a + 24 = 0 \Rightarrow a^2 - 7a + 12 = 0$

$\Rightarrow \; a = 3$ or $4 \quad \Rightarrow b = 4$ or 3

$\therefore$ The possible values of a and b are $a = 3$ and $b = 4$

or, $a = 4$ and $b = 3$

11. **(c)** First n even natural numbers be $2, 4, 6, 8, ..., 2n$

$\therefore \quad \bar{x} = \frac{2(1+2+3+...+n)}{n}$

$= \frac{2[n(n+1)]}{2n} = (n+1)$

And $Var = \frac{\Sigma(x-\bar{x})^2}{2n} \; = \frac{\Sigma x^2}{n} - (\bar{x})^2$

$= \frac{4\Sigma n^2}{n} - (n+1)^2$

$= \frac{4n(n+1)(2n+1)}{6n} - (n+1)^2$

$= \frac{2(2n+1)(n+1)}{3} - (n+1)^2$

$= (n+1)\left[\frac{4n+2-3n-3}{3}\right]$

$= \frac{(n+1)(n-1)}{3} = \frac{n^2-1}{3}$

$\therefore$ Statement-1 is false. Clearly, statement - 2 is true.

12. **(b)** Mean $= \frac{101 + \text{d}(1+2+3+......+100)}{101}$

$= 1 + \frac{\text{d} \times 100 \times 101}{101 \times 2} = 1 + 50\text{ d}$

Given that mean deviation from the mean = 255

$\Rightarrow \; \frac{1}{101}[|1-(1+50d)| + |(1+d)-(1+50d)|$
$+ |(1+2d) - (1+50d)| + ... +$
$|(1+100d) - (1+50d)|] = 255$

$\Rightarrow \; 2d[1+2+3+...+50] = 101 \times 255$

$\Rightarrow \; 2d \times \frac{50 \times 51}{2} = 101 \times 255$

$\Rightarrow \; d = \frac{101 \times 255}{50 \times 51} = 10.1$

13. **(a)** $\sigma_x^2 = 4, \sigma_y^2 = 5, \bar{x} = 2, \bar{y} = 4$

$\sigma_x^2 = \frac{1}{5}\sum x_i^2 - (2)^2 = 4 \Rightarrow \sum x_i^2 = 40;$

$\sigma_y^2 = \frac{1}{5}\sum y_i^2 - (4)^2 = 5 \Rightarrow \sum y_i^2 = 105$

$\Rightarrow \; \sum x_i^2 + \sum y_i^2 = \sum\left(x_i^2 + y_i^2\right) = 145$

$\Rightarrow \; \sum x_i + \sum y_i = \sum(x_i + y_i)$

$= 5(2) + 5(4) = 30$

Variance of combined data

$= \frac{1}{10}\sum\left(x_i^2 + y_i^2\right) - \left(\frac{1}{10}\sum(x_i + y_i)\right)^2$

$= \frac{145}{10} - 9 = \frac{11}{2}$

14. **(b)** $\because \; n = 50$ (even)

Median $= \frac{25^{\text{th}}\text{ obs.} + 26^{\text{th}}\text{ obs.}}{2}$

$\therefore \; M = \frac{25a + 26a}{2} = 25.5a$

$M.D\,(M) = \frac{\sum|x_i - M|}{N}$

$\Rightarrow 50 = \frac{1}{50}[2 \times |a| \times (0.5 + 1.5 + 2.5 + ...24.5)]$

$\Rightarrow \; 2500 = 2|a| \times \frac{25}{2}(25)$

$\Rightarrow \; |a| = 4$

15. **(a)** We know that if each observation is increase by 2 then mean is increase by 2 but S.D. remains same.

Correct mean = observed mean + 2

$= 30 + 2 = 32$

Correct S. D. = observed S.D. = 2

16. **(d)** A.M. of $2x_1, 2x_2, ..., 2x_n$ is

$\frac{2x_1 + 2x_2 + ... + 2x_n}{n} = 2\left(\frac{x_1 + x_2 + ... + x_n}{n}\right) = 2\bar{x}$

$$\left(\because \quad \text{Mean} = \frac{\text{Sum of observations}}{\text{Number of observations}}\right)$$

So statement-2 is false.
If each observations is multiply by 2 then mean multiply by 2 and variance multiply by 2^2.
variance $(2x_i) = 2^2$ variance $(x_i) = 4\sigma^2$
where $i = 1, 2, ...n$
So statement-1 is true.

17. (d) If initially all marks were x_i then

$$\sigma_1^2 = \frac{\sum_i (x_i - \bar{x})^2}{N}$$

Now each is increased by 10

$$\sigma_1{}^2 = \frac{\sum_i [(x_i+10)-(\bar{x}+10)]^2}{N} = \frac{\sum_i (x_i - \bar{x})^2}{N} = \sigma_1{}^2$$

Hence, variance will not change even after the grace marks were given.

18. (d) First 50 even natural numbers are 2, 4, 6 ....., 100

$$\text{Variance} = \frac{\sum x_i^2}{N} - (\bar{x})^2$$

$$\Rightarrow \quad \sigma^2 = \frac{2^2 + 4^2 + ... + 100^2}{50} - \left(\frac{2+4+...+100}{50}\right)^2$$

$$= \frac{4(1^2 + 2^2 + 3^2 + ... + 50^2)}{50} - (51)^2$$

$$= 4\left(\frac{50 \times 51 \times 101}{50 \times 6}\right) - (51)^2 = 3434 - 2601$$

$\Rightarrow \sigma^2 = 833$

19. (b) Sum of 16 observations $= 16 \times 16 = 256$
Sum of resultant 18 observations
$= 256 - 16 + (3 + 4 + 5) = 252$

Mean of observations $= \dfrac{252}{18} = 14$

20. (d) $\bar{x} = \dfrac{2+3+a+11}{4} = \dfrac{a}{4} + 4$

$$\sigma = \sqrt{\sum \frac{x_i^2}{n} - \left(\bar{x}\right)^2}$$

$$\Rightarrow 3.5 = \sqrt{\frac{4+9+a^2+121}{4} - \left(\frac{a}{4}+4\right)^2}$$

$$\Rightarrow \frac{49}{4} = \frac{4(134+a^2) - (a^2+256+32a)}{16}$$

$\Rightarrow 3a^2 - 32a + 84 = 0$

21. (b) Given $\sum_{i=1}^{9} (x_i - 5) = 9 \Rightarrow \sum_{i=1}^{9} x_i = 54$ ...(i)

Also, $\sum_{i=1}^{9} (x_i - 5)^2 = 45$

$\Rightarrow \quad \sum_{i=1}^{9} x_i{}^2 - 10\sum_{i=1}^{9} x_i + 9(25) = 45$ ...(ii)

From (i) and (ii) we get,

$$\sum_{i=1}^{9} x_i^2 = 360$$

$$\text{Since, variance} = \frac{\sum x_i{}^2}{9} - \left(\frac{\sum x_i}{9}\right)^2$$

$$= \frac{360}{9} - \left(\frac{54}{9}\right)^2 = 40 - 36 = 4$$

$\therefore$ Standard deviation $= \sqrt{\text{Variance}} = 2$

22. (a) $\sigma^2 = \dfrac{\Sigma x^2}{n} - \left(\dfrac{\Sigma x}{n}\right)^2$

$$\Rightarrow \frac{k^2+2}{4} - \left(\frac{k}{4}\right)^2 = 5$$

$\Rightarrow 4k^2 + 8 - k^2 = 80$

$\Rightarrow 3k^2 = 72$

$\Rightarrow k = 2\sqrt{6}$

23. (c) $\because$ Variance $= \sigma^2 = \dfrac{\Sigma x_i^2}{N} - (\bar{x})^2$

$$\Rightarrow 18 = \frac{\Sigma x_i^2}{5} - (150)^2$$

$\Rightarrow \Sigma x_i^2 = 90 + 112590 = 112590$

Then, variance of the height of six students

$$V' = \frac{112590 + (156)^2}{6} - \left(\frac{750+156}{6}\right)^2$$

$= 22821 - 22801 = 20$

24. (18) Var $(1, 2, ..., n) = 10$

$$\Rightarrow \frac{1^2 + 2^2 + ... + n^2}{n} - \left(\frac{1+2+.....+n}{n}\right)^2 = 10$$

$$\Rightarrow \quad \frac{(n+1)(2n+1)}{6} - \left(\frac{n+1}{2}\right)^2 = 10$$

$\Rightarrow \quad n^2 - 1 = 120 \quad \Rightarrow \quad n = 11$

Var $(2, 4, 6, ..., 2m) = 16$

$\Rightarrow$ Var $(1, 2, ..., m) = 4$

$\Rightarrow \quad m^2 - 1 = 48 \Rightarrow m = 7$

$\Rightarrow \quad m + n = 18$

15 Probability

1. A and B are events such that $P(A \cup B)=3/4$, $P(A \cap B) = 1/4$, $P(\overline{A})=2/3$, then $P(\overline{A} \cap B)$ is **[2002]**

(a) 5/12 (b) 3/8
(c) 5/8 (d) 1/4

2. Events A, B, C are mutually exclusive events such that $P(A) = \frac{3x+1}{3}$, $P(B) = \frac{1-x}{4}$ and $P(C) = \frac{1-2x}{2}$. The set of possible values of x are in the interval. **[2003]**

(a) $[0, 1]$ (b) $\left[\frac{1}{3}, \frac{1}{2}\right]$
(c) $\left[\frac{1}{3}, \frac{2}{3}\right]$ (d) $\left[\frac{1}{3}, \frac{13}{3}\right]$

3. Five horses are in a race. Mr. A selects two of the horses at random and bets on them. The probability that Mr. A selected the winning horse is **[2003]**

(a) $\frac{2}{5}$ (b) $\frac{4}{5}$
(c) $\frac{3}{5}$ (d) $\frac{1}{5}$

4. Let A and B be two events such that $P(\overline{A \cup B}) = \frac{1}{6}$, $P(A \cap B) = \frac{1}{4}$ and $P(\overline{A}) = \frac{1}{4}$, where $\overline{A}$ stands for complement of event A. Then events A and B are **[2005]**

(a) equally likely and mutually exclusive
(b) equally likely but not independent
(c) independent but not equally likely
(d) mutually exclusive and independent

5. A die is thrown. Let A be the event that the number obtained is greater than 3. Let B be the event that the number obtained is less than 5. Then $P(A \cup B)$ is **[2008]**

(a) $\frac{3}{5}$ (b) 0
(c) 1 (d) $\frac{2}{5}$

6. Four numbers are chosen at random (without replacement) from the set $\{1, 2, 3, ...20\}$. **[2010]**

Statement -1: The probability that the chosen numbers when arranged in some order will form an AP is $\frac{1}{85}$.

Statement -2 : If the four chosen numbers form an AP, then the set of all possible values of common difference is $(\pm1, \pm2, \pm3, \pm4, \pm5)$.

(a) Statement -1 is true, Statement -2 is true ; Statement -2 is **not** a correct explanation for Statement -1

(b) Statement -1 is true, Statment -2 is false

(c) Statement -1 is false, Statment -2 is true.

(d) Statement -1 is true, Statement -2 is true ; Statement -2 is a correct explanation for Statement -1.

7. For three events A, B and C, P(Exactly one of A or B occurs) = P(Exactly one of B or C occurs) = P(Exactly one of C or A occurs) = $\frac{1}{4}$ and P(All the three events occur simultaneously) = $\frac{1}{16}$.
Then the probability that at least one of the events occurs, is : **[2017]**

(a) $\frac{3}{16}$ (b) $\frac{7}{32}$

(c) $\frac{7}{16}$ (d) $\frac{7}{64}$

Answer Key

1	2	3	4	5	6	7								
(a)	(b)	(a)	(c)	(c)	(b)	(c)								

Solutions

1. **(a)** We know that $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$$\Rightarrow \frac{3}{4} = 1 - P(\overline{A}) + P(B) - \frac{1}{4}$$

$$\left[\because P(A) = 1 - P(\overline{A})\right]$$

$$\Rightarrow 1 = 1 - \frac{2}{3} + P(B) \Rightarrow P(B) = \frac{2}{3};$$

Now, $P(\overline{A} \cap B) = P(B) - P(A \cap B)$

$$= \frac{2}{3} - \frac{1}{4} = \frac{5}{12}.$$

2. **(b)** Given that $P(A) = \frac{3x+1}{3}$, $P(B) = \frac{1-x}{4}$ and $P(C) = \frac{1-2x}{2}$

We know that $0 \le P(E) \le 1$

$$\Rightarrow 0 \le \frac{3x+1}{3} \le 1, \ge -1 \le 3x \le 2$$

$$\Rightarrow -\frac{1}{3} \le x \le \frac{2}{3} \quad \text{...(i)}$$

$$0 \le \frac{1-x}{4} \le 1 \Rightarrow -3 \le x \le 1 \quad \text{...(ii)}$$

and $0 \le \frac{1-2x}{2} \le 1 \Rightarrow -1 \le 2x \le 1$

$$\Rightarrow -\frac{1}{2} \le x \le \frac{1}{2} \quad \text{...(iii)}$$

Also for mutually exclusive events A, B, C,

$$P(A \cup B \cup C) = P(A) + P(B) + P(C)$$

$$\Rightarrow P(A \cup B \cup C) = \frac{3x+1}{3} + \frac{1-x}{4} + \frac{1-2x}{2}$$

$$\therefore 0 \le \frac{1+3x}{3} + \frac{1-x}{4} + \frac{1-2x}{2} \le 1$$

$$0 \le 13 - 3x \le 12 \Rightarrow 1 \le 3x \le 13$$

$$\Rightarrow \frac{1}{3} \le x \le \frac{13}{3} \quad \text{...(iv)}$$

From (i), (ii), (iii) and (iv), we get

$$\frac{1}{3} \le x \le \frac{1}{2} \Rightarrow x \in \left[\frac{1}{3}, \frac{1}{2}\right]$$

3. **(a)** Let 5 horses are H_1, H_2, H_3, H_4 and H_5.
Total ways of selecting pair of horses be

$$= {}^5C_2 = 10 [\text{i.e. } H_1H_2, H_1H_3, H_1H_4, H_1H_5, H_2H_3, H_2H_4, H_2H_5, H_3H_4, H_3H_5, H_4H_5]$$

Any horse can win the race in 4 ways
(e.g. for H_1 : H_1H_2, H_1H_3, H_1H_4, H_1H_5)

Hence required probability = $\frac{4}{10} = \frac{2}{5}$

4. **(c)** Given that $P(\overline{A \cup B}) = \frac{1}{6}, P(A \cap B) = \frac{1}{4}$

and $P(\overline{A}) = \frac{1}{4}$

$\Rightarrow P(A \cup B) = 1 - P(\overline{A \cup B}) = 1 - \frac{1}{6} = \frac{5}{6},$

$P(A) = 1 - P(\overline{A}) = 1 - \frac{1}{4} = \frac{3}{4}$

We know that

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\Rightarrow P(B) = \frac{5}{6} - \frac{3}{4} + \frac{1}{4} = \frac{1}{3}$

$\Rightarrow P(A)\ P(B) = \frac{3}{4} \times \frac{1}{3} = \frac{1}{4} = P(A \cap B)$

Hence A and B are independent but not equally likely.

5. **(c)** A (number is greater than 3) = {4, 5, 6}

$\Rightarrow P(A) = \frac{3}{6} = \frac{1}{2}$

B (number is less than 5) = {1, 2, 3, 4}

$\Rightarrow P(B) = \frac{4}{6} = \frac{2}{3}$

$\therefore A \cap B = \{4\}$

$\Rightarrow P(A \cap B) = \frac{1}{6}$

$\therefore P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= \frac{1}{2} + \frac{2}{3} - \frac{1}{6} = \frac{3+4-1}{6} = 1$

6. **(b)** Four numbers are chosen from {1, 2, 3...20}

$n(S) = {}^{20}C_4$

Statement-1:

Common difference is 1; total number of ways = 17

common difference is 2; total number of ways = 14

common difference is 3; total number of ways = 11

common difference is 4; total number of ways = 8

common difference is 5; total number of ways = 5

common difference is 6; total number of ways = 2

Prob. = $\frac{17+14+11+8+5+2}{{}^{20}C_4} = \frac{1}{85}$

Statement -2 is false, because common difference can be 6 also.

7. **(c)** P (exactly one of A or B occurs)

$= P(A) + P(B) - 2P(A \cap B) = \frac{1}{4}$ ...(1)

P (Exactly one of B or C occurs)

$= P(B) + P(C) - 2P(B \cap C) = \frac{1}{4}$ ...(2)

P (Exactly one of C or A occurs)

$= P(C) + P(A) - 2P(C \cap A) = \frac{1}{4}$ ...(3)

Adding (1), (2) and (3),we get

$2\Sigma P(A) - 2\Sigma P(A \cap B) = \frac{3}{4}$

$\therefore \Sigma P(A) - \Sigma P(A \cap B) = \frac{3}{8}$

Now, $P(A \cap B \cap C) = \frac{1}{16}$

$\therefore P(A \cup B \cup C)$

$= \Sigma P(A) - \Sigma P(A \cap B) + P(A \cap B \cap C)$

$= \frac{3}{8} + \frac{1}{16} = \frac{7}{16}$

16 Relations and Functions

1. The period of $\sin^2 \theta$ is **[2002]**
   (a) π^2 (b) π
   (c) 2π (d) $\pi/2$

2. Which one is not periodic? **[2002]**
   (a) $|\sin 3x| + \sin^2 x$
   (b) $\cos\sqrt{x} + \cos^2 x$
   (c) $\cos 4x + \tan^2 x$
   (d) $\cos 2x + \sin x$

3. The function $f(x) = \log\left(x + \sqrt{x^2+1}\right)$, is **[2003]**
   (a) neither an even nor an odd function
   (b) an even function
   (c) an odd function
   (d) a periodic function.

4. A function f from the set of natural numbers to integers defined by **[2003]**

   $$f(n) = \begin{cases} \dfrac{n-1}{2}, \text{when n is odd} \\ -\dfrac{n}{2}, \text{when n is even} \end{cases} \quad \text{is}$$

   (a) neither one -one nor onto
   (b) one-one but not onto
   (c) onto but not one-one
   (d) one-one and onto both.

5. If $f : R \to S$, defined by

   $f(x) = \sin x - \sqrt{3}\cos x + 1$, is onto, then the interval of S is **[2004]**
   (a) $[-1, 3]$ (b) $[-1, 1]$
   (c) $[0, 1]$ (d) $[0, 3]$

6. Let R = $\{(1,3),(4,2),(2,4),(2,3),(3,1)\}$ be a relation on the set $A = \{1,2,3,4\}$.. The relation R is **[2004]**
   (a) reflexive (b) transitive
   (c) not symmetric (d) a function

7. A real valued function $f(x)$ satisfies the functional equation
   $f(x-y) = f(x)f(y) - f(a-x)f(a+y)$
   where a is a given constant and $f(0) = 1$, $f(2a-x)$ is equal to **[2005]**
   (a) $-f(x)$ (b) $f(x)$
   (c) $f(a)+f(a-x)$ (d) $f(-x)$

8. Let $R = \{(3,3),(6,6),(9,9),(12,12),(6,12),(3,9),(3,12),(3,6)\}$ be a relation on the set $A = \{3, 6, 9, 12\}$. The relation is **[2005]**
   (a) reflexive and transitive only
   (b) reflexive only
   (c) an equivalence relation
   (d) reflexive and symmetric only

9. Let W denote the words in the English dictionary. Define the relation R by $R = \{(x, y) \in W \times W |$ the words x and y have at least one letter in common.$\}$ Then R is **[2006]**
   (a) not reflexive, symmetric and transitive
   (b) relexive, symmetric and not transitive
   (c) reflexive, symmetric and transitive
   (d) reflexive, not symmetric and transitive

10. Let $f: N \to Y$ be a function defined as $f(x) = 4x + 3$ where $Y = \{y \in N : y = 4x + 3$ for some $x \in N\}$. Then, the inverse of $f(x)$ is **[2008]**

(a) $g(y) = \frac{3y+4}{3}$ (b) $g(y) = 4 + \frac{y+3}{4}$

(c) $g(y) = \frac{y+3}{4}$ (d) $g(y) = \frac{y-3}{4}$

11. Let R be the real line. Consider the following subsets of the plane $R \times R$:
$S = \{(x, y) : y = x + 1$ and $0 < x < 2\}$
$T = \{(x, y) : x - y$ is an integer$\}$,
Which one of the following is true? **[2008]**

(a) Neither S nor T is an equivalence relation on R
(b) Both S and T are equivalence relation on R
(c) S is an equivalence relation on R but T is not
(d) T is an equivalence relation on R but S is not

12. **DIRECTIONS :** This question contains two statements:
Statement-1 (Assertion) and Statement-2 (Reason).
This question also has four alternative choices, only one of which is the correct answer. You have to select the correct choice.

Let $f(x) = (x+1)^2 - 1,\ x \geq -1$

Statement -1 : The set $\{x : f(x) = f^{-1}(x) = \{0, -1\}$
Statement-2 : f is a bijection. **[2009]**

(a) Statement-1 is true, Statement-2 is true. Statement-2 is not a correct explanation for Statement-1.
(b) Statement-1 is true, Statement-2 is false.
(c) Statement-1 is false, Statement-2 is true.
(d) Statement-1 is true, Statement-2 is true. Statement-2 is not a correct explanation for Statement-1.

13. For real x, let $f(x) = x^3 + 5x + 1$, then **[2009]**

(a) f is onto R but not one-one
(b) f is one-one and onto R
(c) f is neither one-one nor onto R
(d) f is one-one but not onto R

14. Consider the following relations:
R = $\{(x, y) \mid x, y$ are real numbers and $x = wy$ for some rational number $w\}$;

$S = \{\left(\frac{m}{n}, \frac{p}{q}\right) \mid m, n, p$ and q are integers such that $n, q \neq 0$ and $qm = pn\}$. Then **[2010]**

(a) Neither R nor S is an equivalence relation
(b) S is an equivalence relation but R is not an equivalence relation
(c) R and S both are equivalence relations
(d) R is an equivalence relation but S is not an equivalence relation

15. Let R be the set of real numbers. **[2011]**
Statement-1: $A = \{(x, y) \in R \times R : y - x$ is an integer$\}$ is an equivalence relation on R.
Statement-2: $B = \{(x, y) \in R \times R : x = \alpha y$ for some rational number $\alpha\}$ is an equivalence relation on R.

(a) Statement-1 is true, Statement-2 is true; Statement-2 is not a correct explanation for Statement-1.
(b) Statement-1 is true, Statement-2 is false.
(c) Statement-1 is false, Statement-2 is true.
(d) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1.

16. Let f be a function defined by

$f(x) = (x-1)^2 + 1, (x \geq 1)$. **[2011RS]**

Statement - 1 :

The set $\{x : f(x) = f^{-1}(x)\} = \{1, 2\}$.

Statement - 2:

f is a bijection and $f^{-1}(x) = 1 + \sqrt{x-1}, x \geq 1$.

(a) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1.
(b) Statement-1 is true, Statement-2 is true; Statement-2 is NOT a correct explanation for Statement-1.
(c) Statement-1 is true, Statement-2 is false.
(d) Statement-1 is false, Statement-2 is true.

17. If g is the inverse of a function f and $f'(x) = \frac{1}{1+x^5}$, then $g'(x)$ is equal to: **[2014]**

(a) $\dfrac{1}{1+\{g(x)\}^5}$ (b) $1+\{g(x)\}^5$

(c) $1+x^5$ (d) $5x^4$

18. The function f : R $\rightarrow \left[-\dfrac{1}{2}, \dfrac{1}{2}\right]$ defined as $f(x) = \dfrac{x}{1+x^2}$, is: **[2017]**

(a) neither injective nor surjective

(b) invertible

(c) injective but not surjective

(d) surjective but not injective

19. If the function f: R – {1, –1} → A defined by $f(x) = \dfrac{x^2}{1-x^2}$, is surjective, then A is equal to: **[2019]**

(a) R – {–1} (b) $[0, \infty)$

(c) R – [–1, 0) (d) R – (–1, 0)

20. For $x \in \mathbf{R} - \{0, 1\}$, let $f_1(x) = \dfrac{1}{x}$, $f_2(x) = 1 - x$ and $f_3(x) = \dfrac{1}{1-x}$ be three given functions. If a function, J(x) satisfies (f_2oJof_1) (x) = $f_3(x)$ then J(x) is equal to: **[2019]**

(a) $f_3(x)$ (b) $\dfrac{1}{x} f_3(x)$

(c) $f_2(x)$ (d) $f_1(x)$

21. If $g(x) = x^2 + x - 1$ and $(gof)(x) = 4x^2 - 10x + 5$, then $f\left(\dfrac{5}{4}\right)$ is equal to: **[2020]**

(a) $\dfrac{3}{2}$ (b) $-\dfrac{1}{2}$

(c) $\dfrac{1}{2}$ (d) $-\dfrac{3}{2}$

Answer Key

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
(b)	(b)	(c)	(d)	(a)	(c)	(a)	(a)	(b)	(d)	(d)	(a)	(b)	(b)	(a)
16	17	18	19	20	21									
(a)	(b)	(d)	(c)	(a)	(b)									

Solutions

1. **(b)** We know that $\sin^2\theta = \dfrac{1-\cos 2\theta}{2}$;

Since period of cos 2θ $= \dfrac{2\pi}{2} = \pi$

Hence period of $\sin^2\theta$ is also π.

2. **(b)** We know that $\cos\sqrt{x}$ is non-periodic

$\therefore \cos\sqrt{x} + \cos^2 x$ can not be periodic.

3. **(c)** Given $f(x) = \log(x + \sqrt{x^2+1})$

$$f(-x) = \log\left\{-x + \sqrt{x^2+1}\right\}$$

$$= \log\left\{\frac{x^2+1-x^2}{x+\sqrt{x^2+1}}\right\}$$

$$= -\log(x + \sqrt{x^2+1}) = -f(x)$$

$\Rightarrow f(x)$ is an odd function.

4. **(d)** We have $f : N \rightarrow I$

Let x and y are two even natural numbers,

and $f(x) = f(y) \Rightarrow \dfrac{-x}{2} = \dfrac{-y}{2} \Rightarrow x = y$

$\therefore f(n)$ is one-one for even natural number.

Let x and y are two odd natural numbers and

$f(x)=f(y) \Rightarrow \frac{x-1}{2}=\frac{y-1}{2} \Rightarrow x=y$

$\therefore f(n)$ is one-one for odd natural number.
Hence f is one-one.

Let $y=\frac{n-1}{2} \Rightarrow 2y+1=n$

This shows that n is always odd number for $y \in$ I. ....(i)

and $y=\frac{-n}{2} \Rightarrow -2y=n$

This shows that n is always even number for $y \in$ I. ....(ii)

From (i) and (ii)
Range of f = I = codomain
$\therefore f$ is onto.
Hence f is one one and onto both.

5. **(a)** Given that $f(x)$ is onto

$\therefore$ range of $f(x)$ = codomain = S

Now, $f(x)=\sin x-\sqrt{3}\cos x+1$

$=2\sin\left(x-\frac{\pi}{3}\right)+1$

we know that $-1 \le \sin\left(x-\frac{\pi}{3}\right) \le 1$

$-1 \le 2\sin\left(x-\frac{\pi}{3}\right)+1 \le 3$

$\therefore f(x) \in [-1, 3] = S$

6. **(c)** $\because$ $(1, 1) \notin R \Rightarrow R$ is not reflexive
$\because$ $(2, 3) \in R$ but $(3, 2) \notin R$
$\therefore$ R is not symmetric
$\because$ $(4, 2)$ and $(2, 4) \in R$ but $(4, 4) \notin R$
$\Rightarrow R$ is not transitive

7. **(a)** Given that f(0) = 1 and put
$x=0, y=0,$

$f(0)=f^2(0)-f^2(a)$

$\Rightarrow f^2(a)=0 \Rightarrow f(a)=0$

$f(2a-x)=f(a-(x-a))$
$=f(a)f(x-a)-f(0)f(x)$
$=f(a)f(x-a)-f(x)=-f(x)$

$\Rightarrow f(2a-x)=-f(x)$

8. **(a)** R is reflexive and transitive only.
Here (3, 3), (6, 6), (9, 9), (12, 12) $\in$ R [So, reflexive]
(3, 6), (6, 12), (3, 12) $\in$ R [So, transitive].
$\because$ $(3, 6) \in R$ but $(6, 3) \notin R$ [So, non-symmetric]

9. **(b)** Clearly $(x,x) \in R, \forall x \in W$
$\because$ All letter are common in some word.
So R is reflexive.
Let $(x, y) \in R$, then $(y, x) \in R$ as x and y have at least one letter in common. So, R is symmetric.
But R is not transitive for example
Let x = BOY, y = TOY and z = THREE
then $(x, y) \in R$ (O, Y are common) and $(y, z) \in R$ (T is common) but $(x, z) \notin R$. (as no letter is common)

10. **(d)** Clearly $f(x)=4x+3$ is one one and onto, so it is invertible.
Let $f(x)=4x+3=y$

$\Rightarrow x=\frac{y-3}{4} \quad \therefore g(y)=\frac{y-3}{4}$

11. **(d)** Given that
$S=\{(x, y): y=x+1 \text{ and } 0<x<2\}$
$\because$ $x \ne x+1$ for any $x \in (0, 2)$
$\Rightarrow (x, x) \notin$ S
So, S is not reflexive.
Hence, S in not an equivalence relation.
Given $T=\{x, y): x-y$ is an integer$\}$
$\because x-x=0$ is an integer, $\forall x \in R$
So, T is reflexive.
Let $(x, y) \in$ T $\Rightarrow x-y$ is an integer then $y-x$ is also an integer $\Rightarrow (y, x) \in R$
$\therefore T$ is symmetric
If $x-y$ is an integer and $y-z$ is an integer then
$(x-y)+(y-z)=x-z$ is also an integer.
$\therefore$ T is transitive
Hence T is an equivalence relation.

12. **(a)** Given that $f(x)=(x+1)^2-1,\ x \ge -1$
Clearly $D_f=[-1, \infty)$ but co-domain is not given. Therefore $f(x)$ is onto.
Let $f(x_1)=f(x_2)$
$\Rightarrow$ $(x_1+1)^2-1=(x_2+1)^2-1$
$\Rightarrow$ $x_1=x_2$
$\therefore$ $f(x)$ is one-one, hence $f(x)$ is bijection
$\because$ $(x+1)$ being something +ve, $\forall x>-1$
$\therefore$ $f^{-1}(x)$ will exist.

Let $(x+1)^2-1=y$

$\Rightarrow x+1=\sqrt{y+1}$

(+ve square root as $x+1\geq 0$)

$\Rightarrow x=-1+\sqrt{y+1}$

$\Rightarrow f^{-1}(x)=\sqrt{x+1}-1$

Then $f(x)=f^{-1}(x)$

$\Rightarrow (x+1)^2-1=\sqrt{x+1}-1$

$\Rightarrow (x+1)^2=\sqrt{x+1} \Rightarrow (x+1)^4=(x+1)$

$\Rightarrow (x+1)[(x+1)^3-1]=0 \Rightarrow x=-1,0$

$\therefore$ The statement-1 and statement-2 both are true.

13. (b) Given that $f(x)=x^3+5x+1$

$\therefore \quad f'(x)=3x^2+5>0, \forall x\in R$

$\Rightarrow \quad f(x)$ is strictly increasing on R

So, $f(x)$ is one one

Hence, polynomial $f(x)$ is continuous and increasing on R with

$\lim\limits_{x\to-\infty} f(x)=-\infty$

and $\lim\limits_{x\to\infty} f(x)=\infty$

$\therefore$ Range of $f=(-\infty,\infty)=R$

Hence f is onto also. So, f is one one and onto R.

14. (b) Let $x\,R\,y$.

$\Rightarrow x=wy\Rightarrow y=\dfrac{x}{w}$

$\Rightarrow (y,x)\notin R$

R is not symmetric

Let $S:\dfrac{m}{n}S\dfrac{p}{q}$

$\Rightarrow qm=pn \Rightarrow \dfrac{p}{q}=\dfrac{m}{n}$

$\because \dfrac{m}{n}=\dfrac{m}{n} \quad \therefore$ reflexive.

$\dfrac{m}{n}=\dfrac{p}{q} \Rightarrow \dfrac{p}{q}=\dfrac{m}{n} \quad \therefore$ symmetric

Let $\dfrac{m}{n}S\dfrac{p}{q}, \dfrac{p}{q}S\dfrac{r}{s}$

$\Rightarrow \quad qm=pn, ps=rq$

$\Rightarrow \quad \dfrac{p}{q}=\dfrac{m}{n}=\dfrac{r}{s}$

$\Rightarrow \quad ms=rn$ transitive.

S is an equivalence relation.

15. (a) $\because x-x=0\in I$ ($\therefore R$ is reflexive)

Let $(x,y)\in R$ as $x-y$ and $y-x\in I$

($\because R$ is symmetric)

Now $x-y\in I$ and $y-z\in I\Rightarrow x-z\in I$

So, R is transative.

Hence R is equivalence.

Similarly as $x=\alpha y$ for $\alpha=1$. B is reflexive symmetric and transative. Hence B is equivalence.

Both relations are equivalence but not the correct explanation.

16. (a) Given f is a bijective function

$\therefore \quad f:[1,\infty)\to[1,\infty)$

$f(x)=(x-1)^2+1, x\geq 1$

Let $y=(x-1)^2+1\Rightarrow (x-1)^2=y-1$

$\Rightarrow x=1\pm\sqrt{y-1}\Rightarrow f^{-1}(y)=1\pm\sqrt{y-1}$

$\Rightarrow \quad f^{-1}(x)=1+\sqrt{x-1}\ \{\therefore x\geq 1\}$

Hence statement-2 is correct

Now $f(x)=f^{-1}(x)$

$\Rightarrow f(x)=x\Rightarrow (x-1)^2+1=x$

$\Rightarrow \quad x^2-3x+2=0\Rightarrow x=1,2$

Hence statement-1 is correct

17. (b) Since $f(x)$ and $g(x)$ are inverse of each other

$\therefore \quad g'(f(x))=\dfrac{1}{f'(x)}$

$\Rightarrow \quad g'(f(x))=1+x^5$

$\left(\because \quad f'(x)=\dfrac{1}{1+x^5}\right)$

Here $x=g(y)$

$\therefore \quad g'(y)=1+[g(y)]^5$

$\Rightarrow g'(x)=1+(g(x))^5$

18. (d) We have $f:R\to\left[-\dfrac{1}{2},\dfrac{1}{2}\right]$,

$f(x)=\dfrac{x}{1+x^2}\ \forall x\in R$

$\Rightarrow f'(x) = \dfrac{(1+x^2).1 - x.2x}{(1+x^2)^2} = \dfrac{-(x+1)(x-1)}{(1+x^2)^2}$

sign of $f'(x)$

$\Rightarrow f'(x)$ changes sign in different intervals.

$\therefore$ Not injective

Now $y = \dfrac{x}{1+x^2}$

$\Rightarrow \quad y + yx^2 = x$

$\Rightarrow \quad yx^2 - x + y = 0$

For $y \neq 0$, $D = 1 - 4y^2 \geq 0$

$\Rightarrow \quad y \in \left[\dfrac{-1}{2}, \dfrac{1}{2}\right] - \{0\}$

For $y = 0 \Rightarrow x = 0$

$\therefore \quad$ Range is $\left[\dfrac{-1}{2}, \dfrac{1}{2}\right]$

$\Rightarrow \quad$ Surjective but not injective

19. **(c)** $f(x) = \dfrac{x^2}{1-x^2} \Rightarrow f(-x) = \dfrac{x^2}{1-x^2} = f(x)$

$f'(x) = \dfrac{2x}{(1-x^2)^2}$

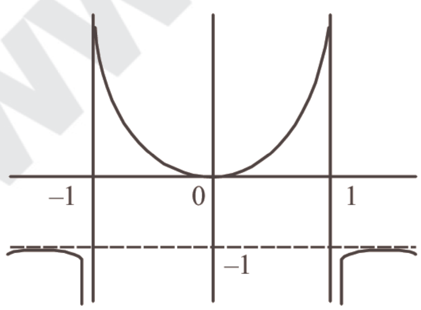
$\therefore f(x)$ increases in $x \in [0, 1) \cup (1, \infty)$

Also, $f(0) = 0$ and

$\lim\limits_{x \to \pm\infty} f(x) = -1$ and $f(x)$ is even function

$\therefore$ Set $A = R - [-1, 0)$

And the graph of function $f(x)$ is



Alternative

For f to be subjective, A = Range of f.

$\dfrac{x^2}{1-x^2} = y \Rightarrow x^2 = y - x^2 y$

$\Rightarrow x = \pm\sqrt{\dfrac{y}{1+y}} \Rightarrow y, (1+y) \geq 0$ and $y \neq -1$

$\Rightarrow y \in (-\infty, -1) \cup [0, \infty) \Rightarrow y \in R - [-1, 0)$

$\Rightarrow A = R - [-1, 0)$

20. **(a)** The given relation is

$(f_2 o J o f_1)(x) = f_3(x) = \dfrac{1}{1-x}$

$\Rightarrow (f_2 o J)(f_1(x)) = \dfrac{1}{1-x}$

$\Rightarrow (f_2 o J)\left(\dfrac{1}{x}\right) = \dfrac{1}{1-x} \quad \left[\because f_1(x) = \dfrac{1}{x}\right]$

$\Rightarrow f_2\left(J\left(\dfrac{1}{x}\right)\right) = \dfrac{1}{1-x}$

$\Rightarrow (f_2 J(x)) = \dfrac{1}{1-\dfrac{1}{x}} = \dfrac{x}{x-1}$

$\left[\dfrac{1}{x}\text{ is replaced by } x\right]$

$\Rightarrow 1 - J(x) = \dfrac{x}{x-1} \qquad [\because f_2(x) = 1 - x]$

$\therefore J(x) = 1 - \dfrac{x}{x-1} = \dfrac{1}{1-x} = f_3(x)$

21. **(b)** $(gof)(x) = g(f(x)) = f^2(x) + f(x) - 1$

$g\left(f\left(\dfrac{5}{4}\right)\right) = 4\left(\dfrac{5}{4}\right)^2 - 10 \cdot \dfrac{5}{4} + 5 = -\dfrac{5}{4}$

$[\because g(f(x)) = 4x^2 - 10x + 5]$

$g\left(f\left(\dfrac{5}{4}\right)\right) = f^2\left(\dfrac{5}{4}\right) + f\left(\dfrac{5}{4}\right) - 1$

$-\dfrac{5}{4} = f^2\left(\dfrac{5}{4}\right) + f\left(\dfrac{5}{4}\right) - 1$

$f^2\left(\dfrac{5}{4}\right) + f\left(\dfrac{5}{4}\right) + \dfrac{1}{4} = 0$

$\left(f\left(\dfrac{5}{4}\right) + \dfrac{1}{2}\right)^2 = 0$

$t\left(\dfrac{5}{4}\right) = -\dfrac{1}{2}$

17 Inverse Trigonometric Functions

1. $\cot^{-1}(\sqrt{\cos\alpha}) - \tan^{-1}(\sqrt{\cos\alpha}) = x$, then $\sin x =$ **[2002]**

(a) $\tan^2\left(\frac{\alpha}{2}\right)$ (b) $\cot^2\left(\frac{\alpha}{2}\right)$

(c) $\tan\alpha$ (d) $\cot\left(\frac{\alpha}{2}\right)$

2. The domain of $\sin^{-1}[\log_3(x/3)]$ is **[2002]**

(a) $[1, 9]$ (b) $[-1, 9]$

(c) $[-9, 1]$ (d) $[-9, -1]$

3. The trigonometric equation $\sin^{-1} x = 2\sin^{-1} a$ has a solution for **[2003]**

(a) $|a| \le \frac{1}{\sqrt{2}}$ (b) $\frac{1}{2} < |a| < \frac{1}{\sqrt{2}}$

(c) all real values of a (d) $|a| < \frac{1}{2}$

4. The domain of the function

$f(x) = \frac{\sin^{-1}(x-3)}{\sqrt{9-x^2}}$ is **[2004]**

(a) $[1, 2]$ (b) $[2, 3)$

(c) $[1, 2]$ (d) $[2, 3]$

5. If $\cos^{-1} x - \cos^{-1}\frac{y}{2} = \alpha$, then

$4x^2 - 4xy\cos\alpha + y^2$ is equal to **[2005]**

(a) $2\sin 2\alpha$ (b) 4

(c) $4\sin^2\alpha$ (d) $-4\sin^2\alpha$

6. Let $f : (-1, 1) \to B$, be a function defined by $f(x) = \tan^{-1}\frac{2x}{1-x^2}$, then f is both one - one and onto when B is the interval **[2005]**

(a) $\left(0, \frac{\pi}{2}\right)$ (b) $\left[0, \frac{\pi}{2}\right)$

(c) $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ (d) $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

7. If $\sin^{-1}\left(\frac{x}{5}\right) + \operatorname{cosec}^{-1}\left(\frac{5}{4}\right) = \frac{\pi}{2}$, then the values of x is **[2007]**

(a) 4 (b) 5

(c) 1 (d) 3

8. The largest interval lying in $\left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$ for which the function,

$f(x) = 4^{-x^2} + \cos^{-1}\left(\frac{x}{2} - 1\right) + \log(\cos x)$,

is defined, is **[2007]**

(a) $\left[-\frac{\pi}{4}, \frac{\pi}{2}\right)$ (b) $\left[0, \frac{\pi}{2}\right)$

(c) $[0, \pi]$ (d) $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

9. The value of $\cot\left(\operatorname{cosec}^{-1}\frac{5}{3}+\tan^{-1}\frac{2}{3}\right)$ is **[2010]**

(a) $\frac{6}{17}$ (b) $\frac{3}{17}$
(c) $\frac{4}{17}$ (d) $\frac{5}{17}$

10. If x, y, z are in A.P. and $\tan^{-1}x, \tan^{-1}y$ and $\tan^{-1}z$ are also in A.P., then **[2013]**

(a) $x = y = z$ (b) $2x = 3y = 6z$
(c) $6x = 3y = 2z$ (d) $6x = 4y = 3z$

11. Let $\tan^{-1}y = \tan^{-1}x + \tan^{-1}\left(\frac{2x}{1-x^2}\right)$, where $|x| < \frac{1}{\sqrt{3}}$. Then a value of y is : **[2015]**

(a) $\frac{3x-x^3}{1+3x^2}$ (b) $\frac{3x+x^3}{1+3x^2}$
(c) $\frac{3x-x^3}{1-3x^2}$ (d) $\frac{3x+x^3}{1-3x^2}$

12. If for $x \in \left(0, \frac{1}{4}\right)$, the derivative of $\tan^{-1}\left(\frac{6x\sqrt{x}}{1-9x^3}\right)$ is $\sqrt{x}.g(x)$, then g(x) equals : **[2017]**

(a) $\frac{3}{1+9x^3}$ (b) $\frac{9}{1+9x^3}$
(c) $\frac{3x\sqrt{x}}{1-9x^3}$ (d) $\frac{3x}{1-9x^3}$

13. If $\cos^{-1}\left(\frac{2}{3x}\right) + \cos^{-1}\left(\frac{3}{4x}\right) = \frac{\pi}{2}\left(x > \frac{3}{4}\right)$, then x is equal to: **[2019]**

(a) $\frac{\sqrt{145}}{12}$ (b) $\frac{\sqrt{145}}{10}$
(c) $\frac{\sqrt{146}}{12}$ (d) $\frac{\sqrt{145}}{11}$

Answer Key

1	2	3	4	5	6	7	8	9	10	11	12	13		
(a)	(a)	(a)	(b)	(c)	(d)	(d)	(b)	(a)	(a)	(c)	(b)	(a)		

Solutions

1. (a) Given that

$\cot^{-1}(\sqrt{\cos\alpha}) - \tan^{-1}(\sqrt{\cos\alpha}) = x$

$\tan^{-1}\left(\frac{1}{\sqrt{\cos\alpha}}\right) - \tan^{-1}(\sqrt{\cos\alpha}) = x$

$$\Rightarrow \tan^{-1}\frac{\frac{1}{\sqrt{\cos\alpha}}-\sqrt{\cos\alpha}}{1+\frac{1}{\sqrt{\cos\alpha}}.\sqrt{\cos\alpha}} = x$$

$$\Rightarrow \tan^{-1}\frac{1-\cos\alpha}{2\sqrt{\cos\alpha}} = x$$

$$\Rightarrow \tan x = \frac{1-\cos\alpha}{2\sqrt{\cos\alpha}}$$

$$\Rightarrow \cot x = \frac{2\sqrt{\cos\alpha}}{1-\cos\alpha} = \frac{B}{P}$$

$P = (1-\cos\alpha)$ and $B = 2\sqrt{\cos\alpha}$

$H = \sqrt{P^2+B^2} = 1+\cos\alpha$

$$\Rightarrow \sin x = \frac{1-\cos\alpha}{1+\cos\alpha} = \frac{1-(1-2\sin^2\alpha/2)}{1+2\cos^2\alpha/2-1}$$

or $\sin x = \tan^2\frac{\alpha}{2}$

2. (a) $f(x)=\sin^{-1}\left(\log_3\left(\frac{x}{3}\right)\right)$

We know that domain of $\sin^{-1}x$ is $x \in [-1, 1]$

$\therefore -1 \le \log_3\left(\frac{x}{3}\right) \le 1 \Rightarrow 3^{-1} \le \frac{x}{3} \le 3^1$

$\Rightarrow 1 \le x \le 9$ or $x \in [1, 9]$

3. **(a)** Given that $\sin^{-1} x = 2\sin^{-1} a$

We know that $-\frac{\pi}{2} \le \sin^{-1} x \le \frac{\pi}{2}$

$\Rightarrow -\frac{\pi}{2} \le 2\sin^{-1} a \le \frac{\pi}{2}$

$\Rightarrow -\frac{\pi}{4} \le \sin^{-1} a \le \frac{\pi}{4} \Rightarrow \frac{-1}{\sqrt{2}} \le a \le \frac{1}{\sqrt{2}}$

$\therefore \ |a| \le \frac{1}{\sqrt{2}}$

4. **(b)** $f(x) = \frac{\sin^{-1}(x-3)}{\sqrt{9-x^2}}$ is defined

When $-1 \le x-3 \le 1 \Rightarrow 2 \le x \le 4$ ....(i)

and $9 - x^2 > 0 \Rightarrow -3 < x < 3$ ....(ii)

from (i) and (ii),

we get $2 \le x < 3$ $\therefore$ Domain $= [2, 3)$

5. **(c)** $\cos^{-1} x - \cos^{-1}\frac{y}{2} = \alpha$

$\Rightarrow \cos^{-1}\left(\frac{xy}{2} + \sqrt{(1-x^2)\left(1-\frac{y^2}{4}\right)}\right) = \alpha$

$\Rightarrow \cos^{-1}\left(\frac{xy + \sqrt{4 - y^2 - 4x^2 + x^2y^2}}{2}\right) = \alpha$

$\Rightarrow xy + \sqrt{4 - y^2 - 4x^2 + x^2y^2} = 2\cos\alpha$

$\Rightarrow \sqrt{4 - y^2 - 4x^2 + x^2y^2} = 2\cos\alpha - xy$

Squaring both sides, we get

$\Rightarrow 4 - y^2 - 4x^2 + x^2y^2$

$= 4\cos^2\alpha + x^2y^2 - 4xy\cos\alpha$

$\Rightarrow 4x^2 + y^2 - 4xy\cos\alpha = 4\sin^2\alpha \cdot$

6. **(d)** Given that $f(x) = \tan^{-1}\left(\frac{2x}{1-x^2}\right) = 2\tan^{-1}x$

for $x \in (-1, 1)$

If $x \in (-1, 1) \Rightarrow \tan^{-1} x \in \left(\frac{-\pi}{4}, \frac{\pi}{4}\right)$

$\Rightarrow 2\tan^{-1} x \in \left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$

So, range of $f(x) = \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

For f to be onto, codomain = range

$\therefore$ Codomain of function $= B = \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$.

7. **(d)** $\sin^{-1}\left(\frac{x}{5}\right) + \text{cosec}^{-1}\left(\frac{5}{4}\right) = \frac{\pi}{2}$

$\Rightarrow \sin^{-1}\left(\frac{x}{5}\right) = \frac{\pi}{2} - \text{cosec}^{-1}\left(\frac{5}{4}\right)$

$\Rightarrow \sin^{-1}\left(\frac{x}{5}\right) = \frac{\pi}{2} - \sin^{-1}\left(\frac{4}{5}\right)$

$[\because \sin^{-1} x + \cos^{-1} x = \pi/2]$

$\Rightarrow \sin^{-1}\left(\frac{x}{5}\right) = \cos^{-1}\left(\frac{4}{5}\right)$

$\sin^{-1}\frac{x}{5} = \sin^{-1}\sqrt{1-\left(\frac{4}{5}\right)^2}$

$\left[\because \cos^{-1}x = \sin^{-1}\sqrt{1-x^2}\right]$

$\Rightarrow \sin^{-1}\frac{x}{5} = \sin^{-1}\frac{3}{5} \Rightarrow \frac{x}{5} = \frac{3}{5}$

$\Rightarrow x = 3$

8. **(b)** Given that

$f(x) = 4^{-x^2} + \cos^{-1}\left(\frac{x}{2} - 1\right) + \log(\cos x)$

$f(x)$ is defined if $-1 \le \left(\frac{x}{2} - 1\right) \le 1$ and

$\cos x > 0$

$\Rightarrow 0 \le \frac{x}{2} \le 2$ and $-\frac{\pi}{2} < x < \frac{\pi}{2}$

$\Rightarrow 0 \le x \le 4$ and $-\frac{\pi}{2} < x < \frac{\pi}{2}$

$\therefore x \in \left[0, \frac{\pi}{2}\right)$

9. **(a)** $\cot\left(\text{cosec}^{-1}\frac{5}{3} + \tan^{-1}\frac{2}{3}\right)$

$\left[\because \text{cosec}^{-1}x = \tan^{-1}\left(\frac{1}{\sqrt{x^2-1}}\right)\right]$

$$= \cot\left[\tan^{-1}\frac{3}{4} + \tan^{-1}\frac{2}{3}\right]$$

$$= \cot\left[\tan^{-1}\left(\frac{\frac{3}{4}+\frac{2}{3}}{1-\frac{3}{4}\times\frac{2}{3}}\right)\right]$$

$$= \cot\left[\tan^{-1}\frac{17}{6}\right]$$

$$= \cot\left(\cot^{-1}\frac{6}{17}\right) = \frac{6}{17}$$

10. **(a)** Since, x, y, z are in A.P.

$\therefore \quad 2y = x + z$

Also, we have

$2\tan^{-1} y = \tan^{-1} x + \tan^{-1}(z)$

$$\Rightarrow \tan^{-1}\left(\frac{2y}{1-y^2}\right) = \tan^{-1}\left(\frac{x+z}{1-xz}\right)$$

$$\Rightarrow \frac{x+z}{1-y^2} = \frac{x+z}{1-xz} \qquad (\because \quad 2y = x+z)$$

$$\Rightarrow y^2 = xz \text{ or } x+z=0 \Rightarrow x=y=z=0$$

11. **(c)** Given that

$$\tan^{-1}y = \tan^{-1}x + \tan^{-1}\left[\frac{2x}{1-x^2}\right]$$

$$= \tan^{-1}x + 2\tan^{-1}x = 3\tan^{-1}x$$

$$\tan^{-1}y = \tan^{-1}\left[\frac{3x-x^3}{1-3x^2}\right]$$

$$\Rightarrow y = \frac{3x-x^3}{1-3x^2}$$

12. **(b)** Let $F(x) = \tan^{-1}\left(\frac{6x\sqrt{x}}{1-9x^3}\right)$

where $x \in \left(0, \frac{1}{4}\right)$.

$$= \tan^{-1}\left(\frac{2.(3x^{3/2})}{1-(3x^{3/2})^2}\right) = 2\tan^{-1}(3x^{3/2})$$

As $3x^{3/2} \in \left(0, \frac{3}{8}\right)$

$$\left[\because 0 < x < \frac{1}{4} \Rightarrow 0 < x^{3/2} < \frac{1}{8} \Rightarrow 0 < 3x^{3/2} < \frac{3}{8}\right]$$

So $\frac{dF(x)}{dx} = 2 \times \frac{1}{1+9x^3} \times 3 \times \frac{3}{2} \times x^{1/2}$

$$= \frac{9}{1+9x^3}\sqrt{x}$$

On comparing

$$\therefore g(x) = \frac{9}{1+9x^3}$$

13. **(a)** $\cos^{-1}\left(\frac{2}{3x}\right) + \cos^{-1}\left(\frac{3}{4x}\right) = \frac{\pi}{2}; \left(x > \frac{3}{4}\right)$

$$\Rightarrow \cos^{-1}\left(\frac{2}{3x}\right) = \frac{\pi}{2} - \cos^{-1}\left(\frac{3}{4x}\right)$$

$$\Rightarrow \cos^{-1}\left(\frac{2}{3x}\right) = \sin^{-1}\left(\frac{3}{4x}\right)$$

$$\left[\because \sin^{-1}x + \cos^{-1}x = \frac{\pi}{2}\right]$$

Put $\sin^{-1}\left(\frac{3}{4x}\right) = \theta \Rightarrow \sin\theta = \frac{3}{4x}$

$$\Rightarrow \cos\theta = \sqrt{1-\sin^2\theta} = \sqrt{1-\frac{9}{16x^2}}$$

$$\Rightarrow \theta = \cos^{-1}\left(\frac{\sqrt{16x^2-9}}{4x}\right)$$

$$\therefore \cos^{-1}\left(\frac{2}{3x}\right) = \cos^{-1}\left(\frac{\sqrt{16x^2-9}}{4x}\right)$$

$$\Rightarrow \frac{2}{3x} = \frac{\sqrt{16x^2-9}}{4x} \Rightarrow x^2 = \frac{64+81}{9\times16}$$

$$\Rightarrow x = \pm\sqrt{\frac{145}{144}}$$

$$\Rightarrow x = \frac{\sqrt{145}}{12} \qquad \left(\because x > \frac{3}{4}\right)$$

18 Matrices

1. If $A = \begin{bmatrix} a & b \\ b & a \end{bmatrix}$ and $A^2 = \begin{bmatrix} \alpha & \beta \\ \beta & \alpha \end{bmatrix}$, then **[2003]**

(a) $\alpha = 2ab, \beta = a^2 + b^2$

(b) $\alpha = a^2 + b^2,\ \beta = ab$

(c) $\alpha = a^2 + b^2,\ \beta = 2ab$

(d) $\alpha = a^2 + b^2,\ \beta = a^2 - b^2$

2. If $A = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$ and $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, then which one of the following holds for all $n \geq 1$, by the principle of mathematical induction **[2005]**

(a) $A^n = nA - (n-1)\,\mathrm{I}$

(b) $A^n = 2^{n-1}A - (n-1)\,\mathrm{I}$

(c) $A^n = nA + (n-1)\,\mathrm{I}$

(d) $An = 2^{n-1}A + (n-1)\,\mathrm{I}$

3. If A and B are square matrices of size $n \times n$ such that $A^2 - B^2 = (A - B)(A + B)$, then which of the following will be always true? **[2006]**

(a) $A = B$

(b) $AB = BA$

(c) either of A or B is a zero matrix

(d) either of A or B is identity matrix

4. Let $A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$ and $B = \begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix}$, $a, b \in N$. Then **[2006]**

(a) there cannot exist any B such that AB = BA

(b) there exist more than one but finite number of B′s such that AB = BA

(c) there exists exactly one B such that AB = BA

(d) there exist infinitely many B′s such that AB = BA

5. The number of 3×3 non-singular matrices, with four entries as 1 and all other entries as 0, is **[2010]**

(a) 5 (b) 6

(c) at least 7 (d) less than 4

6. Let A and B be two symmetric matrices of order 3. **[2011]**

Statement-1: $A(BA)$ and $(AB)A$ are symmetric matrices.

Statement-2: AB is symmetric matrix if matrix multiplication of A with B is commutative.

(a) Statement-1 is true, Statement-2 is true; Statement-2 is **not a** correct explanation for Statement-1.

(b) Statement-1 is true, Statement-2 is false.

(c) Statement-1 is false, Statement-2 is true.

(d) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1.

7. If $\omega \neq 1$ is the complex cube root of unity and matrix $H = \begin{bmatrix} \omega & 0 \\ 0 & \omega \end{bmatrix}$, then H^{70} is equal to **[2011RS]**

(a) 0 (b) –H

(c) H^2 (d) H

8. If $A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ a & 2 & b \end{bmatrix}$ is a matrix satisfying the equation $AA^T = 9I$, where I is 3×3 identity matrix, then the ordered pair (a, b) is equal to: **[2015]**

(a) (2, 1) (b) (–2, – 1)
(c) (2, – 1) (d) (–2, 1)

9. If $A = \begin{bmatrix} 5a & -b \\ 3 & 2 \end{bmatrix}$ and A adj A = A A^T, then $5a + b$ is equal to : **[2016]**

(a) 4 (b) 13
(c) –1 (d) 5

10. If $A = \begin{bmatrix} 2 & -3 \\ -4 & 1 \end{bmatrix}$, then adj $(3A^2 + 12A)$ is equal to : **[2017]**

(a) $\begin{bmatrix} 72 & -63 \\ -84 & 51 \end{bmatrix}$ (b) $\begin{bmatrix} 72 & -84 \\ -63 & 51 \end{bmatrix}$

(c) $\begin{bmatrix} 51 & 63 \\ 84 & 72 \end{bmatrix}$ (d) $\begin{bmatrix} 51 & 84 \\ 63 & 72 \end{bmatrix}$

11. If $A = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$, then the matrix A^{-50} when $\theta = \frac{\pi}{12}$, is equal to: **[2019]**

(a) $\begin{bmatrix} \frac{1}{2} & -\frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix}$ (b) $\begin{bmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}$

(c) $\begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}$ (d) $\begin{bmatrix} \frac{1}{2} & \frac{\sqrt{3}}{2} \\ -\frac{\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix}$

12. If $\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix} \ldots\ldots\ldots\ldots$

$\begin{bmatrix} 1 & n-1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 78 \\ 0 & 1 \end{bmatrix}$

then the inverse of $\begin{bmatrix} 1 & n \\ 0 & 1 \end{bmatrix}$ is: **[2019]**

(a) $\begin{bmatrix} 1 & 0 \\ 12 & 1 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & -13 \\ 0 & 1 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & -12 \\ 0 & 1 \end{bmatrix}$ (d) $\begin{bmatrix} 1 & 0 \\ 13 & 1 \end{bmatrix}$

13. Let α be a root of the equation $x^2 + x + 1 = 0$ and the matrix $A = \frac{1}{\sqrt{3}} \begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha & \alpha^2 \\ 1 & \alpha^2 & \alpha^4 \end{bmatrix}$, then the matrix A^{31} is equal to: **[2020]**

(a) A (b) I_3
(c) A^2 (d) A^3

Answer Key

1	2	3	4	5	6	7	8	9	10	11	12	13		
(c)	(a)	(b)	(d)	(c)	(a)	(d)	(b)	(d)	(c)	(c)	(b)	(d)		

Solutions

1. (c) $A^2=\begin{bmatrix}\alpha & \beta\\ \beta & \alpha\end{bmatrix} \Rightarrow A\cdot A=\begin{bmatrix}a & b\\ b & a\end{bmatrix}\begin{bmatrix}a & b\\ b & a\end{bmatrix}$

$=\begin{bmatrix}a^2+b^2 & 2ab\\ 2ab & a^2+b^2\end{bmatrix}$

$\alpha=a^2+b^2;\ \beta=2ab$

2. (a) Given that $A=\begin{bmatrix}1 & 0\\ 1 & 1\end{bmatrix}$

$A^2=\begin{bmatrix}1 & 0\\ 2 & 1\end{bmatrix}, A^3=\begin{bmatrix}1 & 0\\ 3 & 1\end{bmatrix}$

Therefore we observed that $A^n=\begin{bmatrix}1 & 0\\ n & 1\end{bmatrix}$

Now $nA-(n-1)I=\begin{bmatrix}n & 0\\ n & n\end{bmatrix}-\begin{bmatrix}n-1 & 0\\ 0 & n-1\end{bmatrix}$

$=\begin{bmatrix}1 & 0\\ n & 1\end{bmatrix}=A^n$

$\therefore nA-(n-1)I=A^n$

3. (b) Given that $A^2-B^2=(A-B)(A+B)$

$A^2-B^2=A^2+AB-BA-B^2$

$\Rightarrow AB=BA$

4. (d) Given that $A=\begin{bmatrix}1 & 2\\ 3 & 4\end{bmatrix}$ $B=\begin{bmatrix}a & 0\\ 0 & b\end{bmatrix}$

$AB=\begin{bmatrix}a & 2b\\ 3a & 4b\end{bmatrix}$

$BA=\begin{bmatrix}a & 0\\ 0 & b\end{bmatrix}\begin{bmatrix}1 & 2\\ 3 & 4\end{bmatrix}=\begin{bmatrix}a & 2a\\ 3b & 4b\end{bmatrix}$

Hence, $AB=BA$ only when $a=b$

$\therefore$ There can be infinitely many B's for which $AB=BA$

5. (c) $\begin{bmatrix}1 & \ldots & \ldots\\ \ldots & 1 & \ldots\\ \ldots & \ldots & 1\end{bmatrix}$ are 6 non-singular matrices because 6 blanks will be filled by 5 zeros and 1 one.

Similarly, $\begin{bmatrix}\ldots & \ldots & 1\\ \ldots & 1 & \ldots\\ 1 & \ldots & \ldots\end{bmatrix}$ are 6 non-singular matrices.

Total $=6+6=12$

So, required cases are more than 7, non-singular 3×3 matrices.

6. (a) Given that A and B are symmetric matrix

$A'=A$

$B'=B$

Now $(A(BA))'=(BA)'A'$

$=(A'B')A'=(AB)A=A(BA)$

($\because$ product of matrices are associative)

Similarly, $((AB)A)'=A'(B'A')$

$=A(BA)=(AB)A$

So, A(BA) and (AB)A are symmetric matrices.

Again $(AB)'=B'A'=BA$

Now if BA = AB, then AB is symmetric matrix.

7. (d) $H^2=\begin{bmatrix}\omega & 0\\ 0 & \omega\end{bmatrix}\begin{bmatrix}\omega & 0\\ 0 & \omega\end{bmatrix}=\begin{bmatrix}\omega^2 & 0\\ 0 & \omega^2\end{bmatrix}$

We observed that $H^k=\begin{bmatrix}\omega^k & 0\\ 0 & \omega\end{bmatrix}$

$\therefore H^{70}=\begin{bmatrix}\omega^{70} & 0\\ 0 & \omega^{70}\end{bmatrix}=\begin{bmatrix}\omega^{69}\omega & 0\\ 0 & \omega^{69}\omega\end{bmatrix}=\begin{bmatrix}\omega & 0\\ 0 & \omega\end{bmatrix}=H$

$[\because \omega^{3n}=1]$

8. (b) Given that $AA^T=9I$

$\begin{bmatrix}1 & 2 & 2\\ 2 & 1 & -2\\ a & 2 & b\end{bmatrix}\begin{bmatrix}1 & 2 & a\\ 2 & 1 & 2\\ 2 & -2 & b\end{bmatrix}=\begin{bmatrix}9 & 0 & 0\\ 0 & 9 & 0\\ 0 & 0 & 9\end{bmatrix}$

$\Rightarrow\begin{bmatrix}1+4+4 & 2+2-4 & a+4+2b\\ 2+2-4 & 4+1+4 & 2a+2-2b\\ a+4+2b & 2a+2-2b & a^2+4+b^2\end{bmatrix}=\begin{bmatrix}9 & 0 & 0\\ 0 & 9 & 0\\ 0 & 0 & 9\end{bmatrix}$

$\Rightarrow a+4+2b=0 \Rightarrow a+2b=-4$ ...(i)

$2a+2-2b=0 \Rightarrow 2a-2b=-2$

$\Rightarrow a-b=-1$ ...(ii)

Subtract (ii) from (i)

$a+2b=-4$

$\underline{\begin{array}{l}a - b = -1\\ - \quad + \quad\ \ +\end{array}}$

$3b=-3$

$b=-1$

and $a=-2$

$(a, b)=(-2,-1)$

9. **(d)** Given that A(adj A) = A A^T

Pre-multiply by A^{-1} both side, we get

$\Rightarrow A^{-1}A\text{ (adj A)} = A^{-1}A\, A^T$

adj A = A^T

$$\Rightarrow \begin{bmatrix} 2 & b \\ -3 & 5a \end{bmatrix} = \begin{bmatrix} 5a & 3 \\ -b & 2 \end{bmatrix}$$

$\Rightarrow a = \dfrac{2}{5}$ and $b = 3$

$\Rightarrow 5a + b = 5$

10. **(c)** Given that $A = \begin{bmatrix} 2 & -3 \\ -4 & 1 \end{bmatrix}$

$$\Rightarrow A^2 = \begin{bmatrix} 2 & -3 \\ -4 & 1 \end{bmatrix}\begin{bmatrix} 2 & -3 \\ -4 & 1 \end{bmatrix} = \begin{bmatrix} 16 & -9 \\ -12 & 13 \end{bmatrix}$$

$$\Rightarrow 3A^2 = \begin{bmatrix} 48 & -27 \\ -36 & 39 \end{bmatrix}$$

Also $12A = \begin{bmatrix} 24 & -36 \\ -48 & 12 \end{bmatrix}$

$\therefore\ 3A^2 + 12A$

$$= \begin{bmatrix} 48 & -27 \\ -36 & 39 \end{bmatrix} + \begin{bmatrix} 24 & -36 \\ -48 & 12 \end{bmatrix}$$

$$= \begin{bmatrix} 72 & -63 \\ -84 & 51 \end{bmatrix}$$

$$\text{adj}\,(3A^2 + 12A) = \begin{bmatrix} 51 & 63 \\ 84 & 72 \end{bmatrix}$$

11. **(c)** $A = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \Rightarrow |A| = 1$

$$\text{adj}(A) = \begin{bmatrix} +\cos\theta & -\sin\theta \\ +\sin\theta & +\cos\theta \end{bmatrix}^T$$

$$= \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$$

$$\Rightarrow A^{-1} = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} = B$$

$$B^2 = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$$

$$= \begin{bmatrix} \cos 2\theta & \sin 2\theta \\ -\sin 2\theta & \cos 2\theta \end{bmatrix}$$

$$\Rightarrow B^3 = \begin{bmatrix} \cos 3\theta & \sin 3\theta \\ -\sin 3\theta & \cos 3\theta \end{bmatrix}$$

$$\Rightarrow A^{-50} = B^{50} = \begin{bmatrix} \cos(50\ \theta) & \sin(50\ \theta) \\ -\sin(50\ \theta) & \cos(50\ \theta) \end{bmatrix}$$

$$(A^{-50})_{\theta=\frac{\pi}{12}} = \begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}$$

$$\left[\because \cos\left(\frac{50\pi}{12}\right) = \cos\left(4\pi + \frac{\pi}{6}\right) = \cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}\right]$$

12. **(b)** $\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 4 \\ 0 & 1 \end{bmatrix}\cdots\begin{bmatrix} 1 & n-1 \\ 0 & 1 \end{bmatrix}$

$$= \begin{bmatrix} 1 & 78 \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & 1+2+3+\ldots+(n-1) \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 78 \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow \frac{(n-1)n}{2} = 78 \Rightarrow n^2 - n - 156 = 0$$

$\Rightarrow n = 13$

Now, the matrix $\begin{bmatrix} 1 & n \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 13 \\ 0 & 1 \end{bmatrix}$

Then, the required inverse of

$$\begin{bmatrix} 1 & 13 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -13 \\ 0 & 1 \end{bmatrix}$$

13. **(d)** Solution of $x^2 + x + 1 = 0$ is ω, ω^2

So, $\alpha = \omega$ and

$\omega^4 = \omega^3.\omega = 1.\omega = \omega$

$$A^2 = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \end{bmatrix}\begin{bmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

$\Rightarrow\ A^4 = I$

$\Rightarrow\ A^{30} = A^{28} \times A^3 = A^3$

19 Determinants

1. If $a > 0$ and discriminant of $ax^2 + 2bx + c$ is –ve, then $\begin{vmatrix} a & b & ax+b \\ b & c & bx+c \\ ax+b & bx+c & 0 \end{vmatrix}$ is equal to **[2002]**
   (a) +ve
   (b) $(ac-b^2)(ax^2+2bx+c)$
   (c) –ve
   (d) 0

2. If the system of linear equations **[2003]**
   $x + 2ay + az = 0$; $x + 3by + bz = 0$;
   $x + 4cy + cz = 0$ has a non - zero solution, then a, b, c.
   (a) satisfy $a + 2b + 3c = 0$ (b) are in A.P
   (c) are in G.P (d) are in H.P.

3. If $1, \omega, \omega^2$ are the cube roots of unity, then
   $$\Delta = \begin{vmatrix} 1 & \omega^n & \omega^{2n} \\ \omega^n & \omega^{2n} & 1 \\ \omega^{2n} & 1 & \omega^n \end{vmatrix}$$ is equal to **[2003]**
   (a) ω^2 (b) 0
   (c) 1 (d) ω

4. Let $A = \begin{pmatrix} 0 & 0 & -1 \\ 0 & -1 & 0 \\ -1 & 0 & 0 \end{pmatrix}$. The only correct statement about the matrix A is **[2004]**
   (a) $A^2 = I$
   (b) $A = (-1)I$, where I is a unit matrix
   (c) A^{-1} does not exist
   (d) A is a zero matrix

5. Let $A = \begin{pmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{pmatrix}$ and $10\,B = \begin{pmatrix} 4 & 2 & 2 \\ -5 & 0 & \alpha \\ 1 & -2 & 3 \end{pmatrix}$.
   If B is the inverse of matrix A, then α is **[2004]**
   (a) 5 (b) –1
   (c) 2 (d) –2

6. If $a_1, a_2, a_3, ..., a_n, ...$ are in G.P., then the value of the determinant **[2004]**
   $$\begin{vmatrix} \log a_n & \log a_{n+1} & \log a_{n+2} \\ \log a_{n+3} & \log a_{n+4} & \log a_{n+5} \\ \log a_{n+6} & \log a_{n+7} & \log a_{n+8} \end{vmatrix}, \text{ is}$$
   (a) –2 (b) 1
   (c) 2 (d) 0

7. The system of equations
   $\alpha x + y + z = \alpha - 1$
   $x + \alpha y + z = \alpha - 1$
   $x + y + \alpha z = \alpha - 1$
   has infinite solutions, if α is **[2005]**
   (a) – 2 (b) either – 2 or 1
   (c) not – 2 (d) 1

8. If $a^2+b^2+c^2=-2$ and **[2005]**

$$f(x)=\begin{vmatrix} 1+a^2x & (1+b^2)x & (1+c^2)x \\ (1+a^2)x & 1+b^2x & (1+c^2)x \\ (1+a^2)x & (1+b^2)x & 1+c^2x \end{vmatrix},$$

then f(x) is a polynomial of degree

(a) 1 (b) 0
(c) 3 (d) 2

9. If $a_1, a_2, a_3, ..., a_n, ...$ are in G. P., then the determinant

$$\Delta=\begin{vmatrix} \log a_n & \log a_{n+1} & \log a_{n+2} \\ \log a_{n+3} & \log a_{n+4} & \log a_{n+5} \\ \log a_{n+6} & \log a_{n+7} & \log a_{n+8} \end{vmatrix}$$

is equal to **[2005]**

(a) 1 (b) 0
(c) 4 (d) 2

10. If $A^2-A+I=0$, then the inverse of A is **[2005]**

(a) $A+I$ (b) A
(c) $A-I$ (d) $I-A$

11. If $D=\begin{vmatrix} 1 & 1 & 1 \\ 1 & 1+x & 1 \\ 1 & 1 & 1+y \end{vmatrix}$ for $x \neq 0, y \neq 0$, then D is **[2007]**

(a) divisible by x but not y
(b) divisible by y but not x
(c) divisible by neither x nor y
(d) divisible by both x and y

12. Let $A=\begin{vmatrix} 5 & 5\alpha & \alpha \\ 0 & \alpha & 5\alpha \\ 0 & 0 & 5 \end{vmatrix}$. If $|A^2|=25$, then $|\alpha|$ equals **[2007]**

(a) 1/5 (b) 5
(c) 5^2 (d) 1

13. Let A be a 2×2 matrix with real entries. Let I be the 2×2 identity matrix. Denote by tr(A), the sum of diagonal entries of A. Assume that $A^2=I$. **[2008]**

Statement-1 : If $A \neq I$ and $A \neq -I$, then det $(A)=-1$

Statement-2 : If $A \neq I$ and $A \neq -I$, then tr $(A) \neq 0$.

(a) Statement -1 is false, Statement-2 is true
(b) Statement -1 is true, Statement-2 is true; Statement -2 is a correct explanation for Statement-1
(c) Statement -1 is true, Statement-2 is true; Statement -2 is not a correct explanation for Statement-1
(d) Statement -1 is true, Statement-2 is false

14. Let a, b, c be any real numbers. Suppose that there are real numbers x, y, z not all zero such that $x=cy+bz$, $y=az+cx$, and $z=bx+ay$. Then $a^2+b^2+c^2+2abc$ is equal to **[2008]**

(a) 2 (b) –1
(c) 0 (d) 1

15. Let A be a square matrix all of whose entries are integers. Then which one of the following is true? **[2008]**

(a) If det $A=\pm 1$, then A^{-1} exists but all its entries are not necessarily integers
(b) If det $A \neq \pm 1$, then A^{-1} exists and all its entries are non integers
(c) If det $A=\pm 1$, then A^{-1} exists but all its entries are integers
(d) If det $A=\pm 1$, then A^{-1} need not exists

16. Let A be a 2×2 matrix

Statement -1 : adj (adj A) = A

Statement -2 : |adj A| = |A| **[2009]**

(a) Statement-1 is true, Statement-2 is true. Statement-2 is not a correct explanation for Statement-1.
(b) Statement-1 is true, Statement-2 is false.
(c) Statement -1 is false, Statement-2 is true.
(d) Statement-1 is true, Statement -2 is true. Statement-2 is a correct explanation for Statement-1.

17. Let a, b, c be such that $b(a+c) \neq 0$ if **[2009]**

$$\begin{vmatrix} a & a+1 & a-1 \\ -b & b+1 & b-1 \\ c & c-1 & c+1 \end{vmatrix}+$$

$$\begin{vmatrix} a+1 & b+1 & c-1 \\ a-1 & b-1 & c+1 \\ (-1)^{n+2}a & (-1)^{n+1}b & (-1)^n c \end{vmatrix} = 0,$$

then the value of n is :

(a) any even integer
(b) any odd integer
(c) any integer
(d) zero

18. Let A be a 2×2 matrix with non-zero entries and let $A^2 = I$, where I is 2×2 identity matrix. Define $\text{Tr}(A)$ = sum of diagonal elements of A and $|A|$ = determinant of matrix A.

Statement - 1 : $\text{Tr}(A) = 0$.

Statement -2 : $|A| = 1$. **[2010]**

(a) Statement -1 is true, Statement -2 is true ; Statement -2 is **not** a correct explanation for Statement -1.
(b) Statement -1 is true, Statement -2 is false.
(c) Statement -1 is false, Statement -2 is true .
(d) Statement - 1 is true, Statement 2 is true ; Statement -2 is a correct explanation for Statement -1.

19. Consider the system of linear equations; **[2010]**

$x_1 + 2x_2 + x_3 = 3$
$2x_1 + 3x_2 + x_3 = 3$
$3x_1 + 5x_2 + 2x_3 = 1$

The system has

(a) exactly 3 solutions
(b) a unique solution
(c) no solution
(d) infinite number of solutions

20. The number of values of k for which the linear equations $4x + ky + 2z = 0$, $kx + 4y + z = 0$ and $2x + 2y + z = 0$ possess a non-zero solution is **[2011]**

(a) 2 (b) 1
(c) zero (d) 3

21. If the trivial solution is the only solution of the system of equations **[2011RS]**

$x - ky + z = 0$

$kx + 3y - kz = 0$

$3x + y - z = 0$

then the set of all values of k is :

(a) $R - \{2, -3\}$ (b) $R - \{2\}$
(c) $R - \{-3\}$ (d) $\{2, -3\}$

22 **Statement - 1:**
Determinant of a skew-symmetric matrix of order 3 is zero.
Statement - 2 :
For any matrix A, $\det(A)^T = \det(A)$ and $\det(-A) = -\det(A)$.
Where det (B) denotes the determinant of matrix B. Then : **[2011RS]**

(a) Both statements are true
(b) Both statements are false
(c) Statement-1 is false and statement-2 is true
(d) Statement-1 is true and statement-2 is false

23. Consider the following relation R on the set of real square matrices of order 3. **[2011RS]**

$R = \{(A, B) | A = P^{-1}BP$ for some invertible matrix $P\}$

Statement-1 : R is equivalence relation.

Statement-2 : For any two invertible 3×3 matrices M and N, $(MN)^{-1} = N^{-1}M^{-1}$.

(a) Statement-1 is true, statement-2 is true and statement-2 is a correct explanation for statement-1.
(b) Statement-1 is true, statement-2 is true; statement-2 is not a correct explanation for statement-1.
(c) Statement-1 is true, stement-2 is false.
(d) Statement-1 is false, statement-2 is true.

24. Let $A = \begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 2 & 1 \end{pmatrix}$. If u_1 and u_2 are column matrices such that $Au_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$ and $Au_2 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$, then $u_1 + u_2$ is equal to : **[2012]**

(a) $\begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix}$ (b) $\begin{pmatrix} -1 \\ 1 \\ -1 \end{pmatrix}$

(c) $\begin{pmatrix} -1 \\ -1 \\ 0 \end{pmatrix}$ (d) $\begin{pmatrix} 1 \\ -1 \\ -1 \end{pmatrix}$

25. Let P and Q be 3×3 matrices $P \neq Q$. If $P^3 = Q^3$ and $P^2Q = Q^2P$ then determinant of $(P^2 + Q^2)$ is equal to : **[2012]**

(a) -2 (b) 1
(c) 0 (d) -1

26. The number of values of k, for which the system of equations :

$(k+1)x + 8y = 4k$
$kx + (k+3)y = 3k - 1$

has no solution, is **[2013]**

(a) infinite (b) 1
(c) 2 (d) 3

27. If $P = \begin{bmatrix} 1 & \alpha & 3 \\ 1 & 3 & 3 \\ 2 & 4 & 4 \end{bmatrix}$ is the adjoint of a 3×3 matrix A and $|A| = 4$, then α is equal to : **[2013]**

(a) 4 (b) 11
(c) 5 (d) 0

28. If $\alpha, \beta \neq 0$, and $f(n) = \alpha^n + \beta^n$ and

$$\begin{vmatrix} 3 & 1+f(1) & 1+f(2) \\ 1+f(1) & 1+f(2) & 1+f(3) \\ 1+f(2) & 1+f(3) & 1+f(4) \end{vmatrix} = K(1-\alpha)^2(1-\beta)^2(\alpha-\beta)^2,$$

then K is equal to: **[2014]**

(a) 1 (b) -1
(c) $\alpha\beta$ (d) $\frac{1}{\alpha\beta}$

29. If A is an 3×3 non-singular matrix such that $AA' = A'A$ and $B = A^{-1}A'$, then BB' equals: **[2014]**

(a) B^{-1} (b) $(B^{-1})'$
(c) $I + B$ (d) I

30. The set of all values of λ for which the system of linear equations : **[2015]**

$2x_1 - 2x_2 + x_3 = \lambda x_1$
$2x_1 - 3x_2 + 2x_3 = \lambda x_2$
$-x_1 + 2x_2 = \lambda x_3$

has a non-trivial solution,

(a) contains two elements.
(b) contains more than two elements
(c) is an empty set.
(d) is a singleton

31. The system of linear equations **[2016]**

$x + \lambda y - z = 0$
$\lambda x - y - z = 0$
$x + y - \lambda z = 0$

has a non-trivial solution for:

(a) exactly two values of λ.
(b) exactly three values of λ.
(c) infinitely many values of λ.
(d) exactly one value of λ.

32. Let k be an integer such that triangle with vertices (k, –3k), (5, k) and (–k, 2) has area 28 sq. units. Then the orthocentre of this triangle is at the point : **[2017]**

(a) $\left(2, \frac{1}{2}\right)$ (b) $\left(2, -\frac{1}{2}\right)$
(c) $\left(1, \frac{3}{4}\right)$ (d) $\left(1, -\frac{3}{4}\right)$

33. If S is the set of distinct values of 'b' for which the following system of linear equations **[2017]**

$x + y + z = 1$
$x + ay + z = 1$
$ax + by + z = 0$

has no solution, then S is :

(a) a singleton
(b) an empty set
(c) an infinite set
(d) a finite set containing two or more elements

34. Let ω be a complex number such that $2\omega + 1 = z$ where $z = \sqrt{-3}$. If $\begin{vmatrix} 1 & 1 & 1 \\ 1 & -\omega^2 - 1 & \omega^2 \\ 1 & \omega^2 & \omega^7 \end{vmatrix} = 3k$, then k is equal to : **[2017]**

(a) 1 (b) $-z$
(c) z (d) -1

35. If $\begin{vmatrix} x-4 & 2x & 2x \\ 2x & x-4 & 2x \\ 2x & 2x & x-4 \end{vmatrix} = (A + Bx)(x - A)^2$, then the ordered pair (A, B) is equal to :

(a) $(-4, 3)$ (b) $(-4, 5)$
(c) $(4, 5)$ (d) $(-4, -5)$

36. If the system of linear equations

$x + ky + 3z = 0$
$3x + ky - 2z = 0$
$2x + 4y - 3z = 0$

has a non-zero solution (x, y, z), then $\frac{xz}{y^2}$ is equal to :

(a) 10 (b) – 30
(c) 30 (d) – 10

37. The system of linear equations
$x+y+z=2$
$2x+3y+2z=5$
$2x+3y+(a^2-1)z=a+1$
(a) is inconsistent when $a=4$
(b) has a unique solution for $|a|=\sqrt{3}$
(c) has infinitely many solutions for $a=4$
(d) is inconsistent when $|a|=\sqrt{3}$

38. Let α and β be the roots of the equation $x^2+x+1=0$. Then for $y \neq 0$ in R,

$$\begin{vmatrix} y+1 & \alpha & \beta \\ \alpha & y+\beta & 1 \\ \beta & 1 & y+\alpha \end{vmatrix}$$ is equal to:

(a) $y(y^2-1)$ (b) $y(y^2-3)$
(c) y^3 (d) y^3-1

39. If the system of linear equations
$2x+2ay+az=0$
$2x+3by+bz=0$
$2x+4cy+cz=0,$
where $a, b, c \in \boldsymbol{R}$ are non-zero and distinct; has a non-zero solution, then:

(a) $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are in A.P. (b) a, b, c are in G.P.
(c) $a+b+c=0$ (d) a, b, c are in A.P.

Answer Key

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
(c)	(d)	(b)	(a)	(a)	(d)	(a)	(d)	(b)	(d)	(d)	(a)	(d)	(d)	(c)
16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
(a)	(b)	(b)	(c)	(a)	(a)	(d)	(b)	(d)	(c)	(b)	(b)	(a)	(d)	(a)
31	32	33	34	35	36	37	38	39						
(b)	(a)	(a)	(b)	(b)	(a)	(d)	(c)	(a)						

Solutions

1. (c) Given that $\begin{vmatrix} a & b & ax+b \\ b & c & bx+c \\ ax+b & bx+c & 0 \end{vmatrix}$

Applying $R_3 \to R_3-(xR_1+R_2)$;

$$= \begin{vmatrix} a & b & ax+b \\ b & c & bx+c \\ 0 & 0 & -(ax^2+2bx+c) \end{vmatrix}$$

$=(ax^2+2bx+c)(b^2-ac)=(+)(-)=$ –ve.

[Given that discriminant of $ax^2+2bx+c$ is –ve
$\therefore 4b^2-4ac<0 \Rightarrow b^2-ac<0$]

2. (d) For homogeneous system of equations to have non zero solution, $\Delta=0$

$$\begin{vmatrix} 1 & 2a & a \\ 1 & 3b & b \\ 1 & 4c & c \end{vmatrix}=0$$

Applying $C_2 \to C_2-2C_3$

$$\Rightarrow \begin{vmatrix} 1 & 0 & a \\ 1 & b & b \\ 1 & 2c & c \end{vmatrix}=0 \quad R_3 \to R_3-R_1, R_2 \to R_2-R_1$$

$$\Rightarrow \begin{vmatrix} 1 & 0 & a \\ 0 & b & b-a \\ 0 & 2c & c-a \end{vmatrix}=0$$

$\Rightarrow bc - ab = 2bc - 2ac$

$\Rightarrow \frac{2}{b} = \frac{1}{a} + \frac{1}{c}$

$\therefore a, b, c$ are in Harmonic Progression.

3. **(b)** $\Delta = \begin{vmatrix} 1 & \omega^n & \omega^{2n} \\ \omega^n & \omega^{2n} & 1 \\ \omega^{2n} & 1 & \omega^n \end{vmatrix}$

Expand through R_1

$= 1(\omega^{3n} - 1) - \omega^n(\omega^{2n} - \omega^{2n}) + \omega^{2n}(\omega^n - \omega^{4n})$

$= \omega^{3n} - 1 - 0 + \omega^{3n} - \omega^{6n}$

$= 1 - 1 + 1 - 1 = 0 \left[\because \omega^{3n} = 1\right]$

4. **(a)** Given that $A = \begin{bmatrix} 0 & 0 & -1 \\ 0 & -1 & 0 \\ -1 & 0 & 0 \end{bmatrix}$

clearly $A \neq 0$. Also $|A| = -1 \neq 0$

$\therefore A^{-1}$ exists, further

$(-1)I = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix} \neq A$

Also $A^2 = \begin{bmatrix} 0 & 0 & -1 \\ 0 & -1 & 0 \\ -1 & 0 & 0 \end{bmatrix}\begin{bmatrix} 0 & 0 & -1 \\ 0 & -1 & 0 \\ -1 & 0 & 0 \end{bmatrix}$

$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I$

5. **(a)** Given that $10B = \begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & \alpha \\ 1 & -2 & 3 \end{bmatrix}$

$\Rightarrow B = \frac{1}{10}\begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & \alpha \\ 1 & -2 & 3 \end{bmatrix}$

Given that $B = A^{-1} \Rightarrow AB = I$

$\Rightarrow \frac{1}{10}\begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{bmatrix}\begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & \alpha \\ 1 & -2 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$\Rightarrow \frac{1}{10}\begin{bmatrix} 10 & 0 & 5-2 \\ 0 & 10 & -5+\alpha \\ 0 & 0 & 5+\alpha \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$\Rightarrow \frac{5-\alpha}{10} = 0 \Rightarrow \alpha = 5$

6. **(d)** Let r be the common ratio of an G.P., then

$\begin{vmatrix} \log a_n & \log a_{n+1} & \log a_{n+2} \\ \log a_{n+3} & \log a_{n+4} & \log a_{n+5} \\ \log a_{n+6} & \log a_{n+7} & \log a_{n+8} \end{vmatrix}$

$= \begin{vmatrix} \log a_1 r^{n-1} & \log a_1 r^n & \log a_1 r^{n+1} \\ \log a_1 r^{n+2} & \log a_1 r^{n+3} & \log a_1 r^{n+4} \\ \log a_1 r^{n+5} & \log a_1 r^{n+6} & \log a_1 r^{n+7} \end{vmatrix}$

$= \begin{vmatrix} \log a_1 + (n-1)\log r & \log a_1 + n\log r & \log a_1 + (n+1)\log r \\ \log a_1 + (n+2)\log r & \log a_1 + (n+3)\log r & \log a_1 + (n+4)\log r \\ \log a_1 + (n+5)\log r & \log a_1 + (n+6)\log r & \log a_2 + (n+7)\log r \end{vmatrix}$

Applying $C_3 \to C_3 + C_1$, we get

$= \begin{vmatrix} \log a_1 + (n-1)\log r & \log a_1 + n\log r & 2[\log a_1 + n\log r] \\ \log a_1 + (n+2)\log r & \log a_1 + (n+3)\log r & 2[\log a_1 + (n+3)\log r] \\ \log a_1 + (n+5)\log r & \log a_1 + (n+6)\log r & 2[\log a_1 + (n+6)\log r] \end{vmatrix}$

$= 0$

7. **(a)** $\alpha x + y + z = \alpha - 1;$
$x + \alpha y + z = \alpha - 1;$
$x + y + z\alpha = \alpha - 1$

$\Delta = \begin{vmatrix} \alpha & 1 & 1 \\ 1 & \alpha & 1 \\ 1 & 1 & \alpha \end{vmatrix}$

$= \alpha(\alpha^2 - 1) - 1(\alpha - 1) + 1(1 - \alpha)$

$= \alpha(\alpha - 1)(\alpha + 1) - 1(\alpha - 1) - 1(\alpha - 1)$

$= (\alpha - 1)[\alpha^2 + \alpha - 1 - 1]$

$= (\alpha - 1)[\alpha^2 + \alpha - 2]$

$= (\alpha - 1)\ [\alpha^2 + 2\alpha - \alpha - 2]$

$= (\alpha - 1)[\alpha(\alpha + 2) - 1(\alpha + 2)]$

$= (\alpha - 1)^2(\alpha + 2)$

$\because$ Equations has infinite solutions

$\therefore \Delta = 0$

$\Rightarrow \quad (\alpha - 1) = 0,\ \alpha + 2 = 0$

$\Rightarrow \quad \alpha = -2, 1;$

But $\alpha \neq 1$.

$\therefore \alpha = -2$

8. (d) Applying, $C_1 \to C_1 + C_2 + C_3$, we get

$$f(x) = \begin{vmatrix} 1+(a^2+b^2+c^2+2)x & (1+b^2)x & (1+c^2)x \\ 1+(a^2+b^2+c^2+2)x & 1+b^2x & (1+c^2)x \\ 1+(a^2+b^2+c^2+2)x & (1+b^2)x & 1+c^2x \end{vmatrix}$$

$[\because a^2+b^2+c^2 = -2]$

$$= \begin{vmatrix} 1 & (1+b^2)x & (1+c^2)x \\ 1 & 1+b^2x & (1+c^2)x \\ 1 & (1+b^2)x & 1+c^2x \end{vmatrix}$$

Applying, $R_2 \to R_2 - R_1,\ R_3 \to R_3 - R_1$

$$\therefore f(x) = \begin{vmatrix} 1 & (1+b^2)x & (1+c^2)x \\ 0 & 1-x & 0 \\ 0 & 0 & 1-x \end{vmatrix}$$

$f(x) = (x-1)^2$

Hence degree = 2.

9. (b) Let r be the common ratio of an G.P., then

$$\begin{vmatrix} \log a_n & \log a_{n+1} & \log a_{n+2} \\ \log a_{n+3} & \log a_{n+4} & \log a_{n+5} \\ \log a_{n+6} & \log a_{n+7} & \log a_{n+8} \end{vmatrix}$$

$$= \begin{vmatrix} \log a_1 r^{n-1} & \log a_1 r^n & \log a_1 r^{n+1} \\ \log a_1 r^{n+2} & \log a_1 r^{n+3} & \log a_1 r^{n+4} \\ \log a_1 r^{n+5} & \log a_1 r^{n+6} & \log a_1 r^{n+7} \end{vmatrix}$$

$$= \begin{vmatrix} \log a_1 + (n-1)\log r & \log a_1 + n\log r & \log a_1 + (n+1)\log r \\ \log a_1 + (n+2)\log r & \log a_1 + (n+3)\log r & \log a_1 + (n+4)\log r \\ \log a_1 + (n+5)\log r & \log a_1 + (n+6)\log r & \log a_2 + (n+7)\log r \end{vmatrix}$$

Applying $C_3 \to C_3 + C_1$, we get

$$= \begin{vmatrix} \log a_1 + (n-1)\log r & \log a_1 + n\log r & 2[\log a_1 + n\log r] \\ \log a_1 + (n+2)\log r & \log a_1 + (n+3)\log r & 2[\log a_1 + (n+3)\log r] \\ \log a_1 + (n+5)\log r & \log a_1 + (n+6)\log r & 2[\log a_1 + (n+6)\log r] \end{vmatrix}$$

$= 0$

10. (d) Given that $A^2 - A + I = 0$

Pre-multiply by A^{-1} both side, we get

$A^{-1}A^2 - A^{-1}A + A^{-1}.I = A^{-1}.0$

$\Rightarrow A - I + A^{-1} = 0$ or $A^{-1} = I - A$.

11. (d) Given that, $D = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 1+x & 1 \\ 1 & 1 & 1+y \end{vmatrix}$

Applying $R_2 \to R_2 - R_1$ and $R_3 \to R_3 - R_1$

$\therefore D = \begin{vmatrix} 1 & 1 & 1 \\ 0 & x & 0 \\ 0 & 0 & y \end{vmatrix} = xy$

Hence, D is divisible by both x and y

12. (a) Given that $A = \begin{bmatrix} 5 & 5\alpha & \alpha \\ 0 & \alpha & 5\alpha \\ 0 & 0 & 5 \end{bmatrix}$ and $|A^2| = 25$

$\therefore A^2 = \begin{bmatrix} 5 & 5\alpha & \alpha \\ 0 & \alpha & 5\alpha \\ 0 & 0 & 5 \end{bmatrix}\begin{bmatrix} 5 & 5\alpha & \alpha \\ 0 & \alpha & 5\alpha \\ 0 & 0 & 5 \end{bmatrix}$

$= \begin{bmatrix} 25 & 25\alpha + 5\alpha^2 & 5\alpha + 25\alpha^2 + 5\alpha \\ 0 & \alpha^2 & 5\alpha^2 + 25\alpha \\ 0 & 0 & 25 \end{bmatrix}$

$\therefore |A^2| = 25\ (25\alpha^2)$

$\therefore 25 = 25\ (25\alpha^2) \Rightarrow |\alpha| = \frac{1}{5}$

13. (d) Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

Given that $A^2 = \mathrm{I}$

$\Rightarrow \quad \begin{bmatrix} a^2 + bc & ab + bd \\ ac + cd & bc + d^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\Rightarrow \quad a^2 + bc = 1$ and $ab + bd = 0$

$ac + cd = 0$ and $bc + d^2 = 1$

From these four equations,

$a^2 + bc = bc + d^2 \Rightarrow a^2 = d^2$

and $\quad b(a+d) = 0 = c(a+d) \Rightarrow a = -d$

$|A| = ad - bc = -a^2 - bc = -1$

Also if $A \neq I$ then $tr(A) = a + d = 0$

$\therefore$ Statement 2 is false.

14. (d) The given equations are

$-x + cy + bz = 0$

$cx - y + az = 0$

$bx + ay - z = 0$

Given that x, y, z are not all zero

$\therefore$ The above system have non-zero solution

$$\Rightarrow \Delta = 0 \Rightarrow \begin{vmatrix} -1 & c & b \\ c & -1 & a \\ b & a & -1 \end{vmatrix} = 0$$

$$\Rightarrow -1(1-a^2) - c(-c-ab) + b(ac+b) = 0$$
$$\Rightarrow -1 + a^2 + b^2 + c^2 + 2abc = 0$$
$$\Rightarrow a^2 + b^2 + c^2 + 2abc = 1$$

15. (c) Given that all entries of square matrix A are integers, therefore all cofactors should also be integers.
If det $A = \pm 1$ then A^{-1} exists. Also all entries of A^{-1} are integers.

16. (a) We know that if A is square matrix of order n then
$adj\,(adj\,A) = |A|^{n-2} A.$
$= |A|^0 A = A$
Also $|adj\,A| = |A|^{n-1} = |A|^{2-1} = |A|$
$\therefore$ Both the statements are true but statement-2 is not a correct explanation for statement-1 .

17. (b) Given that $\begin{vmatrix} a & a+1 & a-1 \\ -b & b+1 & b-1 \\ c & c-1 & c+1 \end{vmatrix} +$

$$\begin{vmatrix} a+1 & b+1 & c-1 \\ a-1 & b-1 & c+1 \\ (-1)^{n+2} a & (-1)^{n+1} b & (-1)^n c \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} a & a+1 & a-1 \\ -b & b+1 & b-1 \\ c & c-1 & c+1 \end{vmatrix} +$$

$$\begin{vmatrix} a+1 & a-1 & (-1)^{n+2} a \\ b+1 & b-1 & (-1)^{n+1} b \\ c-1 & c+1 & (-1)^n c \end{vmatrix} = 0$$

(Taking transpose of second determinant)

Applying $C_1 \Leftrightarrow C_3$ in second determinant

$$\Rightarrow \begin{vmatrix} a & a+1 & a-1 \\ -b & b+1 & b-1 \\ c & c-1 & c+1 \end{vmatrix} -$$

$$\begin{vmatrix} (-1)^{n+2} a & a-1 & a+1 \\ (-1)^{n+2}(-b) & b-1 & b+1 \\ (-1)^{n+2} c & c+1 & c-1 \end{vmatrix} = 0$$

Applying $C_2 \Leftrightarrow C_3$ in second determinant

$$\Rightarrow \begin{vmatrix} a & a+1 & a-1 \\ -b & b+1 & b-1 \\ c & c-1 & c+1 \end{vmatrix}$$

$$+(-1)^{n+2} \begin{vmatrix} a & a+1 & a-1 \\ -b & b+1 & b-1 \\ c & c-1 & c+1 \end{vmatrix} = 0$$

$$\Rightarrow \left[1+(-1)^{n+2}\right] \begin{vmatrix} a & a+1 & a-1 \\ -b & b+1 & b-1 \\ c & c-1 & c+1 \end{vmatrix} = 0$$

Applying $C_2 \to C_2 - C_1, C_3 \to C_3 - C_1$

$$\Rightarrow \left[1+(-1)^{n+2}\right] \begin{vmatrix} a & 1 & -1 \\ -b & 2b+1 & 2b-1 \\ c & -1 & 1 \end{vmatrix} = 0$$

Applying $R_1 \to R_1 + R_3$

$$\Rightarrow \left[1+(-1)^{n+2}\right] \begin{vmatrix} a+c & 0 & 0 \\ -b & 2b+1 & 2b-1 \\ c & -1 & 1 \end{vmatrix} = 0$$

$$\Rightarrow [1+(-1)^{n+2}](a+c)(2b+1+2b-1) = 0$$

$$\Rightarrow 4b(a+c)[1+(-1)^{n+2}] = 0$$
$\Rightarrow 1+(-1)^{n+2} = 0$ but given that $b(a+c) \neq 0$
$\Rightarrow$ n should be an odd integer.

18. (b) Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ where a, b, c, d $\neq 0$

$$A^2 = \begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} a & b \\ c & d \end{bmatrix} = I$$

$$\Rightarrow A^2 = \begin{bmatrix} a^2+bc & ab+bd \\ ac+cd & bc+d^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow a^2 + bc = 1, bc + d^2 = 1$$
$$ab + bd = ac + cd = 0$$

$c \neq 0$ and $b \neq 0 \Rightarrow a+d=0$

$|A| = ad - bc = -a^2 - bc = -1$

Also if $A \neq I$, then $tr(A) = a + d = 0$.

$\therefore$ Statement-1 true and statement-2 false.

19. **(c)** $D = \begin{vmatrix} 1 & 2 & 1 \\ 2 & 3 & 1 \\ 3 & 5 & 2 \end{vmatrix} = 0$

$D_x = \begin{vmatrix} 3 & 2 & 1 \\ 3 & 3 & 1 \\ 1 & 5 & 2 \end{vmatrix} \neq 0$

$\Rightarrow$ Given system, does not have any solution.

$\Rightarrow$ No solution

20. **(a)** Given that system of equations have non-zero solution $\Delta = 0$

$\Rightarrow \begin{vmatrix} 4 & k & 2 \\ k & 4 & 1 \\ 2 & 2 & 1 \end{vmatrix} = 0$

$\Rightarrow 4(4-2) - k(k-2) + 2(2k-8) = 0$

$\Rightarrow 8 - k^2 + 2k + 4k - 16 = 0$

$k^2 - 6k + 8 = 0$

$\Rightarrow (k-4)(k-2) = 0 \Rightarrow k = 4, 2$

21. **(a)** $x - ky + z = 0$

$kx + 3y - kz = 0$

$3x + y - z = 0$

The given that system of equations have trivial solution,

$\therefore \begin{vmatrix} 1 & -k & 1 \\ k & 3 & -k \\ 3 & 1 & -1 \end{vmatrix} \neq 0$

$\Rightarrow 1(-3+k) + k(-k+3k) + 1(k-9) \neq 0$

$\Rightarrow k - 3 + 2k^2 + k - 9 \neq 0$

$\Rightarrow k^2 + k - 6 \neq 0$

$\Rightarrow k = -3, k \neq 2$

So, the equation will have only trivial solution,

when $k \in R - \{2, -3\}$

22. **(d)** We know that determinant of skew symmetric matrix of odd order is zero.

So, statement-1 is true.

We know that $\det\left(A^T\right) = \det(A)$.

$\det(-A) = -(-1)^n \det(A)$.

where A is a $n \times n$ order matrix.

So, statement-2 is false.

23. **(b)** **For reflexive**

$A = P^{-1}AP$ is true,

For $P = I$, which is an invertible matrix.

$(A, A) \in R$

$\therefore$ R is reflexive.

For symmetry

As $(A, B) \in R$ for matrix P

$A = P^{-1}BP$

$\Rightarrow PAP^{-1} = B$

$\Rightarrow B = PAP^{-1}$

$\Rightarrow B = \left(P^{-1}\right)^{-1} A (P^{-1})$

$\therefore$ $(B, A) \in R$ for matrix P^{-1}

$\therefore$ R is symmetric.

For transitivity

$A = P^{-1}BP$

and $B = P^{-1}CP$

$\Rightarrow A = P^{-1}\left(P^{-1}CP\right)P$

$\Rightarrow A = \left(P^{-1}\right)^2 CP^2$

$\Rightarrow A = \left(P^2\right)^{-1} C\left(P^2\right)$

$\therefore$ $(A, C) \in R$ for matrix P^2

$\therefore$ R is transitive.

So R is equivalence.

So, statement-1 is true.

We know that if A and B are two invertible matrices of order n, then

$(AB)^{-1} = B^{-1} A^{-1}$

So, statement-2 is true.

24. **(d)** Let $Au_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$ and $Au_2 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$

Then, $Au_1 + Au_2 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$

$$\Rightarrow \quad A(u_1+u_2)=\begin{pmatrix}1\\1\\0\end{pmatrix} \quad ...(1)$$

Given that $A=\begin{pmatrix}1&0&0\\2&1&0\\3&2&1\end{pmatrix}$

$\Rightarrow \quad |A| = 1(1) - 0\,(2) + 0\,(4-3) = 1$

$C_{11}=1 \quad C_{21}=0 \quad C_{31}=0$
$C_{12}=-2 \quad C_{22}=1 \quad C_{32}=0$
$C_{13}=1 \quad C_{23}=-2 \quad C_{33}=1$

$$\therefore adjA=\begin{bmatrix}1&0&0\\-2&0&0\\1&-2&1\end{bmatrix}$$

We know,

$$A^{-1}=\frac{1}{|A|}adjA$$

$\Rightarrow A^{-1}=adj(A) \quad (\because |A|=1)$

Now, from equation (1), we have

$$u_1+u_2=A^{-1}\begin{pmatrix}1\\1\\0\end{pmatrix}$$

$$=\begin{bmatrix}1&0&0\\-2&1&0\\1&-2&1\end{bmatrix}\begin{pmatrix}1\\1\\0\end{pmatrix}=\begin{bmatrix}1\\-1\\-1\end{bmatrix}$$

25. (c) Given that $P^3 = Q^3$ ...(1)
and $P^2Q = Q^2P$ ...(2)
Subtracting (1) and (2), we get
$P^3 - P^2Q = Q^3 - Q^2P$
$\Rightarrow \quad P^2(P-Q) + Q^2(P-Q) = 0$
$\Rightarrow \quad (P^2+Q^2)(P-Q) = 0$
$\because P \neq Q, \therefore P^2 + Q^2 = 0$
Hence $|P^2 + Q^2| = 0$

26. (b) Since, system of equations have no solution

$$\therefore \frac{k+1}{k}=\frac{8}{k+3}\neq\frac{4k}{3k-1}$$
($\because$ System has no solution)

$\Rightarrow \quad k^2+4k+3=8k \Rightarrow k^2-4k+3=0$
$\Rightarrow \quad k=1,3$

If $k=1$ then $\frac{8}{1+3}\neq\frac{4.1}{2}$ which is false

and if $k=3$ then $\frac{8}{6}\neq\frac{4.3}{9-1}$ which is true,

therefore $k=3$
Hence for only one value of k. System has no solution.

27. (b) $|P| = 1(12-12) - \alpha(4-6) + 3(4-6) = 2\alpha - 6$
Now, adj A = P $\Rightarrow |\text{adj } A| = |P|$
$\Rightarrow |A|^2 = |P|$
$\Rightarrow |P| = 16$
$\Rightarrow \quad 2\alpha - 6 = 16$
$\Rightarrow \quad \alpha = 11$

28. (a) Consider

$$\begin{vmatrix}3&1+f(1)&1+f(2)\\1+f(1)&1+f(2)&1+f(3)\\1+f(2)&1+f(3)&1+f(4)\end{vmatrix}$$

$$=\begin{vmatrix}1+1+1&1+\alpha+\beta&1+\alpha^2+\beta^2\\1+\alpha+\beta&1+\alpha^2+\beta^2&1+\alpha^3+\beta^3\\1+\alpha^2+\beta^2&1+\alpha^3+\beta^3&1+\alpha^4+\beta^4\end{vmatrix}$$

$$=\begin{vmatrix}1&1&1\\1&\alpha&\beta\\1&\alpha^2&\beta^2\end{vmatrix}\times\begin{vmatrix}1&1&1\\1&\alpha&\beta\\1&\alpha^2&\beta^2\end{vmatrix}$$
$[\because |A| = |A^1|]$

$$=\begin{vmatrix}1&1&1\\1&\alpha&\beta\\1&\alpha^2&\beta^2\end{vmatrix}^2$$

$= [(1-\alpha)(1-\beta)(\alpha-\beta)]^2$

So, $\boxed{K=1}$

29. (d) $BB' = B(A^{-1}A')' = B(A')'(A^{-1})'$
$= BA\,(A^{-1})'$
$= (A^{-1}A')(A(A^{-1})')$
$= A^{-1}A\,.A'.(A^{-1})'$ $\quad$ {as $AA' = A'A$}
$= I(A^{-1}A)' = I.I = I^2 = I$

30. (a)
$$\left.\begin{aligned}2x_1-2x_2+x_3&=\lambda x_1\\2x_1-3x_2+2x_3&=\lambda x_2\\-x_1+2x_2&=\lambda x_3\end{aligned}\right\}$$

$\Rightarrow\quad (2-\lambda)x_1-2x_2+x_3=0$

$2x_1-(3+\lambda)x_2+2x_3=0$

$-x_1+2x_2-\lambda x_3=0$

For non-trivial solution,

$\Delta=0$

i.e. $\begin{vmatrix}2-\lambda & -2 & 1\\ 2 & -(3+\lambda) & 2\\ -1 & 2 & -\lambda\end{vmatrix}=0$

$\Rightarrow(2-\lambda)[\lambda(3+\lambda)-4]+2[-2\lambda+2]$
$+1[4-(3+\lambda)]=0$

$\Rightarrow\lambda^3+\lambda^2-5\lambda+3=0$

$\Rightarrow\lambda=1,1,3$

Hence λ has 2 values.

31. (b) For non-trivial solution,

$\begin{vmatrix}1 & \lambda & -1\\ \lambda & -1 & -1\\ 1 & 1 & -\lambda\end{vmatrix}=0$

$\Rightarrow-\lambda(\lambda+1)(\lambda-1)=0$

$\Rightarrow\lambda=0,+1,-1$

32. (a) Let A (k, –3k), B(5, k) and C(–k + 2), we have

$\frac{1}{2}\left\|\begin{matrix}k & -3k & 1\\ 5 & k & 1\\ -k & 2 & 1\end{matrix}\right\|=28$

$\Rightarrow5k^2+13k-46=0$

or $5k^2+13k+66=0$

Now, $5k^2+13k-46=0$

$\Rightarrow k=\frac{-13\pm\sqrt{1089}}{10}\ \therefore\ k=\frac{-23}{5};k=2$

since k is an integer, $\therefore$ k = 2

Also $5k^2+13k+66=0$

$\Rightarrow k=\frac{-13\pm\sqrt{-1151}}{10}$

So no real solution exist

A(2, –6), B(5, 2) and C(–2, 2)

For orthocentre H (α,β)

$BH\perp AC$

$\therefore\quad\left(\frac{\beta-2}{\alpha-5}\right)\left(\frac{8}{-4}\right)=-1$

$\Rightarrow\quad\alpha-2\beta=1$ ...(1)

Also $CH\perp AB$

$\therefore\quad\left(\frac{\beta-2}{\alpha+2}\right)\left(\frac{8}{3}\right)=-1$

$\Rightarrow\quad3\alpha+8\beta=1$ ... (2)

Solving (1) and (2), we get

$\alpha=2,\beta=\frac{1}{2}$

orthocentre is $\left(2,\frac{1}{2}\right)$

33. (a) $D=\begin{vmatrix}1 & 1 & 1\\ 1 & a & 1\\ a & b & 1\end{vmatrix}=0$

$\Rightarrow1[a-b]-1[1-a]+1[b-a^2]=0$

$\Rightarrow(a-1)^2=0$

$\Rightarrow a=1$

For $a=1$, First two equations are identical i.e., $x+y+z=1$

To have no solution with $x+by+z=0$

$b=1$

So $b=\{1\}\Rightarrow$ It is singleton set.

34. (b) Given $2\omega+1=z$;

and $z=\sqrt{3}i\Rightarrow\omega=\frac{\sqrt{3}i-1}{2}$

$\Rightarrow\omega$ is complex cube root of unity

Applying $R_1\to R_1+R_2+R_3$

$=\begin{vmatrix}3 & 0 & 0\\ 1 & -\omega^2-1 & \omega^2\\ 1 & \omega^2 & \omega\end{vmatrix}$

$=3(-1-\omega-\omega)=-3(1+2\omega)=-3z$

$\Rightarrow k=-z$

35. (b) Here, $\begin{vmatrix}x-4 & 2x & 2x\\ 2x & x-4 & 2x\\ 2x & 2x & x-4\end{vmatrix}=(A+Bx)(x-A)^2$

Put $x=0\Rightarrow\begin{vmatrix}-4 & 0 & 0\\ 0 & -4 & 0\\ 0 & 0 & -4\end{vmatrix}=A^3\Rightarrow A^3=(-4)^3$

$\Rightarrow A=-4$

$\Rightarrow\begin{vmatrix}x-4 & 2x & 2x\\ 2x & x-4 & 2x\\ 2x & 2x & x-4\end{vmatrix}=(Bx-4)(x+4)^2$

Now take x common from both the sides

$$\therefore \begin{vmatrix} 1-\frac{4}{x} & 2x & 2x \\ 2x & 1-\frac{4}{x} & 2x \\ 2x & 2x & 1-\frac{4}{x} \end{vmatrix} = (B-\frac{4}{x})(1+\frac{4}{x})^2$$

Now take $x \to \infty$, then $\frac{1}{x} \to 0$

$$\Rightarrow \begin{vmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{vmatrix} = B \Rightarrow B = 5$$

$\therefore$ ordered pair (A, B) is (–4, 5)

36. (a) For non-zero solution of the system of linear equations;

$$\begin{vmatrix} 1 & k & 3 \\ 3 & k & -2 \\ 2 & 4 & -3 \end{vmatrix} = 0$$

$\Rightarrow k = 11$

Now equations become

$x + 11y + 3z = 0$ ...(1)

$3x + 11y - 2z = 0$ ...(2)

$2x + 4y - 3z = 0$ ...(3)

Adding equations (1) & (3) we get

$3x + 15y = 0$

$\Rightarrow x = -5y$

Now put $x = -5y$ in equation (1), we get

$-5y + 11y + 3z = 0$

$\Rightarrow z = -2y$

$$\therefore \frac{xz}{y^2} = \frac{(-5y)(-2y)}{y^2} = 10$$

37. (d) Since the system of linear equations are

$x + y + z = 2$ ...(1)

$2x + 3y + 2z = 5$ ...(2)

$2x + 3y + (a^2 - 1)z = a + 1$ ...(3)

If $a^2 = 3$, then plane represented by eqn (2) and eqn (3) are $2x + 3y + 2z = 5$ and $2x + 3y + 2z = \pm\sqrt{3} + 1$ which are parallel, i.e., have no solution.

Hence, the given system of equations is inconsistent, for $|a| = \sqrt{3}$

38. (c) Let $\alpha = \omega$ and $\beta = \omega^2$ are roots of $x^2 + x + 1 = 0$

$$\& \text{ Let } \Delta = \begin{vmatrix} y+1 & \omega & \omega^2 \\ \omega & y+\omega^2 & 1 \\ \omega^2 & 1 & y+\omega \end{vmatrix}$$

Applying $C_1 \to C_1 + C_2 + C_3$, we get

$$\Delta = \begin{vmatrix} y+1+\omega+\omega^2 & \omega & \omega^2 \\ y+1+\omega+\omega^2 & y+\omega^2 & 1 \\ 1+\omega+\omega^2+y & 1 & y+\omega \end{vmatrix}$$

$$\Delta = \begin{vmatrix} y & \omega & \omega^2 \\ y & y+\omega^2 & 1 \\ y & 1 & y+\omega \end{vmatrix} \quad (\because 1+\omega+\omega^2 = 0)$$

$$\Delta = y\begin{vmatrix} 1 & \omega & \omega^2 \\ 1 & y+\omega^2 & 1 \\ 1 & 1 & y+\omega \end{vmatrix}$$

Applying $R_2 \to R_2 - R_1$ & $R_3 \to R_3 - R_1$, we get

$$\Delta = y\begin{vmatrix} y+\omega^2-\omega & 1-\omega^2 \\ 1-\omega & y+\omega-\omega^2 \end{vmatrix}$$

$\Rightarrow \Delta = y \times$

$[(y-(\omega-\omega^2))(y+(\omega-\omega^2)) - (1-\omega)(1-\omega^2)]$

$\Rightarrow \Delta = y[y^2 - (2-\omega^2)^2 - 1 + \omega^2 + \omega - \omega^3]$

$\Rightarrow \Delta = y\,[y^2 - \omega^2 - \omega^4 + 2\omega^3 - 1 + \omega^2 + \omega^4 - \omega^3]$

$(\because \omega^4 = \omega)$

$\Rightarrow \Delta = y\,(y^2) = y^3$

39. (a) For non-zero solution

$$\begin{vmatrix} 2 & 2a & a \\ 2 & 3b & b \\ 2 & 4c & c \end{vmatrix} = 0 \Rightarrow \begin{vmatrix} 1 & 2a & a \\ 1 & 3b & b \\ 1 & 4c & c \end{vmatrix} = 0$$

$\Rightarrow (3bc - 4bc) - (2ac - 4ac) + (2ab - 3ab) = 0$

$\Rightarrow -bc + 2ac - ab = 0$

$\Rightarrow ab + bc + 2ac$

$\Rightarrow \frac{2}{b} = \frac{1}{a} + \frac{1}{c} \Rightarrow \frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ in A.P.

20 Continuity and Differentiability

1. f is defined in [-5, 5] as **[2002]**
$f(x) = x$ if x is rational
$= -x$ if x is irrational. Then
(a) $f(x)$ is continuous at every x, except $x = 0$
(b) $f(x)$ is discontinuous at every x, except $x = 0$
(c) $f(x)$ is continuous everywhere
(d) $f(x)$ is discontinuous everywhere

2. If $f(x+y) = f(x).f(y) \forall x.y$ and $f(5) = 2$, $f'(0) = 3$, then $f'(5)$ is **[2002]**
(a) 0 (b) 1
(c) 6 (d) 2

3. If $y = (x + \sqrt{1+x^2})^n$, then $(1+x^2)\dfrac{d^2y}{dx^2} + x\dfrac{dy}{dx}$ is
(a) n^2y (b) $-n^2y$ **[2002]**
(c) $-y$ (d) $2x^2y$

4. Let $f(a) = g(a) = k$ and their nth derivatives $f^n(a), g^n(a)$ exist and are not equal for some n. Further if
$$\lim_{x \to a} \frac{f(a)g(x) - f(a) - g(a)f(x) + f(a)}{g(x) - f(x)} = 4$$
then the value of k is **[2003]**
(a) 0 (b) 4
(c) 2 (d) 1

5. If $f(x) = \begin{cases} xe^{-\left(\frac{1}{|x|}+\frac{1}{x}\right)} & , x \neq 0 \\ 0 & , x = 0 \end{cases}$ then f(x) is **[2003]**
(a) discontinuous every where
(b) continuous as well as differentiable for all x
(c) continuous for all x but not differentiable at $x = 0$
(d) neither differentiable nor continuous at $x = 0$

6. If $f(x) = x^n$, then the value of **[2003]**
$$f(1) - \frac{f'(1)}{1!} + \frac{f''(1)}{2!} - \frac{f'''(1)}{3!} + \ldots\ldots\ldots \frac{(-1)^n f^n(1)}{n!}$$ is
(a) 1 (b) 2^n
(c) $2^n - 1$ (d) 0

7. Let $f(x)$ be a polynomial function of second degree. If $f(1) = f(-1)$ and a, b, c are in A. P, then $f'(a), f'(b), f'(c)$ are in **[2003]**
(a) Arithmetic -Geometric Progression
(b) A.P
(c) G.P
(d) H.P.

8. Let $f(x) = \dfrac{1 - \tan x}{4x - \pi}, x \neq \dfrac{\pi}{4}, x \in \left[0, \dfrac{\pi}{2}\right]$.
If $f(x)$ is continuous in $\left[0, \dfrac{\pi}{2}\right]$, then $f\left(\dfrac{\pi}{4}\right)$ is **[2004]**
(a) -1 (b) $\dfrac{1}{2}$
(c) $-\dfrac{1}{2}$ (d) 1

9. If $x = e^{y+e^{y+\cdots \text{to} \infty}}, x > 0$, then $\frac{dy}{dx}$ is **[2004]**

(a) $\frac{1+x}{x}$ (b) $\frac{1}{x}$

(c) $\frac{1-x}{x}$ (d) $\frac{x}{1+x}$

10. Suppose $f(x)$ is differentiable at $x = 1$ and $\lim_{h \to 0} \frac{1}{h} f(1+h) = 5$, then $f'(1)$ equals **[2005]**

(a) 3 (b) 4

(c) 5 (d) 6

11. Let f be differentiable for all x. If $f(1) = -2$ and $f'(x) \geq 2$ for $x \in [1, 6]$, then **[2005]**

(a) $f(6) \geq 8$ (b) $f(6) < 8$

(c) $f(6) < 5$ (d) $f(6) = 5$

12. If f is a real valued differentiable function satisfying $|f(x) - f(y)| \leq (x-y)^2, x, y \in R$ and $f(0) = 0$, then $f(1)$ equals **[2005]**

(a) -1 (b) 0

(c) 2 (d) 1

13. The set of points where $f(x) = \frac{x}{1+|x|}$ is differentiable is **[2006]**

(a) $(-\infty, 0) \cup (0, \infty)$

(b) $(-\infty, -1) \cup (-1, \infty)$

(c) $(-\infty, \infty)$

(d) $(0, \infty)$

14. If $x^m . y^n = (x+y)^{m+n}$, then $\frac{dy}{dx}$ is **[2006]**

(a) $\frac{y}{x}$ (b) $\frac{x+y}{xy}$

(c) xy (d) $\frac{x}{y}$

15. Let $f : R \to R$ be a function defined by $f(x) = \min \{x+1, |x|+1\}$. Then which of the following is true?

(a) $f(x)$ is differentiable everywhere **[2007]**

(b) $f(x)$ is not differentiable at $x = 0$

(c) $f(x) \geq 1$ for all $x \in R$

(d) $f(x)$ is not differentiable at $x = 1$

16. The function $f : R - \{0\} \to R$ given by **[2007]**

$$f(x) = \frac{1}{x} - \frac{2}{e^{2x} - 1}$$

can be made continuous at $x = 0$ by defining $f(0)$ as

(a) 0 (b) 1

(c) 2 (d) -1

17. Let $f(x) = \begin{cases} (x-1)\sin \frac{1}{x-1} & \text{if } x \neq 1 \\ 0 & \text{if } x = 1 \end{cases}$ **[2008]**

Then which one of the following is true?

(a) f is neither differentiable at $x = 0$ nor at $x = 1$

(b) f is differentiable at $x = 0$ and at $x = 1$

(c) f is differentiable at $x = 0$ but not at $x = 1$

(d) f is differentiable at $x = 1$ but not at $x = 0$

18. Let y be an implicit function of x defined by $x^{2x} - 2x^x \cot y - 1 = 0$. Then $y'(1)$ equals **[2009]**

(a) 1 (b) $\log 2$

(c) $-\log 2$ (d) -1

19. Let $f : (-1, 1) \to$ R be a differentiable function with $f(0) = -1$ and $f'(0) = 1$. Let $g(x) = [f(2f(x)+2)]^2$. Then $g'(0) =$ **[2010]**

(a) -4 (b) 0

(c) -2 (d) 4

20. The values of p and q for which the function **[2011]**

$$f(x) = \begin{cases} \frac{\sin(p+1)x + \sin x}{x} & , x < 0 \\ q & , x = 0 \\ \frac{\sqrt{x+x^2} - \sqrt{x}}{x^{3/2}} & , x > 0 \end{cases}$$ is continuous for all x in R, are

(a) $p = \frac{5}{2}, q = \frac{1}{2}$

(b) $p = -\frac{3}{2}, q = \frac{1}{2}$

(c) $p = \frac{1}{2}, q = \frac{3}{2}$

(d) $p = \frac{1}{2}, q = -\frac{3}{2}$

21. $\frac{d^2x}{dy^2}$ equals : **[2011]**

(a) $-\left(\frac{d^2y}{dx^2}\right)^{-1}\left(\frac{dy}{dx}\right)^{-3}$

(b) $\left(\frac{d^2y}{dx^2}\right)\left(\frac{dy}{dx}\right)^{-2}$

(c) $-\left(\frac{d^2y}{dx^2}\right)\left(\frac{dy}{dx}\right)^{-3}$

(d) $\left(\frac{d^2y}{dx^2}\right)^{-1}$

22. Define $f(x)$ as the product of two real function **[2011RS]**

$$f_1(x)=x, x\in R, \text{and } f_2(x)=\begin{cases}\sin\frac{1}{x}, & \text{if } x\neq 0\\ 0, & \text{if } x=0\end{cases}$$

as follows :

$$f(x)=\begin{cases}f_1(x).f_2(x), & \text{if } x\neq 0\\ 0 & \text{if } x=0\end{cases}$$

Statement - 1 : $f(x)$ is continuous on R.

Statement - 2 : $f_1(x)$ and $f_2(x)$ are continuous on R.

(a) Statement -1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1.
(b) Statement-1 is true, Statement-2 is true; Statement-2 is NOT a correct explanation for Statement-1
(c) Statement-1 is true, Statement-2 is false
(d) Statement-1 is false, Statement-2 is true

23. If function $f(x)$ is differentiable at $x = a$,

then $\lim_{x\to a}\frac{x^2f(a)-a^2f(x)}{x-a}$ is : **[2011RS]**

(a) $-a^2f'(a)$

(b) $a f(a)-a^2f'(a)$

(c) $2af(a)-a^2f'(a)$

(d) $2af(a)+a^2f'(a)$

24. If $f : R \to R$ is a function defined by $f(x) = [x] \cos\left(\frac{2x-1}{2}\right)\pi$, where $[x]$ denotes the greatest integer function, then f is . **[2012]**
(a) continuous for every real x.
(b) discontinuous only at $x = 0$
(c) discontinuous only at non-zero integral values of x.
(d) continuous only at $x = 0$.

25. Consider the function, $f(x)=|x-2|+|x-5|$, $x\in R$.
Statement-1 : $f'(4)=0$
Statement-2 : f is continuous in [2,5], differentiable in (2,5) and $f(2)=f(5)$. **[2012]**
(a) Statement-1 is false, Statement-2 is true.
(b) Statement-1 is true, statement-2 is true; statement-2 is a correct explanation for Statement-1.
(c) Statement-1 is true, statement-2 is true; statement-2 is **not** a correct explanation for Statement-1.
(d) Statement-1 is true, statement-2 is false.

26. If $y = \sec(\tan^{-1}x)$, then $\frac{dy}{dx}$ at $x = 1$ is equal to :

(a) $\frac{1}{\sqrt{2}}$ (b) $\frac{1}{2}$ **[2013]**
(c) 1 (d) $\sqrt{2}$

27. If f and g are differentiable functions in [0, 1] satisfying $f(0) = 2 = g(1)$, $g(0) = 0$ and $f(1) = 6$, then for some $c \in]0,1[$ **[2014]**
(a) $f'(c)=g'(c)$ (b) $f'(c)=2g'(c)$
(c) $2f'(c)=g'(c)$ (d) $2f'(c)=3g'(c)$

28. If the function.

$$g(x)=\begin{cases}k\sqrt{x+1}, & 0\le x\le 3\\ mx+2, & 3<x\le 5\end{cases}$$ is differentiable,

then the value of k + m is : **[2015]**

(a) $\frac{10}{3}$ (b) 4

(c) 2 (d) $\frac{16}{5}$

29. For $x \in R$, $f(x) = |\log 2 - \sin x|$ and $g(x) = f(f(x))$, then : **[2016]**

(a) $g'(0) = -\cos(\log 2)$

(b) g is differentiable at x = 0 and $g'(0) = -\sin(\log 2)$

(c) g is not differentiable at x = 0

(d) $g'(0) = \cos(\log 2)$

30. Let $S = \{t \in R : f(x) = |x-\pi|(e^{|x|}-1)\sin|x|$ is not differentiable at t$\}$. Then the set S is equal to : **[2018]**

(a) {a} (b) {b}

(c) $\{0, \pi\}$ (d) ϕ (an empty set)

31. Let $f: \mathbf{R} \to \mathbf{R}$ be a function defined as **[2019]**

$$f(x)=\begin{cases}5, & \text{if} \quad x\le 1\\ a+bx, & \text{if} \quad 1<x<3\\ b+5x, & \text{if} \quad 3\le x<5\\ 30, & \text{if} \quad x\ge 5\end{cases}$$

Then, f is :

(a) continuous if $a = 5$ and b = 5

(b) continuous if $a = -5$ and b = 10

(c) continous if $a = 0$ and b = 5

(d) not continuous for any values of a and b

32. If the function f defined on $\left(\frac{\pi}{6}, \frac{\pi}{3}\right)$ by **[2019]**

$$f(x)=\begin{cases}\dfrac{\sqrt{2}\cos x-1}{\cot x-1}, & x\ne\dfrac{\pi}{4}\\ k, & x=\dfrac{\pi}{4}\end{cases}$$

is continuous, then k is equal to:

(a) 2 (b) $\frac{1}{2}$ (c) 1 (d) $\frac{1}{\sqrt{2}}$

33. Let $f(x) = 15 - |x - 10|$; $x \in R$. Then the set of all values of x, at which the function, $g(x) = f(f(x))$ is not differentiable, is: **[2019]**

(a) {5, 10, 15} (b) {10, 15}

(c) {5, 10, 15, 20} (d) {10}

34. If $y(\alpha)=\sqrt{2\left(\dfrac{\tan\alpha+\cot\alpha}{1+\tan^2\alpha}\right)+\dfrac{1}{\sin^2\alpha}}$, **[2020]**

$\alpha \in \left(\frac{3\pi}{4}, \pi\right)$, then $\frac{dy}{d\alpha}$ at $\alpha = \frac{5\pi}{6}$ is:

(a) 4 (b) $\frac{4}{3}$

(c) –4 (d) $-\frac{1}{4}$

35. Let $x^k + y^k = a^k$, $(a, k > 0)$ and $\frac{dy}{dx}+\left(\frac{y}{x}\right)^{\frac{1}{3}}=0$, then k is: **[2020]**

(a) $\frac{3}{2}$ (b) $\frac{4}{3}$

(c) $\frac{2}{3}$ (d) $\frac{1}{3}$

36. Let the function, $f: [-7, 0] \to R$ be continuous on $[-7, 0]$ and differentiable on $(-7, 0)$. If $f(-7) = -3$ and $f'(x) \le 2$, for all $x \in (-7, 0)$, then for all such functions f, $f'(-1) + f(0)$ lies in the interval: **[2020]**

(a) $(-\infty, 20]$ (b) $[-3, 11]$

(c) $(-\infty, 11]$ (d) $[-6, 20]$

37. Let S be the set of points where the function, $f(x) = |2 - |x - 3||$, $x \in \boldsymbol{R}$, is not differentiable. Then $\sum_{x\in S} f(f(x))$ is equal to ______. **[2020]**

Answer Key

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
(b)	(c)	(a)	(b)	(c)	(d)	(b)	(c)	(c)	(c)	(a)	(b)	(c)	(a)	(a)
16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
(b)	(c)	(d)	(a)	(b)	(c)	(c)	(c)	(a)	(c)	(a)	(b)	(c)	(d)	(d)
31	32	33	34	35	36	37								
(d)	(b)	(a)	(a)	(c)	(a)	(3)								

Solutions

1. (b) Let a is a rational number other than 0, in $[-5, 5]$, then $f(a) = a$ and $\lim\limits_{x \to a} f(x) = -a$

$\therefore x \to a^-$ and $x \to a^+$ is tends to irrational number

$\therefore f(x)$ is discontinuous at any rational number

If a is irrational number, then

$f(a) = -a$ and $\lim\limits_{x \to a} f(x) = a$

$\therefore f(x)$ is not continuous at any irrational number. For $x = 0$, $\lim\limits_{x \to 0} f(x) = f(0) = 0$

$\therefore f(x)$ is continuous at $x = 0$

2. (c) Given that $f(x+y) = f(x) \times f(y)$

Differentiate with respect to x, treating y as constant

$f'(x+y) = f'(x) f(y)$

Putting $x = 0$ and $y = x$, we get $f'(x) = f'(0) f(x)$;

$\Rightarrow f'(5) = 3f(5) = 3 \times 2 = 6.$

3. (a) Given that $y = (x + \sqrt{1+x^2})^n$ ...(i)

Differentiating both sides w.r. to x

$$\frac{dy}{dx} = n(x + \sqrt{1+x^2})^{n-1}\left(1 + \frac{1}{2}(1+x^2)^{-1/2} \cdot 2x\right)$$

$$\frac{dy}{dx} = n(x + \sqrt{1+x^2})^{n-1} \frac{(\sqrt{1+x^2} + x)}{\sqrt{1+x^2}}$$

$$= \frac{n(\sqrt{1+x^2} + x)^n}{\sqrt{1+x^2}}$$

or $\sqrt{1+x^2}\dfrac{dy}{dx} = ny$ [from (i)]

$\Rightarrow \sqrt{1+x^2}\, y_1 = ny \quad (\because y_1 = \dfrac{dy}{dx})$

Squaring both sides, we get

$(1+x^2){y_1}^2 = n^2 y^2$

Differentiating it w.r. to x,

$(1+x^2) 2 y_1 y_2 + {y_1}^2 . 2x = n^2 . 2 y y_1$

$\Rightarrow (1+x^2) y_2 + x y_1 = n^2 y$

4. (b) $\lim\limits_{x \to a} \dfrac{f(a)g'(x) - g(a)f'(x)}{g'(x) - f'(x)} = 4$

(By Applying L' Hospital rule)

$\lim\limits_{x \to a} \dfrac{k\, g'(x) - k\, f'(x)}{g'(x) - f'(x)} = 4$

$\therefore k = 4.$

5. (c) Given that $f(0) = 0$; $f(x) = xe^{-\left(\frac{1}{|x|} + \frac{1}{x}\right)}$

R.H.L. $= \lim\limits_{h \to 0} (0+h) e^{-2/h} = \lim\limits_{h \to 0} \dfrac{h}{e^{2/h}} = 0$

L.H.L. $= \lim\limits_{h \to 0} (0-h) e^{-\left(\frac{1}{h} - \frac{1}{h}\right)} = 0$

therefore, $f(x)$ is continuous at $x = 0$.

Now, R.H.D $= \lim\limits_{h \to 0} \dfrac{(0+h)e^{-\left(\frac{1}{h}+\frac{1}{h}\right)} - 0}{h} = 0$

L.H.D. $= \lim\limits_{h \to 0} \dfrac{(0-h)e^{-\left(\frac{1}{h}-\frac{1}{h}\right)} - 0}{-h} = 1$

therefore, L.H.D. $\neq$ R.H.D.

$f(x)$ is not differentiable at $x = 0$.

6. **(d)** Given that $f(x)=x^n \Rightarrow f(1)=1$

$f'(x)=nx^{n-1} \Rightarrow f'(1)=n$

$f''(x)=n(n-1)x^{n-2} \Rightarrow f''(1)=n(n-1)$

....

$f^n(x)=n! \Rightarrow f^n(1)=n!$

$$f(1)-\frac{f'(1)}{1!}+\frac{f''(1)}{2!}-\frac{f'''(1)}{3!}+\ldots+\frac{(-1)^n f^n(1)}{n!}$$

$$=1-\frac{n}{1!}+\frac{n(n-1)}{2!}-\frac{n(n-1)(n-2)}{3!}+\ldots+(-1)^n\frac{n!}{n!}$$

$$={}^nC_0-{}^nC_1+{}^nC_2-{}^nC_3+\ldots+(-1)^n\,{}^nC_n=(1-1)^n=0$$

7. **(b)** $f(x)=ax^2+bx+c$

$f(1)=f(-1)$

$\Rightarrow a+b+c=a-b+c \text{ or } b=0$

$\therefore f(x)=ax^2+c \text{ or } f'(x)=2ax$

Now $f'(a); f'(b)$ and $f'(c)$

are $2a(a); 2a(b); 2a(c)$

i.e. $2a^2, 2ab, 2ac$.

$\Rightarrow$ If a, b, c are in A.P. then $f'(a); f'(b)$ and $f'(c)$ are also in A.P.

8. **(c)** Given that $f(x)=\dfrac{1-\tan x}{4x-\pi}$ is continuous in $\left[0,\dfrac{\pi}{2}\right]$

$$\therefore f\left(\frac{\pi}{4}\right)=\lim_{x\to\frac{\pi^-}{4}} f(x)=\lim_{x\to\frac{\pi^+}{4}} f(x)$$

$$\lim_{x\to\frac{\pi^-}{4}} f(x)=\lim_{h\to 0} f\left(\frac{\pi}{4}+h\right)$$

$$=\lim_{h\to 0}\frac{1-\tan\left(\frac{\pi}{4}+h\right)}{4\left(\frac{\pi}{4}+h\right)-\pi}, h>0$$

$$=\lim_{h\to 0}\frac{1-\dfrac{1+\tan h}{1-\tan h}}{4h}$$

$$=\lim_{h\to 0}\frac{-2}{1-\tan h}\cdot\frac{\tan h}{4h}=\frac{-2}{4}=-\frac{1}{2}$$

$$\left[\because \lim_{\theta\to 0}\frac{\tan 0}{\theta}=1\right]$$

9. **(c)** Given that $x=e^{y+e^{y+\cdots\infty}} \Rightarrow x=e^{y+x}$.
Taking log both sides.

$\log x=y+x$ differentiating both side

$$\Rightarrow \frac{1}{x}=\frac{dy}{dx}+1$$

$$\therefore \frac{dy}{dx}=\frac{1}{x}-1=\frac{1-x}{x}$$

10. **(c)** $f'(1)=\lim\limits_{h\to 0}\dfrac{f(1+h)-f(1)}{h}$;

Given that function is differentiable so it is continuous also

and $\lim\limits_{h\to 0}\dfrac{f(1+h)}{h}=5$ and hence $f(1)=0$

Hence, $f'(1)=\lim\limits_{h\to 0}\dfrac{f(1+h)}{h}=5$

11. **(a)** Given that $f(1)=-2$ &

$f'(x)\ \geq 2\ \forall\ x\in[1,6]$

Therefore f(x) is differentiable on (1,6) and continuous on [1,6].

Applying Lagrange's mean value theorem

$$\frac{f(6)-f(1)}{5}=f'(c)\geq 2$$

$\Rightarrow\ f(6)\geq 10+f(1)$

$\Rightarrow\ f(6)\geq 10-2 \Rightarrow\ f(6)\geq 8.$

12. **(b)** Given that $|f(x)-f(y)|\leq(x-y)^2, x,y\in R$ ...(i) and $f(0)=0$

$$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

$$|f'(x)|=\lim_{h\to 0}\left|\frac{f(x+h)-f(x)}{h}\right|$$

$$\leq \lim_{h\to 0}\left|\frac{(h)^2}{h}\right|$$

$\Rightarrow\ |f'(x)|\leq 0\ \Rightarrow\ f'(x)=0$

$\Rightarrow\ f(x)=$ constant

As $f(0)=0$

$\Rightarrow\ f(1)=0.$

13. **(c)** $$f(x)=\begin{cases}\dfrac{x}{1-x}, & x<0\\[2ex] \dfrac{x}{1+x}, & x\geq 0\end{cases}$$

$f(x) = \dfrac{x}{1-x}$ is not define at $x \neq 1$ but here

$x < 0$ and $f(x) = \dfrac{x}{1+x}$ is not define at $x = -1$

but here $x > 0$. So, $f(x)$ is continuous for $x \in R$

and $f'(x) = \begin{cases} \dfrac{x}{(1-x)^2}, & x < 0 \\ \dfrac{x}{(1+x)^2}, & x \geq 0 \end{cases}$

$\therefore$ $f'(x)$ exist at everywhere.

14. **(a)** $x^m . y^n = (x+y)^{m+n}$

taking log both sides

$\Rightarrow$ $m \ln x + n \ln y = (m+n) \ln (x+y)$

Differentiating both sides, we get

$$\therefore \frac{m}{x} + \frac{n}{y}\frac{dy}{dx} = \frac{m+n}{x+y}\left(1 + \frac{dy}{dx}\right)$$

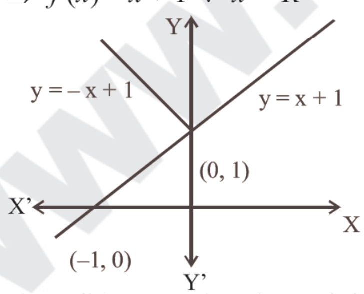
$$\Rightarrow \left(\frac{m}{x} - \frac{m+n}{x+y}\right) = \left(\frac{m+n}{x+y} - \frac{n}{y}\right)\frac{dy}{dx}$$

$$\Rightarrow \frac{my - nx}{x(x+y)} = \left(\frac{my - nx}{y(x+y)}\right)\frac{dy}{dx}$$

$$\Rightarrow \frac{dy}{dx} = \frac{y}{x}$$

15. **(a)** $f(x) = \min\{x+1, |x|+1\}$

$\Rightarrow f(x) = x + 1 \ \forall \ x \in R$



Since $f(x) = x + 1$ is polynomial function

Hence, $f(x)$ is differentiable everywhere for all $x \in R$.

16. **(b)** Given, $f(x) = \dfrac{1}{x} - \dfrac{2}{e^{2x}-1}$ is continuous at $x = 0$

$$\Rightarrow f(0) = \lim_{x\to 0} \frac{1}{x} - \frac{2}{e^{2x}-1}$$

$$= \lim_{x\to 0} \frac{(e^{2x}-1)-2x}{x(e^{2x}-1)}; \left[\frac{0}{0} \text{ form}\right]$$

$\therefore$ Applying, L'Hospital rule

Differentiate two times, we get

$$f(0) = \lim_{x\to 0} \frac{4e^{2x}}{2(xe^{2x}2 + e^{2x}.1) + e^{2x}.2}$$

$$= \lim_{x\to 0} \frac{4e^{2x}}{4xe^{2x} + 2e^{2x} + 2e^{2x}} \left[\frac{0}{0} \text{ form}\right]$$

$$= \lim_{x\to 0} \frac{4e^{2x}}{4(xe^{2x} + e^{2x})} = \frac{4.e^0}{4(0 + e^0)} = 1$$

17. **(c)** Given that

$$f(x) = \begin{cases} (x-1)\sin\left(\dfrac{1}{x-1}\right), & \text{if } x \neq 1 \\ 0 & \text{, if } x = 1 \end{cases}$$

At $x = 1$

$$\text{R.H.D.} = \lim_{h\to 0} \frac{f(1+h) - f(1)}{h}$$

$$= \lim_{h\to 0} \frac{h\sin\frac{1}{h} - 0}{h} = \lim_{h\to 0} \sin\frac{1}{h}$$

= a finite number

Let this finite number be l

$$\text{L.H.D.} = \lim_{h\to 0} \frac{f(1-h) - f(1)}{-h}$$

$$= \lim_{h\to 0} \frac{-h\sin\left(\frac{1}{-h}\right)}{-h} = \lim_{h\to 0} \sin\left(\frac{1}{-h}\right)$$

$$= -\lim_{h\to 0} \sin\left(\frac{1}{h}\right)$$

= – (a finite number) = $-l$

Thus R.H.D $\neq$ L.H.D

$\therefore$ f is not differentiable at x = 1

At $x = 0$

$$f'(0) = \sin\frac{1}{(x-1)} - \frac{x-1}{(x-1)^2}\cos\left(\frac{1}{x-1}\right)\Bigg]_{x=0}$$

$= -\sin 1 + \cos 1$

$\therefore$ f is differentiable at x = 0

18. **(d)** $x^{2x} - 2x^x \cot y - 1 = 0$

$\Rightarrow$ $2 \cot y = x^x - x^{-x}$

Let $u = x^x$

$\Rightarrow 2\cot y = u - \dfrac{1}{u}$

Differentiating both sides with respect to x, we get

$-2\text{cosec}^2 y \dfrac{dy}{dx} = \left(1+\dfrac{1}{u^2}\right)\dfrac{du}{dx}$

Now $u = x^x$ Taking log both sides

$\Rightarrow \log u = x\log x$

$\Rightarrow \dfrac{1}{u}\dfrac{du}{dx} = 1+\log x$

$\Rightarrow \dfrac{du}{dx} = x^x(1+\log x)$

$\therefore$ We get

$-2\text{ cosec}^2 y \dfrac{dy}{dx}$

$= (1+x^{-2x}).x^x(1+\log x)$

$$\Rightarrow \frac{dy}{dx} = \frac{\left(x^x + x^{-x}\right)(1+\log x)}{-2(1+\cot^2 y)} \quad \ldots\text{(i)}$$

Put $n = 1$ in eqn. $x^{2x} - 2x^x \cot y - 1 = 0$, gives

$1 - 2\cot y - 1 = 0$

$\Rightarrow \cot y = 0$

$\therefore$ Putting $x = 1$ and $\cot y = 0$ in eqn. (i), we get

$$y'(1) = \frac{(1+1)(1+0)}{-2(1+0)} = -1$$

19. (a) Given that $g(x) = [f(2f(x))+2]^2$

$\therefore g'(x) = 2\left(f(2f(x)+2)\right)\left(\dfrac{d}{dx}\left(f(2f(x)+2)\right)\right)$

$= 2f(2f(x)+2)f'(2f(x))+2).(2f'(x))$

$\Rightarrow g'(0) = 2f(2f(0)+2.f'(2f(0)+2)$

$.2f'(0) = 4f(0)(f'(0))^2 = 4(-1)(1)^2 = -4$

20. (b) $\underset{(at\, x=0)}{L.H.L} = \lim_{x\to 0^-} f(x)$

$= \lim_{h\to 0}\dfrac{\sin\{(p+1)(-h)\} - \sinh}{-h}$

$= p+1+1 = p+2$

$\underset{(at\, x=0)}{R.H.L} = \lim_{x\to 0^+} f(x)$

$= \lim_{h\to 0}\dfrac{\sqrt{x+x^2}-\sqrt{x}}{x^{3/2}} \times \dfrac{\sqrt{x+x^2}+\sqrt{x}}{\sqrt{x+x^2}+\sqrt{x}}$

$= \dfrac{1}{1+1} = \dfrac{1}{2}$

$f(0) = 2$

Given that f(x) is continuous at $x = 0$

$\therefore p+2 = q = \dfrac{1}{2}$

$\Rightarrow p = -\dfrac{3}{2}, q = \dfrac{1}{2}$

21. (c) $\dfrac{d^2x}{dy^2} = \dfrac{d}{dy}\left(\dfrac{dx}{dy}\right) = \dfrac{d}{dx}\left(\dfrac{dx}{dy}\right)\dfrac{dx}{dy}$

$= \dfrac{d}{dx}\left(\dfrac{1}{dy/dx}\right)\dfrac{dx}{dy}$

$$= -\frac{1}{\left(\frac{dy}{dx}\right)^2}.\frac{d^2y}{dx^2}.\frac{1}{\frac{dy}{dx}}\left[\because \frac{d\left(\frac{1}{x}\right)}{dx} = -\frac{1}{x^2}\right]$$

$$= -\frac{1}{\left(\frac{dy}{dx}\right)^3}\frac{d^2y}{dx^2}$$

22. (c) Given that $f(x) = \begin{cases} x\sin(1/x), & x\neq 0 \\ 0, & x = 0\end{cases}$

At $x = 0$

LHL $= \lim_{h\to 0^-}\left\{-h\sin\left(-\dfrac{1}{h}\right)\right\}$

$= 0 \times$ a finite quantity between -1 and $1 = 0$

RHL $= \lim_{h\to 0^+} h\sin\dfrac{1}{h} = 0$

Also, $f(0) = 0$

Thus LHL = RHL = $f(0)$

$\therefore$ $f(x)$ is continuous on R.

but $f_2(x)$ is not continuous at $x = 0$

23. (c) $\lim_{x\to a}\dfrac{x^2 f(a) - a^2 f(x)}{x-a}$

Applying L-Hospital rule

$= \lim_{x\to a}\dfrac{2xf(a) - a^2 f'(x)}{1}$

$= 2af(a) - a^2 f'(a)$

24. **(a)** Let $f(x)=[x]\cos\left(\frac{2x-1}{2}\right)$

We know that $[x]$ is discontinuous at all integral points and cos x is continuous at $x \in$ R.

So, check at $x = n$, $n \in I$

L.H.L $= \lim\limits_{x\to n^-}[x]\cos\left(\frac{2x-1}{2}\right)\pi$

$=(n-1)\cos\left(\frac{2n-1}{2}\right)\pi=0$

($\because$ $[x]$ is the greatest integer function)

R.H.L $= \lim\limits_{x\to n^+}[x]\cos\left(\frac{2x-1}{2}\right)\pi$

$= n\cos\left(\frac{2n-1}{2}\right)\pi=0$

Now, value of the function at $x = n$ is

$f(n)=0$

Since, L.H.L = R.H.L. = $f(n)$

$\therefore f(x)=[x]\cos\left(\frac{2x-1}{2}\right)$ is continuous for every real x.

25. **(c)** $f(x)=|x-2|=\begin{cases} x-2 & , \ x-2\ge 0 \\ 2-x & , \ x-2\le 0\end{cases}$

$=\begin{cases} x-2 & , \ x\ge 2 \\ 2-x & , \ x\le 2\end{cases}$

Similarly,

$f(x)=|x-5|=\begin{cases} x-5 & , \ x\ge 5 \\ 5-x & , \ x\le 5\end{cases}$

$\therefore \quad f(x)=|x-2|+|x-5|$

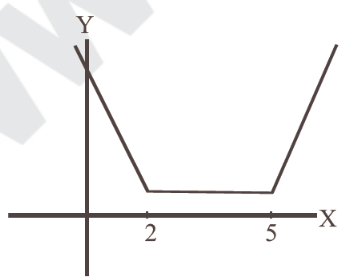
$=\{x-2+5-x=3,\ 2\le x\le 5\}$

Thus $f(x)=3$, $2\le x\le 5$

$f'(x)=0$, $2<x<5$

$f'(4)=0$

$\because$ Statement-1 is true



Since $f(x)=3$, $2\le x\le 5$ is constant function. So, it continuous in 2, 5 and differentiable in (2, 5)

$\because f(2)=0+|2-5|=3$

and $f(5)=|5-2|+0=3$

statement-2 is also true.

26. **(a)** Let $y=\sec(\tan^{-1}x)$

$=\sec\left(\sec^{-1}\sqrt{1+x^2}\right)$

$\Rightarrow \quad y=\sqrt{1+x^2}$

$\Rightarrow \quad \frac{dy}{dx}=\frac{1}{2\sqrt{1+x^2}}\cdot 2x$

At $\quad x=1$,

$\frac{dy}{dx}=\frac{1}{\sqrt{2}}$.

27. **(b)** Since, f and g both are continuous function on [0, 1] and differentiable on (0, 1) then $\exists\, c\in(0,1)$ such that

$f'(c)=\frac{f(1)-f(0)}{1}=\frac{6-2}{1}=4$

and $g'(c)=\frac{g(1)-g(0)}{1}=\frac{2-0}{1}=2$

Thus, we get $\quad f'(c)=2g'(c)$

28. **(c)** Since g (x) is differentiable, it will be continuous at x = 3

$\therefore \lim\limits_{x\to 3^-} g(x)=\lim\limits_{x\to 3^+} g(x)$

$2k=3m+2 \qquad ...(1)$

Also g(x) is differentiable at x = 0

$\therefore \lim\limits_{x\to 3^-} g'(x)=\lim\limits_{x\to 3^+} g'(x)$

$\frac{k}{2\sqrt{3+1}}=m$

$k=4m \qquad ...(2)$

Solving (1) and (2), we get

$m=\frac{2}{5}$, $k=\frac{8}{5}$

$k+m=2$

29. **(d)** g (x) = f(f(x))

In the neighbourhood of x = 0,

f(x) = | log 2 – sin x| = (log 2 – sin x)

$\therefore$ g (x) = |log 2 – sin| log 2 – sin x ||

= (log 2 – sin(log 2 – sin x))

$\therefore$ g (x) is differentiable

and g'(x) = – cos(log 2 – sin x) (– cos x)

$\Rightarrow$ g'(0) = cos (log 2)

30. **(d)** $f(x)=|x-\pi|\,(e^{|x|}-1)\sin|x|$

Check differentiability of f(x) at x = π

and x = 0

at x = π

$$\text{R.H.D.} = \lim_{h\to 0}\frac{|\pi+h-\pi|(e^{|\pi+h|}-1)\sin|\pi+h|-0}{h} = 0$$

$$\text{L.H.D} = \lim_{h\to 0}\frac{|\pi-h-\pi|(e^{|\pi-h|}-1)\sin|\pi-h|-0}{-h} = 0$$

$\because$ RHD = LHD

Therefore, function is differentiable at $x = \pi$

at x = 0:

$$\text{R.H.D.} = \lim_{h\to 0}\frac{|h-\pi|(e^{|h|}-1)\sin|h|-0}{h} = 0$$

$$\text{L.H.D.} = \lim_{h\to 0}\frac{|-h-\pi|(e^{|-h|}-1)\sin|-h|-0}{-h} = 0$$

$\therefore$ RHD = LHD

Therefore, function is differentiable.

at $x = 0$.

Since, the function f(x) is differentiable at all the points including π and 0.

i.e., f(x) is every where differentiable .

Therefore, there is no element in the set S.

$\Rightarrow S = \phi$ (an empty set)

31. **(d)** Let $f(x)$ is continuous at $x = 1$, then

$f(1^-) = f(1) = f(1^+)$

$\Rightarrow \quad 5 = a + b \quad$ ...(1)

Let $f(x)$ is continuous at $x = 3$, then

$f(3^-) = f(3) = f(3^+)$

$\Rightarrow a + 3b = b + 15 \Rightarrow a + 2b = 15 \quad$ ...(2)

Solving (1) & (2) we get $b = 10$, $a = -5$

Now $f(x)$ is continuous at $x = 5$, then

$f(5^-) = f(5) = f(5^+)$

$\Rightarrow b + 25 = 30$

Which is not satisfied by $a = -5$ and $b = 10$.

Hence, $f(x)$ is not continuous for any values of a and b

32. **(b)** Since, f(x) is continuous, then

$$\lim_{x\to\frac{\pi}{4}} f(x) = f\left(\frac{\pi}{4}\right)$$

$$\lim_{x\to\frac{\pi}{4}}\frac{\sqrt{2}\cos x-1}{\cot x-1} = k$$

Now by L- hospital's rule

$$\lim_{x\to\frac{\pi}{4}}\frac{\sqrt{2}\sin x}{\operatorname{cosec}^2 x} = k \Rightarrow \frac{\sqrt{2}\left(\frac{1}{\sqrt{2}}\right)}{(\sqrt{2})^2} = k$$

$$\Rightarrow k = \frac{1}{2}$$

33. **(a)** Since, $f(x) = 15 - |(10 - x)|$

$\therefore g(x) = f(f(x)) = 15 - |10 - [15 - |10 - x|]|$

$= 15 - ||10 - x| - 5|$

$\therefore$ Then, the points where function g(x) is Non-differentiable are

$10 - x = 0$ and $|10 - x| = 5$

$\Rightarrow x = 10$ and $x - 10 = \pm 5$

$\Rightarrow x = 10$ and $x = 15, 5$

34. **(a)** $$y(\alpha) = \sqrt{\frac{\frac{2\sin\alpha}{\cos\alpha}+\frac{\cos\alpha}{\sin\alpha}}{\sec^2\alpha}}$$

$$= \sqrt{\frac{2\cos^2\alpha}{\sin\alpha\cos\alpha}+\frac{1}{\sin^2\alpha}}$$

$$= \sqrt{2\cot\alpha+\operatorname{cosec}^2\alpha} = \sqrt{2\cot\alpha+1+\cot^2\alpha}$$

$$= |1+\cot\alpha| = -1-\cot\alpha \left[\because \alpha \in \left(\frac{3\pi}{4},\pi\right)\right]$$

$$\frac{dy}{d\alpha} = \operatorname{cosec}^2\alpha \Rightarrow \left(\frac{dy}{d\alpha}\right)_{\alpha=\frac{5\pi}{6}} = 4$$

35. **(c)** $$k.x^{k-1}+k.y^{k-1}\frac{dy}{dx} = 0$$

$$\Rightarrow \quad \frac{dy}{dx} = -\left(\frac{x}{y}\right)^{k-1}$$

$$\Rightarrow \quad \frac{dy}{dx}+\left(\frac{x}{y}\right)^{k-1} = 0$$

$$\Rightarrow \quad k-1 = -\frac{1}{3}$$

$$\Rightarrow \quad k = 1-\frac{1}{3} = \frac{2}{3}$$

36. **(a)** From, LMVT for $x \in [-7, -1]$

$$\frac{f(-1)-f(-7)}{(-1+7)} \le 2 \Rightarrow \frac{f(-1)+3}{6} \le 2$$

$\Rightarrow f(-1) \le 9$

From, LMVT for $x \in [-7, 0]$

$$\frac{f(0)-f(-7)}{(0+7)} \le 2$$

$$\frac{f(0)+3}{7} \le 2 \Rightarrow f(0) \le 11$$

$\therefore f(0) + f(-1) < 20$

37. **(3)** $\because f(x)$ is non differentiable at $x = 1, 3, 5$

$[\because |x - 3|$ is not differentiable at $x = 3]$

$\Sigma f(f(x)) = f(f(1) + f(f(3)) + f(f(5))$

$= 1 + 1 + 1 = 3$

21 Application of Derivatives

1. The maximum distance from origin of a point on the curve $x = a \sin t - b \sin\left(\frac{at}{b}\right)$, $y = a\cos t - b\cos\left(\frac{at}{b}\right)$, both $a, b > 0$ is **[2002]**
(a) $a - b$ (b) $a + b$
(c) $\sqrt{a^2 + b^2}$ (d) $\sqrt{a^2 - b^2}$

2. If $2a + 3b + 6c = 0$, $(a, b, c \in R)$ then the quadratic equation $ax^2 + bx + c = 0$ has **[2002]**
(a) at least one root in [0, 1]
(b) at least one root in [2, 3]
(c) at least one root in [4, 5]
(d) none of these

3. If the function $f(x) = 2x^3 - 9ax^2 + 12a^2x + 1$, where $a > 0$, attains its maximum and minimum at p and q respectively such that $p^2 = q$, then a equals **[2003]**
(a) $\frac{1}{2}$ (b) 3
(c) 1 (d) 2

4. The real number x when added to its inverse gives the minimum value of the sum at x equal to **[2003]**
(a) –2 (b) 2
(c) 1 (d) –1

5. A point on the parabola $y^2 = 18x$ at which the ordinate increases at twice the rate of the abscissa is **[2004]**
(a) $\left(\frac{9}{8}, \frac{9}{2}\right)$ (b) $(2, -4)$
(c) $\left(\frac{-9}{8}, \frac{9}{2}\right)$ (d) $(2, 4)$

6. The normal to the curve $x = a(1 + \cos\theta)$, $y = a\sin\theta$ at 'θ' always passes through the fixed point **[2004]**
(a) (a, a) (b) $(0, a)$
(c) $(0, 0)$ (d) $(a, 0)$

7. If $2a + 3b + 6c = 0$, then at least one root of the equation $ax^2 + bx + c = 0$ lies in the interval **[2004]**
(a) $(1, 3)$ (b) $(1, 2)$
(c) $(2, 3)$ (d) $(0, 1)$

8. Area of the greatest rectangle that can be inscribed in the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is **[2005]**
(a) $2ab$ (b) ab
(c) $\sqrt{ab}$ (d) $\frac{a}{b}$

9. The normal to the curve **[2005]**
$x = a(\cos\theta + \theta\sin\theta)$, $y = a(\sin\theta - \theta\cos\theta)$ at any point θ is such that
(a) it passes through the origin
(b) it makes an angle $\frac{\pi}{2} + \theta$ with the x-axis
(c) it passes through $\left(a\frac{\pi}{2}, -a\right)$
(d) It is at a constant distance from the origin

10. A spherical iron ball 10 cm in radius is coated with a layer of ice of uniform thickness that melts at a rate of 50 cm^3/min. When the thickness of ice is 5 cm, then the rate at which the thickness of ice decreases is **[2005]**

(a) $\frac{1}{36\pi}$ cm/min. (b) $\frac{1}{18\pi}$ cm/min.

(c) $\frac{1}{54\pi}$ cm/min. (d) $\frac{5}{6\pi}$ cm/min

11. If the equation $a_n x^n + a_{n-1} x^{n-1} + \ldots\ldots\ldots\ldots + a_1 x = 0$

$a_1 \neq 0, n \geq 2$, has a positive root $x = \alpha$, then the equation $n a_n x^{n-1} + (n-1) a_{n-1} x^{n-2} + \ldots\ldots + a_1 = 0$ has a positive root, which is **[2005]**

(a) greater than α

(b) smaller than α

(c) greater than or equal to α

(d) equal to α

12. A function is matched below against an interval where it is supposed to be increasing. Which of the following pairs is incorrectly matched? **[2005]**

	Interval	*Function*
(a)	$(-\infty, \infty)$	$x^3 - 3x^2 + 3x + 3$
(b)	$[2, \infty)$	$2x^3 - 3x^2 - 12x + 6$
(c)	$\left(-\infty, \frac{1}{3}\right]$	$3x^2 - 2x + 1$
(d)	$(-\infty, -4)$	$x^3 + 6x^2 + 6$

13. The function $f(x) = \frac{x}{2} + \frac{2}{x}$ has a local minimum at **[2006]**

(a) $x = 2$ (b) $x = -2$

(c) $x = 0$ (d) $x = 1$

14. A triangular park is enclosed on two sides by a fence and on the third side by a straight river bank. The two sides having fence are of same length x. The maximum area enclosed by the park is **[2006]**

(a) $\frac{3}{2}x^2$ (b) $\sqrt{\frac{x^3}{8}}$

(c) $\frac{1}{2}x^2$ (d) πx^2

15. Value of c for which conclusion of Mean Value Theorem holds for the function $f(x) = \log_e x$ on the interval [1, 3] is **[2007]**

(a) $\log_3 e$ (b) $\log_e 3$

(c) $2 \log_3 e$ (d) $\frac{1}{2}\log_3 e$

16. The function $f(x) = \tan^{-1}(\sin x + \cos x)$ is an increasing function in **[2007]**

(a) $\left(0, \frac{\pi}{2}\right)$ (b) $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

(c) $\left(\frac{\pi}{4}, \frac{\pi}{2}\right)$ (d) $\left(-\frac{\pi}{2}, \frac{\pi}{4}\right)$

17. If p and q are positive real numbers such that $p^2 + q^2 = 1$, then the maximum value of $(p+q)$ is **[2007]**

(a) $\frac{1}{2}$ (b) $\frac{1}{\sqrt{2}}$

(c) $\sqrt{2}$ (d) 2

18. Suppose the cubic $x^3 - px + q$ has three distinct real roots where $p > 0$ and $q > 0$. Then which one of the following holds? **[2008]**

(a) The cubic has minima at $\sqrt{\frac{p}{3}}$ and maxima at $-\sqrt{\frac{p}{3}}$

(b) The cubic has minima at $-\sqrt{\frac{p}{3}}$ and maxima at $\sqrt{\frac{p}{3}}$

(c) The cubic has minima at both $\sqrt{\frac{p}{3}}$ and $-\sqrt{\frac{p}{3}}$

(d) The cubic has maxima at both $\sqrt{\frac{p}{3}}$ and $-\sqrt{\frac{p}{3}}$

19. How many real solutions does the equation $x^7 + 14x^5 + 16x^3 + 30x - 560 = 0$ have? **[2008]**

(a) 7 (b) 1
(c) 3 (d) 5

20. The differential equation of the family of circles with fixed radius 5 units and centre on the line $y = 2$ is **[2009]**

(a) $(x-2)y'^2 = 25-(y-2)^2$
(b) $(y-2)y'^2 = 25-(y-2)^2$
(c) $(y-2)^2y'^2 = 25-(y-2)^2$
(d) $(x-2)^2 y'^2 = 25-(y-2)^2$

21. Let $f(x) = x\,|\,x\,|$ and $g(x) = \sin x$.
Statement-1 : gof is differentiable at $x = 0$ and its derivative is continuous at that point.
Statement-2 : gof is twice differentiable at $x = 0$. **[2009]**

(a) Statement-1 is true, Statement-2 is true; Statement-2 is not a correct explanation for Statement-1.
(b) Statement-1 is true, Statement-2 is false.
(c) Statement-1 is false, Statement-2 is true.
(d) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1.

22. Given $P(x) = x^4 + ax^3 + bx^2 + cx + d$ such that $x = 0$ is the only real root of $P'(x) = 0$. If $P(-1) < P(1)$, then in the interval $[-1, 1]$: **[2009]**

(a) P(–1) is not minimum but P(1) is the maximum of P
(b) P(–1) is the minimum but P(1) is not the maximum of P
(c) Neither P(–1) is the minimum nor P(1) is the maximum of P
(d) P(–1) is the minimum and P(1) is the maximum of P

23. The equation of the tangent to the curve $y = x + \dfrac{4}{x^2}$, that is parallel to the x-axis, is **[2010]**

(a) $y = 1$ (b) $y = 2$
(c) $y = 3$ (d) $y = 0$

24. Let $f : R \to R$ be defined by

$$f(x) = \begin{cases} k-2x, & \text{if } x \le -1 \\ 2x+3, & \text{if } x > -1 \end{cases}$$

If f has a local minimum at $x = -1$, then a possible value of k is **[2010]**

(a) 0 (b) $-\dfrac{1}{2}$
(c) –1 (d) 1

25. Let $f : R \to R$ be a continuous function defined by $f(x) = \dfrac{1}{e^x + 2e^{-x}}$ **[2010]**

Statement -1 : $f(c) = \dfrac{1}{3}$, for some $c \in R$.

Statement -2 : $0 < f(x) \le \dfrac{1}{2\sqrt{2}}$, for all $x \in R$

(a) Statement -1 is true, Statement -2 is true ; Statement -2 is **not** a correct explanation for Statement -1.
(b) Statement -1 is true, Statement -2 is false.
(c) Statement -1 is false, Statement -2 is true .
(d) Statement - 1 is true, Statement 2 is true ; Statement -2 is a correct explanation for Statement -1.

26. Let f be a function defined by - **[2011RS]**

$$f(x) = \begin{cases} \dfrac{\tan x}{x}, & x \ne 0 \\ 1, & x = 0 \end{cases}$$

Statement - 1 : $x = 0$ is point of minima of f

Statement - 2 : $f'(0) = 0$.

(a) Statement-1 is true, statement-2 is true; statement-2 is a correct explanation for statement-1.
(b) Statement-1 is true, statement-2 is true; statement-2 is NOT a correct explanation for statement-1.
(c) Statement-1 is true, statement-2 is false.
(d) Statement-1 is false, statement-2 is true.

27. A spherical balloon is filled with 4500π cubic meters of helium gas. If a leak in the balloon causes the gas to escape at the rate of 72π cubic meters per minute, then the rate (in meters per minute) at which the radius of the balloon decreases 49 minutes after the leakage began is: **[2012]**

(a) $\dfrac{9}{7}$ (b) $\dfrac{7}{9}$
(c) $\dfrac{2}{9}$ (d) $\dfrac{9}{2}$

28. Let $a, b \in R$ be such that the function f given by $f(x) = ln|x| + bx^2 + ax, x \neq 0$ has extreme values at $x = -1$ and $x = 2$

Statement-1 : f has local maximum at $x = -1$ and at $x = 2$.

Statement-2 : $a = \frac{1}{2}$ and $b = \frac{-1}{4}$ **[2012]**

(a) Statement-1 is false, Statement-2 is true.
(b) Statement-1 is true, statement-2 is true; statement-2 is a correct explanation for Statement-1.
(c) Statement-1 is true, statement-2 is true; statement-2 is **not** a correct explanation for Statement-1.
(d) Statement-1 is true, statement-2 is false.

29. The real number k for which the equation, $2x^3 + 3x + k = 0$ has two distinct real roots in [0, 1] **[2013]**

(a) lies between 1 and 2
(b) lies between 2 and 3
(c) lies between .1 and 0
(d) does not exist.

30. If $x = -1$ and $x = 2$ are extreme points of $f(x) = \alpha \log|x| + \beta x^2 + x$ then **[2014]**

(a) $\alpha = 2, \beta = -\frac{1}{2}$ (b) $\alpha = 2, \beta = \frac{1}{2}$
(c) $\alpha = -6, \beta = \frac{1}{2}$ (d) $\alpha = -6, \beta = -\frac{1}{2}$

31. The normal to the curve, $x^2 + 2xy - 3y^2 = 0$, at (1, 1) **[2015]**

(a) meets the curve again in the third quadrant.
(b) meets the curve again in the fourth quadrant.
(c) does not meet the curve again.
(d) meets the curve again in the second quadrant.

32. Consider

$f(x) = \tan^{-1}\left(\sqrt{\frac{1+\sin x}{1-\sin x}}\right), x \in \left(0, \frac{\pi}{2}\right).$

A normal to y = f(x) at $x = \frac{\pi}{6}$ a so passes through the point : **[2016]**

(a) $\left(\frac{\pi}{6}, 0\right)$ (b) $\left(\frac{\pi}{4}, 0\right)$
(c) $(0, 0)$ (d) $\left(0, \frac{2\pi}{3}\right)$

33. A wire of length 2 units is cut into two parts which are bent respectively to form a square of side = x units and a circle of radius = r units. If the sum of the areas of the square and the circle so formed is minimum, then: **[2016]**

(a) $x = 2r$ (b) $2x = r$
(c) $2x = (\pi + 4)r$ (d) $(4 - \pi)x = \pi r$

34. The normal to the curve $y(x - 2)(x - 3) = x + 6$ at the point where the curve intersects the y-axis passes through the point: **[2016]**

(a) $\left(\frac{1}{2}, \frac{1}{3}\right)$ (b) $\left(-\frac{1}{2}, -\frac{1}{2}\right)$
(c) $\left(\frac{1}{2}, \frac{1}{2}\right)$ (d) $\left(\frac{1}{2}, -\frac{1}{3}\right)$

35. Twenty metres of wire is available for fencing off a flower-bed in the form of a circular sector. Then the maximum area (in sq. m) of the flower-bed, is : **[2017]**

(a) 30 (b) 12.5
(c) 10 (d) 25

36. The eccentricity of an ellipse whose centre is at the origin is $\frac{1}{2}$. If one of its directices is x = – 4, then the equation of the normal to it at $\left(1, \frac{3}{2}\right)$ is : **[2017]**

(a) $x + 2y = 4$ (b) $2y - x = 2$
(c) $4x - 2y = 1$ (d) $4x + 2y = 7$

37. Let $f(x) = x^2 + \frac{1}{x^2}$ and $g(x) = x - \frac{1}{x}$, $x \in R - \{-1, 0, 1\}$. If $h(x) = \frac{f(x)}{g(x)}$, then the local minimum value of h(x) is : **[2018]**

(a) -3 (b) $-2\sqrt{2}$
(c) $2\sqrt{2}$ (d) 3

38. If the curves $y^2 = 6x, 9x^2 + by^2 = 16$ intersect each other at right angles, then the value of b is : **[2018]**

(a) $\frac{7}{2}$ (b) 4
(c) $\frac{9}{2}$ (d) 6

39. The maximum volume (in cu.m) of the right circular cone having slant height 3 m is: **[2019]**

(a) 6π (b) $3\sqrt{3}\pi$

(c) $\frac{4}{3}\pi$ (d) $2\sqrt{3}\pi$

40. Let $0 < \theta < \frac{\pi}{2}$. If the eccentricity of the hyperbola **[2019]**

$\frac{x^2}{\cos^2\theta} - \frac{y^2}{\sin^2\theta} = 1$ is greater than 2, then the length of its latus rectum lies in the interval:

(a) $(3, \infty)$ (b) $(3/2, 2]$

(c) $(2, 3]$ (d) $(1, 3/2]$

41. If θ denotes the acute angle between the curves, $y = 10 - x^2$ and $y = 2 + x^2$ at a point of their intersection, then $|\tan\theta|$ is equal to: **[2019]**

(a) $\frac{4}{9}$ (b) $\frac{8}{15}$

(c) $\frac{7}{17}$ (d) $\frac{8}{17}$

42. If the tangent to the curve, $y = x^3 + ax - b$ at the point $(1, -5)$ is perpendicular to the line, $-x + y + 4 = 0$, then which one of the following points lies on the curve? **[2019]**

(a) $(-2, 1)$ (b) $(-2, 2)$

(c) $(2, -1)$ (d) $(2, -2)$

43. Let S be the set of all values of x for which the tangent to the curve $y = f(x) = x^3 - x^2 - 2x$ at (x, y) is parallel to the line segment joining the points $(1, f(1))$ and $(-1, f(-1))$, then S is equal to: **[2019]**

(a) $\left\{\frac{1}{3}, 1\right\}$ (b) $\left\{-\frac{1}{3}, -1\right\}$

(c) $\left\{\frac{1}{3}, -1\right\}$ (d) $\left\{-\frac{1}{3}, 1\right\}$

Answer Key

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
(b)	(a)	(d)	(c)	(a)	(d)	(d)	(a)	(d)	(b)	(b)	(c)	(a)	(c)	(c)
16	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**
(d)	(c)	(a)	(b)	(c)	(b)	(a)	(c)	(c)	(d)	(b)	(c)	(b)	(d)	(a)
31	**32**	**33**	**34**	**35**	**36**	**37**	**38**	**39**	**40**	**41**	**42**	**43**		
(b)	(d)	(a)	(c)	(d)	(c)	(c)	(c)	(d)	(a)	(b)	(d)	(d)		

Solutions

1. **(b)** We know that distance of origin from (x, y)

$= \sqrt{x^2 + y^2}$

$= \sqrt{a^2 + b^2 - 2ab\cos\left(t - \frac{at}{b}\right)}$;

$\le \sqrt{a^2 + b^2 + 2ab}$

$\left[\left\{\cos\left(t - \frac{at}{b}\right)\right\}_{\min} = -1\right]$

$= a + b$

$\therefore$ Maximum distance from origin $= a + b$

2. **(a)** Let $f(x) = \frac{ax^3}{3} + \frac{bx^2}{2} + cx$

$\Rightarrow f(0) = 0$ and

$f(1) = \frac{a}{3} + \frac{b}{2} + c = \frac{2a + 3b + 6c}{6} = 0$

$\therefore x = 0$ and $x = 1$ are two zeroes of $f(x)$. Also $f(x)$ is continuous and differentiable in [0, 1] and (0, 1). So by Rolle's theorem, $f'(x) = 0$.

i.e $ax^2 + bx + c = 0$ has at least one root in (0, 1).

3. **(d)** $f(x) = 2x^3 - 9ax^2 + 12a^2x + 1$

$f'(x) = 6x^2 - 18ax + 12a^2$;

For maxima or minima.

$6x^2 - 18ax + 12a^2 = 0 \Rightarrow x^2 - 3ax + 2a^2 = 0$

$\Rightarrow x = a$ or $x = 2a$.

$f''(x) = 12x - 18a$

$f''(a) = -6a < 0 \therefore f(x)$ is max. at $x = a$,

$f''(2a) = 6a > 0$

$\therefore f(x)$ is min. at $x = 2a$

$\therefore p = a$ and $q = 2a$

ATQ, $p^2 = q$

$\therefore a^2 = 2a \Rightarrow a = 2$ or $a = 0$

but $a > 0$, therefore, $a = 2$.

4. **(c)** ATQ, $y = x + \frac{1}{x} \Rightarrow \frac{dy}{dx} = 1 - \frac{1}{x^2}$

For maxima. or minima.,

$1 - \frac{1}{x^2} = 0 \Rightarrow x = \pm 1$

$\frac{d^2y}{dx^2} = \frac{2}{x^3} \Rightarrow \left(\frac{d^2y}{dx^2}\right)_{x=1} = 2 > 0$

$\therefore y$ is minimum at $x = 1$

5. **(a)** Given $y^2 = 18x \Rightarrow 2y\frac{dy}{dx} = 18 \Rightarrow \frac{dy}{dx} = \frac{9}{y}$

ATQ $\frac{dy}{dt} = \frac{2dx}{dt} \Rightarrow \frac{dy}{dx} = 2$

$\Rightarrow \frac{9}{y} = 2 \Rightarrow y = \frac{9}{2}$

Putting in $y^2 = 18x \Rightarrow x = \frac{9}{8}$

$\therefore$ Required point is $\left(\frac{9}{8}, \frac{9}{2}\right)$

6. **(d)** Since, x = a (1 + cosθ)

$\Rightarrow \frac{dx}{d\theta} = -a\sin\theta$ and $y = a\sin\theta$

$\Rightarrow \frac{dy}{d\theta} = a\cos\theta$

$\therefore \frac{dy}{dx} = -\cot\theta.$

$\therefore$ The slope of the normal at $\theta = \tan\theta$

$\therefore$ The equation of the normal at θ is

$y - a\sin\theta = \tan\theta(x - a - a\cos\theta)$

$\Rightarrow y\cos\theta - a\sin\theta\cos\theta = x\sin\theta - a\sin\theta - a\sin\theta\cos\theta$

$\Rightarrow x\sin\theta - y\cos\theta = a\sin\theta$

$\Rightarrow y = (x - a)\tan\theta$

which always passes through $(a, 0)$

7. **(d)** Let $f(x) = \frac{ax^3}{3} + \frac{bx^2}{2} + cx$

$f(0) = 0$ and $f(1) = \frac{a}{3} + \frac{b}{2} + c$

$\Rightarrow f(1) = \frac{2a + 3b + 6c}{6} = 0$ (given)

$\therefore f(0) = f(1)$

$f(x)$ is polynomial, so, it is continuous and differentiable, also,

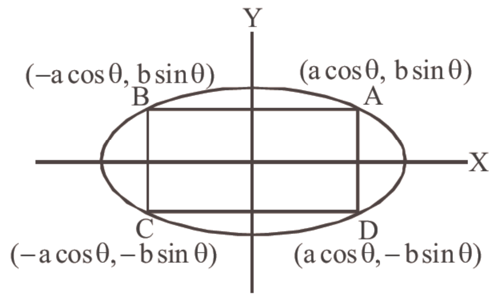
$\therefore f(x)$ satisfies all conditions of Rolle's theorem therefore $f'(x) = 0$ has a root in (0, 1)

i.e. $ax^2 + bx + c = 0$ has at lease one root in (0, 1)

8. **(a)** Area of rectangle

$ABCD = 2a\cos\theta\,(2b\sin\theta)$

$A = 2ab\sin 2\theta$



$\frac{dA}{d\theta} = 2ab\cos 2\theta.2 = 4ab\cos 2\theta$

for maxima and minima

$4ab\cos 2\theta = 0 \Rightarrow \cos 2\theta = 0$

$\Rightarrow \theta = \frac{\pi}{4}$ or $\theta = \frac{3\pi}{4}$

$\frac{d^2A}{d\theta^2} = -8ab\sin 2\theta$

$\left(\frac{d^2A}{d\theta^2}\right)_{\frac{\pi}{4}} = -8ab < 0$

$\therefore$ Area is maximum at $x = \frac{\pi}{4}$

$A = 2ab\sin\frac{\pi}{2} = 2ab$.

9. **(d)** Given $x = a(\cos\theta + \theta\sin\theta)$

$\Rightarrow \frac{dx}{d\theta} = a(-\sin\theta + \sin\theta + \theta\cos\theta)$

$\Rightarrow \frac{dx}{d\theta} = a\theta\cos\theta$ .....(1)

$y = a(\sin\theta - \theta\cos\theta)$

$\frac{dy}{d\theta} = a[\cos\theta - \cos\theta + \theta\sin\theta]$

$\Rightarrow \frac{dy}{d\theta} = a\theta\sin\theta$ .....(2)

From equations (1) and (2) we get

$\frac{dy}{dx} = \tan\theta \Rightarrow$ Slope of normal $= -\cot\theta$

Equation of normal at 'θ' is

$y - a(\sin\theta - \theta\cos\theta)$

$= -\cot\theta\ (x - a(\cos\theta + \theta\sin\theta))$

$\Rightarrow y\sin\theta - a\sin^2\theta + a\theta\cos\theta\sin\theta$

$= -x\cos\theta + a\cos^2\theta + a\theta\sin\theta\cos\theta$

$\Rightarrow x\cos\theta + y\sin\theta = a$

Clearly this is an equation of straight line which is at a constant distance 'a' from origin.

10. **(b)** Given that

Total radius $r = 10 + 5 = 15$ cm

$\frac{dv}{dt} = 50$ cm^3/min $\Rightarrow \frac{d}{dt}\left(\frac{4}{3}\pi r^3\right) = 50$

$\Rightarrow 4\pi r^2\frac{dr}{dt} = 50$

$\Rightarrow \frac{dr}{dt} = \frac{50}{4\pi(15)^2} = \frac{1}{18\pi}$ cm/min

11. **(b)** Let $f(x) = a_n x^n + a_{n-1}x^{n-1} + \ldots\ldots + a_1 x = 0$

Given second equation,

$na_n x^{n-1} + (n-1)a_{n-1}x^{n-2} + \ldots + a_1 = 0 = f'(x)$

Given $a_1 \neq 0 \Rightarrow f(0) = 0$

Again $f(x)$ has root α, $\Rightarrow f(\alpha) = 0$

$\therefore\quad f(0) = f(\alpha)$

$\therefore\quad$ By Rolle's theorem $f'(x) = 0$ has root between $(0, \alpha)$

Hence $f'(x)$ has a positive root smaller than α.

12. **(c)** From option (c), $f(x) = 3x^2 - 2x + 1$ is increasing when $f'(x) = 6x - 2 \geq 0$

$\Rightarrow\quad x \in [1/3, \infty)$

$\therefore f(x)$ is incorrectly matched with

$\left(-\infty, \frac{1}{3}\right]$

13. **(a)** Given $f(x) = \frac{x}{2} + \frac{2}{x} \Rightarrow f'(x) = \frac{1}{2} - \frac{2}{x^2} = 0$

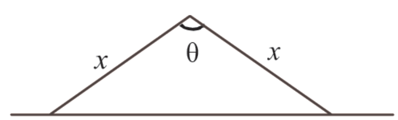
$\Rightarrow x^2 = 4 \Rightarrow x = 2, -2;$

Now, $f''(x) = \frac{4}{x^3}$

$f''(x)\big]_{x=2} = +ve \Rightarrow f(x)$ has local min at $x = 2$.

14. **(c)** Area $= \frac{1}{2}x^2\sin\theta \Rightarrow A = \frac{1}{2}x^2\sin\theta$

(x is constant)



$\frac{dA}{d\theta} = \frac{1}{2}x^2\cos\theta = 0$

$\Rightarrow \theta = \frac{\pi}{2}$

$\frac{dA}{d\theta} = -\frac{1}{2}x^2\sin\theta$

$\left(\frac{dA}{d\theta}\right)_{x=\frac{\pi}{2}} = -ve$

$\therefore A_{max} = \frac{1}{2}x^2$

15. **(c)** We know that Lagrange's Mean Value Theorem

Let $f(x)$ be a function defined on $[a, b]$

then, $f'(c) = \frac{f(b) - f(a)}{b - a}$ ....(i)

$\Rightarrow c \in [a, b]$

$\therefore\quad$ Given $f(x) = \log_e x \quad \therefore f'(x) = \frac{1}{x}$

$\therefore$ From eqn. (i)

$$\frac{1}{c}=\frac{f(3)-f(1)}{3-1}$$

$$\Rightarrow \frac{1}{c}=\frac{\log_e 3-\log_e 1}{2}=\frac{\log_e 3}{2}$$

$$\Rightarrow c=\frac{2}{\log_e 3} \Rightarrow c=2\log_3 e$$

16. (d) Given that $f(x)=\tan^{-1}(\sin x+\cos x)$

Differentiate w.r. to x

$$f'(x)=\frac{1}{1+(\sin x+\cos x)^2}.(\cos x-\sin x)$$

$$=\frac{\sqrt{2}.\left(\frac{1}{\sqrt{2}}\cos x-\frac{1}{\sqrt{2}}\sin x\right)}{1+(\sin x+\cos x)^2}$$

$$=\frac{\sqrt{2}\left(\cos\frac{\pi}{4}.\cos x-\sin\frac{\pi}{4}.\sin x\right)}{1+(\sin x+\cos x)^2}$$

$$\therefore \quad f'(x)=\frac{\sqrt{2}\cos\left(x+\frac{\pi}{4}\right)}{1+(\sin x+\cos x)^2}$$

Given that $f(x)$ is increasing

$$\therefore f'(x)>0 \Rightarrow \cos\left(x+\frac{\pi}{4}\right)>0$$

$$\Rightarrow -\frac{\pi}{2}<x+\frac{\pi}{4}<\frac{\pi}{2}$$

$$\Rightarrow -\frac{3\pi}{4}<x<\frac{\pi}{4}$$

Hence, $f(x)$ is increasing when

$$n\in\left(-\frac{\pi}{2},\frac{\pi}{4}\right)$$

17. (c) Given that $p^2+q^2=1$

$\therefore$ $p=\cos\theta$ and $q=\sin\theta$ satisfy the given equation

Then $p+q=\cos\theta+\sin\theta$

We know that

$$-\sqrt{a^2+b^2}\le a\cos\theta+b\sin\theta\le\sqrt{a^2+b^2}$$

$$\therefore -\sqrt{2}\le\cos\theta+\sin\theta\le\sqrt{2}$$

Hence max. value of $p+q$ is $\sqrt{2}$

18. (a) Let $y=x^3-px+q \Rightarrow \frac{dy}{dx}=3x^2-p$

For maxima and minima

$$\frac{dy}{dx}=0 \Rightarrow 3x^2-p=0 \Rightarrow x=\pm\sqrt{\frac{p}{3}}$$

$$\frac{d^2y}{dx^2}=6x$$

$$\left.\frac{d^2y}{dx^2}\right|_{x=\sqrt{\frac{p}{3}}}=+ve \text{ and } \left.\frac{d^2y}{dx^2}\right|_{x=-\sqrt{\frac{p}{3}}}=-ve$$

$\therefore$ y has minimum at $x=\sqrt{\frac{p}{3}}$ and maximum at $x=-\sqrt{\frac{p}{3}}$

19. (b) Let $f(x)=x^7+14x^5+16x^3+30x-560$

$\Rightarrow f'(x)=7x^6+70x^4+48x^2+30>0, \forall\, x\in R$ ...(i)

$\Rightarrow$ f is an increasing function on R

Also $\lim\limits_{x\to\infty} f(x)=\infty$ and

$\lim\limits_{x\to-\infty} f(x)=-\infty$ ...(ii)

From (i) and (ii) clear that the curve $y=f(x)$ crosses x-axis only once.

$\therefore f(x)=0$ has exactly one real root.

20. (c) Let the centre of the circle be $(h, 2)$

$\therefore$ Equation of circle is

$(x-h)^2+(y-2)^2=25$ ...(1)

Differentiating with respect to x, we get

$$2(x-h)+2(y-2)\frac{dy}{dx}=0$$

$$\Rightarrow x-h=-(y-2)\frac{dy}{dx}$$

Squaring both sides

$$(x-h)^2=(y-2)^2\left(\frac{dy}{dx}\right)^2$$

Substituting in equation (1), we get

$$(y-2)^2\left(\frac{dy}{dx}\right)^2+(y-2)^2=25$$

$$\Rightarrow (y-2)^2(y')^2=25-(y-2)^2$$

21. (b) Given that $f(x)=x|x|$ and $g(x)=\sin x$

$\therefore gof(x)=g(f(x))=g(x|x|)=\sin x|x|$

$$=\begin{cases}\sin(-x^2), & \text{if } x<0\\ \sin(x^2), & \text{if } x\ge 0\end{cases}$$

$$=\begin{cases}-\sin x^2, & \text{if } x<0\\ \sin x^2, & \text{if } x\ge 0\end{cases}$$

$$\therefore (go f)'(x) = \begin{cases} -2x\cos x^2, & \text{if } x < 0 \\ 2x\cos x^2, & \text{if } x \geq 0 \end{cases}$$

Here we observe that

LHD $(go f)'(0) = 0 =$ RHD $(go f)'(0)$

$\Rightarrow$ $go f$ is differentiable at $x = 0$

and $(go f)'$ is continuous at $x = 0$

Now $(go f)''(x)$

$$= \begin{cases} -2\cos x^2 + 4x^2\sin x^2, & x < 0 \\ 2\cos x^2 - 4x^2\sin x^2, & x \geq 0 \end{cases}$$

Here we observe that

LHD $(go f)''(0) = -2$ and RHD $(go f)''(0) = 2$

$\because$ LHD $(go f)''(0) \neq$ RHD $(go f)''(0)$

$\Rightarrow$ $go f(x)$ is not twice differentiable at $x = 0$.

$\therefore$ Statement-1 is true but statement-2 is false.

22. **(a)** Given that $P(x) = x^4 + ax^3 + bx^2 + cx + d$

$\Rightarrow P'(x) = 4x^3 + 3ax^2 + 2bx + c$

But given $P'(0) = 0 \Rightarrow c = 0$

$\therefore P(x) = x^4 + ax^3 + bx^2 + d$

Again given that $P(-1) < P(1)$

$\Rightarrow 1 - a + b + d < 1 + a + b + d$

$\Rightarrow a > 0$

Now $P'(x) = 4x^3 + 3ax^2 + 2bx$

$= x(4x^2 + 3ax + 2b)$

As $P'(x) = 0$, there is only one solution $x = 0$, therefore $4x^2 + 3ax + 2b = 0$ should not have any real roots i.e. $D < 0$

$\Rightarrow 9a^2 - 32b < 0 \Rightarrow b > \dfrac{9a^2}{32} > 0$

Hence $a, b > 0$

$\Rightarrow P'(x) = 4x^3 + 3ax^2 + 2bx > 0$

$\forall x > 0$

$\therefore$ $P(x)$ is an increasing function on $(0,1)$

$\therefore P(0) < P(a)$

Similarly we can prove $P(x)$ is decreasing on $(-1, 0)$

$\therefore P(-1) > P(0)$

So we can conclude that

Max $P(x) = P(1)$ and Min $P(x) = P(0)$

$\Rightarrow$ $P(-1)$ is not minimum but $P(1)$ is the maximum of P.

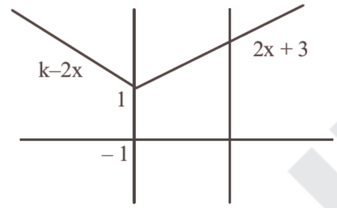
23. **(c)** Since the tangent is parallel to x-axis,

$\therefore \dfrac{dy}{dx} = 0 \Rightarrow 1 - \dfrac{8}{x^3} = 0 \Rightarrow x = 2 \Rightarrow y = 3$

Equation of the tangent is $y - 3 = 0(x - 2)$

$\Rightarrow y = 3$

24. **(c)** $f(x) = \begin{cases} k - 2x, & \text{if } x \leq -1 \\ 2x + 3, & \text{if } x > -1 \end{cases}$



Clear that $f(x)$ is minimum at $(-1, 1)$

$\therefore f(-1) = 1$

$1 = k + 2 \Rightarrow k = -1$

25. **(d)** Given $f(x) = \dfrac{1}{e^x + 2e^{-x}} = \dfrac{e^x}{e^{2x} + 2}$

$$f'(x) = \frac{(e^{2x} + 2)e^x - 2e^{2x}.e^x}{(e^{2x} + 2)^2}$$

$f'(x) = 0 \Rightarrow e^{2x} + 2 = 2e^{2x}$

$\Rightarrow e^{2x} = 2 \Rightarrow e^x = \sqrt{2}$

$\because f''(\sqrt{2}) = +ve$

$\therefore$ Maximum values of $f(x) = \dfrac{\sqrt{2}}{4} = \dfrac{1}{2\sqrt{2}}$

$\Rightarrow 0 < f(x) \leq \dfrac{1}{2\sqrt{2}} \quad \forall x \in R$

Since, $0 < \dfrac{1}{3} < \dfrac{1}{2\sqrt{2}}$

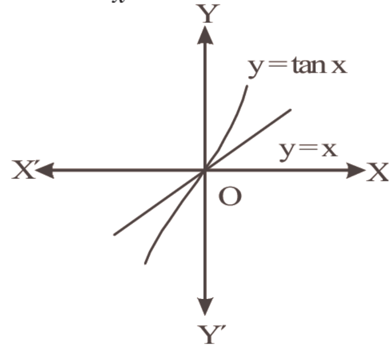
$\Rightarrow$ for some $c \in R$, $f(c) = \dfrac{1}{3}$

26. **(b)** $f(x) = \begin{cases} \dfrac{\tan x}{x}, & x \neq 0 \\ 1, & x = 0 \end{cases}$

For $x > 0$

$\tan x > x$

$\dfrac{\tan x}{x} > 1$



For $x < 0 \Rightarrow \tan x < x$

$\Rightarrow \dfrac{\tan x}{x} > 1$

$f(0) = 1$ at $x = 0$

$\Rightarrow \quad x = 0$ is the point of minima

So, Statement 1 is true. Statement 2 is also true.

27. (c) Volume of spherical balloon

$= V = \dfrac{4}{3}\pi r^3$

Differentiate both the side, w.r.t 't' we get,

$$\frac{dV}{dt} = 4\pi r^2\left(\frac{dr}{dt}\right) \quad ...(i)$$

$\therefore$ After 49 min,

Volume $= (4500 - 49 \times 72)\pi$

$= (4500 - 3528)\pi = 972\,\pi\,m^3$

$\Rightarrow \quad V = 972\,\pi\,m^3$

$\therefore\ 972\pi = \dfrac{4}{3}\pi r^3$

$\Rightarrow \quad r^3 = 3 \times 243 = 3 \times 3^5 = 3^6 = (3^2)^3$

$\Rightarrow \quad r = 9$

Given $\dfrac{dV}{dt} = 72\pi$

Putting $\dfrac{dV}{dt} = 72\pi$ and $r = 9$, we get

$\therefore \quad 72\pi = 4\pi \times 9 \times 9\left(\dfrac{dr}{dt}\right)$

$\Rightarrow \dfrac{dr}{dt} = \left(\dfrac{2}{9}\right)$

28. (b) Given that, $f(x) = ln|x| + bx^2 + ax$

$\therefore \quad f'(x) = \dfrac{1}{x} + 2bx + a$

At $x = -1$, $f'(x) = -1 - 2b + a = 0$

$\Rightarrow \quad a - 2b = 1 \quad ...(i)$

At $x = 2$, $f'(x) = \dfrac{1}{2} + 4b + a = 0$

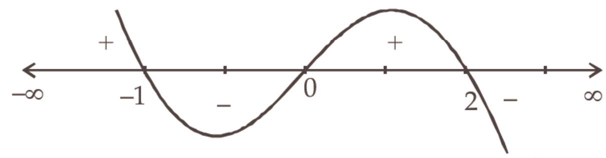
$\Rightarrow \quad a + 4b = -\dfrac{1}{2} \quad ...(ii)$

On solving (i) and (ii) we get $a = \dfrac{1}{2}, b = -\dfrac{1}{4}$

Thus, $f'(x) = \dfrac{1}{x} - \dfrac{x}{2} + \dfrac{1}{2} = \dfrac{2 - x^2 + x}{2x}$

$= \dfrac{-x^2 + x + 2}{2x} = \dfrac{-(x^2 - x - 2)}{2x}$

$= \dfrac{-(x+1)(x-2)}{2x}$



So maxima at $x = -1, 2$

29. (d) $f(x) = 2x^3 + 3x + k$

$f'(x) = 6x^2 + 3 > 0\ \forall\, x \in R \quad (\because \quad x^2 > 0)$

$\Rightarrow \quad f(x)$ is strictly increasing function

$\Rightarrow \quad f(x) = 0$ has only one real root, so two roots are not possible.

30. (a) Let $f(x) = \alpha \log|x| + \beta x^2 + x$

Differentiate both side,

$$f'(x) = \frac{\alpha}{x} + 2\beta x + 1$$

Since $x = -1$ and $x = 2$ are extreme points therefore $f'(x) = 0$ at these points.

Put $x = -1$ and $x = 2$ in $f'(x)$, we get

$-\alpha - 2\beta + 1 = 0 \Rightarrow \alpha + 2\beta = 1$ ...(i)

$\dfrac{\alpha}{2} + 4\beta + 1 = 0 \Rightarrow \alpha + 8\beta = -2$ ...(ii)

On solving (i) and (ii), we get

$6\beta = -3 \Rightarrow \beta = -\dfrac{1}{2}$

$\therefore\ \alpha = 2$

31. (b) Given curve is

$x^2 + 2xy - 3y^2 = 0 \quad ...(1)$

Differentiatew.r.t. x

$$2x + 2x\frac{dy}{dx} + 2y - 6y\frac{dy}{dx} = 0$$

$$\left(\frac{dy}{dx}\right)_{(1,1)} = 1$$

Equation of normal at (1, 1) is

$y = 2 - x \quad ...(2)$

Solving eqs. (1) and (2), we get

$x = 1, 3$

Point of intersection (1, 1), (3, –1)

Normal cuts the curve again in 4th quadrant.

32. (d) $f(x) = \tan^{-1}\left(\sqrt{\dfrac{1+\sin x}{1-\sin x}}\right)$

$$= \tan^{-1}\left(\sqrt{\frac{\left(\sin\frac{x}{2} + \cos\frac{x}{2}\right)^2}{\left(\sin\frac{x}{x} - \cos\frac{x}{2}\right)^2}}\right) = \tan^{-1}\left(\frac{1 + \tan\frac{x}{2}}{1 - \tan\frac{x}{2}}\right)$$

$$= \tan^{-1}\left(\tan\left(\frac{\pi}{4} + \frac{x}{2}\right)\right)$$

$\Rightarrow \ y = \frac{\pi}{4} + \frac{x}{2}$

$\Rightarrow \ \frac{dy}{dx} = \frac{1}{2}$

Slope of normal $= \frac{-1}{\left(\frac{dy}{dx}\right)} = -2$

Equation of normal at $\left(\frac{\pi}{6}, \frac{\pi}{4} + \frac{\pi}{12}\right)$

$y - \left(\frac{\pi}{4} + \frac{\pi}{12}\right) = -2\left(x - \frac{\pi}{6}\right)$

$y - \frac{4\pi}{12} = -2x + \frac{2\pi}{6}$

$y - \frac{\pi}{3} = -2x + \frac{\pi}{3}$

$y = -2x + \frac{2\pi}{3}$

This equation is satisfied only by the point $\left(0, \frac{2\pi}{3}\right)$

33. (a) $4x + 2\pi r = 2 \quad \Rightarrow 2x + \pi r = 1$

$S = x^2 + \pi r^2$

$S = \left(\frac{1 - \pi r}{2}\right)^2 + \pi r^2$

$\frac{dS}{dr} = 2\left(\frac{1 - \pi r}{2}\right)\left(\frac{-\pi}{2}\right) + 2\pi r$

$\Rightarrow \frac{-\pi}{2} + \frac{\pi^2 r}{2} + 2\pi r = 0 \quad \Rightarrow r = \frac{1}{\pi + 4}$

$\Rightarrow x = \frac{2}{\pi + 4} \quad \Rightarrow x = 2r$

34. (c) We have $y = \frac{x + 6}{(x - 2)(x - 3)}$

At y-axis, $x = 0 \Rightarrow y = 1$

On differentiating, we get

$\frac{dy}{dx} = \frac{(x^2 - 5x + 6)(1) - (x + 6)(2x - 5)}{(x^2 - 5x + 6)^2}$

$\frac{dy}{dx} = 1$ at point (0, 1)

$\therefore$ Slope of normal $= -1$

Now equation of normal is $y - 1 = -1(x - 0)$

$\Rightarrow y - 1 = -x$

$x + y = 1$

$\therefore \left(\frac{1}{2}, \frac{1}{2}\right)$ satisfy it.

35. (d) We have

Total length $= r + r + r\theta = 20$

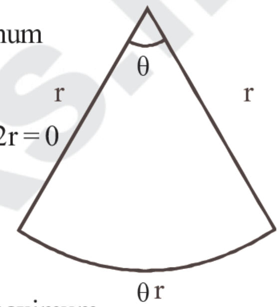
$\Rightarrow 2r + r\,\theta = 20$

$\Rightarrow \theta = \frac{20 - 2r}{r}$ ...(1)

$A = \text{Area} = \frac{\theta}{2\pi} \times \pi r^2 = \frac{1}{2} r^2 \theta = \frac{1}{2} r^2 \left(\frac{20 - 2r}{r}\right)$

$A = 10r - r^2$

For A to be maximum



$\frac{dA}{dr} = 0 \ \Rightarrow 10 - 2r = 0$

$\Rightarrow r = 5$

$\frac{d^2A}{dr^2} = -2 < 0$

$\therefore$ For r = 5 A is maximum

From (1)

$\theta = \frac{20 - 2(5)}{5} = \frac{10}{5} = 2$

$A = \frac{2}{2\pi} \times \pi(5)^2 = 25$ sq. m

36. (c) Eccentricity of ellipse $= \frac{1}{2}$

Now, $-\frac{a}{e} = -4 \Rightarrow a = 4 \times \frac{1}{2} = 2 \Rightarrow a = 2$

We have $b^2 = a^2(1 - e^2) = a^2\left(1 - \frac{1}{4}\right)$

$= 4 \times \frac{3}{4} = 3$

$\therefore$ Equation of ellipse is

$\frac{x^2}{4} + \frac{y^2}{3} = 1$

Now differentiating, we get

$\Rightarrow \frac{x}{2} + \frac{2y}{3} \times y' = 0 \ \Rightarrow y' = -\frac{3x}{4y}$

$y'\Big|_{(1,3/2)}\Big| = -\frac{3}{4} \times \frac{2}{3} = -\frac{1}{2}$

Slope of normal = 2

$\therefore$ Equation of normal at $\left(1, \frac{3}{2}\right)$ is

$y - \frac{3}{2} = 2(x - 1) \Rightarrow 2y - 3 = 4x - 4$

$\therefore 4x - 2y = 1$

37. (c) Here, $h(x)=\dfrac{x^2+\dfrac{1}{x^2}}{x-\dfrac{1}{x}}=\left(x-\dfrac{1}{x}\right)+\dfrac{2}{x-\dfrac{1}{x}}$

Let $x-\dfrac{1}{x}=t$

$h_1(t)=t+\dfrac{2}{t}$

$h_1'(t)=1-\dfrac{2}{t^2}=0$

$t=\pm\sqrt{2}$

$h_1''(t)=\dfrac{4}{t^3}$

$h_1''(-\sqrt{2})=\dfrac{-4}{(\sqrt{2})3}<0$

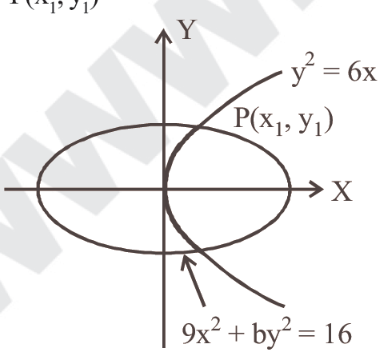
$h_1''(\sqrt{2})=\dfrac{4}{(\sqrt{2})3}>0$

$\therefore$ $h_1(t)$ is minimum at $t=\sqrt{2}$

$\therefore$ $h(\sqrt{2})=2\sqrt{2}$

Hence, $2\sqrt{2}$ will be local minimum value of h(x).

38. (c) Let curve intersect each other at point $P(x_1, y_1)$



Since, point of intersection is on both the curves, then

$y_1^2=6x_1$ ...(i)

and $9x_1^2+by_1^2=16$ ...(ii)

Now, find the slope of tangent to both the curves at the point of intersection $P(x_1, y_1)$

For slope of curves:

curve (i):

$\left(\dfrac{dy}{dx}\right)_{(x_1,y_1)}=m_1=\dfrac{3}{y_1}$

curve (ii):

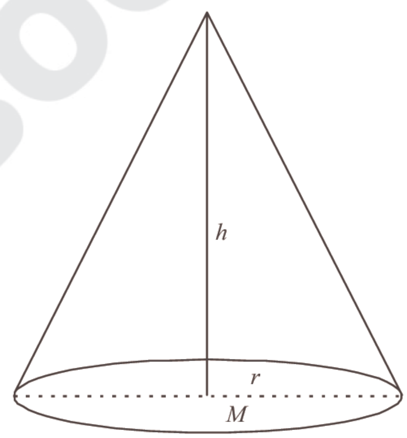
and $\left(\dfrac{dy}{dx}\right)_{(x_1,y_1)}=m_2=-\dfrac{9x_1}{by_1}$

Since, both the curves intersect each other at right angle then,

$m_1m_2=-1\Rightarrow\dfrac{27x_1}{by_1^2}=1\ \Rightarrow b=27\dfrac{x_1}{y_1^2}$

$\therefore$ from equation (i), $b=27\times\dfrac{1}{6}=\dfrac{9}{2}$

39. (d)



$h^2+r^2=\ell^2=9$ ...(1)

Volume of cone

$V=\dfrac{1}{3}\pi r^2h$ ...(2)

From (1) and (2),

$\Rightarrow V=\dfrac{1}{3}\pi(9-h^2)h$

$\Rightarrow V=\dfrac{1}{3}\pi(9h-h^3)$

$\Rightarrow \dfrac{dV}{dh}=\dfrac{1}{3}\pi\left(9-3h^2\right)$

For maxima/minima,

$\dfrac{dV}{dh}=0\Rightarrow\dfrac{1}{3}\pi(9-3h^2)=0$

$\Rightarrow h=\pm\sqrt{3}\Rightarrow h=\sqrt{3}$ $(\because h>0)$

Now; $\frac{d^2V}{dh^2} = \frac{1}{3}\pi(-6h)$

Here, $\left(\frac{d^2V}{dh^2}\right)_{\text{at } h=\sqrt{3}} < 0$

Then, $h=\sqrt{3}$ is point of maxima

Hence, the required maximum volume is,

$$V = \frac{1}{3}\pi(9-3)\sqrt{3} = 2\sqrt{3}\pi$$

40. (a) $\because a^2 = \cos^2\theta, b^2 = \sin^2\theta$

and $e > 2 \Rightarrow e^2 > 4 \Rightarrow 1 + b^2/a^2 > 4$

$\Rightarrow 1 + \tan^2\theta > 4$

$\Rightarrow \sec^2\theta > 4 \Rightarrow \theta \in \left(\frac{\pi}{3}, \frac{\pi}{2}\right)$

Latus rectum,

$$LR = \frac{2b^2}{a} = \frac{2\sin^2\theta}{\cos\theta} = 2(\sec\theta - \cos\theta)$$

$$\Rightarrow \frac{d(LR)}{d\theta} = 2(\sec\theta\tan\theta + \sin\theta)$$

$$\frac{d^2(\text{LR})}{d\theta^2} = 2[\sec^2\theta\cdot\tan\theta + \sec^3\theta + \cos\theta] > 0$$

$\forall\, \theta \in \left(\frac{\pi}{3}, \frac{\pi}{2}\right)$

$\therefore$ min (LR)

$$= 2\left(\sec\frac{\pi}{3} - \cos\frac{\pi}{3}\right) = 2\left(2 - \frac{1}{2}\right) = 3$$

max (LR) tends to infinity as $\theta \to \frac{\pi}{2}$

Hence, length of latus rectum lies in the interval $(3, \infty)$.

41. (b) Since, the equation of curves are

$y = 10 - x^2$ ...(1)

$y = 2 + x^2$ ...(2)

Adding eqs. (1) and (2), we get

$2y = 12 \Rightarrow y = 6$

Then, from eqn (1)

$x = \pm 2$

Differentiate equation (1) with respect to x

$$\frac{dy}{dx} = -2x \Rightarrow \left(\frac{dy}{dx}\right)_{(2,6)} = -4 \text{ and } \left(\frac{dy}{dx}\right)_{(-2,6)} = 4$$

Differentiate equation (2) with respect to x

$$\frac{dy}{dx} = 2x \Rightarrow \left(\frac{dy}{dx}\right)_{(2,6)} = 4 \text{ and } \left(\frac{dy}{dx}\right)_{(-2,6)} = -4$$

At (2, 6) $\tan\theta = \left(\frac{(-4)-(4)}{1+(-4)\times(4)}\right) = \frac{-8}{15}$

At (–2, 6), $\tan\theta = \frac{(4)-(-4)}{1+(4)(-4)} = \frac{8}{-15}$

$\therefore |\tan\theta| = \frac{8}{15}$

42. (d) $y = x^3 + ax - b$

Since, the point (1, –5) lies on the curve.

$\Rightarrow 1 + a - b = -5$

$\Rightarrow a - b = -6$ ...(1)

Now, $\frac{dy}{dx} = 3x^2 + a \Rightarrow \left(\frac{dy}{dx}\right)_{\text{at } x=1} = 3 + a$

Since, tangent is perpendicular to $y = x - 4$, then slope of tangent at the point P (1, –5) = –1

$\therefore 3 + a = -1 \Rightarrow a = -4 \Rightarrow b = 2$

$\therefore$ the equation of the curve is $y = x^3 - 4x - 2$

$\Rightarrow$ (2, –2) lies on the curve.

43. (d) $y = f(x) = x^3 - x^2 - 2x$

$$\Rightarrow \frac{dy}{dx} = 3x^2 - 2x - 2$$

$f(1) = 1 - 1 - 2 = -2$, $f(-1) = -1 - 1 + 2 = 0$

Since the tangent to the curve is parallel to the line segment joining the points (1, –2) and (–1, 0)

And their slopes are equal.

$$\Rightarrow 3x^2 - 2x - 2 = \frac{-2-0}{2} \Rightarrow x = 1, \frac{-1}{3}$$

Hence, the required set $S = \left\{\frac{-1}{3}, 1\right\}$

22 Integrals

1. $\int_0^{10\pi} |\sin x|\, dx$ is **[2002]**
   (a) 20 (b) 8
   (c) 10 (d) 18

2. $I_n = \int_0^{\pi/4} \tan^n x\, dx$ then $\lim_{n\to\infty} n[I_n + I_{n+2}]$ equals **[2002]**
   (a) $\frac{1}{2}$ (b) 1
   (c) ∞ (d) zero

3. $\int_0^2 [x^2] dx$ is **[2002]**
   (a) $2-\sqrt{2}$ (b) $2+\sqrt{2}$
   (c) $\sqrt{2}-1$ (d) $-\sqrt{2}-\sqrt{3}+5$

4. $\int_{-\pi}^{\pi} \frac{2x(1+\sin x)}{1+\cos^2 x} dx$ is **[2002]**
   (a) $\frac{\pi^2}{4}$ (b) π^2
   (c) zero (d) $\frac{\pi}{2}$

5. If $f(a+b-x) = f(x)$ then $\int_a^b x f(x) dx$ is equal to **[2003]**
   (a) $\frac{a+b}{2}\int_a^b f(a+b+x)dx$
   (b) $\frac{a+b}{2}\int_a^b f(b-x)dx$
   (c) $\frac{a+b}{2}\int_a^b f(x)dx$
   (d) $\frac{b-a}{2}\int_a^b f(x)dx$.

6. Let $f(x)$ be a function satisfying $f'(x) = f(x)$ with $f(0)=1$ and $g(x)$ be a function that satisfies $f(x) + g(x) = x^2$. Then the value of the integral $\int_0^1 f(x)\, g(x) dx$, is **[2003]**
   (a) $e+\frac{e^2}{2}+\frac{5}{2}$ (b) $e-\frac{e^2}{2}-\frac{5}{2}$
   (c) $e+\frac{e^2}{2}-\frac{3}{2}$ (d) $e-\frac{e^2}{2}-\frac{3}{2}$.

7. The value of the integral $I = \int_0^1 x(1-x)^n dx$ is **[2003]**
   (a) $\frac{1}{n+1}+\frac{1}{n+2}$ (b) $\frac{1}{n+1}$
   (c) $\frac{1}{n+2}$ (d) $\frac{1}{n+1}-\frac{1}{n+2}$.

8. $\lim_{n\to\infty} \sum_{r=1}^{n} \frac{1}{n} e^{\frac{r}{n}}$ is **[2004]**
   (a) $e+1$ (b) $e-1$
   (c) $1-e$ (d) e

9. The value of $\int_{-2}^{3}|1-x^2|dx$ is [2004]

(a) $\frac{1}{3}$ (b) $\frac{14}{3}$

(c) $\frac{7}{3}$ (d) $\frac{28}{3}$

10. The value of $I=\int_{0}^{\pi/2}\frac{(\sin x+\cos x)^2}{\sqrt{1+\sin 2x}}dx$ is [2004]

(a) 3 (b) 1

(c) 2 (d) 0

11. If $\int_{0}^{\pi}xf(\sin x)dx=A\int_{0}^{\pi/2}f(\sin x)dx$, then A is [2004]

(a) 2π (b) π

(c) $\frac{\pi}{4}$ (d) 0

12. If $f(x)=\frac{e^x}{1+e^x}$, $I_1=\int_{f(-a)}^{f(a)}xg\{x(1-x)\}dx$ and $I_2=\int_{f(-a)}^{f(a)}g\{x(1-x)\}dx$, then the value of $\frac{I_2}{I_1}$ is [2004]

(a) 1 (b) –3

(c) –1 (d) 2

13. If $\int\frac{\sin x}{\sin(x-\alpha)}dx=Ax+B\log\sin(x-\alpha)+C$, then value of (A, B) is [2004]

(a) $(-\cos\alpha, \sin\alpha)$ (b) $(\cos\alpha, \sin\alpha)$

(c) $(-\sin\alpha, \cos\alpha)$ (d) $(\sin\alpha, \cos\alpha)$

14. $\int\frac{dx}{\cos x-\sin x}$ is equal to [2004]

(a) $\frac{1}{\sqrt{2}}\log\left|\tan\left(\frac{x}{2}+\frac{3\pi}{8}\right)\right|+C$

(b) $\frac{1}{\sqrt{2}}\log\left|\cot\left(\frac{x}{2}\right)\right|+C$

(c) $\frac{1}{\sqrt{2}}\log\left|\tan\left(\frac{x}{2}-\frac{3\pi}{8}\right)\right|+C$

(d) $\frac{1}{\sqrt{2}}\log\left|\tan\left(\frac{x}{2}-\frac{\pi}{8}\right)\right|+C$

15. $\int\left\{\frac{(\log x-1)}{1+(\log x)^2}\right\}^2 dx$ is equal to [2005]

(a) $\frac{\log x}{(\log x)^2+1}+C$ (b) $\frac{x}{x^2+1}+C$

(c) $\frac{xe^x}{1+x^2}+C$ (d) $\frac{x}{(\log x)^2+1}+C$

16. If $I_1=\int_{0}^{1}2^{x^2}dx$, $I_2=\int_{0}^{1}2^{x^3}dx$, $I_3=\int_{1}^{2}2^{x^2}dx$ and $I_4=\int_{1}^{2}2^{x^3}dx$ then [2005]

(a) $I_2>I_1$ (b) $I_1>I_2$

(c) $I_3=I_4$ (d) $I_3>I_4$

17. The value of $\int_{-\pi}^{\pi}\frac{\cos^2 x}{1+a^x}dx$, a > 0, is [2005]

(a) a π (b) $\frac{\pi}{2}$

(c) $\frac{\pi}{a}$ (d) 2π

18. The value of integral, $\int_{3}^{6}\frac{\sqrt{x}}{\sqrt{9-x}+\sqrt{x}}dx$ is [2006]

(a) $\frac{1}{2}$ (b) $\frac{3}{2}$

(c) 2 (d) 1

19. $\int_0^{\pi} xf(\sin x)dx$ is equal to **[2006]**

(a) $\pi\int_0^{\pi} f(\cos x)dx$ (b) $\pi\int_0^{\pi} f(\sin x)dx$

(c) $\frac{\pi}{2}\int_0^{\pi/2} f(\sin x)dx$ (d) $\pi\int_0^{\pi/2} f(\cos x)dx$

20. $\int_{-\frac{3\pi}{2}}^{-\frac{\pi}{2}} [(x+\pi)^3+\cos^2(x+3\pi)]dx$ is equal to **[2006]**

(a) $\frac{\pi^4}{32}$ (b) $\frac{\pi^4}{32}+\frac{\pi}{2}$

(c) $\frac{\pi}{2}$ (d) $\frac{\pi}{4}-1$

21. The value of $\int_1^a [x]f'(x)dx$, $a > 1$ where $[x]$ denotes the greatest integer not exceeding x is **[2006]**

(a) $af(a)-\{f(1)+f(2)+.............f([a])\}$

(b) $[a]f(a)-\{f(1)+f(2)+.............f([a])\}$

(c) $[a]f([a])-\{f(1)+f(2)+.............f(a)\}$

(d) $af([a])-\{f(1)+f(2)+.............f(a)\}$

22. $\int \frac{dx}{\cos x+\sqrt{3}\sin x}$ equals **[2007]**

(a) $\log\tan\left(\frac{x}{2}+\frac{\pi}{12}\right)+C$

(b) $\log\tan\left(\frac{x}{2}-\frac{\pi}{12}\right)+C$

(c) $\frac{1}{2}\log\tan\left(\frac{x}{2}+\frac{\pi}{12}\right)+C$

(d) $\frac{1}{2}\log\tan\left(\frac{x}{2}-\frac{\pi}{12}\right)+C$

23. Let $F(x)=f(x)+f\left(\frac{1}{x}\right)$, where $f(x)=\int_l^x \frac{\log t}{1+t}dt$, Then $F(e)$ equals **[2007]**

(a) 1 (b) 2

(c) 1/2 (d) 0

24. The solution for x of the equation

$\int_{\sqrt{2}}^{x} \frac{dt}{t\sqrt{t^2-1}}=\frac{\pi}{2}$ is **[2007]**

(a) $\frac{\sqrt{3}}{2}$ (b) $2\sqrt{2}$

(c) 2 (d) None of these

25. Let $I=\int_0^1 \frac{\sin x}{\sqrt{x}}dx$ and $J=\int_0^1 \frac{\cos x}{\sqrt{x}}dx$. Then which one of the following is true? **[2008]**

(a) $I>\frac{2}{3}$ and $J>2$ (b) $I<\frac{2}{3}$ and $J<2$

(c) $I<\frac{2}{3}$ and $J>2$ (d) $I>\frac{2}{3}$ and $J<2$

26. The value of $\sqrt{2}\int \frac{\sin x dx}{\sin\left(x-\frac{\pi}{4}\right)}$ is **[2008]**

(a) $x+\log\left|\cos\left(x-\frac{\pi}{4}\right)\right|+c$

(b) $x-\log\left|\sin\left(x-\frac{\pi}{4}\right)\right|+c$

(c) $x+\log\left|\sin\left(x-\frac{\pi}{4}\right)\right|+c$

(d) $x-\log\left|\cos\left(x-\frac{\pi}{4}\right)\right|+c$

27. $\int_0^{\pi}[\cot x]dx$, where [.] denotes the greatest integer function, is equal to : **[2009]**

(a) 1 (b) –1

(c) $-\frac{\pi}{2}$ (d) $\frac{\pi}{2}$

28. Let $p(x)$ be a function defined on **R** such that $p'(x) = p'(1-x)$, for all $x \in [0, 1]$, $p(0) = 1$ and $p(1) = 41$. Then $\int_0^1 p(x)\,dx$ equals **[2010]**

(a) 21 (b) 41
(c) 42 (d) $\sqrt{41}$

29. The value of $\int_0^1 \frac{8\log(1+x)}{1+x^2}\,dx$ is **[2011]**

(a) $\frac{\pi}{8}\log 2$ (b) $\frac{\pi}{2}\log 2$
(c) $\log 2$ (d) $\pi \log 2$

30. Let [.] denote the greatest integer function then the value of $\int_0^{1.5} x\left[x^2\right]dx$ is :. **[2011 RS]**

(a) 0 (b) $\frac{3}{2}$
(c) $\frac{3}{4}$ (d) $\frac{5}{4}$

31. If the
$$\int \frac{5\tan x}{\tan x - 2}\,dx = x + a\ln|\sin x - 2\cos x| + k,$$
then a is equal to : **[2012]**

(a) -1 (b) -2
(c) 1 (d) 2

32. If $g(x) = \int_0^x \cos 4t\,dt$, then $g(x+\pi)$ equals **[2012]**

(a) $\frac{g(x)}{g(\pi)}$ (b) $g(x) + g(\pi)$
(c) $g(x) - g(\pi)$ (d) $g(x) \cdot g(\pi)$

33. If $\int f(x)dx = \psi(x)$, then $\int x^5 f(x^3)dx$ is equal to **[2013]**

(a) $\frac{1}{3}\left[x^3\psi(x^3) - \int x^2\psi(x^3)dx\right] + C$

(b) $\frac{1}{3}x^3\psi(x^3) - 3\int x^3\psi(x^3)dx + C$

(c) $\frac{1}{3}x^3\psi(x^3) - \int x^2\psi(x^3)dx + C$

(d) $\frac{1}{3}\left[x^3\psi(x^3) - \int x^3\psi(x^3)dx\right] + C$

34. **Statement-1 :** The value of the integral
$$\int_{\pi/6}^{\pi/3} \frac{dx}{1+\sqrt{\tan x}}$$ is equal to $\pi/6$

Statement-2 : $\int_a^b f(x)dx = \int_a^b f(a+b-x)dx.$

[2013]

(a) Statement-1 is true; Statement-2 is true; Statement-2 is a correct explanation for Statement-1.
(b) Statement-1 is true; Statement-2 is true; Statement-2 is not a correct explanation for Statement-1.
(c) Statement-1 is true; Statement-2 is false.
(d) Statement-1 is false; Statement-2 is true.

35. The intercepts on x-axis made by tangents to the curve, $y = \int_0^x |t|\,dt,\ x \in \mathrm{R}$, which are parallel to the line $y = 2x$, are equal to : **[2013]**

(a) ± 1 (b) ± 2
(c) ± 3 (d) ± 4

36. The integral $\int\left(1 + x - \frac{1}{x}\right)e^{x+\frac{1}{x}}dx$ is equal to **[2014]**

(a) $(x+1)e^{x+\frac{1}{x}} + c$ (b) $-xe^{x+\frac{1}{x}} + c$
(c) $(x-1)e^{x+\frac{1}{x}} + c$ (d) $xe^{x+\frac{1}{x}} + c$

37. The integral $\int_0^{\pi}\sqrt{1 + 4\sin^2\frac{x}{2} - 4\sin\frac{x}{2}}\,dx$ equals: **[2014]**

(a) $4\sqrt{3} - 4$ (b) $4\sqrt{3} - 4 - \frac{\pi}{3}$
(c) $\pi - 4$ (d) $\frac{2\pi}{3} - 4 - 4\sqrt{3}$

38. The integral $\int \frac{dx}{x^2(x^4+1)^{3/4}}$ equals : **[2015]**

(a) $-(x^4+1)^{\frac{1}{4}}+c$ (b) $-\left(\frac{x^4+1}{x^4}\right)^{\frac{1}{4}}+c$

(c) $\left(\frac{x^4+1}{x^4}\right)^{\frac{1}{4}}+c$ (d) $(x^4+1)^{\frac{1}{4}}+c$

39. The integral $\int_2^4 \frac{\log x^2}{\log x^2+\log(36-12x+x^2)}dx$ is equal to : **[2015]**

(a) 1 (b) 6
(c) 2 (d) 4

40. The integral $\int \frac{2x^{12}+5x^9}{\left(x^5+x^3+1\right)^3}dx$ is equal to : **[2016]**

(a) $\frac{x^5}{2\left(x^5+x^3+1\right)^2}+C$

(b) $\frac{-x^{10}}{2\left(x^5+x^3+1\right)^2}+C$

(c) $\frac{-x^5}{\left(x^5+x^3+1\right)^2}+C$

(d) $\frac{x^{10}}{2\left(x^5+x^3+1\right)^2}+C$

where C is an arbitrary constant.

41. The integral $\int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \frac{dx}{1+\cos x}$ is equal to : **[2017]**

(a) –1 (b) –2
(c) 2 (d) 4

42. Let $I_n = \int \tan^n x\, dx, (n>1)$. $I_4+I_6 = a\tan^5 x + bx^5 + C$, where C is constant of integration, then the ordered pair (a, b) is equal to : **[2017]**

(a) $\left(-\frac{1}{5},0\right)$ (b) $\left(-\frac{1}{5},1\right)$

(c) $\left(\frac{1}{5},0\right)$ (d) $\left(\frac{1}{5},-1\right)$

43. The integral

$$\int \frac{\sin^2 x\cos^2 x}{(\sin^5 x+\cos^3 x\sin^2 x+\sin^3 x\cos^2 x+\cos^5 x)^2}dx$$

is equal to : **[2018]**

(a) $\frac{-1}{3(1+\tan^3 x)}+C$ (b) $\frac{1}{1+\cot^3 x}+C$

(c) $\frac{-1}{1+\cot^3 x}+C$ (d) $\frac{1}{3(1+\tan^3 x)}+C$

(where C is a constant of integration)

44. The value of $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{\sin^2 x}{1+2^x}dx$ is : **[2018]**

(a) $\frac{\pi}{2}$ (b) 4π

(c) $\frac{\pi}{4}$ (d) $\frac{\pi}{8}$

45. The value of $\int_0^{\pi} |\cos x|^3\, dx$ is: **[2019]**

(a) 0 (b) $\frac{4}{3}$

(c) $\frac{2}{3}$ (d) $-\frac{4}{3}$

46. For $x^2 \neq n\pi + 1$, $n \in N$ (the set of natural numbers), the integral

$$\int x\sqrt{\frac{2\sin(x^2-1)-\sin 2\left(x^2-1\right)}{2\sin(x^2-1)+\sin 2\left(x^2-1\right)}}\,dx$$ equal is to: **[2019]**

(a) $\log_e\left|\frac{1}{2}\sec^2\left(x^2-1\right)\right|+c$

(b) $\frac{1}{2}\log_e|\sec(x^2-1)|+c$

(c) $\frac{1}{2}\log_e\left|\sec^2\left(\frac{x^2-1}{2}\right)\right|+c$

(d) $\log_e\left|\sec\left(\frac{x^2-1}{2}\right)\right|+c$

(where c is a constant of integration)

47. If f(x) is a non-zero polynomial of degree four, having local extreme points at $x = -1, 0, 1$; then the set $S = \{x \in R : f(x) = f(0)\}$ contains exactly: **[2019]**

(a) four irrational numbers.
(b) four rational numbers.
(c) two irrational and two rational numbers.
(d) two irrational and one rational number.

48. The integral $\int \sec^{2/3} x \operatorname{cosec}^{4/3} x\, dx$ is equal to: **[2019]**

(a) $-3\tan^{-1/3} x + C$ (b) $-\frac{3}{4}\tan^{-4/3} x + C$
(c) $-3\cot^{-1/3} x + C$ (d) $3\tan^{-1/3} x + C$
(Here C is a constant of integration)

49. The value of $\int_0^{\pi/2} \frac{\sin^3 x}{\sin x + \cos x} dx$ is: **[2019]**

(a) $\frac{\pi - 2}{8}$ (b) $\frac{\pi - 1}{4}$
(c) $\frac{\pi - 2}{4}$ (d) $\frac{\pi - 1}{2}$

50. If $f(a + b + 1 - x) = f(x)$, for all x, where a and b are fixed positive real numbers, **[2020]**

then $\frac{1}{a+b}\int_a^b x(f(x) + f(x+1))dx$ is equal to:

(a) $\int_{a+1}^{b+1} f(x)dx$ (b) $\int_{a-1}^{b-1} f(x)dx$
(c) $\int_{a-1}^{b-1} f(x+1)dx$ (d) $\int_{a+1}^{b+1} f(x+1)dx$

Answer Key

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
(a)	(b)	(d)	(b)	(c)	(d)	(d)	(b)	(d)	(c)	(b)	(d)	(b)	(a)	(d)
16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
(b)	(b)	(b)	(d)	(c)	(b)	(c)	(c)	(d)	(b)	(c)	(c)	(a)	(d)	(c)
31	32	33	34	35	36	37	38	39	40	41	42	43	44	45
(d)	(b, c)	(c)	(d)	(a)	(d)	(b)	(b)	(a)	(d)	(c)	(c)	(a)	(c)	(b)
46	47	48	49	50										
(c, d)	(d)	(a)	(b)	(c)										

Solutions

1. (a) $I = \int_0^{10\pi} |\sin x|\, dx = 10\int_0^{\pi} |\sin x|\, dx$

$[\because \sin(10\pi - x) = \sin x]$

$= 10\int_0^{\pi} \sin x\, dx$

$\because \sin x > 0$, for $0 < x < \pi$.
as $\sin(\pi - x) = \sin x$

$I = 20\int_0^{\pi/2} \sin x\, dx = 20[-\cos x]_0^{\pi/2} = 20$

2. (b) $I_n + I_{n+2} = \int_0^{\pi/4} \tan^n x(1 + \tan^2 x)dx$

$= \int_0^{\pi/4} \tan^n x \sec^2 x\, dx = \left[\frac{\tan^{n+1} x}{n+1}\right]_0^{\pi/4}$

$\left[\because \int x^n dx = \frac{x^{n+1}}{n+1}\right]$

$= \frac{1-0}{n+1} = \frac{1}{n+1}$

$\therefore I_n + I_{n+2} = \frac{1}{n+1} \Rightarrow \lim_{n\to\infty} n[I_n + I_{n+2}]$

$$= \lim_{n\to\infty} n.\frac{1}{n+1} = \lim_{n\to\infty}\frac{n}{n+1}$$

$$= \lim_{n\to\infty}\frac{n}{n\left(1+\frac{1}{n}\right)} = 1$$

3. **(d)** We know that $[x]$ is greatest integer function less than equal to x

$$\therefore \int_0^2 \left[x^2\right]dx = \int_0^1 \left[x^2\right]dx + \int_1^{\sqrt{2}} \left[x^2\right]dx + \int_{\sqrt{2}}^{\sqrt{3}} \left[x^2\right] + \int_{\sqrt{3}}^{2} \left[x^2\right]dx$$

$$= \int_0^1 0dx + \int_1^{\sqrt{2}} 1dx + \int_{\sqrt{2}}^{\sqrt{3}} 2dx + \int_{\sqrt{3}}^{2} 3dx$$

$$= [x]_1^{\sqrt{2}} + [2x]_{\sqrt{2}}^{\sqrt{3}} + [3x]_{\sqrt{3}}^{2}$$

$$= \sqrt{2} - 1 + 2\sqrt{3} - 2\sqrt{2} + 6 - 3\sqrt{3}$$

$$= 5 - \sqrt{3} - \sqrt{2}$$

4. **(b)** $\int_{-\pi}^{\pi}\frac{2x\,(1+\sin\,x)}{1+\cos^2 x}dx$

$$= \int_{-\pi}^{\pi}\frac{2x\,dx}{1+\cos^2 x} + 2\int_{-\pi}^{\pi}\frac{x\,\sin\,x}{1+\cos^2 x}dx$$

$$= 0 + 4\int_0^{\pi}\frac{x\,\sin x\,dx}{1+\cos^2 x};$$

We know that

$$\because \int_{-a}^{a} f(x)\,dx = 0 \quad , \text{ if } f(x) \text{ is odd.}$$

$$= 2\int_0^a f(x)dx \qquad , \text{ if } f(x) \text{ is even}$$

$$I = 4\int_0^{\pi}\frac{(\pi - x)\,\sin\,(\pi - x)}{1+\cos^2\,(\pi - x)}dx$$

$$I = 4\int_0^{\pi}\frac{(\pi - x)\,\sin\,x}{1+\cos^2\,x}dx$$

$$\Rightarrow I = 4\pi\int_0^{\pi}\frac{\sin\,x\,dx}{1+\cos^2\,x} - 4\int\frac{x\,\sin\,x\,dx}{1+\cos^2\,x}$$

$$\Rightarrow 2I = 4\pi\int_0^{\pi}\frac{\sin\,x}{1+\cos^2\,x}dx$$

put $\cos x = t \;\Rightarrow -\sin x\,dx = dt$

when $x = 0$, $t = 1$ and when $x = \pi$, $t = -1$

$$\therefore\; I = -2\pi\int_1^{-1}\frac{1}{1+t^2}dt = 2\pi\int_{-1}^{1}\frac{1}{1+t^2}dt$$

$$= 2\pi\left[\tan^{-1} t\right]_{-1}^{1}$$

$$= 2\pi\left[\tan^{-1} 1 - \tan^{-1}(-1)\right]$$

$$= 2\pi\left[\frac{\pi}{4} - \left(\frac{-\pi}{4}\right)\right] = 2\pi.\frac{\pi}{2} = \pi^2$$

5. **(c)** $I = \int_a^b xf(x)dx = \int_a^b (a+b-x)f(a+b-x)dx$

We know that

$$\because \int_a^b f(x)dx = \int_a^b f(a+b-x)dx$$

$$= (a+b)\int_a^b f(a+b-x)dx - \int_a^b xf(a+b-x)dx$$

$$= (a+b)\int_a^b f(x)\,dx - \int_a^b xf(x)dx$$

[$\because$ Given that $f(a+b-x) = f(x)$]

$$2I = (a+b)\int_a^b f(x)dx$$

$$\Rightarrow \quad I = \frac{(a+b)}{2}\int_a^b f(x)dx$$

6. **(d)** Given that $f'(x) = f(x) \Rightarrow \frac{f'(x)}{f(x)} = 1$

Integrating both side we get

$$\log f(x) = x + c \Rightarrow f(x) = e^{x+c}$$

$$f(0) = 1 \Rightarrow f(x) = e^x$$

$$\therefore g(x) = x^2 - f(x) = x^2 - e^x$$

$$\therefore \int_0^1 f(x)g(x)dx = \int_0^1 e^x(x^2 - e^x)dx$$

$$= \int_0^1 x^2 e^x dx - \int_0^1 e^{2x}dx$$

$$= \left[x^2 e^x\right]_0^1 - 2\left[xe^x - e^x\right]_0^1 - \frac{1}{2}\left[e^{2x}\right]_0^1$$

$$= e - \left[\frac{e^2}{2} - \frac{1}{2}\right] - 2[e - e + 1] = e - \frac{e^2}{2} - \frac{3}{2}$$

7. (d) $I=\int_0^1 x(1-x)^n\,dx=\int_0^1 (1-x)(1-1+x)^n\,dx$

$=\int_0^1 (1-x)x^n dx=\int_0^1 (x^n-x^{n+1})dx$

$=\left[\frac{x^{n+1}}{n+1}-\frac{x^{n+2}}{n+2}\right]_0^1 \quad =\frac{1}{n+1}-\frac{1}{n+2}$

8. (b) $\lim_{n\to\infty}\sum_{r=1}^{n}\frac{1}{n}e^{\frac{r}{n}}$ [Using definite integrals as limit of sum]

$=\int_0^1 e^x dx=e-1$

9. (d) $\int_{-2}^{3} |1-x^2|\,dx=\int_{-2}^{3} |x^2-1|\,dx$

We know that $x^2-1=0\Rightarrow x=1,-1$

Now $|x^2-1|=\begin{cases} x^2-1 & if \quad x\le -1 \\ 1-x^2 & if \quad -1\le x\le 1 \\ x^2-1 & if \quad x\ge 1\end{cases}$

$\therefore$ Integral is

$\int_{-2}^{-1}(x^2-1)dx+\int_{-1}^{1}(1-x^2)dx+\int_{1}^{3}(x^2-1)dx$

$=\left[\frac{x^3}{3}-x\right]_{-2}^{-1}+\left[x-\frac{x^3}{3}\right]_{-1}^{1}+\left[\frac{x^3}{3}-x\right]_{1}^{3}$

$=\left(-\frac{1}{3}+1\right)-\left(-\frac{8}{3}+2\right)+\left(2-\frac{2}{3}\right)$

$+\left(\frac{27}{3}-3\right)-\left(\frac{1}{3}-1\right)$

$=\frac{2}{3}+\frac{2}{3}+\frac{4}{3}+6+\frac{2}{3}=\frac{28}{3}$

10. (c) $I=\int_0^{\frac{\pi}{2}}\frac{(\sin x+\cos x)^2}{\sqrt{1+\sin 2x}}dx$

$\int_0^{\pi/2}\frac{(\sin x+\cos x)^2}{\sqrt{(\sin x+\cos x)^2}}$

$I=\int_0^{\frac{\pi}{2}}\frac{(\sin x+\cos x)^2}{(\sin x+\cos x)}dx$

$=\int_0^{\frac{\pi}{2}}(\sin x+\cos x)dx$

$\left[\because \sin x+\cos x>0 \ if\, 0<x<\frac{\pi}{2}\right]$

or $I=[-\cos x+\sin x]_0^{\frac{\pi}{2}}=2$

11. (b) Let $I=\int_0^{\pi} xf(\sin x)dx$ ...(i)

We know that

$\int_0^a f(x)dx=\int_0^a f(a-x)dx$

$=\int_0^{\pi}(\pi-x)f(\sin x)dx$ ...(ii)

Adding (i) and (ii)

$\therefore 2I=\pi\int_2^{\pi} f(\sin x)dx \ =\pi.2\int_0^{\frac{\pi}{2}} f(\sin x)dx$

$[\because \sin(\pi-x)=\sin x]$

$\therefore I=\pi\int_0^{\frac{\pi}{2}} f(\sin x)dx\Rightarrow A=\pi$

Let $\log x=t\Rightarrow e^t=x$

$\Rightarrow \frac{1}{x}dx=dt$

$\Rightarrow dx=xdt\Rightarrow e^t dt.$

12. (d) $f(x)=\frac{e^x}{1+e^x}$

$\Rightarrow f(-x)=\frac{e^{-x}}{1+e^{-x}}=\frac{1}{e^x+1}$

$\therefore f(x)+f(-x)=1 \ \forall \ x \in R$

Now $I_1 = \int_{f(-a)}^{f(a)} xg\{x(1-x)\}dx$

$= \int_{f(-a)}^{f(a)} (1-x)g\{x(1-x)\}dx$

$\left[\text{using} \int_a^b f(x)\,dx\, a = \int_a^b f(a+b-x)\,dx\right]$

$\Rightarrow \int_{f(-a)}^{f(a)} g\{x(1-x)\}dx - \int_{f(-a)}^{f(a)} xg(x(1-x)dx$

$= I_2 - I_1 \Rightarrow 2I_1 = I_2$

13. (b) $\int \frac{\sin x}{\sin(x-\alpha)}dx = \int \frac{\sin(x-\alpha+\alpha)}{\sin(x-\alpha)}dx$

$= \int \frac{\sin(x-\alpha)\cos\alpha + \cos(x-\alpha)\sin\alpha}{\sin(x-\alpha)}dx$

$= \int \{\cos\alpha + \sin\alpha \cot(x-\alpha)\}dx$

$= (\cos\alpha)x + (\sin\alpha)\log\sin(x-\alpha) + C$

Comparing with $Ax + B\log\sin(x-\alpha) + c$

$\therefore A = \cos\alpha,\ B = \sin\alpha$

14. (a) $\int \frac{dx}{\cos x - \sin x} = \int \frac{dx}{\sqrt{2}\left(\frac{1}{\sqrt{2}}\cos x - \frac{1}{\sqrt{2}}\sin x\right)}$

$= \int \frac{dx}{\sqrt{2}\cos\left(x+\frac{\pi}{4}\right)}$

$= \frac{1}{\sqrt{2}}\int \sec\left(x+\frac{\pi}{4}\right)dx$

$= \frac{1}{\sqrt{2}}\log\left|\tan\left(\frac{\pi}{4}+\frac{x}{2}+\frac{\pi}{8}\right)\right| + C$

$\left[\because \int \sec x\,dx = \log\left|\tan\left(\frac{\pi}{4}+\frac{x}{2}\right)\right|\right]$

$= \frac{1}{\sqrt{2}}\log\left|\tan\left(\frac{x}{2}+\frac{3\pi}{8}\right)\right| + C$

15. (d) $\int \frac{(\log x - 1)^2}{(1+(\log x)^2)^2}dx$

$= \int \frac{1+(\log x)^2 - 2\log x}{\left[1+(\log x)^2\right]^2}dx$

$= \int \left[\frac{1}{(1+(\log x)^2)} - \frac{2\log x}{(1+(\log x)^2)^2}\right]dx$

$\therefore \mathrm{I} = \int \left[\frac{e^t}{1+t^2} - \frac{2te^t}{(1+t^2)^2}\right]dt$

$= \int e^t\left[\frac{1}{1+t^2} - \frac{2t}{(1+t^2)^2}\right]dt$

$\left[\text{Which is of the form} \int e^x\left(f(x)+f'(x)\right)dx = f(x)\cdot e^x + c\right]$

$= \frac{e^t}{1+t^2} + c = \frac{x}{1+(\log x)^2} + c$

16. (b) $I_1 = \int_0^1 2^{x^2}dx,\ I_2 = \int_0^1 2^{x^3}dx,$

$I_3 = \int_0^1 2^{x^2}dx,\ I_4 = \int_0^1 2^{x^3}dx$

$\because\ 2^{x^3} < 2^{x^2},\ 0 < x < 1$

$\Rightarrow \int_0^1 2^{x^2}dx > \int_0^1 2^{x^3}dx \ \Rightarrow\ I_1 > I_2$

and $2^{x^3} > 2^x,\ x > 1$

$\Rightarrow \mathrm{I}_4 > \mathrm{I}_3$

17. (b) Let $I = \int_{-\pi}^{\pi} \frac{\cos^2 x}{1+a^x}dx$ ....(1)

$= \int_{-\pi}^{\pi} \frac{\cos^2(-x)}{1+a^{-x}}dx$

$\left[\text{Using} \int_a^b f(x)\,dx = \int_a^b f(a+b-x)\,dx\right]$

$= \int_{-\pi}^{\pi} \frac{a^x\cos^2 x}{1+a^x}dx$ ...(2)

Adding equations (1) and (2) we get

$2I = \int_{-\pi}^{\pi} \cos^2 x\left(\frac{1+a^x}{1+a^x}\right)dx = \int_{-\pi}^{\pi} \cos^2 x\,dx$

$= 2\int_0^{\pi} \cos^2 x\,dx \ \ [\because f(\pi - x) = f(x)]$

$$= 2\times 2\int_0^{\frac{\pi}{2}} \cos^2 x\,dx = 4\int_0^{\frac{\pi}{2}} \sin^2 x\,dx$$

$$\left[\because f\left(\frac{\pi}{2}-x\right) = f(x)\right]$$

$$\Rightarrow I = 2\int_0^{\frac{\pi}{2}} \sin^2 x\,dx = 2\int_0^{\frac{\pi}{2}} \left(1-\cos^2 x\right)dx$$

$$\Rightarrow I = 2\int_0^{\frac{\pi}{2}} dx - 2\int_0^{\frac{\pi}{2}} \cos^2 x\,dx$$

$$\Rightarrow I + I = 2\left(\frac{\pi}{2}\right) = \pi \Rightarrow I = \frac{\pi}{2}$$

18. **(b)** $I = \int_3^6 \frac{\sqrt{x}}{\sqrt{9-x}+\sqrt{x}}dx \quad \ldots(1)$

$I = \int_3^6 \frac{\sqrt{9-x}}{\sqrt{9-x}+\sqrt{x}}dx \quad \ldots(2)$

$[\because \int_a^b f(x)dx = \int_a^b f(a+b-x)dx]$

Adding equation (1) and (2)

$2I = \int_3^6 dx = [x]_3^6 = 3 \Rightarrow I = \frac{3}{2}$

19. **(d)** $I = \int_0^{\pi} xf(\sin x)dx = \int_0^{\pi} (\pi - x)f(\sin x)dx$

$= \pi\int_0^{\pi} f(\sin x)dx - I$

$\Rightarrow 2I = \pi\int_0^{\pi} f(\sin x)dx$

$I = \frac{\pi}{2}\int_0^{\pi} f(\sin x)dx = \pi\int_0^{\pi/2} f(\sin x)dx$

$[\because \sin(\pi - x) = \sin x]$

$= \pi\int_0^{\pi/2} f(\cos x)dx$

20. **(c)** $I = \int_{-\frac{3\pi}{2}}^{-\frac{\pi}{2}} [(x+\pi)^3 + \cos^2(x+3\pi)]dx$

Put $x + \pi = t$

$I = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} [t^3 + \cos^2 t]dt = 2\int_0^{\frac{\pi}{2}} \cos^2 t\,dt$

[$\because t^3$ is odd and $\cos^2 t$ is even function]

$= \int_0^{\frac{\pi}{2}} (1+\cos 2t)dt = \frac{\pi}{2} + 0$

21. **(b)** Let $a = k + h$ where k is an integer such that and $0 \le h < 1$

$\Rightarrow [a] = k$

$\therefore \int_1^a [x]f'(x)dx = \int_1^2 1f'(x)dx + \int_2^3 2f'(x)dx +$

$\ldots \int_{k-1}^{k} (k-1)f'(x)dx + \int_k^{k+h} kf'(x)dx$

$= \{f(2) - f(1)\} + 2\{f(3) - f(2)\} + 3\{f(4) - f(3)\} + \ldots\ldots\ldots + (k-1)\{f(k) - f(k-1)\} + k\{f(k+h) - f(k)\}$

$= -f(1) - f(2) - f(3) \ldots\ldots\ldots - f(k) + kf(k+h)$

$= [a]f(a) - \{f(1) + f(2) + f(3) + \ldots f([a])\}$

22. **(c)** $I = \int \frac{dx}{\cos x + \sqrt{3}\sin x}$

$\Rightarrow I = \int \frac{dx}{2\left[\frac{1}{2}\cos x + \frac{\sqrt{3}}{2}\sin x\right]}$

$= \frac{1}{2}\int \frac{dx}{\left[\sin\frac{\pi}{6}\cos x + \cos\frac{\pi}{6}\sin x\right]}$

$= \frac{1}{2}.\int \frac{dx}{\sin\left(x + \frac{\pi}{6}\right)}$

$\Rightarrow I = \frac{1}{2}.\int \operatorname{cosec}\left(x + \frac{\pi}{6}\right)dx$

We know that

$\int \operatorname{cosec} x\,dx = \log|(\tan x/2)| + C$

$\therefore \quad I = \frac{1}{2}.\log\tan\left(\frac{x}{2} + \frac{\pi}{12}\right) + C$

23. **(c)** Given that $F(x) = f(x) + f\left(\frac{1}{x}\right)$, where

$$f(x) = \int_1^x \frac{\log t}{1+t}\,dt$$

$$\therefore F(e) = f(e) + f\left(\frac{1}{e}\right)$$

$$\Rightarrow F(e) = \int_1^e \frac{\log t}{1+t}\,dt + \int_1^{1/e} \frac{\log t}{1+t}\,dt \quad ...(1)$$

Let $I = \int_1^{1/e} \frac{\log t}{1+t}\,dt$

$\therefore$ Put $\frac{1}{t} = z \Rightarrow -\frac{1}{t^2}dt = dz \Rightarrow dt = -\frac{dz}{z^2}$

when $t = 1 \Rightarrow z = 1$ and when $t = 1/e$
$\Rightarrow z = e$

$$\therefore \quad I = \int_1^e \frac{\log\left(\frac{1}{z}\right)}{1+\frac{1}{z}}\left(-\frac{dz}{z^2}\right)$$

$$= \int_1^e \frac{(\log 1 - \log z).z}{z+1}\left(-\frac{dz}{z^2}\right)$$

$$= \int_1^e -\frac{\log z}{(z+1)}\left(-\frac{dz}{z}\right) \qquad [\because \log 1 = 0]$$

$$= \int_1^e \frac{\log z}{z(z+1)}\,dz$$

$$\therefore I = \int_1^e \frac{\log t}{t(t+1)}\,dt$$

[By property $\int_a^b f(t)dt = \int_a^b f(x)dx$]

Now from eqn. (1)

$$F(e) = \int_1^e \frac{\log t}{1+t}\,dt + \int_1^e \frac{\log t}{t(1+t)}\,dt$$

$$= \int_1^e \frac{t.\log t + \log t}{t(1+t)}\,dt = \int_1^e \frac{(\log t)(t+1)}{t(1+t)}\,dt$$

$$\Rightarrow F(e) = \int_1^e \frac{\log t}{t}\,dt$$

Let $\log t = x$ $\qquad \therefore \frac{1}{t}dt = dx$

[when $t = 1, x = 0$ and when $t = e, x = \log e = 1$]

$$\therefore F(e) = \int_0^1 x\,dx \quad F(e) = \left[\frac{x^2}{2}\right]_0^1$$

$$\Rightarrow F(e) = \frac{1}{2}$$

24. **(d)** $\int_{\sqrt{2}}^x \frac{dt}{t\sqrt{t^2-1}} = \frac{\pi}{2}$

$$\therefore \left[\sec^{-1} t\right]_{\sqrt{2}}^x = \frac{\pi}{2}$$

$$\left[\because \int \frac{dx}{x\sqrt{x^2-1}} = \sec^{-1} x\right]$$

$$\Rightarrow \sec^{-1} x - \sec^{-1}\sqrt{2} = \frac{\pi}{2}$$

$$\Rightarrow \sec^{-1} x - \frac{\pi}{4} = \frac{\pi}{2} \Rightarrow \sec^{-1} x = \frac{\pi}{2} + \frac{\pi}{4}$$

$$\Rightarrow \sec^{-1} x = \frac{3\pi}{4} \Rightarrow x = \sec\frac{3\pi}{4} = \sec\left(\pi - \frac{\pi}{4}\right)$$

$$\Rightarrow x = -\sec\frac{\pi}{4} \Rightarrow x = -\sqrt{2}$$

25. **(b)** We know that $\frac{\sin x}{x} < 1$, for x $\in (0, 1)$

$$\Rightarrow \frac{\sin x}{\sqrt{x}} < \sqrt{x} \text{ on } x \in (0, 1)$$

$$\Rightarrow \int_0^1 \frac{\sin x}{\sqrt{x}}dx < \int_0^1 \sqrt{x}dx = \left[\frac{2x^{3/2}}{3}\right]_0^1$$

$$\Rightarrow \int_0^1 \frac{\sin x}{\sqrt{x}}dx < \frac{2}{3} \Rightarrow I < \frac{2}{3}$$

Also $\frac{\cos x}{\sqrt{x}} < \frac{1}{\sqrt{x}}$ $\qquad$ for $x \in (0, 1)$

$$\Rightarrow \int_0^1 \frac{\cos x}{\sqrt{x}}dx < \int_0^1 x^{-1/2}dx = \left[2\sqrt{x}\right]_0^1 = 2$$

$$\Rightarrow \int_0^1 \frac{\cos x}{\sqrt{x}}dx < 2 \Rightarrow J < 2$$

26. **(c)** Let $I = \sqrt{2}\int \frac{\sin x\,dx}{\sin\left(x - \frac{\pi}{4}\right)}$

Let $x - \frac{\pi}{4} = t \Rightarrow dx = dt$

$$\Rightarrow I=\sqrt{2}\int\frac{\sin\left(t+\frac{\pi}{4}\right)}{\sin t}dt$$

$$=\frac{\sqrt{2}}{\sqrt{2}}\int\left(\frac{\sin t+\cos t}{\sin t}\right)dt$$

$$\Rightarrow \quad I=\int(1+\cot t)dt=t+\log|\sin t|+c_1$$

$$=x-\frac{\pi}{4}+\log\left|\sin\left(x-\frac{\pi}{4}\right)\right|+c_1$$

$$=x+\log\left|\sin\left(x-\frac{\pi}{4}\right)\right|+c \quad \left(\text{where } c=c_1-\frac{\pi}{4}\right)$$

27. (c) Let $I=\int_0^{\pi}[\cot x]\,dx$ ....(1)

$=\int_0^{\pi}[\cot(\pi-x)]\,dx=\int_0^{\pi}[-\cot x]\,dx$ ....(2)

Adding eq^n s (1) & (2),
We get

$$2I=\int_0^{\pi}([\cot x]+[-\cot x])dx$$

$$=\int_0^{\pi}(-1)dx$$

$[\because [x]+[-x]=-1,\ \text{if } x\notin z$ and $[x]+[-x]=0,\ \text{if } x\in z]$

$$=[-x]_0^{\pi}=-\pi \quad \Rightarrow I=-\frac{\pi}{2}$$

28. (a) $p'(x)=p'(1-x)$

$\Rightarrow p(x)=-p(1-x)+c$

at $x=0$

$p(0)=-p(1)+c\Rightarrow 42=c$

Now, $p(x)=-p(1-x)+42$

$\Rightarrow p(x)+p(1-x)=42$

Let $I=\int_0^1 p(x)dx$ ...(i)

$\Rightarrow I=\int_0^1 p(1-x)dx$ ...(ii)

Adding eqn. (i) and (ii),

$$2I=\int_0^1(42)\,dx \Rightarrow I=21$$

29. (d) $I=\int_0^1\frac{8\log(1+x)}{1+x^2}dx$

Put $x=\tan\theta$,

$\therefore \quad dx=\sec^2\theta d\theta$

$$\therefore I=8\int_0^{\pi/4}\frac{\log(1+\tan\theta)}{1+\tan^2\theta}.\sec^2\theta\,d\theta$$

$$I=8\int_0^{\pi/4}\log(1+\tan\theta)d\theta \quad ...(i)$$

Applying $\int_0^a f(x)dx=\int_0^a f(a-x)dx$

$$=8\int_0^{\pi/4}\log\left[1+\tan\left(\frac{\pi}{4}-\theta\right)\right]d\theta$$

$$=8\int_0^{\pi/4}\log\left[1+\frac{1-\tan\theta}{1+\tan\theta}\right]d\theta$$

$$=8\int_0^{\pi/4}\log\left[\frac{2}{1+\tan\theta}\right]d\theta$$

$$=8\int_0^{\pi/4}[\log 2-\log(1+\tan\theta)]d\theta$$

$$=8\log 2\int_0^{\pi 14}1d\theta-8\int_0^{\pi 14}\log(1+\tan\theta)\,d\theta$$

$$I=8.(\log 2)[x]_0^{\pi/4}-8\int_0^{\pi/4}\log(1+\tan\theta)d\theta$$

$I=8.\frac{\pi}{4}.\log 2-I$ [From equation (i)]

$\Rightarrow 2I=2\pi\log 2$

$\therefore I=\pi\log 2$

30. (c)

$$\int_0^{1.5}x\left[x^2\right]dx=\int_0^1 x[x^2]dx+\int_1^{\sqrt{2}}x\left[x^2\right]dx+\int_{\sqrt{2}}^{1.5}x\left[x^2\right]dx$$

$$=\int_0^1 x.0\,dx+\int_1^{\sqrt{2}}xdx+\int_{\sqrt{2}}^{1.5}2xdx$$

$$=0+\left[\frac{x^2}{2}\right]_1^{\sqrt{2}}+\left[x^2\right]_{\sqrt{2}}^{1.5}$$

$$=\frac{1}{2}(2-1)+(2.25-2)=\frac{1}{2}+0.25$$

$$=\frac{1}{2}+\frac{1}{4}=\frac{3}{4}$$

31. (d) $\int \frac{5\tan x}{\tan x-2}dx = \int \frac{5\frac{\sin x}{\cos x}}{\frac{\sin x}{\cos x}-2}dx$

$= \int\left(\frac{5\sin x}{\cos x}\times\frac{\cos x}{\sin x-2\cos x}\right)dx$

$= \int \frac{5\sin x\,dx}{\sin x-2\cos x}$

$= \int\left(\frac{4\sin x+\sin x+2\cos x-2\cos x}{\sin x-2\cos x}\right)dx$

$= \int \frac{(\sin x-2\cos x)+(4\sin x+2\cos x)}{\sin x-2\cos x}dx$

$= \int \frac{(\sin x-2\cos x)+2(\cos x+2\sin x)}{(\sin x-2\cos x)}dx$

$= \int \frac{\sin x-2\cos x}{\sin x-2\cos x}dx + 2\int\left(\frac{\cos x+2\sin x}{\sin x-2\cos x}\right)dx$

$= \int dx + 2\int \frac{\cos x+2\sin x}{\sin x-2\cos x}dx$

$= I_1 + I_2$

where

$I_1 = \int dx$ and $I_2 = 2\int \frac{\cos x+2\sin x}{\sin x-2\cos x}dx$

Let $\sin x - 2\cos x = t$

$\Rightarrow \quad (\cos x + 2\sin x)\,dx = dt$

$\therefore\ I_2 = 2\int \frac{dt}{t} = 2\ln t + C$

$= 2\,ln\,(\sin x – 2\cos x) + C$

Hence,

$I_1 + I_2 = \int dx + 2\ln(\sin x - 2\cos x) + c$

$= x + 2ln\,|(\sin x - 2\cos x)| + k \Rightarrow a = 2$

32. (b, c) $g(x+\pi) = \int_0^{x+\pi}\cos 4t\,dt$

$= \int_0^x \cos 4t\,dt + \int_x^{\pi+x}\cos 4t\,dt$

$= g(x) + \int_0^{\pi}\cos 4t\,dt$

(it is clear from graph of cos 4t)

$\int_x^{\pi+x}\cos 4t\,dt = \int_0^{\pi}\cos 4t\,dt$

$= g(x) + g(\pi) = g(x) - g(\pi)$

($\because$ From graph of cos 4t, $g(\pi) = 0$)

33. (c) Let $\int f(x)dx = \psi(x)$

Let $I = \int x^5 f(x^3)dx$

put $x^3 = t$

$\Rightarrow \quad 3x^2dx = dt$

$I = \frac{1}{3}\int 3\cdot x^2\cdot x^3\cdot f(x^3)\cdot dx$

$= \frac{1}{3}\int tf(t)dt = \frac{1}{3}\left[t\int f(t)dt - \int f(t)dt\right]$

$= \frac{1}{3}\left[t\psi(t) - \int \psi(t)dt\right]$

$= \frac{1}{3}\left[x^3\psi(x^3) - 3\int x^2\psi(x^3)dx\right] + c$

$= \frac{1}{3}x^3\psi(x^3) - \int x^2\psi(x^3)dx + c$

34. (d) Let $I = \int_{\pi/6}^{\pi/3}\frac{dx}{1+\sqrt{\tan x}}$

$= \int_{\pi/6}^{\pi/3}\frac{dx}{1+\sqrt{\tan\left(\frac{\pi}{2}-x\right)}}$

$= \int_{\pi/6}^{\pi/3}\frac{\sqrt{\tan x}\,dx}{1+\sqrt{\tan x}}$ ...(1)

Also, given

$I = \int_{\pi/6}^{\pi/3}\frac{\sqrt{\tan x}\,dx}{1+\sqrt{\tan x}}$ ...(2)

By adding (1) and (2), we get

$2I = \int_{\pi/6}^{\pi/3}dx$

$\Rightarrow \quad I = \frac{1}{2}\left[\frac{\pi}{3} - \frac{\pi}{6}\right] = \frac{\pi}{12}$,

Statement-1 is false

$\because \quad \int_a^b f(x)dx = \int_a^b f(a+b-x)dx$

It is fundamental property.

Statement -2 is true.

35. (a) Since, $y = \int_0^x |t|\,dt,\ x \in R$

therefore $\frac{dy}{dx} = |x|$

But from $y = 2x$, $\therefore\ \frac{dy}{dx} = 2$

$\Rightarrow \quad |x| = 2 \quad \Rightarrow \quad x = \pm 2$

Points $y = \int_0^{\pm 2} |t|\, dt = \pm 2$

$\therefore$ Equation of tangent is
$y - 2 = 2(x - 2)$ or $y + 2 = 2(x + 2)$

$\Rightarrow$ x-intercept $= \pm 1$.

36. (d) Let $I = \int \left(1 + x - \frac{1}{x}\right) e^{x + 1/x} dx$

$$= \int e^{x+1/x} dx + \int \left(x - \frac{1}{x}\right) e^{x+1/x} dx$$

$$= x.e^{x+1/x} - \int x\left(1 - \frac{1}{x^2}\right) e^{x+1/x} dx + \int \left(x - \frac{1}{x}\right) e^{x+1/x} dx$$

$$= x.e^{x+\frac{1}{x}} - \int \left(x - \frac{1}{x}\right) e^{x+1/x} dx + \int \left(x - \frac{1}{x}\right) e^{x+1/x} dx$$

$$= xe^{x+\frac{1}{x}} + C$$

37. (b) Let $I = \int_0^{\pi} \sqrt{1 + 4\sin^2 \frac{x}{2} - 4\sin \frac{x}{2}}\, dx$

$$= \int_0^{\pi} \left|2\sin \frac{x}{2} - 1\right| dx$$

$$= \int_0^{\pi/3} \left(1 - 2\sin \frac{x}{2}\right) dx + \int_{\pi/3}^{\pi} \left(2\sin \frac{x}{2} - 1\right) dx$$

$$\left[\because \sin \frac{x}{2} = \frac{1}{2} \Rightarrow \frac{x}{2} = \frac{\pi}{6} \Rightarrow x = \frac{\pi}{3}, \frac{x}{2} = \frac{5\pi}{6} \Rightarrow x = \frac{5\pi}{3} > \pi\right]$$

$$= \left[x + 4\cos \frac{x}{2}\right]_0^{\pi/3} + \left[-4\cos \frac{x}{2} - x\right]_{\pi/3}^{\pi}$$

$$= \frac{\pi}{3} + 4\frac{\sqrt{3}}{2} - 4 + \left(0 - \pi + 4\frac{\sqrt{3}}{2} + \frac{\pi}{3}\right)$$

$$= 4\sqrt{3} - 4 - \frac{\pi}{3}$$

38. (b) $I = \int \frac{dx}{x^2(x^4+1)^{3/4}} = \int \frac{dx}{x^3(1+x^{-4})^{3/4}}$

Let $x^{-4} = y$

$\Rightarrow -4x^{-3}\, dx = dy$

$\Rightarrow dx = \frac{-1}{4} x^3 dy$

$$\therefore I = \frac{-1}{4} \int \frac{x^3 dy}{x^3(1+y)^{3/4}} = \frac{-1}{4} \int \frac{dy}{(1+y)^{3/4}}$$

$$= \frac{-1}{4} \times 4(1+y)^{1/4} = -(1+x^{-4})^{1/4} + C$$

$$= -\left(\frac{x^4+1}{x^4}\right)^{1/4} + C$$

39. (a) $I = \int_2^4 \frac{\log x^2}{\log x^2 + \log(36 - 12x + x^2)} dx$

$$I = \int_2^4 \frac{\log x^2}{\log x^2 + \log(6-x)^2} dx. \quad \text{...(i)}$$

$$I = \int_2^4 \frac{\log(6-x)^2}{\log(6-x)^2 + \log x^2} dx \quad \text{...(ii)}$$

Adding (i) and (ii)

$$2I = \int_2^4 dx = [x]_2^4 = 2$$

$I = 1$

40. (d) $\int \frac{2x^{12} + 5x^9}{(x^5 + x^3 + 1)^3} dx$

Dividing by x^{15} in numerator and denominator

$$\int \frac{\frac{2}{x^3} + \frac{5}{x^6}\, dx}{\left(1 + \frac{1}{x^2} + \frac{1}{x^5}\right)^3}$$

Let $1 + \frac{1}{x^2} + \frac{1}{x^5} = t$

$$\Rightarrow \left(\frac{-2}{x^3} - \frac{5}{x^6}\right) dx = dt$$

$$\Rightarrow \left(\frac{2}{x^3} + \frac{5}{x^6}\right) dx = -dt$$

This gives,

$$\int \frac{\frac{2}{x^3}+\frac{5}{x^6}dx}{\left(1+\frac{1}{x^2}+\frac{1}{x^5}\right)^3} = \int \frac{-dt}{t^3} = \frac{1}{2t^2}+C$$

$$= \frac{1}{2\left(1+\frac{1}{x^2}+\frac{1}{x^5}\right)^2}+C = \frac{x^{10}}{2(x^5+x^3+1)^2}+C$$

41. **(c)** $I = \int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \frac{dx}{1+\cos x}$ ...(i)

$I = \int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \frac{dx}{1-\cos x}$ ...(ii)

Using $\int_a^b f(x)dx = \int_a^b f(a+b-x)\,dx$

Adding (i) and (ii)

$2I = \int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \frac{2}{\sin^2 x}dx$; $I = \int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \operatorname{cosec}^2 x\,dx$

$I = -(\cot x)_{\pi/4}^{3\pi/4} = -\left[\cot\frac{3\pi}{4} - \cot\frac{\pi}{4}\right] = 2$

42. **(c)** $I_n = \int \tan^n x\,dx, n > 1$

Let $I = I_4 + I_6$

$= \int (\tan^4 x + \tan^6 x)dx = \int \tan^4 x \sec^2 x\,dx$

Let $\tan x = t$

$\Rightarrow \sec^2 x\,dx = dt$

$\therefore I = \int t^4 dt = \frac{t^5}{5}+C$

$= \frac{1}{5}\tan^5 x + C \Rightarrow$ On comparing, we have

$a = \frac{1}{5}, b = 0$

43. **(a)** Let I

$$= \int \frac{\sin^2 x\cos^2 x}{(\sin^5 x+\cos^3 x\sin^2 x+\sin^3 x\cos^2 x+\cos^5 x)^2}dx$$

$$= \int \frac{\sin^2 x\cos^2 x}{[(\sin^2 x+\cos^2 x)(\sin^3 x+\cos^3 x)]^2}dx$$

$$= \int \frac{\sin^2 x\cos^2 x}{(\sin^3 x+\cos^3 x)^2}dx$$

$$= \int \frac{\tan^2 x\cdot\sec^2 x}{(1+\tan^3 x)^2}\,dx$$

Now, put $(1+\tan^3 x) = t$

$\Rightarrow 3\tan^2 x\sec^2 x\,dx = dt$

$\therefore I = \frac{1}{3}\int \frac{dt}{t^2} = -\frac{1}{3t}+C = \frac{-1}{3(1+\tan^3 x)}+C$

44. **(c)** Let, $I = \int_{-\pi/2}^{\pi/2} \frac{\sin^2 x}{1+2^x}dx$ ...(i)

Using, $\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$, we get :

$I = \int_{-\pi/2}^{\pi/2} \frac{\sin^2 x}{1+2^{-x}}dx$ ...(ii)

Adding (i) and (ii), we get;

$2I = \int_{-\pi/2}^{\pi/2} \sin^2 x\,dx$

$\Rightarrow 2I = 2\cdot\int_0^{x/2} \sin^2 x\,dx$

$\Rightarrow 2I = 2\times\frac{\pi}{4} \Rightarrow I = \frac{\pi}{4}$

45. **(b)** $I = \int_0^{\pi} |\cos x|^3\,dx$

$= 2\int_0^{\pi/2} \cos^3 x\,dx$

$= \frac{2}{4}\int_0^{\pi/2} (3\cos x + \cos 3x)dx$

$[\because \cos 3\theta = 4\cos^3\theta - 3\cos\theta]$

$= \frac{1}{2}\left[3\sin x + \frac{\sin 3x}{3}\right]_0^{\pi/2}$

$= \frac{1}{2}\left(3-\frac{1}{3}\right) = \frac{4}{3}$

46. (c, d) Consider the given integral

$$I=\int x\sqrt{\frac{2\sin(x^2-1)-2\sin(x^2-1)\cos(x^2-1)}{2\sin(x^2-1)+2\sin(x^2-1)\cos(x^2-1)}}dx$$

$(\therefore \sin 2\theta = 2\sin\theta\cos\theta)$

$$\Rightarrow I=\int x\sqrt{\frac{1-\cos(x^2-1)}{1+\cos(x^2-1)}}dx$$

$$\Rightarrow I=,\int x\left|\tan\left(\frac{x^2-1}{2}\right)\right|dx,$$

Now let $\frac{x^2-1}{2}=t \Rightarrow \frac{2x}{2}dx=dt$

$\therefore \quad I=\int|\tan(t)|\,dt=\ln|\sec t|+C$

or $\quad I=\ln\left|\sec\left(\frac{x^2-1}{2}\right)\right|+c=\frac{1}{2}\ln\left|\sec^2\left(\frac{x^2-1}{2}\right)\right|+c$

47. (d) Since, function $f(x)$ have local extreem points at $x=-1, 0, 1$. Then

$f'(x)=K(x+1)x(x-1)$

$=K(x^3-x)$

$\Rightarrow f(x)=K\left(\frac{x^4}{4}-\frac{x^2}{2}\right)+C$

(using integration)

$\Rightarrow f(0)=C$

$\because f(x)=f(0)\Rightarrow K\left(\frac{x^4}{4}-\frac{x^2}{2}\right)=0$

$\Rightarrow \frac{x^2}{2}\left(\frac{x^2}{2}-1\right)=0\Rightarrow x=0,0,\sqrt{2},-\sqrt{2}$

$\therefore S=\{0,-\sqrt{2},\sqrt{2}\}$

48. (a) $I=\int \sec^{\frac{2}{3}}x.\mathrm{cosec}^{\frac{4}{3}}dx$

$$I=\int\frac{\sec^2 x dx}{\tan^{\frac{4}{3}}x}$$

Put $\tan x = z$

$\Rightarrow \sec^2 x\, dx = dz$

$$\Rightarrow I=\int z^{-\frac{4}{3}}\cdot dz=\frac{z^{\frac{-1}{3}}}{\left(\frac{-1}{3}\right)}+C$$

$$\Rightarrow I=-3(\tan x)^{\frac{-1}{3}}+C$$

49. (b) Let $I=\int_0^{\pi/2}\frac{\sin^3 x dx}{\sin x+\cos x}$ ...(1)

Use the property

$\int_0^a f(x)dx=\int_0^a f(a-x)dx$

$\Rightarrow \quad I=\int_0^{\pi/2}\frac{\cos^3 x\,dx}{\sin x+\cos x}$ ...(2)

Adding equations (1) and (2), we get

$$2I=\int_0^{\pi/2}\left(1-\frac{1}{2}\sin(2x)\right)dx$$

$$\Rightarrow I=\frac{1}{2}\left[x+\frac{1}{4}\cos 2x\right]_0^{\pi/2}\Rightarrow I=\frac{\pi-1}{4}$$

50. (c) $I=\frac{1}{(a+b)}\int_a^b x[f(x)+f(x+1)]dx$ ...(i)

$x\to a+b-x$

$$I=\frac{1}{(a+b)}\int_a^b(a+b-x)[f(a+b-x)+f(a+b+1-x)]dx$$

$$I=\frac{1}{(a+b)}\int_a^b(a+b-x)[f(x+1)+f(x)]dx$$ ...(ii)

$[\because$ put $x\to x+1$ in $f(a+b+1-x)=f(x)]$

Add (i) and (ii)

$$2I=\int_a^b[f(x+1)+f(x)]dx$$

$$2I=\int_a^b f(x+1)dx+\int_a^b f(x)dx$$

$$=\int_a^b f(a+b+1-x)dx+\int_a^b f(x)dx$$

$$2I=2\int_a^b f(x)dx$$

$\therefore \int_{a-1}^{b-1} f(x+1)dx$ $\quad[\because$ Put $x\to x+1]$

23 Applications of Integrals

1. If $y = f(x)$ makes +ve intercept of 2 and 0 unit on x and y axes and encloses an area of 3/4 square unit with the axes then $\int_0^2 xf'(x)dx$ is **[2002]**

(a) 3/2 (b) 1

(c) 5/4 (d) –3/4

2. The area bounded by the curves $y = \ln x$, $y = \ln|x|, y = |\ln x|$ and $y = |\ln|x||$ is **[2002]**

(a) 4sq. units (b) 6 sq. units

(c) 10 sq. units (d) none of these

3. The area of the region bounded by the curves $y = |x-1|$ *and* $y = 3 - |x|$ is **[2003]**

(a) 6 sq. units (b) 2 sq. units

(c) 3 sq. units (d) 4 sq. units.

4. The area of the region bounded by the curves $y = |x-2|, x = 1, x = 3$ and the x-axis is **[2004]**

(a) 4 (b) 2

(c) 3 (d) 1

5. The area enclosed between the curve $y = \log_e(x+e)$ and the coordinate axes is **[2005]**

(a) 1 (b) 2

(c) 3 (d) 4

6. The parabolas $y^2 = 4x$ and $x^2 = 4y$ divide the square region bounded by the lines $x = 4, y = 4$ and the coordinate axes. If S_1, S_2, S_3 are respectively the areas of these parts numbered from top to bottom; then $S_1 : S_2 : S_3$ is **[2005]**

(a) 1 : 2 : 1 (b) 1 : 2 : 3

(c) 2 : 1 : 2 (d) 1 : 1 : 1

7. Let $f(x)$ be a non – negative continuous function such that the area bounded by the curve $y = f(x)$, x - axis and the ordinates $x = \frac{\pi}{4}$ and $x = \beta > \frac{\pi}{4}$ is $\left(\beta\sin\beta + \frac{\pi}{4}\cos\beta + \sqrt{2}\beta\right)$. Then $f\left(\frac{\pi}{2}\right)$ **[2005]**

(a) $\left(\frac{\pi}{4} + \sqrt{2} - 1\right)$ (b) $\left(\frac{\pi}{4} - \sqrt{2} + 1\right)$

(c) $\left(1 - \frac{\pi}{4} - \sqrt{2}\right)$ (d) $\left(1 - \frac{\pi}{4} + \sqrt{2}\right)$

8. The area enclosed between the curves $y^2 = x$ and $y = |x|$ is **[2007]**

(a) 1/6 (b) 1/3

(c) 2/3 (d) 1

9. The area of the plane region bounded by the curves $x + 2y^2 = 0$ and $x + 3y^2 = 1$ is equal to **[2008]**

(a) $\frac{5}{3}$ (b) $\frac{1}{3}$

(c) $\frac{2}{3}$ (d) $\frac{4}{3}$

10. The area of the region bounded by the parabola $(y-2)^2 = x - 1$, the tangent of the parabola at the point (2, 3) and the x-axis is: **[2009]**

(a) 6 (b) 9

(c) 12 (d) 3

11. The area bounded by the curves $y = \cos x$ and $y = \sin x$ between the ordinates $x = 0$ and $x = \frac{3\pi}{2}$ is **[2010]**

(a) $4\sqrt{2}+2$ (b) $4\sqrt{2}-1$

(c) $4\sqrt{2}+1$ (d) $4\sqrt{2}-2$

12. The area of the region enclosed by the curves $y = x, x = e, y = \frac{1}{x}$ and the positive x-axis is **[2011]**

(a) 1 square unit (b) $\frac{3}{2}$ square units

(c) $\frac{5}{2}$ square units (d) $\frac{1}{2}$ square unit

13. The area bounded by the curves **[2011 RS]**

$y^2 = 4x$ and $x^2 = 4y$ is:

(a) $\frac{32}{3}$ sq units (b) $\frac{16}{3}$ sq units

(c) $\frac{8}{3}$ sq. units (d) 0 sq. units

14. The area between the parabolas $x^2 = \frac{y}{4}$ and $x^2 = 9y$ and the straight line $y = 2$ is: **[2012]**

(a) $20\sqrt{2}$ (b) $\frac{10\sqrt{2}}{3}$

(c) $\frac{20\sqrt{2}}{3}$ (d) $10\sqrt{2}$

15. The area (in square units) bounded by the curves $y = \sqrt{x}$, $2y - x + 3 = 0$, x-axis, and lying in the first quadrant is: **[2013]**

(a) 9 (b) 36

(c) 18 (d) $\frac{27}{4}$

16. The area of the region described by $A = \left\{(x, y) : x^2 + y^2 \le 1 \text{ and } y^2 \le 1 - x\right\}$ is: **[2014]**

(a) $\frac{\pi}{2}-\frac{2}{3}$ (b) $\frac{\pi}{2}+\frac{2}{3}$

(c) $\frac{\pi}{2}+\frac{4}{3}$ (d) $\frac{\pi}{2}-\frac{4}{3}$

17. The area (in sq. units) of the region described by $\{(x, y) : y^2 \le 2x \text{ and } y \ge 4x - 1\}$ is **[2015]**

(a) $\frac{15}{64}$ (b) $\frac{9}{32}$

(c) $\frac{7}{32}$ (d) $\frac{5}{64}$

18. The area (in sq. units) of the region $\{(x, y) : y^2 \ge 2x \text{ and } x^2 + y^2 \le 4x, x \ge 0, y \ge 0\}$ is: **[2016]**

(a) $\pi - \frac{4\sqrt{2}}{3}$ (b) $\frac{\pi}{2} - \frac{2\sqrt{2}}{3}$

(c) $\pi - \frac{4}{3}$ (d) $\pi - \frac{8}{3}$

19. The area (in sq. units) of the region $\{(x, y) : x \ge 0, x + y \le 3, x^2 \le 4y \text{ and } y \le 1 + \sqrt{x}\}$ is: **[2017]**

(a) $\frac{5}{2}$ (b) $\frac{59}{12}$

(c) $\frac{3}{2}$ (d) $\frac{7}{3}$

20. Let $g(x) = \cos x^2, f(x) = \sqrt{x}$, and $\alpha, \beta\ (\alpha < \beta)$ be the roots of the quadratic equation $18x^2 - 9\pi x + \pi^2 = 0$. Then the area (in sq. units) bounded by the curve $y = (gof)(x)$ and the lines $x = \alpha, x = \beta$ and $y = 0$, is: **[2018]**

(a) $\frac{1}{2}(\sqrt{3}+1)$ (b) $\frac{1}{2}(\sqrt{3}-\sqrt{2})$

(c) $\frac{1}{2}(\sqrt{2}-1)$ (d) $\frac{1}{2}(\sqrt{3}-1)$

21. The area (in sq. units) bounded by the parabola $y = x^2 - 1$, the tangent at the point (2, 3) to it and the y-axis is: **[2019]**

(a) $\frac{8}{3}$ (b) $\frac{32}{3}$

(c) $\frac{56}{3}$ (d) $\frac{14}{3}$

22. The area (in sq. units) of the region **[2019]**
$A = \{(x, y) : x^2 \le y \le x + 2\}$ is:

(a) $\frac{10}{3}$ (b) $\frac{9}{2}$

(c) $\frac{31}{6}$ (d) $\frac{13}{6}$

23. The area of the region, enclosed by the circle $x^2 + y^2 = 2$ which is not common to the region bounded by the parabola $y^2 = x$ and the straight line $y = x$, is: **[2020]**

(a) $\frac{1}{6}(24\pi - 1)$ (b) $\frac{1}{3}(6\pi - 1)$

(c) $\frac{1}{3}(12\pi - 1)$ (d) $\frac{1}{6}(12\pi - 1)$

Answer Key

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
(d)	(a)	(d)	(d)	(a)	(d)	(d)	(a)	(d)	(b)	(d)	(b)	(b)	(c)	(a)
16	17	18	19	20	21	22	23							
(c)	(b)	(d)	(a)	(d)	(a)	(b)	(d)							

Solutions

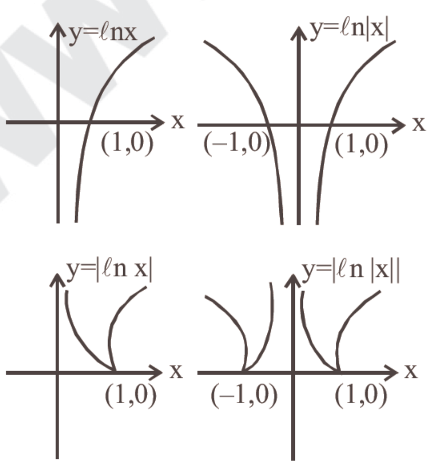
1. **(d)** Given that $\int_0^2 f(x)dx = \frac{3}{4}$; Now,

$$\int_0^2 xf'(x)dx = x\int_0^2 f'(x)dx - \int_0^2 f(x)dx$$

$$= [x\ f(x)]_0^2 - \frac{3}{4} = 2f(2) - \frac{3}{4}$$

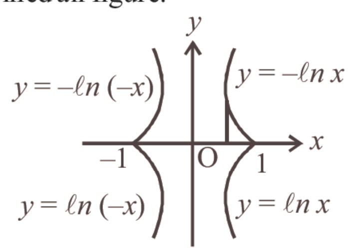
$$= 0 - \frac{3}{4}\ (\because f(2) = 0) = -\frac{3}{4}.$$

2. **(a)** Separate graph of each curve



[Note: Graph of $y = |f(x)|$ can be obtained from the graph of the curve $y = f(x)$ by drawing the mirror image of the portion of the graph below x-axis, with respect to x-axis.

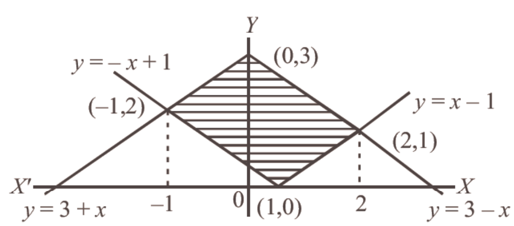
Hence the bounded area is as shown by combined all figure.



Required area $= 4\int_0^1 (-\ell nx)dx$

$$= -4\left[x\ \ell n\ x - x\right]_0^1 = 4 \text{ sq. units}$$

3. **(d)** Intersection point of $y = x - 1$ and $y = 3 - x$ is (2, 1) and eqns. $y = -x + 1$ and $y = 3 + x$ is (–1, 2)



$$A=\int_{-1}^{0}\{(3+x)-(-x+1)\}\,dx+$$

$$\int_{0}^{1}\{(3-x)-(-x+1)\}\,dx+\int_{1}^{2}\{(3-x)-(x-1)\}\,dx$$

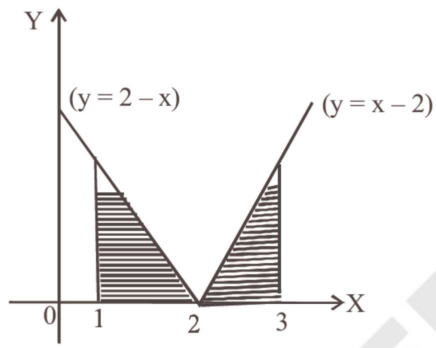
$$=\int_{-1}^{0}(2+2x)dx+\int_{0}^{1}2dx+\int_{1}^{2}(4-2x)dx$$

$$=\left[2x+x^2\right]_{-1}^{0}+\left[2x\right]_{0}^{1}+\left[4x-x^2\right]_{1}^{2}$$

$$=0-(-2+1)+(2-0)+(8-4)-(4-1)$$

$$=1+2+4-3=4\ sq.\ \text{units}$$

4. **(d)**

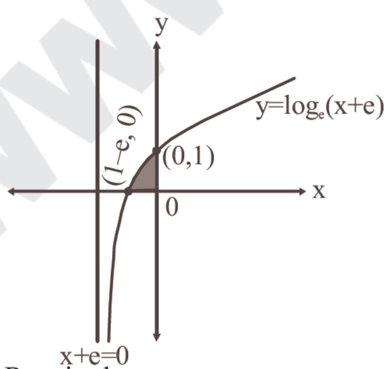


Required Area

$$A=2\int_{2}^{3}(x-2)dx$$

$$=2\left[\frac{x^2}{2}-2x\right]_{2}^{3}=1$$

5. **(a)**



Required area

$$A=\int_{1-e}^{0}ydx=\int_{1-e}^{0}\log_e(x+e)dx$$

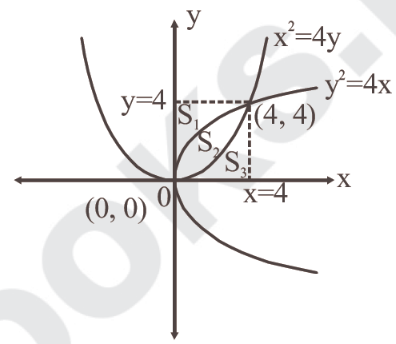
put $x+e=t\Rightarrow dx=dt$ also when $x=1-e$, $t=1$ and when $x=0$, $t=e$

$$\therefore A=\int_{1}^{e}\log_e t\,dt=\left[t\log_e t-t\right]_{1}^{e}$$

$e-e-0+1=1$

Hence the required area is 1 square unit.

6. **(d)** On solving, we get intersection points of $x^2=4y$ and $y^2=4x$ are (0, 0) and (4, 4).



By symmetry, we observe

$$S_1=S_3=\int_{0}^{4}ydx$$

$$=\int_{0}^{4}\frac{x^2}{4}dx=\left[\frac{x^3}{12}\right]_{0}^{4}=\frac{16}{3}\text{ sq. units}$$

Also $S_2=\int_{0}^{4}\left(2\sqrt{x}-\frac{x^2}{4}\right)dx=\left[\frac{2x^{\frac{3}{2}}}{\frac{3}{2}}-\frac{x^3}{12}\right]_{0}^{4}$

$$=\frac{4}{3}\times 8-\frac{16}{3}=\frac{16}{3}\text{ sq. units}$$

$\therefore S_1:S_2:S_3=1:1:1$

7. **(d)** From given condition

$$\int_{\pi/4}^{\beta}f(x)dx=\beta\sin\beta+\frac{\pi}{4}\cos\beta+\sqrt{2}\beta$$

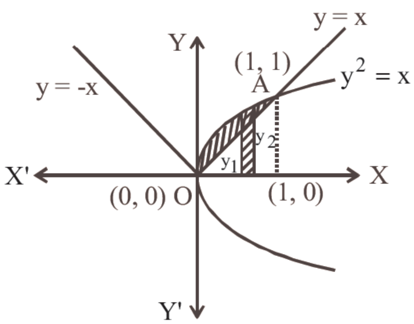
Differentiating w. r . t β, we get

$$f(\beta)=\beta\cos\beta+\sin\beta-\frac{\pi}{4}\sin\beta+\sqrt{2}$$

$$f\left(\frac{\pi}{2}\right)=\beta\cdot 0+\left(1-\frac{\pi}{4}\right)\sin\frac{\pi}{2}+\sqrt{2}=1-\frac{\pi}{4}+\sqrt{2}$$

8. **(a)** It is clear from the figure, area lies between $y^2=x$ and $y=x$

Intersection point $y=x$ and $y^2=x$ is (1, 1)



$\therefore$ Required area $= \int_0^1 (y_2 - y_1)dx$

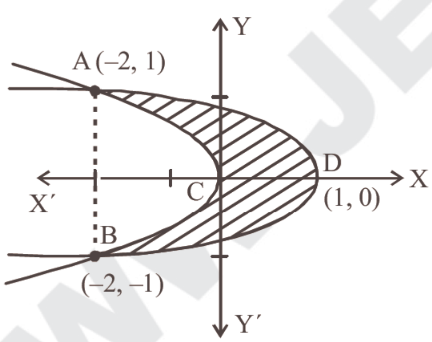
$$= \int_0^1 (\sqrt{x} - x)dx = \left[\frac{x^{3/2}}{3/2} - \frac{x^2}{2}\right]_0^1$$

$$= \frac{2}{3}\left[x^{3/2}\right]_0^1 - \frac{1}{2}\left[x^2\right]_0^1 = \frac{2}{3} - \frac{1}{2} = \frac{1}{6}$$

9. **(d)** Given $x + 2y^2 = 0 \Rightarrow y^2 = -\frac{x}{2}$

and $x + 3y^2 = 1 \Rightarrow y^2 = -\frac{1}{3}(x-1)$

On solving these two equations we get the points of intersection as (–2, 1), (–2, –1)



The required area is ACBDA, given by

$$A = 2\left\{\int_{-2}^{1} \frac{1}{\sqrt{3}}\sqrt{1-x}\ dx - \frac{1}{\sqrt{2}}\int_{-2}^{0}\sqrt{-x}\ dx\right\}$$

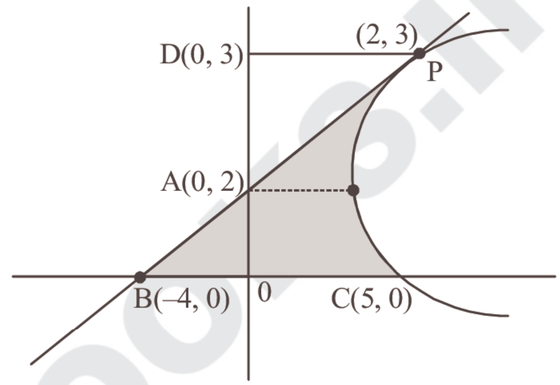
$$\Rightarrow 2\left\{\frac{1}{\sqrt{3}}\left[\frac{2}{3}(1-x)^{3/2}\right]_{-2}^{1} - \frac{1}{\sqrt{2}}\left[\frac{2}{3}(-x)^{3/2}\right]_{-2}^{0}\right\}$$

$$\Rightarrow 2\left\{\left[-\frac{1}{\sqrt{3}}\times\frac{2}{3}\left(0-3^{3/2}\right)\right] - \left[\frac{-1}{\sqrt{2}}\times\frac{2}{3}\left(0-2^{3/2}\right)\right]\right\}$$

$$\Rightarrow 2\left\{\frac{2}{\cancel{3}\sqrt{\cancel{3}}}\times\cancel{3}\sqrt{\cancel{3}} - \frac{1}{\sqrt{\cancel{2}}}\times\frac{2}{3}\cdot 2\sqrt{\cancel{2}}\right\}$$

$$\Rightarrow 2\left\{2 - \frac{4}{3}\right\} = 2\left\{\frac{6-4}{3}\right\} = \frac{4}{3} \text{ sq. units}$$

10. **(b)**



For slope of tangents at (2, 3)
$(y-2)^2 = x - 1$

$$2(y-2)\frac{dy}{dx} = 1 \Rightarrow \frac{dy}{dx} = \frac{1}{2(y-2)}$$

$$m = \left(\frac{dy}{dx}\right)_{(2,3)} = \frac{1}{2(3-2)} = \frac{1}{2}$$

Equation of tangent

$$y - 3 = \frac{1}{2}(x-2)$$

$$\Rightarrow x - 2y + 4 = 0 \qquad ...(i)$$

The given parabola is $(y-2)^2 = x - 1$ ...(ii)
vertex (1, 2) and it meets x-axis at (5, 0)
Then required area = Ar ΔBOA + Ar (OCPD) – Ar (ΔAPD)

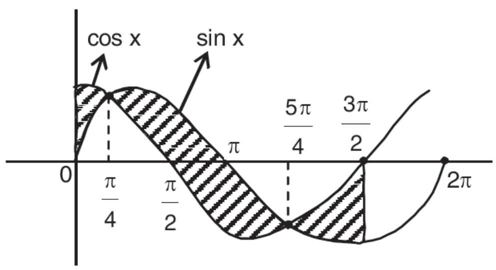
$$= \frac{1}{2}\times 4\times 2 + \int_0^3 x dy - \frac{1}{2}\times 2\times 1$$

$$= 3 + \int_0^3 (y-2)^2 + 1\ dy$$

$$= 3 + \left[\frac{(y-2)^3}{3} + y\right]_0^3$$

$$= 3 + \left[\frac{1}{3} + 3 + \frac{8}{3}\right] = 3 + 6 = 9 \text{ sq. units}$$

11. (d)



Area above x-axis = Area below x-axis

$\therefore$ Required area

$$= 2\left[\int_0^{\pi/4}(\cos x - \sin x)dx + \int_{\pi/4}^{\pi}\sin x dx - \int_{\pi/4}^{\pi/2}\cos x dx\right]$$

$$= 2\left[(\sin x + \cos x)_0^{\pi/4} + (-\cos x)_{\pi/4}^{\pi} - (\sin x)_{\pi/4}^{\pi/2}\right]$$

$$= 2\left[\left(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}\right) - (0+1) + \left(1 + \frac{1}{\sqrt{2}}\right) - \left(1 - \frac{1}{\sqrt{2}}\right)\right]$$

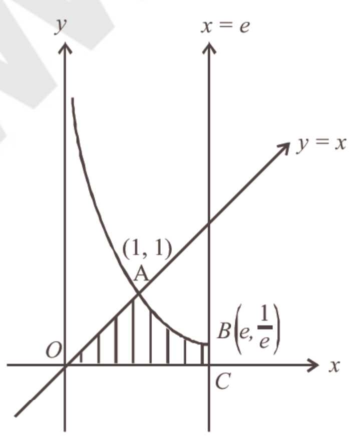
$$= 2\left[\sqrt{2} - 1 + 1 + \frac{1}{\sqrt{2}} - 1 + \frac{1}{\sqrt{2}}\right]$$

$$= 2\left[\sqrt{2} + \sqrt{2} - 1\right] = 4\sqrt{2} - 2$$

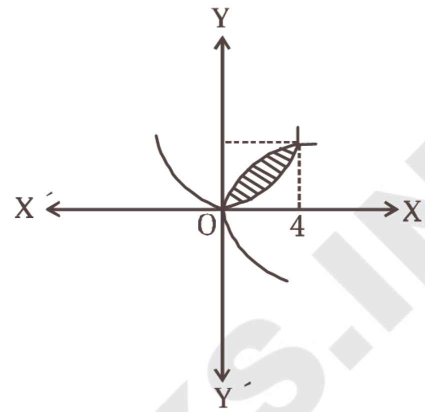
12. (b) Area of required region $AOCBO$

$$= \int_0^1 x dx + \int_1^e \frac{1}{x}dx = \left[\frac{x^2}{2}\right]_0^1 + [\log x]_1^e$$

$$= \frac{1}{2} + 1 = \frac{3}{2} \text{ sq. units}$$



13. (b)

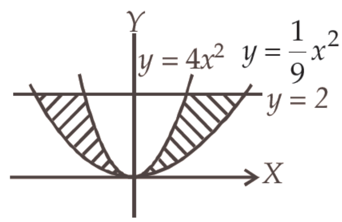


$$\text{Required area} = \int_0^4\left(2\sqrt{x} - \frac{x^2}{4}\right)dx$$

$$= \left[2\left(\frac{x^{3/2}}{3/2}\right) - \frac{x^3}{12}\right]_0^4 = \frac{4}{3}\times 8 - \frac{64}{12}$$

$$= \frac{32}{3} - 16 = \frac{16}{3} \text{ sq. units}$$

14. (c)



$$\text{Required area} = 2\int_0^2\left(\sqrt{9y} - \sqrt{\frac{y}{4}}\right)dy$$

$$= 2\int_0^2\left(3\sqrt{y} - \frac{\sqrt{y}}{2}\right)dy$$

$$= 2\left[\frac{2}{3}\times 3.y^{\frac{3}{2}} - \frac{1}{2}\times\frac{2}{3}.y^{\frac{3}{2}}\right]_0^2$$

$$= 2\left[2y^{\frac{3}{2}} - \frac{1}{3}y^{\frac{3}{2}}\right]_0^2 = 2\times\left[\frac{5}{3}y^{\frac{3}{2}}\right]_0^2$$

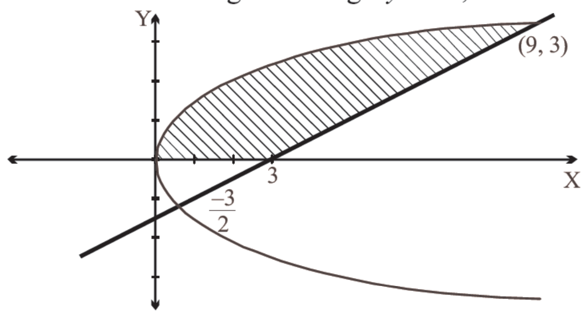
$$= 2.\frac{5}{3}2\sqrt{2} = \frac{20\sqrt{2}}{3}$$

15. (a) Given curves are

$y = \sqrt{x}$ ...(1)

and $2y - x + 3 = 0$ ...(2)

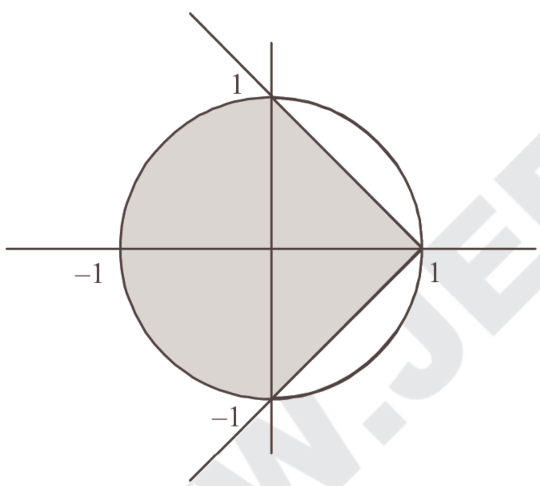
On solving both we get $y = -1, 3$



Required area $= \int_0^3 \left\{(2y+3) - y^2\right\} dy$

$= \left[y^2 + 3y - \frac{y^3}{3}\right]_0^3 = 9.$

16. **(c)** Given curves are $x^2 + y^2 = 1$ and $y^2 = 1 - x$.
Intersecting points are $x = 0, 1$



Area of shaded portion is the required area.
So, Required Area = Area of semi-circle + Area bounded by parabola

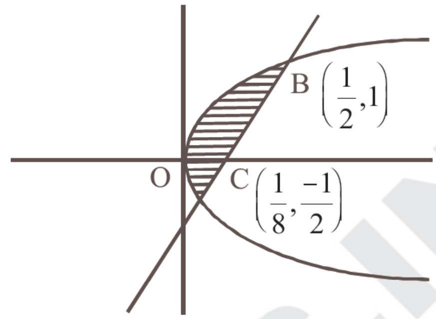
$= \frac{\pi r^2}{2} + 2\int_0^1 \sqrt{1-x}\,dx$

$= \frac{\pi}{2} + 2\int_0^1 \sqrt{1-x}\,dx$ ($\because$ radius of circle = 1)

$= \frac{\pi}{2} + 2\left[\frac{(1-x)^{3/2}}{-3/2}\right]_0^1$

$= \frac{\pi}{2} - \frac{4}{3}(-1) = \frac{\pi}{2} + \frac{4}{3}$ sq. unit

17. **(b)** Required area

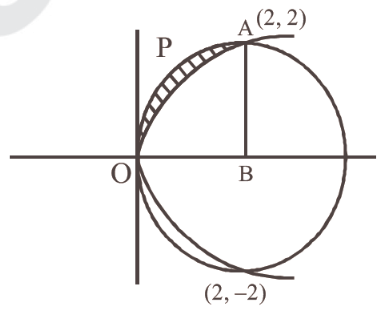


$= \int_{-1/2}^{1} \frac{y+1}{4}dy - \int_{-1/2}^{1} \frac{y^2}{2}dy$

$= \frac{1}{4}\left[\frac{y^2}{2} + y\right]_{-1/2}^{1} - \frac{1}{2}\left[\frac{y^3}{3}\right]_{-1/2}^{1}$

$= \frac{1}{4}\left[\frac{3}{2} + \frac{3}{8}\right] - \frac{9}{48} = \frac{15}{32} - \frac{9}{48} = \frac{27}{96} = \frac{9}{32}$

18. **(d)**

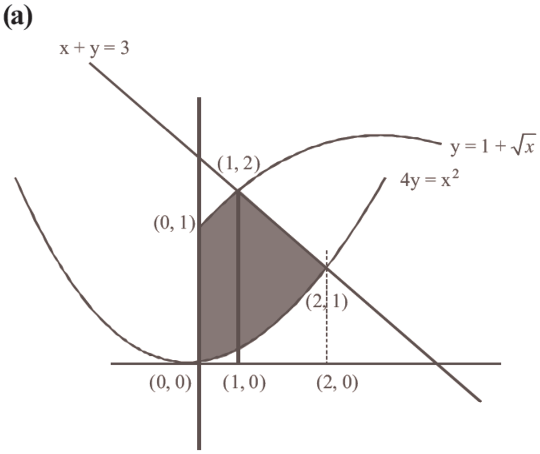


Points of intersection of the two curves are (0, 0), (2, 2) and (2, –2)

Area = Area (OPAB) – area under parabola (0 to 2)

$= \frac{\pi \times (2)^2}{4} - \int_0^2 \sqrt{2}\sqrt{x}\,dx \quad = \pi - \frac{8}{3}$

19. **(a)**



Area of shaded region

$$= \int_0^1 (1+\sqrt{x})dx + \int_1^2 (3-x)dx - \int_0^2 \frac{x^2}{4}dx$$

$$= [x]_0^1 + \left[\frac{x^{\frac{3}{2}}}{\frac{3}{2}}\right]_0^1 + [3x]_1^2 - \left[\frac{x^2}{2}\right]_1^2 - \left[\frac{x^3}{12}\right]_0^2 = \frac{5}{2}\text{sq.units}$$

20. **(d)** Here, $18x^2 - 9\pi x + \pi^2 = 0$

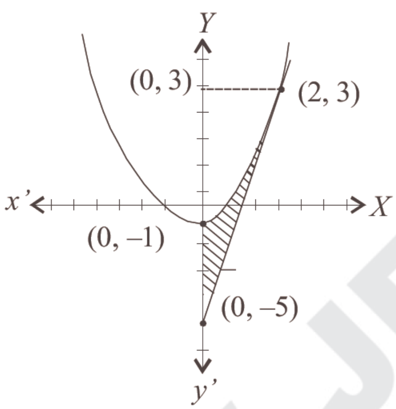
$\Rightarrow (3x-\pi)(6x-\pi) = 0$

$\Rightarrow \alpha = \frac{\pi}{6}, \beta = \frac{\pi}{3}$

Also, $gof(x) = \cos x$

$\therefore$ Req. area $= \int_{\pi/6}^{\pi/3} \cos x dx = \frac{\sqrt{3}-1}{2}$

21. **(a)**



$\because$ Curve is given as :

$y = x^2 - 1$

$\Rightarrow \frac{dy}{dx} = 2x$

$\Rightarrow \left(\frac{dy}{dx}\right)_{(2,3)} = 4$

$\therefore$ Equation of tangent at (2, 3)

$(y-3) = 4(x-2)$

$\Rightarrow y = 4x - 5$

but $x = 0$

$\Rightarrow y = -5$

$\because$ The tangent cuts Y–axis at (0, –5)

$\therefore$ Required area

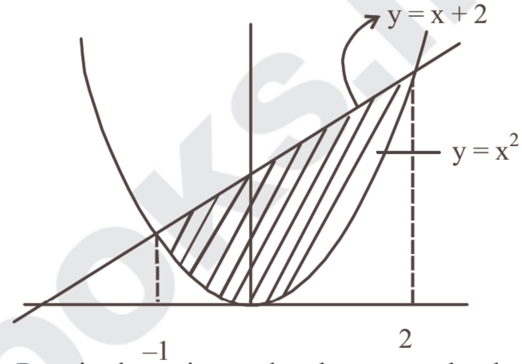
$$= \frac{1}{4}\int_{-5}^{3}(y+5)dy - \int_{-1}^{3}\sqrt{y+1}\,dy$$

$$= \frac{1}{4}\left[\frac{y^2}{2}+5y\right]_{-5}^{3} - \frac{2}{3}\left[(y+1)^{3/2}\right]_{-1}^{3}$$

$$= \frac{1}{4}\left[\frac{9}{2}+15-\frac{25}{2}+25\right] - \frac{2}{3}\left[4^{3/2}-0\right]$$

$$= \frac{32}{4} - \frac{16}{3} = \frac{8}{3} \text{ sq-units.}$$

22. **(b)**



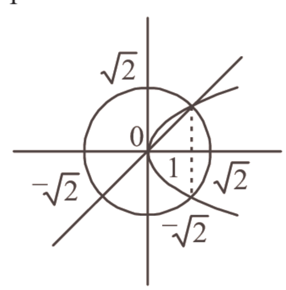
Required area is equal to the area under the curves $y \ge x^2$ and $y \le x + 2$

$\therefore$ requried area $\int_{-1}^{2}((x+2)-x^2)dx$

$$= \left(\frac{x^2}{2}+2x-\frac{x^3}{3}\right)_{-1}^{2}$$

$$= \left(2+4-\frac{8}{3}\right) - \left(\frac{1}{2}-2+\frac{1}{3}\right) = \frac{9}{2}$$

23. **(d)** Total area – enclosed area between line and parabola



$$= 2\pi - \int_0^1 \sqrt{x} - x dx$$

$$= 2\pi - \left(\frac{2x^{3/2}}{3} - \frac{x^2}{2}\right)_0^1$$

$$= 2\pi - \left(\frac{2}{3}-\frac{1}{2}\right) = 2\pi - \left(\frac{1}{6}\right) = \frac{12\pi - 1}{6}$$

24 Differential Equations

1. The order and degree of the differential equation $\left(1+3\frac{dy}{dx}\right)^{2/3} = 4\frac{d^3y}{dx^3}$ are **[2002]**

(a) $(1, \frac{2}{3})$ (b) $(3, 1)$
(c) $(3, 3)$ (d) $(1, 2)$

2. The solution of the equation $\frac{d^2y}{dx^2} = e^{-2x}$ **[2002]**

(a) $\frac{e^{-2x}}{4}$ (b) $\frac{e^{-2x}}{4} + cx + d$
(c) $\frac{1}{4}e^{-2x} + cx^2 + d$ (d) $\frac{1}{4}e^{-4x} + cx + d$

3. The degree and order of the differential equation of the family of all parabolas whose axis is x - axis, are respectively.

(a) 2, 3 (b) 2, 1 **[2003]**
(c) 1, 2 (d) 3, 2

4. The solution of the differential equation $(1+y^2)+(x-e^{\tan^{-1}y})\frac{dy}{dx} = 0$, is **[2003]**

(a) $xe^{2\tan^{-1}y} = e^{\tan^{-1}y} + k$
(b) $(x-2) = ke^{2\tan^{-1}y}$
(c) $2xe^{\tan^{-1}y} = e^{2\tan^{-1}y} + k$
(d) $xe^{\tan^{-1}y} = \tan^{-1}y + k$

5. The differential equation for the family of circle $x^2+y^2-2ay=0$, where a is an arbitrary constant is **[2004]**

(a) $(x^2+y^2)y' = 2xy$
(b) $2(x^2+y^2)y' = xy$
(c) $(x^2-y^2)y' = 2xy$
(d) $2(x^2-y^2)y' = xy$

6. Solution of the differential equation $ydx+(x+x^2y)dy = 0$ is **[2004]**

(a) $\log y = Cx$ (b) $-\frac{1}{xy} + \log y = C$
(c) $\frac{1}{xy} + \log y = C$ (d) $-\frac{1}{xy} = C$

7. A function $y = f(x)$ has a second order derivative $f''(x) = 6(x-1)$. If its graph passes through the point (2,1) and at that point the tangent to the graph is $y = 3x - 5$, then the function is **[2004]**

(a) $(x+1)^2$ (b) $(x-1)^3$
(c) $(x+1)^3$ (d) $(x-1)^2$

8. The differential equation representing the family of curves $y^2 = 2c(x+\sqrt{c})$, where $c > 0$, is a parameter, is of order and degree as follows : **[2005]**

(a) order 1, degree 2 (b) order 1, degree 1
(c) order 1, degree 3 (d) order 2, degree 2

9. If $x\dfrac{dy}{dx} = y(\log y - \log x + 1)$, then the solution of the equation is **[2005]**

(a) $y\log\left(\dfrac{x}{y}\right) = cx$ (b) $x\log\left(\dfrac{y}{x}\right) = cy$

(c) $\log\left(\dfrac{y}{x}\right) = cx$ (d) $\log\left(\dfrac{x}{y}\right) = cy$

10. The differential equation whose solution is $Ax^2 + By^2 = 1$ where A and B are arbitrary constants is of **[2006]**

(a) second order and second degree
(b) first order and second degree
(c) first order and first degree
(d) second order and first degree

11. The differential equation of all circles passing through the origin and having their centres on the x-axis is **[2007]**

(a) $y^2 = x^2 + 2xy\dfrac{dy}{dx}$ (b) $y^2 = x^2 - 2xy\dfrac{dy}{dx}$

(c) $x^2 = y^2 + xy\dfrac{dy}{dx}$ (d) $x^2 = y^2 + 3xy\dfrac{dy}{dx}$.

12. The solution of the differential equation $\dfrac{dy}{dy} = \dfrac{x+y}{x}$ satisfying the condition $y(1) = 1$ is **[2008]**

(a) $y = \ln x + x$ (b) $y = x\ln x + x^2$
(c) $y = xe^{(x-1)}$ (d) $y = x\ln x + x$

13. The differential equation which represents the family of curves $y = c_1 e^{c_2 x}$, where c_1 and c_2 are arbitrary constants, is **[2009]**

(a) $y'' = y'y$ (b) $yy'' = y'$
(c) $yy'' = (y')^2$ (d) $y' = y^2$

14. Solution of the differential equation

$\cos x\, dy = y(\sin x - y)\, dx,\ 0 < x < \dfrac{\pi}{2}$ is **[2010]**

(a) $y\sec x = \tan x + c$ (b) $y\tan x = \sec x + c$
(c) $\tan x = (\sec x + c)y$ (d) $\sec x = (\tan x + c)y$

15. If $\dfrac{dy}{dx} = y + 3 > 0$ and $y(0) = 2$, then $y(\ln 2)$ is equal to : **[2011]**

(a) 5 (b) 13
(c) -2 (d) 7

16. Let I be the purchase value of an equipment and $V(t)$ be the value after it has been used for t years. The value $V(t)$ depreciates at a rate given by differential equation $\dfrac{dV(t)}{dt} = -k(T-t)$, where $k > 0$ is a constant and T is the total life in years of the equipment. Then the scrap value $V(T)$ of the equipment is **[2011]**

(a) $I - \dfrac{kT^2}{2}$ (b) $I - \dfrac{k(T-t)^2}{2}$

(c) e^{-kT} (d) $T^2 - \dfrac{1}{k}$

17. The curve that passes through the point (2, 3), and has the property that the segment of any tangent to it lying between the coordinate axes is bisected by the point of contact is given by : **[2011RS]**

(a) $2y - 3x = 0$ (b) $y = \dfrac{6}{x}$

(c) $x^2 + y^2 = 13$ (d) $\left(\dfrac{x}{2}\right)^2 + \left(\dfrac{y}{3}\right)^2 = 2$

18. Consider the differential equation **[2011RS]**

$y^2 dx + \left(x - \dfrac{1}{y}\right)dy = 0$. If $y(1) = 1$, then x is given by :

(a) $4 - \dfrac{2}{y} - \dfrac{e^{\frac{1}{y}}}{e}$ (b) $3 - \dfrac{1}{y} + \dfrac{e^{\frac{1}{y}}}{e}$

(c) $1 + \dfrac{1}{y} - \dfrac{e^{\frac{1}{y}}}{e}$ (d) $1 - \dfrac{1}{y} + \dfrac{e^{\frac{1}{y}}}{e}$

19. The population $p(t)$ at time t of a certain mouse species satisfies the differential equation $\dfrac{dp(t)}{dt} = 0.5\,p(t) - 450$. If $p(0) = 850$, then the time at which the population becomes zero is : **[2012]**

(a) $2 ln 18$ (b) $ln 9$

(c) $\frac{1}{2} ln 18$ (d) $ln 18$

20. At present, a firm is manufacturing 2000 items. It is estimated that the rate of change of production P w.r.t. additional number of workers x is given by $\frac{dP}{dx} = 100 - 12\sqrt{x}$. If the firm employs 25 more workers, then the new level of production of items is **[2013]**

(a) 2500 (b) 3000

(c) 3500 (d) 4500

21. Let the population of rabbits surviving at time t be governed by the differential equation $\frac{dp(t)}{dt} = \frac{1}{2}p(t) - 200$. If $p(0) = 100$, then $p(t)$ equals: **[2014]**

(a) $600 - 500\, e^{t/2}$ (b) $400 - 300\, e^{-t/2}$

(c) $400 - 300\, e^{t/2}$ (d) $300 - 200\, e^{-t/2}$

22. Let y(x) be the solution of the differential equation $(x \log x)\frac{dy}{dx} + y = 2x \log x, (x \geq 1)$. Then y (e) is equal to: **[2015]**

(a) 2 (b) 2e

(c) e (d) 0

23. If a curve $y = f(x)$ passes through the point $(1, -1)$ and satisfies the differential equation, $y(1 + xy)\, dx = x\, dy$, then $f\left(-\frac{1}{2}\right)$ is equal to: **[2016]**

(a) $\frac{2}{5}$ (b) $\frac{4}{5}$

(c) $-\frac{2}{5}$ (d) $-\frac{4}{5}$

24. If $(2 + \sin x)\frac{dy}{dx} + (y + 1)\cos x = 0$ and $y(0) = 1$, then $y\left(\frac{\pi}{2}\right)$ is equal to : **[2017]**

(a) $\frac{4}{3}$ (b) $\frac{1}{3}$

(c) $-\frac{2}{3}$ (d) $-\frac{1}{3}$

25. Let $y = y(x)$ be the solution of the differential equation $\sin x \frac{dy}{dx} + y \cos x = 4x, x \in (0, \pi)$. If $y\left(\frac{\pi}{2}\right) = 0$, then $y\left(\frac{\pi}{6}\right)$ is equal to : **[2018]**

(a) $\frac{-8}{9\sqrt{3}}\pi^2$ (b) $-\frac{8}{9}\pi^2$

(c) $-\frac{4}{9}\pi^2$ (d) $\frac{4}{9\sqrt{3}}\pi^2$

26. If $y = y(x)$ is the solution of the differential equation, $x\frac{dy}{dx} + 2y = x^2$ satisfying $y(1) = 1$, then $y\left(\frac{1}{2}\right)$ is equal to: **[2019]**

(a) $\frac{7}{64}$ (b) $\frac{1}{4}$

(c) $\frac{49}{16}$ (d) $\frac{13}{16}$

27. The solution of the differential equation $x\frac{dy}{dx} + 2y = x^2\ (x \neq 0)$ with $y(1) = 1$, is: **[2019]**

(a) $y = \frac{4}{5}x^3 + \frac{1}{5x^2}$ (b) $y = \frac{x^3}{5} + \frac{1}{5x^2}$

(c) $y = \frac{x^2}{4} + \frac{3}{4x^2}$ (d) $y = \frac{3}{4}x^2 + \frac{1}{4x^2}$

28. If $y = y(x)$ is the solution of the differential equation, $e^y\left(\frac{dy}{dx} - 1\right) = e^x$ such that $y(0) = 0$, then $y(1)$ is equal to: **[2020]**

(a) $1 + \log_e 2$ (b) $2 + \log_e 2$

(c) $2e$ (d) $\log_e 2$

Answer Key

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
(c)	(b)	(c)	(c)	(c)	(b)	(b)	(c)	(c)	(d)	(a)	(d)	(c)	(d)	(d)
16	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**	**25**	**26**	**27**	**28**		
(a)	(b)	(c)	(a)	(c)	(c)	(a)	(b)	(b)	(b)	(c)	(c)	(a)		

Solutions

1. **(c)** $\left(1+3\frac{dy}{dx}\right)^2=\left(\frac{4d^3y}{dx^3}\right)^3$

$\Rightarrow \left(1+3\frac{dy}{dx}\right)^2=16\left(\frac{d^3y}{dx^3}\right)^3$

Order = 3, degree 3

2. **(b)** $\frac{d^2y}{dx^2}=e^{-2x}$; on integration $\frac{dy}{dx}=\frac{e^{-2x}}{-2}+c$;

Again integrate we get $y=\frac{e^{-2x}}{4}+cx+d$

3. **(c)** $y^2=4a(x-h)$,

Differentiating $2yy_1=4a \Rightarrow yy_1=2a$

Again differentiating, we get

$\Rightarrow {y_1}^2+yy_2=0$

Degree $=1$, order $=2$.

4. **(c)** $(1+y^2)+(x-e^{\tan^{-1}y})\frac{dy}{dx}=0$

$\Rightarrow \frac{dx}{dy}+\frac{x}{(1+y^2)}=\frac{e^{\tan^{-1}y}}{(1+y^2)}$

It is form of linear differential equation.

$I.F=e^{\int\frac{1}{(1+y^2)}dy}=e^{\tan^{-1}y}$

$x(e^{\tan^{-1}y})=\int\frac{e^{\tan^{-1}y}}{1+y^2}e^{\tan^{-1}y}\,dy$

$x(e^{\tan^{-1}y})=\frac{e^{2\tan^{-1}y}}{2}+C$

$\left[\because \int e^{2x}dx=\frac{e^{2x}}{2}\right]$

$\therefore\ 2xe^{\tan^{-1}y}=e^{2\tan^{-1}y}+k$

5. **(c)** $x^2+y^2-2ay=0$ ....(1)

Differentiate w.r. to x,

$2x+2y\frac{dy}{dx}-2a\frac{dy}{dx}=0 \Rightarrow a=\frac{x+yy'}{y'}$

Putting in (1) we get,

$x^2+y^2-2\left(\frac{x+yy'}{y'}\right)y=0$

$\Rightarrow (x^2+y^2)y'-2xy-2y^2y'=0$

$\Rightarrow (x^2-y^2)y'=2xy$

6. **(b)** $ydx+(x+x^2y)dy=0$

$\Rightarrow \frac{dx}{dy}=-\frac{x}{y}-x^2 \Rightarrow \frac{dx}{dy}+\frac{x}{y}=-x^2$,

It is Bernoulli form. Divide by x^2

$x^{-2}\frac{dx}{dy}+x^{-1}\left(\frac{1}{y}\right)=-1.$

put $x^{-1}=t, -x^{-2}\frac{dx}{dy}=\frac{dt}{dy}$ we get,

$-\frac{dt}{dy}+t\left(\frac{1}{y}\right)=-1 \Rightarrow \frac{dt}{dy}-\left(\frac{1}{y}\right)t=1$

It is linear differential eqn. in t.

$\text{I.F}=e^{\int-\frac{1}{y}dy}=e^{-\log y}=y^{-1}$

$\therefore$ Solution is $t(y^{-1})=\int(y^{-1})dy+C$

$\Rightarrow \frac{1}{x}.\frac{1}{y}=\log y+C \Rightarrow \log y-\frac{1}{xy}=C$

7. **(b)** Given $f''(x)=6(x-1)$.

Inegrating both sides, we get

$f'(x)=3x^2-6x+c$

Slope of tangent $y=3x-5$ be 3

Slope of tangent of $f(x)$ at (2, 1)

$=f'(2)=c=3$

$\therefore f'(x)=3x^2-6x+3=3(x-1)^2$

Inegrating again both sides, we get

$f(x)=(x-1)^3+D$

The curve passes through (2, 1)

$\Rightarrow 1=(2-1)^3+D \Rightarrow D=0$

$\therefore f(x)=(x-1)^3$

8. **(c)** $y^2 = 2c(x+\sqrt{c})$ .... (i)

Differentite it w.r. to x

$2yy' = 2c.1$ or $yy' = c$ .... (ii)

[On putting value of c from (ii) in (i)]

$\Rightarrow \quad y^2 = 2yy'(x+\sqrt{yy'})$

On simplifying, we get

$(y-2xy')^2 = 4yy'^3$ .... (iii)

Hence equation (iii) is of order 1 and degree 3.

9. **(c)** $\frac{xdy}{dx} = y(\log y - \log x + 1)$

$\frac{dy}{dx} = \frac{y}{x}\left(\log\left(\frac{y}{x}\right)+1\right)$

Put $y = vx$

$\frac{dy}{dx} = v + \frac{xdv}{dx} \Rightarrow v + \frac{xdv}{dx} = v(\log v + 1)$

$\frac{xdv}{dx} = v\log v \Rightarrow \int\frac{dv}{v\log v} = \int\frac{dx}{x}$

Put $\log v = z$

$\frac{1}{v}dv = dz \Rightarrow \int\frac{dz}{z} = \int\frac{dx}{x}$

$\ln z = \ln x + \ln c$

$x = cx$ or $\log v = cx$ or $\log\left(\frac{y}{x}\right) = cx.$

10. **(d)** $Ax^2 + By^2 = 1$ ...(i)

Differentiate w.r. to x

$Ax + By\frac{dy}{dx} = 0$ ....(ii)

Again differentiate w.r. to x

$A + By\frac{d^2y}{dx^2} + B\left(\frac{dy}{dx}\right)^2 = 0$ ....(iii)

From (ii) and (iii)

$x\left\{-By\frac{d^2y}{dx^2} - B\left(\frac{dy}{dx}\right)^2\right\} + By\frac{dy}{dx} = 0$

Dividing both sides by –B, we get

$xy\frac{d^2y}{dx^2} + x\left(\frac{dy}{dx}\right)^2 - y\frac{dy}{dx} = 0$

Therefore order 2 and degree 1.

11. **(a)** General equation of circles passing through origin and having their centres on the x-axis is

$x^2 + y^2 + 2gx = 0$ ...(i)

On differentiating w.r.t x, we get

$2x + 2y.\frac{dy}{dx} + 2g = 0 \Rightarrow g = -\left(x + y\frac{dy}{dx}\right)$

Putting in (i)

$x^2 + y^2 + 2\left\{-\left(x + y\frac{dy}{dx}\right)\right\}.x = 0$

$\Rightarrow x^2 + y^2 - 2x^2 - 2x\frac{dy}{dx}.y = 0$

$\Rightarrow y^2 = x^2 + 2xy\frac{dy}{dx}$

12. **(d)** $\frac{dy}{dx} = \frac{x+y}{x} = 1 + \frac{y}{x}$

It is homogeneous differential eqn.

Putting $y = vx$ and $\frac{dy}{dx} = v + x\frac{dv}{dx}$

we get

$v + x\frac{dv}{dx} = 1 + v \Rightarrow \int\frac{dx}{x} = \int dv$

$\Rightarrow v = \ln x + c \Rightarrow y = x\ln x + \mathbf{c}x$

As $y(1) = 1$

$\therefore\ c = 1$ So solution is $y = x\ln x + x$

13. **(c)** We have $y = c_1e^{c_2x}$

Differentiate it w.r. to x

$\Rightarrow \quad y' = c_1c_2e^{c_2x} = c_2y$

$\Rightarrow \quad \frac{y'}{y} = c_2$ Differentiate it w.r. to x

$\Rightarrow \quad \frac{y''y - (y')^2}{y^2} = 0$

$\Rightarrow \quad y''y = (y')^2$

14. **(d)** $\cos x\,dy = y(\sin x - y)dx$

$\frac{dy}{dx} = y\tan x - y^2\sec x$

$\frac{1}{y^2}\frac{dy}{dx} - \frac{1}{y}\tan x = -\sec x$ ...(i)

Let $\frac{1}{y} = t \Rightarrow -\frac{1}{y^2}\frac{dy}{dx} = \frac{dt}{dx}$

Putting in (i)

$-\frac{dt}{dx} - t\tan x = -\sec x$

$\Rightarrow \frac{dt}{dx} + (\tan x)t = \sec x$

$\text{I.F.} = e^{\int \tan x\, dx} = e^{\log|\sec x|} = \sec x$

Solution : $t \sec x = \int \sec x \sec x\, dx$

$\Rightarrow \frac{1}{y}\sec x = \tan x + c$

15. (d) $\frac{dy}{dx} = y + 3 \Rightarrow \int \frac{dy}{y+3} = \int dx$

$\Rightarrow \ell n|y+3| = x + c$

Given $y(0) = 2$, $\therefore \ell n 5 = c$

$\Rightarrow \ell n|y+3| = x + \ell n 5$

Put $x = \ell n 2$, then $\ell n|y+3| = \ell n 2 + \ell n 5$

$\Rightarrow \ell n|y+3| = \ell n 10$

$\therefore\ y + 3 = \pm 10 \Rightarrow y = 7, -13$

16. (a) $\frac{dV(t)}{dt} = -k(T-t)$

$\Rightarrow \int dV(t) = -k\int (T-t)dt$

$V(t) = \frac{k(T-t)^2}{2} + c$

at $t = 0$, $V(t) = I$

$I = \frac{kT^2}{2} + c$

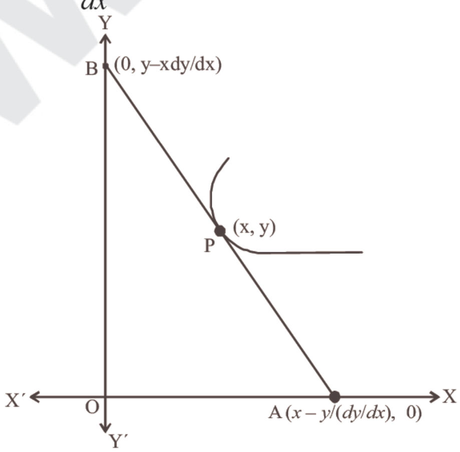
$\Rightarrow c = I - \frac{kT^2}{2}$

$\Rightarrow V(t) = I + \frac{k}{2}(t^2 - 2tT)$

$V(T) = I + \frac{k}{2}(T^2 - 2T^2) = I - \frac{k}{2}T^2$

17. (b) Equation of tangent at P

$Y - y = \frac{dy}{dx}(X - x)$



X-intercept $= x - \frac{y}{dy/dx}$

Y-intercept $= y - \frac{xdy}{dx}$

Since P is mid-point of A and B

$x - \frac{y}{dy} = 2x$ and $y - \frac{xdy}{dx} = 2y$

$\Rightarrow \frac{-y}{\frac{dy}{dx}} = x$ and $\frac{-xdy}{dx} = y$

$\Rightarrow \frac{dx}{x} = -\frac{dy}{y}$

$\ell ny = -\ell nx + \ell nc$

$y = \frac{c}{x}$

Since the above line passes through the point (2, 3).

$\therefore\ c = 6$

Hence $y = \frac{6}{x}$ is the required equation.

18. (c) $\frac{dx}{dy} + \frac{x}{y^2} = \frac{1}{y^3}$

It is linear differential eqn.

$\text{I.F.} = e^{\int \frac{1}{y^2}dy} = e^{-\frac{1}{y}}$

So $\quad x.e^{-\frac{1}{y}} = \int \frac{1}{y^3} e^{-\frac{1}{y}} dy$

Let $\quad \frac{-1}{y} = t$

$\Rightarrow \quad \frac{1}{y^2}dy = dt$

$\Rightarrow \quad I = -\int te^t dt = e^t - te^t$

$= e^{-\frac{1}{y}} + \frac{1}{y}e^{-\frac{1}{y}} + c$

$\Rightarrow \quad xe^{-\frac{1}{y}} = e^{-\frac{1}{y}} + \frac{1}{y}e^{-\frac{1}{y}} + c$

$\Rightarrow \quad x = 1 + \frac{1}{y} + c.e^{1/y}$

Given $y(1) = 1$

$\therefore \quad c = -\frac{1}{e}$

$\Rightarrow \quad x = 1 + \frac{1}{y} - \frac{1}{e}.e^{1/y}$

19. **(a)** Given differential equation is

$$\frac{dp(t)}{dt}=0.5p(t)-450$$

$$\Rightarrow \quad \frac{dp(t)}{dt}=\frac{1}{2}p(t)-450$$

$$\Rightarrow \quad \frac{dp(t)}{dt}=\frac{p(t)-900}{2}$$

$$\Rightarrow \quad 2\frac{dp(t)}{dt}=-[900-p(t)]$$

$$\Rightarrow \quad 2\frac{dp(t)}{900-p(t)}=-dt$$

Integrate both the side, we get

$$-2\int\frac{dp(t)}{900-p(t)}=\int dt$$

Let $900-p(t)=u$

$\Rightarrow \quad -dp(t)=du$

$$2\int\frac{du}{u}=\int dt \Rightarrow 2\ ln\ u=t+c \qquad ...(i)$$

$\Rightarrow \quad 2ln[900-p(t)]=t+c$

Given $t=0, p(0)=850$

$2\ ln(50)=c$

Putting in (i)

$$\Rightarrow \quad 2\left[ln\left(\frac{900-p(t)}{50}\right)\right]=t$$

$$\Rightarrow \quad 900-p(t)=50e^{\frac{t}{2}}$$

$$\Rightarrow \quad p(t)=900-50e^{\frac{t}{2}}$$

let $p(t_1)=0$

$$0=900-50e^{\frac{t_1}{2}} \ \therefore \ t_1=2ln\ 18$$

20. **(c)** Given, Rate of change is $\frac{dP}{dx}=100-12\sqrt{x}$

$\Rightarrow \quad dP=(100-12\sqrt{x})\,dx$

By integrating

$$\int dP=\int(100-12\sqrt{x})\,dx$$

$P=100x-8x^{3/2}+C$

Given when $x=0$ then $P=2000$

$\Rightarrow \quad C=2000$

Now when $x=25$ then

$P=100\times 25-8\times(25)^{3/2}+2000$

$=4500-1000$

$\Rightarrow \quad P=3500$

21. **(c)** Given differential equation is

$$\frac{dp(t)}{dt}=\frac{1}{2}p(t)-200$$

By separating the variable, we get

$$dp(t)=\left[\frac{1}{2}p(t)-200\right]dt$$

$$\Rightarrow \quad \frac{dp(t)}{\frac{1}{2}p(t)-200}=dt$$

Integrate on both the sides,

$$\int\frac{d(p(t))}{\frac{1}{2}p(t)-200}=\int dt$$

Let $\frac{1}{2}p(t)-200=s \Rightarrow \frac{dp(t)}{2}=ds$

So, $$\int\frac{d\,p(t)}{\left(\frac{1}{2}p(t)-200\right)}=\int dt$$

$$\Rightarrow \quad \int\frac{2ds}{s}=\int dt$$

$\Rightarrow \quad 2\log s=t+c$

$$\Rightarrow \ 2\log\left(\frac{p(t)}{2}-200\right)=t+c$$

$$\Rightarrow \quad \frac{p(t)}{2}-200=e^{\frac{t}{2}}k$$

Using given condition $p(t)=400-300\,e^{t/2}$

22. **(a)** Given, $\frac{dy}{dx}+\left(\frac{1}{x\log x}\right)y=2$

$$\text{I.F.}=e^{\int\frac{1}{x\log x}dx}$$

$$=e^{\log(\log x)}=\log x$$

$$y.\log x=\int 2\log x\,dx+c$$

$y\log x=2[x\log x-x]+c$

Put $x=1$, $y.0=-2+c$

$c=2$

Put $x=e$

$y\log e=2e(\log e-1)+c$

$y(e)=c=2$

23. **(b)** $y(1+xy)dx=xdy$

$$\frac{xdy-ydx}{y^2}=xdx$$

$$\Rightarrow \int -d\left(\frac{x}{y}\right)=\int xdx$$

$-\frac{x}{y}=\frac{x^2}{2}+C$ as $y(1)=-1 \Rightarrow C=\frac{1}{2}$

Hence, $y=\frac{-2x}{x^2+1} \Rightarrow f\left(\frac{-1}{2}\right)=\frac{4}{5}$

24. **(b)** We have $(2+\sin x)\frac{dy}{dx}+(y+1)\cos x=0$

$\Rightarrow \frac{d}{dx}(2+\sin x)(y+1)=0$

On integrating, we get

$(2+\sin x)(y+1)=C$

At $x=0, y=1$ we have

$(2+\sin 0)(1+1)=C$

$\Rightarrow C=4$

$\Rightarrow y+1=\frac{4}{2+\sin x}$

$y=\frac{4}{2+\sin x}-1$

Now $y\left(\frac{\pi}{2}\right)=\frac{4}{2+\sin\frac{\pi}{2}}-1$

$=\frac{4}{3}-1=\frac{1}{3}$

25. **(b)** Consider the given differential equation

$\sin x dy+y\cos x dx=4x dx$

$\Rightarrow$ $d(y.\sin x)=4x dx$

Integrate both sides

$\Rightarrow$ $y.\sin x=2x^2+C$ ...(1)

$\Rightarrow$ $y(x)=\frac{2x^2}{\sin x}+\frac{C}{\sin x}$ ...(2)

$\because$ eq. (2) passes through $\left(\frac{\pi}{2},0\right)$

$\Rightarrow 0=\frac{\pi^2}{2}+C$

$\Rightarrow C=-\frac{\pi^2}{2}$

Now, put the value of C in (1)

Then, $y\sin x=2x^2-\frac{\pi^2}{2}$ is the solution

$\therefore\ y\left(\frac{\pi}{6}\right)=\left(2.\frac{\pi^2}{36}-\frac{\pi^2}{2}\right)2=-\frac{8\pi^2}{9}$

26. **(c)** Since, $x\frac{dy}{dx}+2y=x^2$

$\Rightarrow$ $\frac{dy}{dx}+\frac{2}{x}y=x$

I.F. $=e^{\int\frac{2}{x}dx}=e^{2\ln x}=e^{\ln x^2}=x^2.$

Solution of differential equation is:

$y\cdot x^2=\int x\cdot x^2 dx$

$y\cdot x^2=\frac{x^4}{4}+C$ ...(1)

$\because\ y(1)=1$

$\therefore\ C=\frac{3}{4}$

Then, from equation (1)

$y\cdot x^2=\frac{x^4}{4}+\frac{3}{4}$

$\therefore\ y=\frac{x^2}{4}+\frac{3}{4x^2}$

$\therefore\ y\left(\frac{1}{2}\right)=\frac{1}{16}+3=\frac{49}{16}$

27. **(c)** $\frac{dy}{dx}+\frac{2}{x}y=x$ and $y(1)=1$ (given)

Since, the above differential equation is the linear differential equation, then

$I.F=e^{\int\frac{2}{x}dx}=x^2$

Now, the solution of the linear differential equation

$y\times x^2=\int x^3dx \Rightarrow yx^2=\frac{x^4}{4}+C$

$\because y(1)=1\ \therefore 1\times 1=\frac{1}{4}+C \Rightarrow C=\frac{3}{4}$

$\therefore$ solution becomes

$y=\frac{x^2}{4}+\frac{3}{4x^2}$

28. **(a)** Let $e^y=t$

$e^y\frac{dy}{dx}=\frac{dt}{dx}$

$\therefore\ \frac{dt}{dx}-t=e^x\ \left[\because\ e^y\frac{dy}{dx}-e^y=e^x\right]$

I.F. $=e^{\int -1.dx}=e^{-x}$

$t\left(e^{-x}\right)=\int e^x.e^{-x}dx$

$\Rightarrow e^{y-x}=x+c$

Put $x=0, y=0$, then we get $c=1$

$e^{y-x}=x+1$

$y=x+\log_e(x+1)$

Put $x=1\ \therefore\ y=1+\log_e 2$

25 Vector Algebra

1. If $|\vec{a}|=4, |\vec{b}|=2$ and the angle between $\vec{a}$ and $\vec{b}$ is $\pi/6$ then $(\vec{a}\times\vec{b})^2$ is equal to **[2002]**
(a) 48 (b) 16
(c) $\vec{a}$ (d) none of these

2. If $\vec{a}, \vec{b}, \vec{c}$ are vectors such that $[\vec{a}\ \vec{b}\ \vec{c}]=4$ then $[\vec{a}\times\vec{b}\ \ \vec{b}\times\vec{c}\ \ \vec{c}\times\vec{a}]=$ **[2002]**
(a) 16 (b) 64
(c) 4 (d) 8

3. If $\vec{a}, \vec{b}, \vec{c}$ are vectors such that $\vec{a}+\vec{b}+\vec{c}=0$ and $|\vec{a}|=7, |\vec{b}|=5, |\vec{c}|=3$ then angle between vector $\vec{b}$ and $\vec{c}$ is **[2002]**
(a) 60° (b) 30°
(c) 45° (d) 90°

4. If $|\vec{a}|=5,\ |\vec{b}|=4, |\vec{c}|=3$ thus what will be the value of $|\vec{a}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a}|$, given that $\vec{a}+\vec{b}+\vec{c}=0$ **[2002]**
(a) 25 (b) 50
(c) –25 (d) –50

5. If the vectors $\vec{c}, \vec{a}=x\hat{i}+y\hat{j}+z\hat{k}$ and $\hat{b}=\hat{j}$ are such that $\vec{a}, \vec{c}$ and $\vec{b}$ form a right handed system then $\vec{c}$ is: **[2002]**
(a) $z\hat{i}-x\hat{k}$ (b) $\vec{0}$
(c) $y\hat{j}$ (d) $-z\hat{i}+x\hat{k}$

6. $\vec{a}=3\hat{i}-5\hat{j}$ and $\vec{b}=6\hat{i}+3\hat{j}$ are two vectors and $\vec{c}$ is a vector such that $\vec{c}=\vec{a}\times\vec{b}$ then $|\vec{a}|:|\vec{b}|:|\vec{c}|$ **[2002]**
(a) $\sqrt{34}:\sqrt{45}:\sqrt{39}$ (b) $\sqrt{34}:\sqrt{45}:39$
(c) $34:39:45$ (d) $39:35:34$

7. If $\vec{a}\times\vec{b}=\vec{b}\times\vec{c}=\vec{c}\times\vec{a}$ then $\vec{a}+\vec{b}+\vec{c}=$ **[2002]**
(a) abc (b) –1
(c) 0 (d) 2

8. The sum of two forces is 18 N and resultant whose direction is at right angles to the smaller force is 12 N. The magnitude of the two forces are **[2002]**
(a) 13, 5 (b) 12, 6
(c) 14, 4 (d) 11, 7

9. A bead of weight w can slide on smooth circular wire in a vertical plane. The bead is attached by a light thread to the highest point of the wire and in equilibrium, the thread is taut and make an angle θ with the vertical then tension of the thread and reaction of the wire on the bead are **[2002]**
(a) $T=w\cos\theta$ $R=w\tan\theta$
(b) $T=2w\cos\theta$ $R=w$
(c) $T=w$ $R=w\sin\theta$
(d) $T=w\sin\theta$ $R=w\cot\theta$

10. Let $\vec{u}=\hat{i}+\hat{j}, \vec{v}=\hat{i}-\hat{j}$ and $\vec{w}=\hat{i}+2\hat{j}+3\hat{k}$. If $\hat{n}$ is a unit vector such that $\vec{u}.\hat{n}=0$ and $\vec{v}.\hat{n}=0$, then $|\vec{w}.\hat{n}|$ is equal to **[2003]**
(a) 3 (b) 0
(c) 1 (d) 2.

11. A particle acted on by constant forces $4\hat{i}+\hat{j}-3\hat{k}$ and $3\hat{i}+\hat{j}-\hat{k}$ is displaced from the point $\hat{i}+2\hat{j}+3\hat{k}$ to the point $5\hat{i}+4\hat{j}+\hat{k}$. The total work done by the forces is **[2003]**

(a) 50 units (b) 20 units
(c) 30 units (d) 40 units.

12. The vectors $\overrightarrow{AB}=3\hat{i}+4\hat{k}$ & $\overrightarrow{AC}=5\hat{i}-2\hat{j}+4\hat{k}$ are the sides of a triangle ABC. The length of the median through A is **[2003]**

(a) $\sqrt{288}$ (b) $\sqrt{18}$
(c) $\sqrt{72}$ (d) $\sqrt{33}$

13. $\vec{a},\vec{b},\vec{c}$ are 3 vectors, such that $\vec{a}+\vec{b}+\vec{c}=0$, $|\vec{a}|=1,\ |\vec{b}|=2, |\vec{c}|=3,$ then $\vec{a}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a}$ is equal to **[2003]**

(a) 1 (b) 0
(c) –7 (d) 7

14. A tetrahedron has vertices at O(0, 0, 0), A(1, 2, 1) B(2, 1, 3) and C(-1, 1, 2). Then the angle between the faces OAB and ABC will be **[2003]**

(a) 90^o (b) $\cos^{-1}\left(\frac{19}{35}\right)$
(c) $\cos^{-1}\left(\frac{17}{31}\right)$ (d) 30^o

15. If $\begin{vmatrix} a & a^2 & 1+a^3 \\ b & b^2 & 1+b^3 \\ c & c^2 & 1+c^3 \end{vmatrix}=0$ and vectors $(1,a,a^2)$, $(1,b,b^2)$ and $(1,c,c^2)$ are non- coplanar, then the product abc equals **[2003]**

(a) 0 (b) 2
(c) –1 (d) 1

16. Consider points A, B, C and D with position vectors $7\hat{i}-4\hat{j}+7\hat{k}, \hat{i}-6\hat{j}+10\hat{k}, -\hat{i}-3\hat{j}+4\hat{k}$ and $5\hat{i}-\hat{j}+5\hat{k}$ respectively. Then ABCD is a **[2003]**

(a) parallelogram but not a rhombus
(b) square
(c) rhombus (d) rectangle.

17. If $\vec{u},\vec{v}$ and $\vec{w}$ are three non- coplanar vectors, then $(\vec{u}+\vec{v}-\vec{w}).(\vec{u}-\vec{v})\times(\vec{v}-\vec{w})$ equals **[2003]**

(a) $3\vec{u}.\vec{v}\times\vec{w}$ (b) 0
(c) $\vec{u}.(\vec{v}\times\vec{w})$ (d) $\vec{u}.\vec{w}\times\vec{v}$.

18. A couple is of moment $\vec{G}$ and the force forming the couple is $\vec{P}$. If $\vec{P}$ is turned through a right angle the moment of the couple thus formed is $\vec{H}$. If instead , the force $\vec{P}$ are turned through an angle α , then the moment of couple becomes **[2003]**

(a) $\vec{H}\sin\alpha-\vec{G}\cos\alpha$
(b) $\vec{G}\sin\alpha-\vec{H}\cos\alpha$
(c) $\vec{H}\sin\alpha+\vec{G}\cos\alpha$
(d) $\vec{G}\sin\alpha+\vec{H}\cos\alpha$.

19. The resultant of forces $\vec{P}$ and $\vec{Q}$ is $\vec{R}$. If $\vec{Q}$ is doubled then $\vec{R}$ is doubled. If the direction of $\vec{Q}$ is reversed,then $\vec{R}$ is again doubled. Then $P^2:Q^2:R^2$ is **[2003]**

(a) 2 : 3 : 1 (b) 3 : 1 : 1
(c) 2 : 3 : 2 (d) 1 : 2 : 3.

20. A body travels a distance s in t seconds. It starts from rest and ends at rest. In the first part of the journey, it moves with constant acceleration f and in the second part with constant retardation r. The value of t is given by **[2003]**

(a) $\sqrt{2s\left(\frac{1}{f}+\frac{1}{r}\right)}$ (b) $2s\left(\frac{1}{f}+\frac{1}{r}\right)$
(c) $\frac{2s}{\frac{1}{f}+\frac{1}{r}}$ (d) $\sqrt{2s(f+r)}$

21. Two stones are projected from the top of a cliff h metres high, with the same speed u, so as to hit the ground at the same spot. If one of the stones is projected horizontally and the other is projected horizontally and the other is projected at an angle θ to the horizontal then $\tan\theta$ equals **[2003]**

(a) $u\sqrt{\frac{2}{gh}}$ (b) $\sqrt{\frac{2u}{gh}}$

(c) $2g\sqrt{\frac{u}{h}}$ (d) $2h\sqrt{\frac{u}{g}}$

22. Two particles start simultaneously from the same point and move along two straight lines, one with uniform velocity $\vec{u}$ and the other from rest with uniform acceleration $\vec{f}$. Let α be the angle between their directions of motion. The relative velocity of the second particle w.r.t. the first is least after a time **[2003]**

(a) $\frac{u\cos\alpha}{f}$ (b) $\frac{u\sin\alpha}{f}$

(c) $\frac{f\cos\alpha}{u}$ (d) $u\sin\alpha$

23. The upper $\frac{3}{4}$ th portion of a vertical pole subtends an angle $\tan^{-1}\frac{3}{5}$ at a point in the horizontal plane through its foot and at a distance 40 m from the foot. A possible height of the vertical pole is **[2003]**

(a) $80\,m$ (b) $20\,m$

(c) $40\,m$ (d) $60\,m$.

24. Let R_1 and R_2 respectively be the maximum ranges up and down an inclined plane and R be the maximum range on the horizontal plane. Then R_1, R, R_2 are in **[2003]**

(a) H.P (b) A.G.P

(c) A.P (d) G.P.

25. Let $\vec{a}, \vec{b}$ and $\vec{c}$ be three non-zero vectors such that no two of these are collinear. If the vector $\vec{a}+2\vec{b}$ is collinear with $\vec{c}$ and $\vec{b}+3\vec{c}$ is collinear with $\vec{a}$ (l being some non-zero scalar) then $\vec{a}+2\vec{b}+6\vec{c}$ equals **[2004]**

(a) 0 (b) $\lambda\vec{b}$

(c) $\lambda\vec{c}$ (d) $\lambda\vec{a}$

26. A particle is acted upon by constant forces $4\hat{i}+\hat{j}-3\hat{k}$ and $3\hat{i}+\hat{j}-\hat{k}$ which displace it from a point $\hat{i}+2\hat{j}+3\hat{k}$ to the point $5\hat{i}+4\hat{j}+\hat{k}$. The work done in standard units by the forces is given by **[2004]**

(a) 15 (b) 30

(c) 25 (d) 40

27. If $\vec{a}, \vec{b}, \vec{c}$ are non-coplanar vectors and l is a real number, then the vectors $\overline{a}+2\overline{b}+3\overline{c}$, $\lambda\overline{b}+4\overline{c}$ and $(2\lambda-1)\overline{c}$ are non coplanar for **[2004]**

(a) no value of l

(b) all except one value of l

(c) all except two values of l

(d) all values of l

28. Let $\overline{u}, \overline{v}, \overline{w}$ be such that $|\overline{u}|=1, |\overline{v}|=2$, $|\overline{w}|=3$. If the projection $\overline{v}$ along $\overline{u}$ is equal to that of $\overline{w}$ along $\overline{u}$ and $\overline{v}$, $\overline{w}$ are perpendicular to each other then $|\overline{u}-\overline{v}+\overline{w}|$ equals **[2004]**

(a) 14 (b) $\sqrt{7}$

(c) $\sqrt{14}$ (d) 2

29. Let $\overline{a}, \overline{b}$ and $\overline{c}$ be non-zero vectors such that $(\overline{a}\times\overline{b})\times\overline{c}=\frac{1}{3}|\overline{b}||\overline{c}|\overline{a}$. If q is the acute angle between the vectors $\overline{b}$ and $\overline{c}$, then sinq equals **[2004]**

(a) $\frac{2\sqrt{2}}{3}$ (b) $\frac{\sqrt{2}}{3}$

(c) $\frac{2}{3}$ (d) $\frac{1}{3}$

30. With two forces acting at point, the maximum affect is obtained when their resultant is 4N. If they act at right angles, then their resultant is 3N. Then the forces are **[2004]**

(a) $\left(2+\frac{1}{2}\sqrt{3}\right)N$ and $\left(2-\frac{1}{2}\sqrt{3}\right)N$

(b) $\left(2+\sqrt{3}\right)N$ and $\left(2-\sqrt{3}\right)N$

(c) $\left(2+\frac{1}{2}\sqrt{2}\right)N$ and $\left(2-\frac{1}{2}\sqrt{2}\right)N$

(d) $\left(2+\sqrt{2}\right)N$ and $\left(2-\sqrt{2}\right)N$

31. In a right angle ΔABC, $\angle A = 90°$ and sides a, b, c are respectively, 5 cm, 4 cm and 3 cm. If a force $\vec{F}$ has moments 0, 9 and 16 in N cm. units respectively about vertices A, B and C, then magnitude of $\vec{F}$ is **[2004]**

(a) 9 (b) 4

(c) 5 (d) 3

32. Three forces $\vec{P}, \vec{Q}$ and $\vec{R}$ acting along IA, IB and IC, where I is the incentre of a ΔABC are in equilibrium. Then $\vec{P} : \vec{Q} : \vec{R}$ is **[2004]**

(a) $\operatorname{cosec}\frac{A}{2} : \operatorname{cosec}\frac{B}{2} : \operatorname{cosec}\frac{C}{2}$

(b) $\sin\frac{A}{2} : \sin\frac{B}{2} : \sin\frac{C}{2}$

(c) $\sec\frac{A}{2} : \sec\frac{B}{2} : \sec\frac{C}{2}$

(d) $\cos\frac{A}{2} : \cos\frac{B}{2} : \cos\frac{C}{2}$

33. A paticle moves towards east from a point A to a point B at the rate of 4 km/h and then towards north from B to C at the rate of 5km/hr. If $AB = 12$ km and $BC = 5$ km, then its average speed for its journey from A to C and resultant average velocity direct from A to C are respectively **[2004]**

(a) $\frac{13}{9}$ km/h and $\frac{17}{9}$ km/h

(b) $\frac{13}{4}$ km/h and $\frac{17}{4}$ km/h

(c) $\frac{17}{9}$ km/h and $\frac{13}{9}$ km/h

(d) $\frac{17}{4}$ km/h and $\frac{13}{4}$ km/h

34. A velocity $\frac{1}{4}$ m/s is resolved into two components along OA and OB making angles 30° and 45° respectively with the given velocity. Then the component along OB is **[2004]**

(a) $\frac{1}{8}(\sqrt{6}-\sqrt{2})$ m/s (b) $\frac{1}{4}(\sqrt{3}-1)$ m/s

(c) $\frac{1}{4}$ m/s (d) $\frac{1}{8}$ m/s

35. If t_1 and t_2 are the times of flight of two particles having the same initial velocity u and range R on the horizontal, then $t_1^2 + t_2^2$ is equal to **[2004]**

(a) 1 (b) $4u^2/g^2$

(c) $u^2/2g$ (d) u^2/g

36. If C is the mid point of AB and P is any point outside AB, then **[2005]**

(a) $\overrightarrow{PA} + \overrightarrow{PB} = 2\overrightarrow{PC}$

(b) $\overrightarrow{PA} + \overrightarrow{PB} = \overrightarrow{PC}$

(c) $\overrightarrow{PA} + \overrightarrow{PB} + 2\overrightarrow{PC} = \vec{0}$

(d) $\overrightarrow{PA} + \overrightarrow{PB} + \overrightarrow{PC} = \vec{0}$

37. For any vector $\vec{a}$, the value of $(\vec{a}\times\hat{i})^2 + (\vec{a}\times\hat{j})^2 + (\vec{a}\times\hat{k})^2$ is equal to

(a) $3\vec{a}^2$ (b) $\vec{a}^2$ **[2005]**

(c) $2\vec{a}^2$ (d) $4\vec{a}^2$

38. If $\vec{a}, \vec{b}, \vec{c}$ are non coplanar vectors and λ is a real number then **[2005]**

$[\lambda(\vec{a}+\vec{b})\ \lambda^2\vec{b}\ \lambda\vec{c}] = [\vec{a}\ \ \vec{b}+\vec{c}\ \ \vec{b}]$

for

(a) exactly one value of λ

(b) no value of λ

(c) exactly three values of λ

(d) exactly two values of λ

39. Let $\vec{a} = \hat{i} - \hat{k}$, $\vec{b} = x\hat{i} + \hat{j} + (1-x)\hat{k}$ and $\vec{c} = y\hat{i} + x\hat{j} + (1+x-y)\hat{k}$. Then $[\vec{a}, \vec{b}, \vec{c}]$ depends on **[2005]**

(a) only y (b) only x
(c) both x and y (d) neither x nor y

40. ABC is a triangle. Forces $\vec{P}$, $\vec{Q}$, $\vec{R}$ acting along IA, IB, and IC respectively are in equilibrium, where I is the incentre of ΔABC. Then $P : Q : R$ is **[2005]**

(a) $\sin A : \sin B : \sin C$

(b) $\sin \frac{A}{2} : \sin \frac{B}{2} : \sin \frac{C}{2}$

(c) $\cos \frac{A}{2} : \cos \frac{B}{2} : \cos \frac{C}{2}$

(d) $\cos A : \cos B : \cos C$

41. A particle is projected from a point O with velocity u at an angle of 60° with the horizontal. When it is moving in a direction at right angles to its direction at O, its velocity then is given by **[2005]**

(a) $\frac{u}{3}$ (b) $\frac{u}{2}$

(c) $\frac{2u}{3}$ (d) $\frac{u}{\sqrt{3}}$

42. A and B are two like parallel forces. A couple of moment H lies in the plane of A and B and is contained with them. The resultant of A and B after combining is displaced through a distance **[2005]**

(a) $\frac{2H}{A-B}$ (b) $\frac{H}{A+B}$

(c) $\frac{H}{2(A+B)}$ (d) $\frac{H}{A-B}$

43. The resultant R of two forces acting on a particle is at right angles to one of them and its magnitude is one third of the other force. The ratio of larger force to smaller one is: **[2005]**

(a) $2:1$ (b) $3:\sqrt{2}$

(c) $3:2$ (d) $3:2\sqrt{2}$

44. ABC is a triangle, right angled at A. The resultant of the forces acting along $\overline{AB}, \overline{BC}$ with magnitudes $\frac{1}{AB}$ and $\frac{1}{AC}$ respectively is the force along $\overline{AD}$, where D is the foot of the perpendicular from A onto BC. The magnitude of the resultant is **[2006]**

(a) $\frac{AB^2 + AC^2}{(AB)^2(AC)^2}$ (b) $\frac{(AB)(AC)}{AB + AC}$

(c) $\frac{1}{AB} + \frac{1}{AC}$ (d) $\frac{1}{AD}$

45. If $(\bar{a} \times \bar{b}) \times \bar{c} = \bar{a} \times (\bar{b} \times \bar{c})$ where $\bar{a}, \bar{b}$ and $\bar{c}$ are any three vectors such that $\bar{a}.\bar{b} \neq 0$, $\bar{b}.\bar{c} \neq 0$ then $\bar{a}$ and $\bar{c}$ are **[2006]**

(a) inclined at an angle of $\frac{\pi}{3}$ between them

(b) inclined at an angle of $\frac{\pi}{6}$ between them

(c) perpendicular

(d) parallel

46. The values of a, for which points A, B, C with position vectors $2\hat{i} - \hat{j} + \hat{k}$, $\hat{i} - 3\hat{j} - 5\hat{k}$ and $a\hat{i} - 3\hat{j} + \hat{k}$ respectively are the vertices of a right angled triangle with $C = \frac{\pi}{2}$ are **[2006]**

(a) 2 and 1 (b) – 2 and – 1

(c) – 2 and 1 (d) 2 and – 1

47. A particle has two velocities of equal magnitude inclined to each other at an angle θ. If one of them is halved, the angle between the other and the original resultant velocity is bisected by the new resultant. Then θ is **[2006]**

(a) 90° (b) 120°
(c) 45° (d) 60°

48. A body falling from rest under gravity passes a certain point P. It was at a distance of 400 m from P, 4s prior to passing through P. If $g = 10 m/s^2$, then the height above the point P from where the body began to fall is **[2006]**

(a) 720m (b) 900m
(c) 320m (d) 680m

49. If $\hat{u}$ and $\hat{v}$ are unit vectors and θ is the acute angle between them, then $2\hat{u} \times 3\hat{v}$ is a unit vector for **[2007]**

(a) no value of θ
(b) exactly one value of θ
(c) exactly two values of θ
(d) more than two values of θ

50. Let $\vec{a} = \hat{i} + \hat{j} + \hat{k}, \vec{b} = \hat{i} - \hat{j} + 2\hat{k}$ and $\vec{c} = x\hat{i} + (x-2)\hat{j} - \hat{k}$. If the vector $\vec{c}$ lies in the plane of $\vec{a}$ and $\vec{b}$, then x equals **[2007]**

(a) – 4 (b) – 2
(c) 0 (d) 1.

51. The resultant of two forces Pn and 3n is a force of 7n. If the direction of 3n force were reversed, the resultant would be $\sqrt{19}\, n$. The value of P is **[2007]**

(a) 3 n (b) 4 n
(c) 5 n (d) 6 n.

52. A particle just clears a wall of height b at a distance a and strikes the ground at a distance c from the point of projection. The angle of projection is **[2007]**

(a) $\tan^{-1}\dfrac{bc}{a(c-a)}$ (b) $\tan^{-1}\dfrac{bc}{a}$

(c) $\tan^{-1}\dfrac{b}{ac}$ (d) 45°.

53. A body weighing 13 kg is suspended by two strings 5m and 12m long, their other ends being fastened to the extremities of a rod 13m long. If the rod be so held that the body hangs immediately below the middle point, then tensions in the strings are **[2007]**

(a) 5 kg and 12 kg (b) 5 kg and 13 kg
(c) 12 kg and 13 kg (d) 5 kg and 5 kg

54. The non-zero vectors are $\vec{a}$, $\vec{b}$ and $\vec{c}$ are related by $\vec{a} = 8\vec{b}$ and $\vec{c} = -7\vec{b}$. Then the angle between $\vec{a}$ and $\vec{c}$ is **[2008]**

(a) 0 (b) $\dfrac{\pi}{4}$

(c) $\dfrac{\pi}{2}$ (d) p

55. The projections of a vector on the three coordinate axis are 6, –3, 2 respectively. The direction cosines of the vector are : **[2009]**

(a) $\dfrac{6}{5}, \dfrac{-3}{5}, \dfrac{2}{5}$ (b) $\dfrac{6}{7}, \dfrac{-3}{7}, \dfrac{2}{7}$

(c) $\dfrac{-6}{7}, \dfrac{-3}{7}, \dfrac{2}{7}$ (d) $6, -3, 2$

56. If $\vec{u}, \vec{v}, \vec{w}$ are non-coplanar vectors and p, q are real numbers, then the equality $[3\vec{u}\ p\vec{v}\ p\vec{\omega}] - [p\vec{v}\ p\vec{\omega}\ q\vec{u}] - [2\vec{\omega}\ q\vec{v}\ q\vec{u}] = 0$ holds for : **[2009]**

(a) exactly two values of (p, q)
(b) more than two but not all values of (p, q)
(c) all values of (p, q)
(d) exactly one value of (p, q)

57. Let $\vec{a} = \hat{j} - \hat{k}$ and $\vec{c} = \hat{i} - \hat{j} - \hat{k}$. Then the vector $\vec{b}$ satisfying $\vec{a} \times \vec{b} + \vec{c} = \vec{0}$ and $\vec{a}.\vec{b} = 3$ is **[2010]**

(a) $2\hat{i} - \hat{j} + 2\hat{k}$ (b) $\hat{i} - \hat{j} - 2\hat{k}$

(c) $\hat{i} + \hat{j} - 2\hat{k}$ (d) $-\hat{i} + \hat{j} - 2\hat{k}$

58. If the vectors $\vec{a} = \hat{i} - \hat{j} + 2\hat{k}$, $\vec{b} = 2\hat{i} + 4\hat{j} + \tilde{\hat{k}}$ and $\vec{c} = \lambda\hat{i} + \hat{j} + \mu\hat{k}$ are mutually orthogonal, then $(\lambda, \mu) =$ **[2010]**

(a) (2, –3) (b) (–2, 3)
(c) (3, –2) (d) (–3, 2)

59. If $\vec{a} = \dfrac{1}{\sqrt{10}}\left(3\hat{i} + \hat{k}\right)$ and $\vec{b} = \dfrac{1}{7}\left(2\hat{i} + 3\hat{j} - 6\hat{k}\right)$, then the value of $\left(2\vec{a} - \vec{b}\right)\left[\left(\vec{a} \times \vec{b}\right) \times \left(\vec{a} + 2\vec{b}\right)\right]$ is **[2011]**

(a) –3 (b) 5
(c) 3 (d) –5

60. The vectors $\vec{a}$ and $\vec{b}$ are not perpendicular and $\vec{c}$ and $\vec{d}$ are two vectors satisfying $\vec{b}\times\vec{c}=\vec{b}\times\vec{d}$ and $\vec{a}.\vec{d}=0$. Then the vector $\vec{d}$ is equal to **[2011]**

(a) $\vec{c}+\left(\frac{\vec{a}.\vec{c}}{\vec{a}.\vec{b}}\right)\vec{b}$ (b) $\vec{b}+\left(\frac{\vec{b}.\vec{c}}{\vec{a}.\vec{b}}\right)\vec{c}$

(c) $\vec{c}-\left(\frac{\vec{a}.\vec{c}}{\vec{a}.\vec{b}}\right)\vec{b}$ (d) $\vec{b}-\left(\frac{\vec{b}.\vec{c}}{\vec{a}.\vec{b}}\right)\vec{c}$

61. If the $p\hat{i}+\hat{j}+\hat{k},\ \hat{i}+q\hat{j}+\hat{k}$ and $\hat{i}+\hat{j}+r\hat{k}\ (p\neq q\neq r\neq 1)$ vector are coplanar, then the value of $pqr-(p+q+r)$ is **[2011RS]**

(a) 2 (b) 0
(c) – 1 (d) – 2

62. Let $\vec{a},\vec{b},\vec{c}$ be three non-zero vectors which are pairwise non-collinear. If $\vec{a}+3\vec{b}$ is collinear with $\vec{c}$ and $\vec{b}+2\vec{c}$ is collinear with $\vec{a}$, then $\vec{a}+3\vec{b}+6\vec{c}$ is : **[2011RS]**

(a) $\vec{a}$ (b) $\vec{c}$
(c) $\vec{0}$ (d) $\vec{a}+\vec{c}$

63. Let $\hat{a}$ and $\hat{b}$ be two unit vectors. If the vectors $\vec{c}=\hat{a}+2\hat{b}$ and $\vec{d}=5\hat{a}-4\hat{b}$ are perpendicular to each other, then the angle between $\hat{a}$ and $\hat{b}$ is: **[2012]**

(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{2}$
(c) $\frac{\pi}{3}$ (d) $\frac{\pi}{4}$

64. Let $ABCD$ be a parallelogram such that $\overrightarrow{AB}=\vec{q},\overrightarrow{AD}=\vec{p}$ and ÐBAD be an acute angle. If $\vec{r}$ is the vector that coincide with the altitude directed from the vertex B to the side AD, then $\vec{r}$ is given by : **[2012]**

(a) $\vec{r}=3\vec{q}-\frac{3(\vec{p}.\vec{q})}{(\vec{p}.\vec{p})}\vec{p}$ (b) $\vec{r}=-\vec{q}+\frac{(\vec{p}.\vec{q})}{(\vec{p}.\vec{p})}\vec{p}$

(c) $\vec{r}=\vec{q}-\frac{(\vec{p}.\vec{q})}{(\vec{p}.\vec{p})}\vec{p}$ (d) $\vec{r}=-3\vec{q}-\frac{3(\vec{p}.\vec{q})}{(\vec{p}.\vec{p})}\vec{p}$

65. If the vectors $\overrightarrow{AB}=3\hat{i}+4\hat{k}$ and $\overrightarrow{AC}=5\hat{i}-2\hat{j}+4\hat{k}$ are the sides of a triangle ABC, then the length of the median through A is **[2013]**

(a) $\sqrt{18}$ (b) $\sqrt{72}$
(c) $\sqrt{33}$ (d) $\sqrt{45}$

66. If $\left[\vec{a}\times\vec{b}\ \ \vec{b}\times\vec{c}\ \ \vec{c}\times\vec{a}\right]=\lambda\left[\vec{a}\,\vec{b}\,\vec{c}\right]^2$ then λ is equal to **[2014]**

(a) 0 (b) 1
(c) 2 (d) 3

67. Let $\vec{a}$, $\vec{b}$ and $\vec{c}$ be three non-zero vectors such that no two of them are collinear and $(\vec{a}\times\vec{b})\times\vec{c}=\frac{1}{3}|\vec{b}||\vec{c}|\vec{a}$. If q is the angle between vectors $\vec{b}$ and $\vec{c}$, then a value of sin q is : **[2015]**

(a) $\frac{2}{3}$ (b) $\frac{-2\sqrt{3}}{3}$
(c) $\frac{2\sqrt{2}}{3}$ (d) $\frac{-\sqrt{2}}{3}$

68. Let $\vec{a},\vec{b}$ and $\vec{c}$ be three unit vectors such that $\vec{a}\times(\vec{b}\times\vec{c})=\frac{\sqrt{3}}{2}(\vec{b}+\vec{c})$. If $\vec{b}$ is not parallel to $\vec{c}$, then the angle between $\vec{a}$ and $\vec{b}$ is: **[2016]**

(a) $\frac{2\pi}{3}$ (b) $\frac{5\pi}{6}$
(c) $\frac{3\pi}{4}$ (d) $\frac{\pi}{2}$

69. Let $\vec{a}=2\hat{i}+\hat{j}-2\hat{k}$ and $\vec{b}=\hat{i}+\hat{j}$. Let $\vec{c}$ be a vector such that $|\vec{c}-\vec{a}|=3$, $\left|(\vec{a}\times\vec{b})\times\vec{c}\right|=3$ and the angle between $\vec{c}$ and $\vec{a}\times\vec{b}$ be 30°. Then $\vec{a}.\vec{c}$ is equal to : **[2017]**

(a) $\frac{1}{8}$ (b) $\frac{25}{8}$
(c) 2 (d) 5

70. Let $\vec{u}$ be a vector coplanar with the vectors $\vec{a} = 2\hat{i} + 3\hat{j} - \hat{k}$ and $\vec{b} = \hat{j} + \hat{k}$. If $\vec{u}$ is perpendicular to $\vec{a}$ and $\vec{u} \cdot \vec{b} = 24$, then $|\vec{u}|^2$ is equal to : **[2018]**

(a) 315 (b) 256
(c) 84 (d) 336

71. Let $\vec{a} = \hat{i} - \hat{j}, \vec{b} = \hat{i} + \hat{j} + \hat{k}$ and $\vec{c}$ be a vector such that $\vec{a} \times \vec{c} + \vec{b} = \vec{0}$ and $\vec{a}.\vec{c} = 4$, then $|\vec{c}|^2$ is equal to: **[2019]**

(a) $\frac{19}{2}$ (b) 9
(c) 8 (d) $\frac{17}{2}$

72. Let $\vec{\alpha} = 3\hat{i} + \hat{j}$ and $\vec{\beta} = 2\hat{i} - \hat{j} + 3\hat{k}$.
If $\vec{\beta} = \vec{\beta}_1 - \vec{\beta}_2$, where $\vec{\beta}_1$ is parallel to $\vec{\alpha}$ and $\vec{\beta}_2$ is perpendicular to $\vec{\alpha}$, then $\vec{\beta}_1 \times \vec{\beta}_2$ is equal to: **[2019]**

(a) $-3\hat{i} + 9\hat{j} + 5\hat{k}$ (b) $3\hat{i} - 9\hat{j} - 5\hat{k}$
(c) $\frac{1}{2}\left(-3\hat{i} + 9\hat{j} + 5\hat{k}\right)$ (d) $\frac{1}{2}\left(3\hat{i} - 9\hat{j} + 5\hat{k}\right)$

Answer Key

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
(b)	(a)	(a)	(a)	(a)	(b)	(c)	(a)	(b)	(a)	(d)	(d)	(c)	(b)	(c)
16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
(none)	(c)	(c)	(c)	(a)	(a)	(a)	(c)	(a)	(c)	(d)	(c)	(c)	(a)	(c)
31	32	33	34	35	36	37	38	39	40	41	42	43	44	45
(c)	(d)	(d)	(a)	(b)	(a)	(c)	(b)	(d)	(c)	(d)	(b)	(d)	(d)	(d)
46	47	48	49	50	51	52	53	54	55	56	57	58	59	60
(a)	(b)	(a)	(b)	(b)	(c)	(a)	(a)	(d)	(b)	(d)	(d)	(d)	(d)	(c)
61	62	63	64	65	66	67	68	69	70	71	72			
(d)	(c)	(c)	(b)	(c)	(b)	(c)	(b)	(c)	(d)	(a)	(c)			

Solutions

1. **(b)** Since, $\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}| \cos\frac{\pi}{6}$

$= 4 \times 2 \times \frac{\sqrt{3}}{2} = 4\sqrt{3}$.

We know that,

$(\vec{a} \times \vec{b})^2 + (\vec{a} \cdot \vec{b})^2 = |\vec{a}|^2 |\vec{b}|^2$

$\Rightarrow (\vec{a} \times \vec{b})^2 + 48 = 16 \times 4$

$\Rightarrow (\vec{a} \times \vec{b})^2 = 16$

2. **(a)** $[\vec{a} \times \vec{b} \;\; \vec{b} \times \vec{c} \;\; \vec{c} \times \vec{a}]$

$= (\vec{a} \times \vec{b}).\left\{(\vec{b} \times \vec{c}) \times (\vec{c} \times \vec{a})\right\}$

$\because \vec{a} \times (\vec{b} \times \vec{c}) = (\vec{a}.\vec{c})\vec{b} - (\vec{a}.\vec{b})\vec{c}$

$= (\vec{a} \times \vec{b}).\left\{(\vec{m} \cdot \vec{a})\vec{c} - (\vec{m} \cdot \vec{c})\vec{a}\right\}$

(where $\vec{m} = \vec{b} \times \vec{c}$)

$= \{(\vec{a} \times \vec{b}).\vec{c}\}.\{(\vec{a} \cdot (\vec{b} \times \vec{c})\}$

$= [\vec{a} \; \vec{b} \; \vec{c}]^2 = 4^2 = 16 \cdot$

3. **(a)** Given that $\vec{a}+\vec{b}+\vec{c}=0 \Rightarrow \vec{b}+\vec{c}=-\vec{a}$

$$\Rightarrow |\vec{b}+\vec{c}|^2=|\vec{a}|^2=5^2+3^2+2\vec{b}\cdot\vec{c}=7^2$$

$$\Rightarrow 2|\vec{b}||\vec{c}|\cos\theta=49-34=15;$$

$$\Rightarrow 2\times 5\times 3\cos\theta=15;$$

$$\Rightarrow \cos\theta=1/2; \Rightarrow \theta=\frac{\pi}{3}=60^\circ$$

4. **(a)** Given that, $\vec{a}+\vec{b}+\vec{c}=\vec{0}$

$$\Rightarrow |\vec{a}+\vec{b}+\vec{c}|^2=0$$

$$\Rightarrow |\vec{a}|^2+|\vec{b}|^2|+|\vec{c}|^2$$

$$+2(\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a})=0$$

$$\Rightarrow 25+16+9+2(\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a})=0$$

$$\Rightarrow (\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a})=-25\cdot$$

$$\therefore |\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a}|=25\cdot$$

5. **(a)** Given that $\vec{a},\vec{c},\vec{b}$ form a right handed system,

$$\therefore \quad \vec{c}=\vec{b}\times\vec{a}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 0 & 1 & 0\\ x & y & z\end{vmatrix}=z\hat{i}-x\hat{k}$$

6. **(b)** We have $\vec{a}\times\vec{b}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 3 & -5 & 0\\ 6 & 3 & 0\end{vmatrix}$

$$=39\hat{k}=\vec{c}$$

Also $|\vec{a}|=\sqrt{34}, |\vec{b}|=\sqrt{45}, |\vec{c}|=39;$

$$\therefore |\vec{a}|:|\vec{b}|:|\vec{c}|=\sqrt{34}:\sqrt{45}:39\cdot$$

7. **(c)** Let $\vec{a}+\vec{b}+\vec{c}=\vec{r}$. Then

$$\vec{a}\times(\vec{a}+\vec{b}+\vec{c})=\vec{a}\times\vec{r}$$

$$\Rightarrow 0+\vec{a}\times\vec{b}+\vec{a}\times\vec{c}=\vec{a}\times\vec{r}$$

$$\Rightarrow \vec{a}\times\vec{b}-\vec{c}\times\vec{a}=\vec{a}\times\vec{r} \Rightarrow \vec{a}\times\vec{r}=\vec{0}$$

$$[\because\ \vec{a}\times\vec{b}=\vec{c}\times\vec{a}]$$

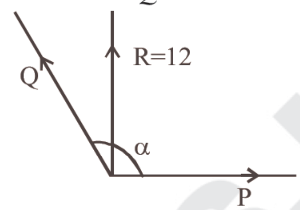
Similarly $\vec{b}\times\vec{r}=\vec{0}$ & $\vec{c}\times\vec{r}=\vec{0}$

Above three conditions can be hold if and only if $\vec{r}=\vec{0}$

8. **(a)** Given that $P+Q=18$ ....(1)

We know that $P^2+Q^2+2PQ\cos\alpha=144$ ......(2)

$$\tan 90^\circ=\frac{Q\sin\alpha}{P+Q\cos\alpha}$$



$\Rightarrow \quad P+Q\cos\alpha=0$ ......(3)

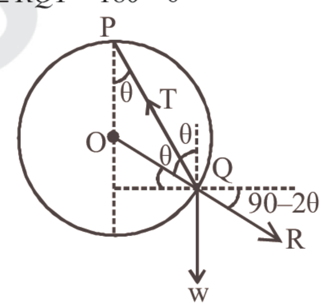
From (2) and (3),

$$Q^2-P^2=144 \Rightarrow (Q-P)(Q+P)=144$$

$$\therefore \quad Q-P=\frac{144}{18}=8$$

From (1), On solving, we get $Q=13, P=5$

9. **(b)** From figure $\angle TQW=180-\theta$; $\angle RQW=2\theta$; $\angle RQT=180-\theta$



Applying Lami's theorem at Q.

$$\frac{T}{\sin 2\theta}=\frac{R}{\sin(180-\theta)}=\frac{W}{\sin(180-\theta)}$$

$\Rightarrow \quad R=W$ and $T=2W\cos\theta$

10. **(a)** Given that $\vec{u}.\hat{n}=0$ and $\vec{v}.\vec{n}=0$

$\Rightarrow \quad \vec{n}$ is perpendicular both $\vec{u}$ *and* $\vec{v}$,

$$\therefore \quad \hat{n}=\frac{\vec{u}\times\vec{v}}{|\vec{u}||\vec{v}|}$$

$$\hat{n}=\frac{\begin{vmatrix}i & j & k\\ 1 & 1 & 0\\ 1 & -1 & 0\end{vmatrix}}{\sqrt{2}\times\sqrt{2}}=\frac{-2\hat{k}}{2}=-\hat{k}$$

$$|\vec{\omega}.\hat{n}|=|(i+2j+3k).(-k)|=|-3|=3$$

11. **(d)** $\vec{F}=\vec{F}_1+\vec{F}_2=7i+2j-4k$

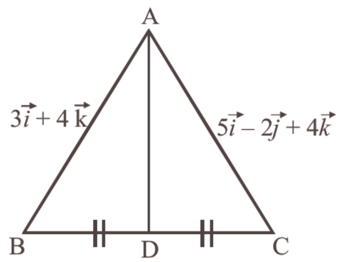
$\vec{d}$ = Positon Vector of $\vec{B}$

– Position Vector of $\vec{A}$

$$=4i+2j-2k$$

$$W=\vec{F}.\vec{d}=28+4+8=40 \text{ unit}$$

12. (d)



Given that AD is median of ΔABC.

$$\therefore\ \overrightarrow{AD}=\frac{(3+5)i+(0-2)j+(4+4)k}{2}$$

$$=4i-j+4k$$

$$\left|\overrightarrow{AD}\right|=\sqrt{16+16+1}=\sqrt{33}$$

13. (c) Given that $\vec{a}+\vec{b}+\vec{c}=0$

$$\Rightarrow(\vec{a}+\vec{b}+\vec{c}).(\vec{a}+\vec{b}+\vec{c})=0$$

$$\Rightarrow|\vec{a}|^2+|\vec{b}|^2+|\vec{c}|^2+2(\vec{a}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a})=0$$

$$1+4+9+2(\vec{a}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a})=0$$

$$\vec{a}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a}=\frac{-1-4-9}{2}=-7$$

14. (b) Normal vector of the face OAB

$$=\overrightarrow{OA}\times\overrightarrow{OB}=\begin{vmatrix}\hat{i}&\hat{j}&\hat{k}\\1&2&1\\2&1&3\end{vmatrix}=5\hat{i}-\hat{j}-3\hat{k}$$

Normal vector of the face ABC

$$=\overrightarrow{AB}\times\overrightarrow{AC}=\begin{vmatrix}\hat{i}&\hat{j}&\hat{k}\\1&-1&2\\-2&-1&1\end{vmatrix}=\hat{i}-5\hat{j}-3\hat{k}$$

Angle between the faces = angle between their normals

$$\cos\theta=\left|\frac{5+5+9}{\sqrt{35}\sqrt{35}}\right|=\frac{19}{35}\text{ or }\theta=\cos^{-1}\left(\frac{19}{35}\right)$$

15. (c) Given $\begin{vmatrix}a&a^2&1+a^3\\b&b^2&1+b^3\\c&c^2&1+c^3\end{vmatrix}=0$

$$\Rightarrow\begin{vmatrix}a&a^2&1\\b&b^2&1\\c&c^2&1\end{vmatrix}+\begin{vmatrix}a&a^2&a^3\\b&b^2&b^3\\c&c^2&c^3\end{vmatrix}=0$$

$$\Rightarrow(1+abc)\begin{vmatrix}1&a&a^2\\1&b&b^2\\1&c&c^2\end{vmatrix}=0$$

Given that $(1, a, a^2)$, $(1, b, b^2)$ and $(1, c, c^2)$ are non-coplanar

$$\therefore\ \begin{vmatrix}1&a&a^2\\1&b&b^2\\1&c&c^2\end{vmatrix}\neq0\ (\textit{given condition})$$

$$\therefore\ 1+abc=0\Rightarrow abc=-1$$

16. (none) Given that

$A=(7,-4,7), B=(1,-6,10), C=(-1,-3,4)$ and $D=(5,-1,5)$

$$\therefore\ AB=\sqrt{(7-1)^2+(-4+6)^2+(7-10)^2}$$

$$=\sqrt{36+4+9}=7$$

Similarly, $BC=7$, $CD=\sqrt{41}$, $DA=\sqrt{17}$

$\therefore$ None of the options is satisfied

17. (c) $(\vec{u}+\vec{v}-\vec{w}).(\vec{u}-\vec{v})\times(\vec{v}-\vec{w})$

$$=(\vec{u}+\vec{v}-\vec{w}).(\vec{u}\times\vec{v}-\vec{u}\times\vec{w}-\vec{v}\times\vec{v}+\vec{v}\times\vec{w})$$

$$=(\vec{u}+\vec{v}-\vec{w}).(\vec{u}\times\vec{v}-\vec{u}\times\vec{w}+\vec{v}\times\vec{w})$$

$$[\because\vec{v}\times\vec{v}=0]$$

$$=\vec{u}.(\vec{u}\times\vec{v})-\vec{u}.(\vec{u}\times\vec{w})+\vec{u}.(\vec{v}\times\vec{w})+\vec{v}.(\vec{u}\times\vec{v})$$
$$-\vec{v}.(\vec{u}\times\vec{w})+\vec{v}.(\vec{v}\times\vec{w})-\vec{w}.(\vec{u}\times\vec{v})$$
$$+\vec{w}.(\vec{u}\times\vec{w})-\vec{w}.(\vec{v}\times\vec{w})$$

We know that $[\vec{a},\vec{a},\vec{b}]=0$

$$=\vec{u}.(\vec{v}\times\vec{w})-\vec{v}.(\vec{u}\times\vec{w})-\vec{w}.(\vec{u}\times\vec{v})$$

$$=[\vec{u}\vec{v}\vec{w})]+[\vec{v}\vec{w}\vec{u})]-[\vec{w}\vec{u}\vec{v}]=\vec{u}.(\vec{v}\times\vec{w})$$

18. (c) We know that $\vec{G}=\vec{r}\times\vec{p}$; $|\vec{G}|=|\vec{r}||\vec{p}|\sin\theta$

$$|\vec{H}|=|\vec{r}||\vec{p}|\cos\theta\ \left[\because\sin(90^o+\theta)=\cos\theta\right]$$

$G = |\vec{r}||\vec{p}|\sin\theta$ .....(1)

$H = |\vec{r}||\vec{p}|\cos\theta$ .....(2)

$x = |\vec{r}||\vec{p}|\sin(\theta+\alpha)$ .....(3)

From (1), (2) & (3),

$x = \vec{G}\cos\alpha + \vec{H}\sin\alpha \cdot$

19. **(c)** $R^2 = P^2 + Q^2 + 2PQ\cos\theta$ .... (1)

When $\vec{Q}$ and $\vec{R}$ are doubled

$4R^2 = P^2 + 4Q^2 + 4PQ\cos\theta$ .... (2)

When $\vec{Q}$ is reversed and $\vec{R}$ is doubled

$4R^2 = P^2 + Q^2 - 2PQ\cos\theta$ .... (3)

Adding (1) and (3), $5R^2 = 2P^2 + 2Q^2$

$\Rightarrow 2P^2 + 2Q^2 - 5R^2 = 0$ .... (4)

Applying $(3)\times 2 + (2)$, $12R^2 = 3P^2 + 6Q^2$

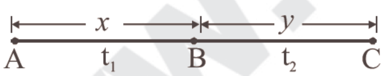
$\Rightarrow 3P^2 + 6Q^2 - 12R^2 = 0$ ....(5)

From (4) and (5)

$$\frac{P^2}{-24+30} = \frac{Q^2}{24-15} = \frac{R^2}{12-6}$$

$$\frac{P^2}{6} = \frac{Q^2}{9} = \frac{R^2}{6} \text{ or } P^2 : Q^2 : R^2 = 2:3:2$$

20. **(a)** Let the body travels from A to B with constant acceleration t and from B to C with constant retardation r.



If $AB = x$, $BC = y$, time taken from A to $B = t_1$ and time taken from B to $C = t_2$, then $s = x + y$ and $t = t_1 + t_2$

For the motion from A to B

$v^2 = u^2 + 2fs \Rightarrow v^2 = 2fx \; (\because u = 0)$

$\Rightarrow \quad x = \dfrac{v^2}{2f}$ ....(1)

and $v = u + ft \quad \Rightarrow v = ft_1$

$\Rightarrow \quad t_1 = \dfrac{v}{f}$ ...(2)

For the motion from B to C

$v^2 = u^2 + 2fs$

$\Rightarrow \quad 0 = v^2 - 2ry \quad \Rightarrow y = \dfrac{v^2}{2r}$ ...(3)

and $v = u + ft \Rightarrow 0 = v - rt_2$

$\Rightarrow \quad t_2 = \dfrac{v}{r}$

Adding equations (1) and (3), we get

$$x + y = \frac{v^2}{2}\left[\frac{1}{f} + \frac{1}{r}\right] = s$$

Adding equations (2) and (4), we get

$$t_1 + t_2 = v\left[\frac{1}{f} + \frac{1}{r}\right] = t$$

$$\therefore \quad \frac{t^2}{2s} = \frac{v^2\left[\frac{1}{f} + \frac{1}{r}\right]^2}{2 \times \frac{v^2}{2}\left(\frac{1}{f} + \frac{1}{r}\right)} = \frac{1}{f} + \frac{1}{r}$$

$$\Rightarrow \quad t = \sqrt{2s\left(\frac{1}{f} + \frac{1}{r}\right)}$$

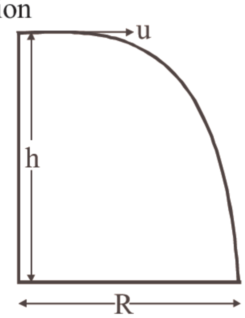
21. **(a)** Given that the stone projected horizontally. For horizontal motion,

Distance = speed × time $\Rightarrow R = ut$

and for vertical motion

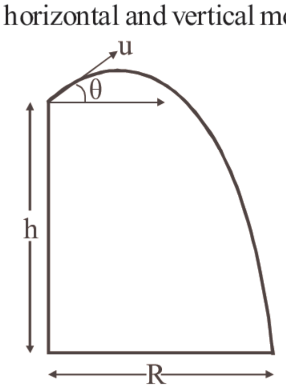
$h = 0 \times t + \dfrac{1}{2}gt^2$

$\Rightarrow t = \sqrt{\dfrac{2h}{g}}$



$\therefore$ We get $R = u\sqrt{\dfrac{2h}{g}}$ ....(1)

When the stone projected at an angle θ, for horizontal and vertical motions, we have



$R = u\cos\theta \times t$ ....(2)

and $h = -u\sin\theta \times t + \frac{1}{2}gt^2$ ....(3)

From eqns. (1) and (2) we get

$$u\sqrt{\frac{2h}{g}} = u\cos\theta \times t$$

$$\Rightarrow t = \frac{1}{\cos\theta}\sqrt{\frac{2h}{g}}$$

Putting the value of t in eq (3) we get

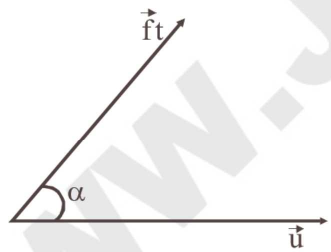
$$h = -\frac{u\sin\theta}{\cos\theta}\sqrt{\frac{2h}{g}} + \frac{1}{2}g\left[\frac{2h}{g\cos^2\theta}\right]$$

$$h = -u\sqrt{\frac{2h}{g}}\tan\theta + h\sec^2\theta$$

$$h = -u\sqrt{\frac{2h}{g}}\tan\theta + h\tan^2\theta + h$$

$$\tan^2\theta - u\sqrt{\frac{2}{hg}}\tan\theta = 0; \therefore \tan\theta = u\sqrt{\frac{2}{hg}}$$

22. (a) Let the two velocities be $\vec{v}_1 = u\hat{i}$ and $\vec{v}_2 = (ft\cos\alpha)\hat{i} + (ft\sin\alpha)\hat{j}$



$\therefore$ Relative velocity of second with respect to first

$$\vec{v} = \vec{v}_2 - \vec{v}_1 = (ft\cos\alpha - u)\hat{i} + ft\sin\alpha\,\hat{j}$$

$$\Rightarrow |\vec{v}|^2 = (ft\cos\alpha - u)^2 + (ft\sin\alpha)^2$$

$$= f^2t^2 + u^2 - 2uft\cos\alpha$$

For $|\vec{v}|$ to be min and max. we should have

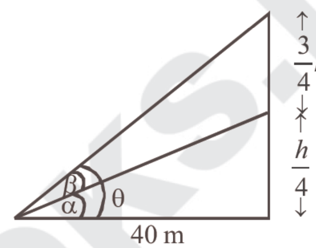
$$\frac{d|v|^2}{dt} = 0 \Rightarrow 2f^2t - 2uf\cos\alpha = 0$$

$$\Rightarrow t = \frac{u\cos\alpha}{f}$$

Also $\frac{d^2|v|^2}{dt^2} = 2f^2 = +ve$

$\therefore$ $|v|^2$ and hence $|v|$ is least at the time $\frac{u\cos\alpha}{f}$

23. (c)



$$0 = \alpha + \beta, \beta = \tan^{-1}\left(\frac{3}{5}\right)$$

or $\beta = \theta - \alpha$

$$\Rightarrow \tan\beta = \tan(\theta - \alpha) = \frac{\tan\theta - \tan\alpha}{1 + \tan\theta.\tan\alpha}$$

or $$\frac{3}{5} = \frac{\frac{h}{40} - \frac{h}{160}}{1 + \frac{h}{40}.\frac{h}{160}}$$

$$h^2 - 200h + 6400 = 0$$

$\Rightarrow h = 40$ or 160 metre

$\therefore$ possible height $= 40$ metre

24. (a) Let β be the inclination of the plane to the horizontal and u be the velocity of projection of the projectile

We have $R_1 = \frac{u^2}{g(1+\sin\beta)}$

and $R_2 = \frac{u^2}{g(1-\sin\beta)}$

Adding above equations

$$\frac{1}{R_1} + \frac{1}{R_2} = \frac{2g}{u^2} \text{ or } \frac{1}{R_1} + \frac{1}{R_2} = \frac{2}{R}\left[\because R = \frac{u^2}{g}\right]$$

$\therefore R_1, R, R_2$ are in H.P.

25. (c) Given that $\vec{a}+2\vec{b}$ is collinear with $\vec{c}$ and $\vec{b}+3\vec{c}$ is collinear with $\vec{a}$

Let $\vec{a}+2\vec{b}=t\vec{c}$ and $\vec{b}+3\vec{c}=s\vec{a}$, where t and s are scalars

$\therefore\ \vec{a}+2\vec{b}+6\vec{c}=t\vec{c}+6\vec{c}$

$=(t+6)\vec{c}$ [using $\vec{a}+2\vec{b}=t\vec{c}$]

$=\lambda\vec{c}$, where $\lambda=t+6$

26. (d) Resultant of forces

$\vec{F}=4\hat{i}+\hat{j}-3\hat{k}+3\hat{i}+\hat{j}-\hat{k}=7\hat{i}+2\hat{j}-4\hat{k}$

Displacement

$\vec{d}=5\hat{i}+4\hat{j}+\hat{k}-(\hat{i}+2\hat{j}+3\hat{k})$

$=4\hat{i}+2\hat{j}-2\hat{k}$

$\therefore$ Work done $=\vec{F}.\vec{d}=28+4+8=40$

27. (c) If vectors $\vec{a}+2\vec{b}+3\vec{c}, \lambda\vec{b}+4\vec{c}$, and $(2\lambda-1)\vec{c}$ are coplanar then

$$\begin{vmatrix}1 & 2 & 3\\ 0 & \lambda & 4\\ 0 & 0 & 2\lambda-1\end{vmatrix}=0$$

$\Rightarrow \lambda(2\lambda-1)=0 \ \Rightarrow \lambda=0$ or $\frac{1}{2}$

$\therefore$ Forces are noncoplanar for all λ, except $\lambda=0, \frac{1}{2}$

28. (c) Projection of $\vec{v}$ along $\vec{u}$ $=\frac{\vec{v}.\vec{u}}{|\vec{u}|}=\vec{v}.\vec{u}$

projection of $\vec{w}$ along $\vec{u}$ $=\frac{\vec{w}.\vec{u}}{|\vec{u}|}=\vec{w}.\vec{u}$

Given $\vec{v}.\vec{u}=\vec{w}.\vec{u}$ ....(1)

Also , $\vec{v}.\vec{w}=0$ [$\because$ $\vec{v}\perp\vec{w}$] ....(2)

Now $|\vec{u}-\vec{v}+\vec{w}|^2$

$=|\vec{u}|^2+|\vec{v}|^2+|\vec{w}|^2-2\vec{u}.\vec{v}-2\vec{v}.\vec{w}+2\vec{u}.\vec{w}$

$=1+4+9+0$ [From (1) and (2)] $=14$

$\therefore |\vec{u}-\vec{v}+\vec{w}|=\sqrt{14}$

29. (a) Given that $(\vec{a}\times\vec{b})\times\vec{c}=\frac{1}{3}|\vec{b}||\vec{c}|\vec{a}$

Clearly $\vec{a}$ and $\vec{b}$ are non collinear

$\Rightarrow (\vec{a}.\vec{c})\vec{b}-(\vec{b}.\vec{c})\vec{a}=\frac{1}{3}|\vec{b}||\vec{c}|\vec{a}$

Comparing both side.

$\therefore\ \vec{a}.\vec{c}=0$ and $-\vec{b}.\vec{c}=\frac{1}{3}|\vec{b}||\vec{c}|$

$\Rightarrow \cos\theta=\frac{-1}{3}$

$\therefore \sin\ \theta=\sqrt{1-\frac{1}{9}}=\frac{2\sqrt{2}}{3}$

[θ is acute angle between $\vec{b}$ and $\vec{c}$]

30. (c) Let forces be P and Q. then $P+Q=4$ ....(1)

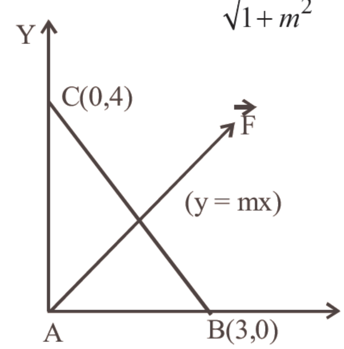
and $P^2+Q^2=3^2$ ....(2)

Solving eqns. (1) and (2), we get the forces

$\left(2+\frac{\sqrt{2}}{2}\right)N$ and $\left(2-\frac{\sqrt{2}}{2}\right)N$

31. (c) Since, the moment about A is zero, hence $\vec{F}$ passes through A. Taking A as origin. Let the line of action of force $\vec{F}$ be $y=mx$. (see figure)

Moment about B $=\frac{3m}{\sqrt{1+m^2}}|\vec{F}|=9$ ....(1)



Moment about C= $\frac{4}{\sqrt{1+m^2}}|\vec{F}|=16$....(2)

Dividing (1) by (2), we get

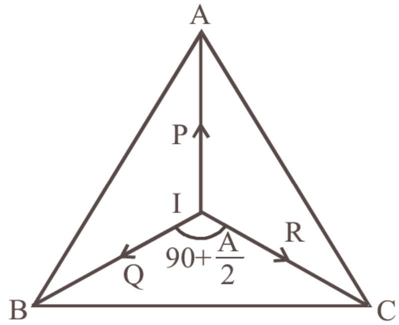
$m=\frac{3}{4}\Rightarrow|\vec{F}|=5N.$

32. (d) Let I is incentre of ΔABC.

$\therefore$ IA, IB, IC are bisectors of the angles A, B and C.

Now $\angle BIC = 180 - \frac{B}{2} - \frac{C}{2} = 90° + \frac{A}{2}$ *etc.*

Applying Lami's theorem at I

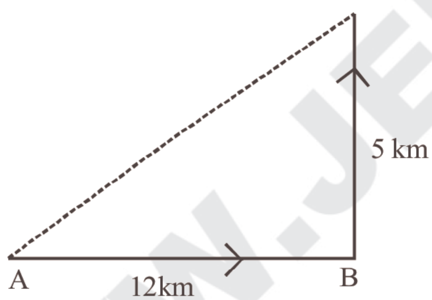


$$\frac{P}{\sin(90° + \frac{A}{2})} = \frac{Q}{\sin\left(90° + \frac{B}{2}\right)} = \frac{R}{\sin\left(90° + \frac{C}{2}\right)}$$

$$\Rightarrow \quad P : Q : R = \cos\frac{A}{2} : \cos\frac{B}{2} : \cos\frac{C}{2}$$

33. (d) Time taken by the particle in complete journey

$$T = \frac{12}{4} + \frac{5}{5} = 4\ hr.$$



$$\therefore \text{ Average speed } = \frac{12+5}{4} = \frac{17}{4}$$

$$\text{Average velocity } = \sqrt{\frac{12^2 + 5^2}{4}} = \frac{13}{4}$$

34. (a) Given $v = \frac{1}{4}$ m/s, component along OB

$$= \frac{v \sin 30°}{\sin(45° + 30°)} = \frac{\frac{1}{4} \times \frac{1}{2}}{\frac{\sqrt{3}+1}{2\sqrt{2}}} = \frac{\sqrt{6} - \sqrt{2}}{8}$$

35. (b) For same horizontal range the angles of projection must be α and $\frac{\pi}{2} - \alpha$

$$\therefore t_1 = \frac{2u \sin\alpha}{g} \qquad \text{.... (i)}$$

$$t_2 = \frac{2u \sin\left(\frac{\pi}{2} - \alpha\right)}{g} = \frac{2u\cos\alpha}{g} \qquad \text{.... (ii)}$$

Squaring and adding eqn. (i) and (ii),

$$\therefore t_1^2 + t_2^2 = \frac{4u^2}{g^2}$$

36. (a) $\overrightarrow{PA} + \overrightarrow{AP} = 0$ and $\overrightarrow{PC} + \overrightarrow{CP} = 0$

$$\Rightarrow \quad \overrightarrow{PA} + \overrightarrow{AC} + \overrightarrow{CP} = 0 \qquad \text{.... (i)}$$

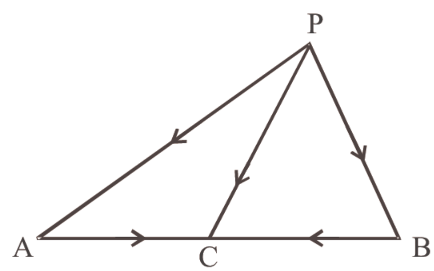
Similarly, $\overrightarrow{PB} + \overrightarrow{BC} + \overrightarrow{CP} = 0$ .... (ii)

Adding eqn. (i) and (ii), we get

$$\overrightarrow{PA} + \overrightarrow{PB} + \overrightarrow{AC} + \overrightarrow{BC} + 2\overrightarrow{CP} = 0.$$

Since $\overrightarrow{AC} = -\overrightarrow{BC}$ & $\overrightarrow{CP} = -\overrightarrow{PC}$

$$\Rightarrow \quad \overrightarrow{PA} + \overrightarrow{PB} - 2\overrightarrow{PC} = 0.$$



37. (c) Let $\vec{a} = x\vec{i} + y\vec{j} + z\vec{k}$

$$\vec{a} \times \vec{i} = z\vec{j} - y\vec{k} \Rightarrow |\vec{a} \times \vec{i}|^2 = y^2 + z^2$$

Similarly,

$$|\vec{a} \times \vec{j}|^2 = x^2 + z^2 \text{ and } |\vec{a} \times \vec{k}|^2 = x^2 + y^2$$

Adding all above equation

$$\Rightarrow |\vec{a} \times \vec{i}|^2 + |\vec{a} \times \vec{j}|^2 + |\vec{a} \times \vec{k}|^2$$

$$= 2(x^2 + y^2 + z^2) = 2|\vec{a}|^2$$

38. (b) Let $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$

$$\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$$

$$\vec{c} = c_1\hat{i} + c_2\hat{j} + c_3\hat{k}$$

Given that

$$\left[\lambda(\vec{a}+\vec{b})\ \lambda^2\vec{b}\ \lambda\vec{c}\right]=\left[\vec{a}\ \vec{b}+\vec{c}\ \vec{b}\right]$$

$$\Rightarrow\begin{vmatrix}\lambda(a_1+b_1) & \lambda(a_2+b_2) & \lambda(a_3+b_3)\\ \lambda^2 b_1 & \lambda^2 b_2 & \lambda^2 b_3\\ \lambda c_1 & \lambda c_2 & \lambda c_3\end{vmatrix}$$

$$=\begin{vmatrix}a_1 & a_2 & a_3\\ b_1+c_1 & b_2+c_2 & b_3+c_3\\ b_1 & b_2 & b_3\end{vmatrix}$$

$$\Rightarrow\lambda^4\begin{vmatrix}a_1+b_1 & a_2+b_2 & a_3+b_3\\ b_1 & b_2 & b_3\\ c_1 & c_2 & c_3\end{vmatrix}$$

$$=\begin{vmatrix}a_1 & a_2 & a_3\\ b_1+c_1 & b_2+c_2 & b_3+c_3\\ b_1 & b_2 & b_3\end{vmatrix}$$

$R_1\to R_1-R_2$ in 1st det.

and $R_2\to R_2-R_3$ in 2nd det.

$$\Rightarrow\lambda^4\begin{vmatrix}a_1 & a_2 & a_3\\ b_1 & b_2 & b_3\\ c_1 & c_2 & c_3\end{vmatrix}=\begin{vmatrix}a_1 & a_2 & a_3\\ c_1 & c_2 & c_3\\ b_1 & b_2 & b_3\end{vmatrix}$$

$\Rightarrow\lambda^4=-1$

Hence λ has no real values.

39. (d) Given that

$\vec{a}=\hat{i}-\hat{k},\ \vec{b}=x\hat{i}+\hat{j}+(1-x)\hat{k}$ and

$\vec{c}=y\hat{i}+x\hat{j}+(1+x-y)\hat{k}$

$$\therefore\ [\vec{a}\ \vec{b}\ \vec{c}]=\vec{a}.(\vec{b}\times\vec{c})=\begin{vmatrix}1 & 0 & -1\\ x & 1 & 1-x\\ y & x & 1+x-y\end{vmatrix}$$

$$=1\left[1+x-y-x+x^2\right]-\left[x^2-y\right]$$

$$=1-y+x^2-x^2+y$$
$$=1$$

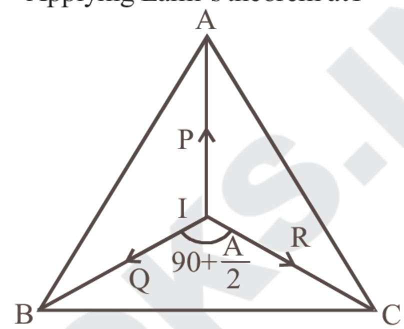
Hence $\left[\vec{a}\ \vec{b}\ \vec{c}\right]$ is independent of x and y both.

40. (c) Let I is incentre of ΔABC.

$\therefore$ IA, IB, IC are bisectors of the angles A, B and C

Now $\angle BIC=180-\frac{B}{2}-\frac{C}{2}=90°+\frac{A}{2}$ etc.

Applying Lami's theorem at I

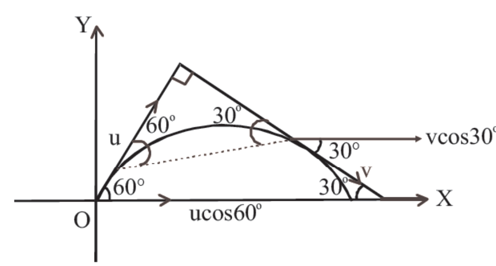


$$\frac{P}{Sin(90°+\frac{A}{2})}=\frac{Q}{\sin\left(90°+\frac{B}{2}\right)}=\frac{R}{\sin\left(90°+\frac{C}{2}\right)}$$

$$\Rightarrow P:Q:R=\cos\frac{A}{2}:\cos\frac{B}{2}:\cos\frac{C}{2}$$

41. (d) As per question $u\cos 60°=v\cos 30°$ (as horizontal component of velocity remains the same)

$$\Rightarrow u\cdot\frac{1}{2}=v\cdot\frac{\sqrt{3}}{2}\quad\text{or}\quad v=\frac{1}{\sqrt{3}}u$$

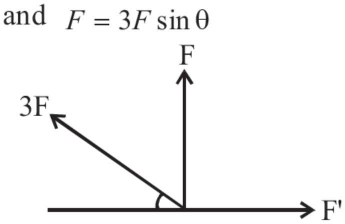


42. (b) Let A and B be displaced by a distance x then Change in moment of $(A+B)$ = applied moments

$$\Rightarrow(A+B)\times x=H\Rightarrow x=\frac{H}{A+B}$$

43. (d) According to question $F'=3F\cos\theta$

and $F=3F\sin\theta$



$\Rightarrow F'=2\sqrt{2}F\Rightarrow F:F'::\ 3:2\sqrt{2}$.

44. **(d)** If we consider unit vectors $\hat{i}$ and $\hat{j}$ in the direction AB and AC respectively and its magnitude $\frac{1}{AB}$ and $\frac{1}{AC}$ respectively, then as per quesiton, forces along AB and AC respectively are

$$\left(\frac{1}{AB}\right)\hat{i} \text{ and } \left(\frac{1}{AC}\right)\hat{j}$$

$\therefore$ Their resultant along AD

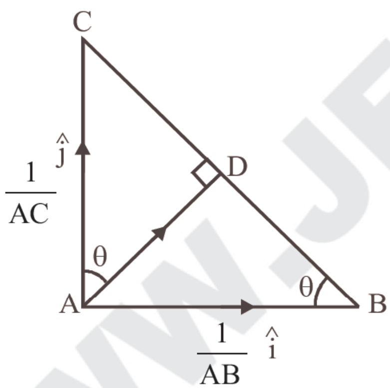
$$= \left(\frac{1}{AB}\right)i + \left(\frac{1}{AC}\right)j$$

$\therefore$ Magnitude of resultant is

$$= \sqrt{\left(\frac{1}{AB}\right)^2 + \left(\frac{1}{AC}\right)^2} = \sqrt{\frac{AC^2 + AB^2}{AB^2 + AC^2}}$$

$$[\because\ AC^2 + AB^2 = BC^2]$$

$$= \frac{BC}{AB.AC}$$



$\because\ \Delta ABC \sim \Delta DBA$

$$\Rightarrow \frac{BC}{AB} = \frac{AC}{AD} \Rightarrow \frac{BC}{AB \times AC} = \frac{1}{AD}$$

$\therefore$ The required magnitude of resultant becomes $\frac{1}{AD}$.

45. **(d)** Given that $(\bar{a} \times \bar{b}) \times \bar{c} = \bar{a} \times (\bar{b} \times \bar{c})$,

$\bar{a}.\bar{b} \neq 0$, $\bar{b}.\bar{c} \neq 0$

$\Rightarrow (\bar{a}.\bar{c}).\bar{b} - (\bar{b}.\bar{c})\bar{a} = (\bar{a}.\bar{c}).\bar{b} - (\bar{a}.\bar{b}).\bar{c}$

$\Rightarrow (\bar{a}.\bar{b}).\bar{c} = (\bar{b}.\bar{c})\bar{a} \Rightarrow \bar{a} \parallel \bar{c}$.

46. **(a)** $\overrightarrow{CA} = (2-a)\hat{i} + 2\hat{j}$;

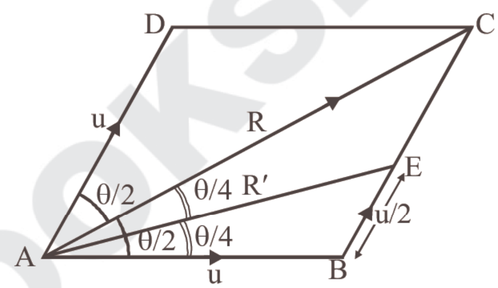
$\overrightarrow{CB} = (1-a)\hat{i} - 6\hat{k}$

$\left[\because\ \overrightarrow{CA} \perp \overrightarrow{CB}\right]$

$\therefore\ \overrightarrow{CA} \cdot \overrightarrow{CB} = 0 \Rightarrow (2-a)(1-a) = 0$

$\Rightarrow a = 2, 1$

47. **(b)** Let two velocities u and u at an angle θ to each other the resultant is given by



$R^2 = u^2 + u^2 + 2u^2 \cos\theta = 2u^2 (1 + \cos\theta)$

$\Rightarrow R^2 = 4u^2 \cos^2 \theta/2$ or $R = 2u\cos\frac{\theta}{2}$

Now in second case, the new resultant AE (i.e., R') bisects $\angle CAB$, therefore using angle bisector theorem in ΔABC, we get

$$\frac{AB}{AC} = \frac{BE}{EC} \Rightarrow \frac{u}{R} = \frac{u/2}{u/2} \Rightarrow R = u$$

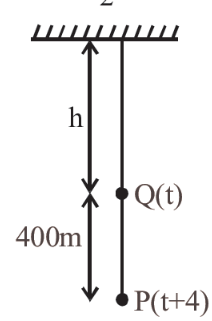
$$\Rightarrow 2u\cos\frac{\theta}{2} = u$$

$$\Rightarrow \quad \cos\frac{\theta}{2} = \frac{1}{2} = \cos 60° \Rightarrow \frac{\theta}{2} = 60°$$

or $\quad \theta = 120°$

48. **(a)** We know that $h = \frac{1}{2}gt^2$

and $h + 400 = \frac{1}{2}g(t+4)^2$



Subtracting, we get $400 = 8g + 4gt$

$\Rightarrow \quad t = 8$ sec

$\therefore \quad h = \frac{1}{2} \times 10 \times 64 = 320m$

$\therefore$ Required height $= 320 + 400 = 720\ m$

49. (b) Given that $|2\hat{u} \times 3\hat{v}| = 1$ and θ is acute angle between $\hat{u}$ and $\hat{v}$, $|\hat{u}| = 1, |\hat{v}| = 1$

$\Rightarrow \quad |2\hat{u} \times 3\hat{v}| = 6|\hat{u}||\hat{v}||\sin\theta| = 1$

$\Rightarrow \quad 6|\sin\theta| = 1 \Rightarrow \sin\theta = \frac{1}{6}$

Hence, there is exactly one value of θ for which $2\hat{u} \times 3\hat{v}$ is a unit vector.

50. (b) Given $\vec{a} = \hat{i} + \hat{j} + \hat{k}$, $\vec{b} = \hat{i} - \hat{j} + 2\hat{k}$ and $\vec{c} = x\hat{i} + (x-2)\hat{j} - \hat{k}$

Given that $\vec{c}$ lies in the plane of $\vec{a}$ and $\vec{b}$, then $\vec{a}, \vec{b}$ and $\vec{c}$ are coplanar

$\therefore \quad [\vec{a}\ \vec{b}\ \vec{c}] = 0$

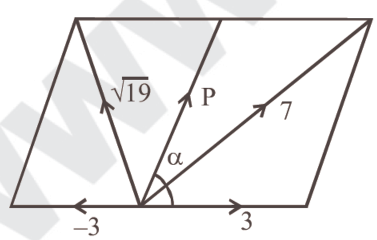
i.e. $\begin{vmatrix} 1 & 1 & 1 \\ 1 & -1 & 2 \\ x & (x-2) & -1 \end{vmatrix} = 0$

$\Rightarrow 1[1-2(x-2)] - 1[-1-2x] + 1[x-2+x] = 0$

$\Rightarrow 1 - 2x + 4 + 1 + 2x + 2x - 2 = 0$

$\Rightarrow 2x = -4 \Rightarrow x = -2$

51. (c) Given that : Force $P = Pn$, $Q = 3n$, resultant $R = 7n$ & $P' = Pn$, $Q' = (-3)n$, $R' = \sqrt{19}\,n$



We know that $R^2 = P^2 + Q^2 + 2PQ\cos\alpha$

$\Rightarrow \quad (7)^2 = P^2 + (3)^2 + 2 \times P \times 3\cos\alpha$

$\Rightarrow \quad 49 = P^2 + 9 + 6P\cos\alpha$

$\Rightarrow \quad 40 = P^2 + 6P\cos\alpha$ .....(i)

and $\left(\sqrt{19}\right)^2 = P^2 + (-3)^2 + 2P \times -3\cos\alpha$

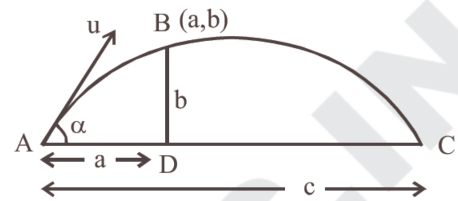
$\Rightarrow \quad 19 = P^2 + 9 - 6P\cos\alpha$

$\Rightarrow \quad 10 = P^2 - 6P\cos\alpha$ .....(ii)

Adding (i) and (ii) $50 = 2P^2$

$\Rightarrow \quad P^2 = 25 \Rightarrow P = 5n.$

52. (a) Let B be the top of the wall whose coordinates will be (a, b). Range $(R) = c$



B lies on the trajectory

$\therefore \quad y = x\tan\alpha - \frac{1}{2}g\frac{x^2}{u^2\cos^2\alpha}$

$\Rightarrow \quad b = a\tan\alpha - \frac{1}{2}g\frac{a^2}{u^2\cos^2\alpha}$

$\Rightarrow \quad b = a\tan\alpha\left[1 - \frac{ga}{2u^2\cos^2\alpha\tan\alpha}\right]$

$= a\tan\alpha\left[1 - \frac{a}{\frac{2u^2}{g}\cos^2\alpha.\frac{\sin\alpha}{\cos\alpha}}\right]$

$= a\tan\alpha\left[1 - \frac{a}{\frac{u^2.2\sin\alpha\cos\alpha}{g}}\right]$

$= a\tan\alpha\left[1 - \frac{a}{\frac{u^2\sin 2\alpha}{g}}\right]$

$= a\tan\alpha\left[1 - \frac{a}{R}\right]$

$\left(\because \quad R = \frac{u^2\sin^2\alpha}{g}\right)$

$\Rightarrow \quad b = a\tan\alpha\left[1 - \frac{a}{c}\right]$

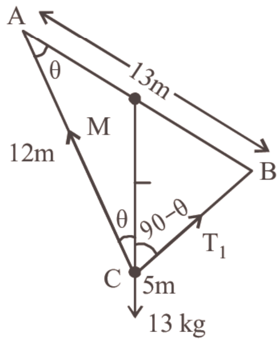
$\Rightarrow \quad b = a\tan\alpha \,.\left(\frac{c-a}{c}\right)$

$\Rightarrow \quad \tan\alpha = \frac{bc}{a(c-a)}$

The angle of projection,

$\alpha = \tan^{-1}\frac{bc}{a(c-a)}$

53. (a)



In ΔABC

$\because\ 13^2 = 5^2 + 12^2 \Rightarrow AB^2 = AC^2 + BC^2$

$\Rightarrow \angle ACB = 90°$

M is mid point of the hypotenuse AB, therefore $MA = MB = MC$

$\Rightarrow \angle A = \angle ACM = \theta$

Applying Lami's theorem at C, we get

$$\frac{T_1}{\sin(180-\theta)} = \frac{T_2}{\sin(90+\theta)} = \frac{13kg}{\sin 90°}$$

$\Rightarrow\ T_1 = 13 \sin\theta$ and $T_2 = 13\cos\theta$

$\Rightarrow\ T_1 = 13 \times \frac{5}{13}$ and $T_2 = 13 \times \frac{12}{13}$

$\Rightarrow\ T_1 = 5$ kg and $T_2 = 12$ kg

54. (d) Clearly $\vec{a} = -\frac{8}{7}\vec{c}$

$\Rightarrow \vec{a} \,||\, \vec{c}$ and are opposite in direction

$\therefore$ Angle between $\vec{a}$ and $\vec{c}$ is π.

55. (b) Given that direction ratios are $6, -3, 2$

$\therefore$ Direction cosines are

$$\frac{6}{\sqrt{6^2+(-3)^2+2^2}}, \frac{-3}{\sqrt{6^2+(-3)^2+2^2}}, \frac{2}{\sqrt{6^2+(-3)^2+2^2}} = \frac{6}{7}, \frac{-3}{7}, \frac{2}{7}$$

56. (d) $\because\ \vec{u}, \vec{v}, \vec{w}$ are non coplanar vectors

$\therefore\ [\vec{u}, \vec{v}, \vec{w}] \neq 0$

Now, $[3\vec{u}, p\vec{v}, p\vec{w}] - [p\vec{v}, p\vec{w}, q\vec{u}] - [2\vec{w}, q\vec{v}, q\vec{u}] = 0$

$\Rightarrow\ 3p^2[\vec{u}, \vec{v}, \vec{w}] - pq[\vec{v}, \vec{w}, \vec{u}] - 2q^2[\vec{w}, \vec{v}, \vec{u}] = 0$

$\Rightarrow\ 3p^2[\vec{u}, \vec{v}, \vec{w}] - pq[\vec{u}, \vec{v}, \vec{w}] + 2q^2[\vec{u}, \vec{v}, \vec{w}]$

$\Rightarrow\ (3p^2 - pq + 2q^2)\,[\vec{u}, \vec{v}, \vec{w}] = 0$

$\Rightarrow\ 3p^2 - pq + 2q^2 = 0$ $\quad (\because [\vec{u}, \vec{v}, \vec{w}] = 0)$

$\Rightarrow\ 2p^2 + p^2 - pq + \frac{q^2}{4} + \frac{7q^2}{4} = 0$

$\Rightarrow\ 2p^2 + \left(p - \frac{q}{2}\right)^2 + \frac{7}{4}q^2 = 0$

$\Rightarrow\ p = 0, q = 0, p = q/2$

This is possible only when $p = 0, q = 0$

$\therefore$ There is exactly one value of (p, q).

57. (d) Given that

$\vec{c} = \vec{b} \times \vec{a} \Rightarrow \vec{b}.\vec{c} = \vec{b}.(\vec{b} \times \vec{a}) \Rightarrow \vec{b}.\vec{c} = 0$

$\Rightarrow \left(b_1\hat{i} + b_2\hat{j} + b_3\hat{k}\right).\left(\hat{i} - \hat{j} - \hat{k}\right) = 0,$

where $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$

$b_1 - b_2 - b_3 = 0$ ...(i)

and $\vec{a}.\vec{b} = 3 \Rightarrow (\hat{j} - \hat{k}).(b_1\hat{i} + b_2\hat{j} + b_3\hat{k}) = 3$

$\Rightarrow b_2 - b_3 = 3$

From equation (i)

$b_1 = b_2 + b_3 = (3 + b_3) + b_3 = 3 + 2b_3$

$\vec{b} = (3 + 2b_3)\hat{i} + (3 + b_3)\hat{j} + b_3\hat{k}$

From the option given, it is clear that b_3 equal to either 2 or -2.

If $b_3 = 2$ then $\vec{b} = 7\hat{i} + 5\hat{j} + 2\hat{k}$ which is not possible

If $b_3 = -2$, then $\vec{b} = -\hat{i} + \hat{j} - 2\hat{k}$

58. (d) Given that, $\vec{a}, \vec{b}$ and $\vec{c}$ are mutually orthogonal

$\therefore\ \vec{a}.\vec{b} = 0,\ \vec{b}.\vec{c} = 0,\ \vec{c}.\vec{a} = 0$

$\Rightarrow 2\lambda + 4 + \mu = 0$ ...(i)

$\lambda - 1 + 2\mu = 0$ ...(ii)

On solving (i) and (ii), we get $\lambda = -3, \mu = 2$

59. (d) $(2\vec{a} - \vec{b}).\left((\vec{a} \times \vec{b}) \times (\vec{a} + 2\vec{b})\right)$

$= (2\vec{a} - \vec{b}).\left((\vec{a} \times \vec{b}) \times \vec{a} + 2(\vec{a} \times \vec{b}) \times \vec{b}\right)$

$$= (2\vec{a}-\vec{b})\begin{pmatrix}(\vec{a}.\vec{a})\vec{b}-(\vec{a}.\vec{b})\vec{a}+2(\vec{a}.\vec{b})\vec{b}\\ -2(\vec{b}.\vec{b})\vec{a}\end{pmatrix}$$

$= (2\vec{a}-\vec{b})(\vec{b}-0+0-2\vec{a})$

From given values we get

$\vec{a}.\vec{b}=0$ and $\vec{b}.\vec{b}=1$

$= -4\vec{a}.\vec{a}-\vec{b}.\vec{b}=-5$

60. **(c)** Given that $\vec{a}.\vec{b}\neq 0$, $\vec{a}.\vec{d}=0$

Now, $\vec{b}\times\vec{c}=\vec{b}\times\vec{d}$

$\Rightarrow \vec{a}\times(\vec{b}\times\vec{c})=\vec{a}\times(\vec{b}\times\vec{d})$

$\Rightarrow (\vec{a}.\vec{c})\vec{b}-(\vec{a}.\vec{b})\vec{c}=(\vec{a}.\vec{d})\vec{b}-(\vec{a}.\vec{b})\vec{d}$

$\Rightarrow (\vec{a}.\vec{b})\vec{d}=-(\vec{a}.\vec{c})\vec{b}+(\vec{a}.\vec{b})\vec{c}$

$$\vec{d}=\vec{c}-\left(\frac{\vec{a}.\vec{c}}{\vec{a}.\vec{b}}\right)\vec{b}$$

61. **(d)** The given vectors are coplanar then

$$\begin{vmatrix}p & 1 & 1\\ 1 & q & 1\\ 1 & 1 & r\end{vmatrix}=0$$

$\Rightarrow \quad p(qr-1)+1(1-r)+1(1-q)=0$

$\Rightarrow \quad pqr-p+1-r+1-q=0$

$\Rightarrow \quad pqr-(p+q+r)=-2$

62. **(c)** As per question

$\vec{a}+3\vec{b}=\lambda\vec{c}$ ....(i)

$\vec{b}+2\vec{c}=\mu\vec{a}$ ....(ii)

On solving equations (i) and (ii)

$(1+3\mu)\vec{a}-(\lambda+6)\vec{c}=0$

As $\vec{a}$ and $\vec{c}$ are non collinear,

$\therefore \quad 1+3\mu=0$ and $\lambda+6=0$

From (i), $\vec{a}+3\vec{b}+6\vec{c}=\vec{0}$

63. **(c)** Given that $\vec{c}=\hat{a}+2\hat{b}$ and $\vec{d}=5\hat{a}-4\hat{b}$ and $|\hat{a}|=|\hat{b}|=1$

Since $\vec{c}$ and $\vec{d}$ are perpendicular to each other

$\therefore \quad \vec{c}\cdot\vec{d}=0$

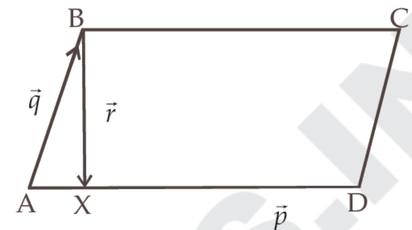
$\Rightarrow \quad (\hat{a}+2\hat{b}).(5\hat{a}-4\hat{b})=0$

$\Rightarrow \quad 5+6\hat{a}\cdot\hat{b}-8=0$

$\Rightarrow \quad \hat{a}\cdot\hat{b}=\frac{1}{2} \Rightarrow \sin\theta=\frac{1}{2} \Rightarrow \theta=\frac{\pi}{3}$

64. **(b)** Let $ABCD$ be a parallelogram such that $\vec{AB}=\vec{q}$, $\vec{AD}=\vec{p}$ and $\angle BAD$ be an acute angle.

We have



$$\overrightarrow{AX}=\left(\frac{\vec{p}\cdot\vec{q}}{|\vec{p}|}\right)\left(\frac{\vec{p}}{|\vec{p}|}\right)=\frac{\vec{p}.\vec{q}}{|\vec{p}|^2}\vec{p}$$

From triangle law

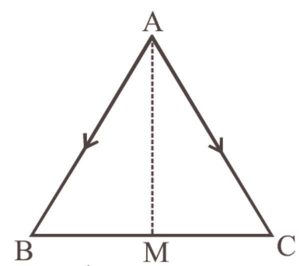
Let $\vec{r}=\overrightarrow{BX}=\overrightarrow{BA}+\overrightarrow{AX}=-\vec{q}+\dfrac{\vec{p}.\vec{q}}{|\vec{p}|^2}\vec{p}$

65. **(c)** We have,

$\overrightarrow{AB}+\overrightarrow{BC}+\overrightarrow{CA}=0\Rightarrow\overrightarrow{BC}=\overrightarrow{AC}-\overrightarrow{AB}$

Let M be mid-point of BC

Now, $\overrightarrow{BM}=\dfrac{\overrightarrow{AC}-\overrightarrow{AB}}{2}\quad\left(\because\quad \overrightarrow{BM}=\dfrac{\overrightarrow{BC}}{2}\right)$



Also, we have

$\overrightarrow{AB}+\overrightarrow{BM}+\overrightarrow{MA}=0$

$\Rightarrow \quad \overrightarrow{AB}+\dfrac{\overrightarrow{AC}-\overrightarrow{AB}}{2}=\overrightarrow{AM}$

$\Rightarrow \quad 1\,\overrightarrow{AM}=\dfrac{\overrightarrow{AB}+\overrightarrow{AC}}{2}=4\hat{i}-\hat{j}+4\hat{k}$

$\Rightarrow \quad |\overrightarrow{AM}|=\sqrt{33}$

66. **(b)** L.H.S $=(\vec{a}\times\vec{b}).[(\vec{b}\times\vec{c})\times(\vec{c}\times\vec{a})]$

$=(\vec{a}\times\vec{b}).[(\vec{b}\times\vec{c}.\vec{a})\vec{c}-(\vec{b}\times\vec{c}.\vec{c})\vec{a}]$

$=(\vec{a}\times\vec{b}).[[\vec{b}\,\vec{c}\,\vec{a}]\vec{c}] \qquad [\because \vec{b}\times\vec{c}.\vec{c}=0]$

$=[\vec{a}\,\vec{b}\,\vec{c}].(\vec{a}\times\vec{b}.\vec{c})=[\vec{a}\,\vec{b}\,\vec{c}]^2$

$=[\vec{a}\times\vec{b}\quad \vec{b}\times\vec{c}\quad \vec{c}\times\vec{a}]=[\vec{a}\,\vec{b}\,\vec{c}]^2$

So $\lambda=1$

67. **(c)** $\left(\vec{a}\times\vec{b}\right)\times\vec{c} = \frac{1}{3}\left|\vec{b}\right|\left|\vec{c}\right|\vec{a}$

$\Rightarrow\ -\vec{c}\times\left(\vec{a}\times\vec{b}\right) = \frac{1}{3}\left|\vec{b}\right|\left|\vec{c}\right|\vec{a}$

$\Rightarrow\ -\left(\vec{c}.\vec{b}\right)\vec{a}+\left(\vec{c}.\vec{a}\right)\vec{b} = \frac{1}{3}\left|\vec{b}\right|\left|\vec{c}\right|\vec{a}$

$\Rightarrow\ -\left|\vec{b}\right|\left|\vec{c}\right|\cos\theta\vec{a}+\left(\vec{c}.\vec{a}\right)\vec{b} = \frac{1}{3}\left|\vec{b}\right|\left|\vec{c}\right|\vec{a}$

$\because$ $\vec{a},\vec{b},\vec{c}$ are non collinear, the above equation is possible only when

$-\cos\theta = \frac{1}{3}$ and $\vec{c}.\vec{a} = 0$

$\Rightarrow\ \cos\theta = -\frac{1}{3}\ \Rightarrow \sin\theta = \frac{2\sqrt{2}}{3}$; $\theta\in$ II quad

68. **(b)** $\vec{a}\times(\vec{b}\times\vec{c}) = \frac{\sqrt{3}}{2}(\vec{b}+\vec{c})$

$\Rightarrow (\vec{a}\cdot\vec{c})\vec{b}-(\vec{a}\cdot\vec{b})\vec{c} = \frac{\sqrt{3}}{2}\vec{b}+\frac{\sqrt{3}}{2}\vec{c}$

On comparing both sides

$\vec{a}\cdot\vec{b} = -\frac{\sqrt{3}}{2} \Rightarrow \cos\theta = -\frac{\sqrt{3}}{2}$

[$\because \vec{a}$ and $\vec{b}$ are unit vectors]

where θ is the angle between $\vec{a}$ and $\vec{b}$

$\theta = \frac{5\pi}{6}$

69. **(c)** Given :

$\vec{a} = 2\hat{i}+\hat{j}-2\hat{k},\ \ \vec{b} = \hat{i}+\hat{j}$

$\Rightarrow |\vec{a}| = 3$

$\therefore\ \vec{a}\times\vec{b} = 2\hat{i}-2\hat{j}+\hat{k}$

$|\vec{a}\times\vec{b}| = \sqrt{2^2+2^2+1^2} = 3$

We have $(\vec{a}\times\vec{b})\times\vec{c} = |\vec{a}\times\vec{b}||\vec{c}|\ \sin 30\ n$

$\Rightarrow |(\vec{a}\times\vec{b})\times\vec{c}| = 3|\vec{c}|.\frac{1}{2} \Rightarrow 3 = 3|\vec{c}|.\frac{1}{2}$

$\therefore\ |\vec{c}| = 2$

Now $|\vec{c}-\vec{a}| = 3$

On squaring, we get

$\Rightarrow |\vec{c}|^2+|\vec{a}|^2-2\vec{a}.\vec{c} = 9 \Rightarrow 4+9-2\,\vec{a}.\vec{c} = 9$

$\Rightarrow \vec{a}.\vec{c} = 2$ $\qquad$ [$\because \vec{c}.\vec{a} = \vec{a}.\vec{c}$]

70. **(d)** $\because \vec{u}, \vec{a}$ & $\vec{b}$ are coplanar

$\therefore \vec{u} = \lambda(\vec{a}\times\vec{b})\times\vec{a}\ \ = \lambda\{\vec{a}^2.\vec{b}-(\vec{a}.\vec{b})\vec{a}\}$

$= \lambda\{-4\hat{i}+8\hat{j}+16\hat{k}\} = \lambda'\{-\hat{i}+2\hat{j}+4\hat{k}\}$.

Also, $\vec{u}.\vec{b} = 24\ \Rightarrow \lambda' = 4$

$\therefore\ \ \vec{u} = -4\hat{i}+8\hat{j}+16\hat{k}$

$\Rightarrow\ \ |\vec{u}|^2 = 336$

71. **(a)** $\because |\vec{a}\times\vec{c}|^2 = |\vec{a}|^2|\vec{c}|^2-(\vec{a}.\vec{c})^2$

$\Rightarrow\ \ |-\vec{b}|^2 = 2|\vec{c}|^2-16$

$\Rightarrow\ \ 3 = 2|\vec{c}|^2-16$

$\Rightarrow\ \ = \frac{19}{2}$

72. **(c)** $\vec{\beta} = \vec{\beta}_1-\vec{\beta}_2$ $\qquad$ ...(1)

Since, $\vec{\beta}_2$ is perpendicular to $\vec{\alpha}$.

$\therefore \vec{\beta}_2.\vec{\alpha} = 0$

Since, $\vec{\beta}_1$ is parallel to $\vec{\alpha}$.

then $\vec{\beta}_1 = \lambda\vec{\alpha}$ (say)

$\vec{\alpha}\cdot\vec{\beta} = \vec{\alpha}\cdot\vec{\beta}_1-\vec{\alpha}\cdot\vec{\beta}_2$

$\Rightarrow 5\ = \lambda|\vec{\alpha}|^2 \Rightarrow 5 = 1\times 10\ \ \left(\because |\vec{\alpha}| = \sqrt{10}\right)$.

$\Rightarrow \lambda = \frac{1}{2}$

$\therefore \vec{\beta}_1 = \frac{\vec{\alpha}}{2}$

Cross product with $\vec{\beta}_1$ in equation (1)

$\Rightarrow \vec{\beta}\times\vec{\beta}_1 = -\vec{\beta}_2\times\vec{\beta}_1$

$\Rightarrow \vec{\beta}\times\vec{\beta}_1 = \vec{\beta}_1\times\vec{\beta}_2\qquad \Rightarrow \vec{\beta}_1\times\vec{\beta}_2 = \frac{\vec{\beta}\times\vec{\alpha}}{2}$

$$\Rightarrow \vec{\beta}_1\times\vec{\beta}_2 = \frac{1}{2}\begin{vmatrix}\hat{i} & j & k\\ 2 & -1 & 3\\ 3 & 1 & 0\end{vmatrix}$$

$= \frac{1}{2}\left[-3\hat{i}-\hat{j}(-9)+\hat{k}(5)\right]$

$= \frac{1}{2}\left[-3\hat{i}+9\hat{j}+5\hat{k}\right]$

26 Three Dimensional Geometry

1. A plane which passes through the point (3, 2, 0) and the line $\frac{x-4}{1}=\frac{y-7}{5}=\frac{z-4}{4}$ is **[2002]**

(a) $x-y+z=1$ (b) $x+y+z=5$
(c) $x+2y-z=1$ (d) $2x-y+z=5$

2. The d.r. of normal to the plane through (1, 0, 0), (0, 1, 0) which makes an angle $\pi/4$ with plane $x+y=3$ are **[2002]**

(a) $1,\sqrt{2},1$ (b) $1,1,\sqrt{2}$
(c) $1,1,2$ (d) $\sqrt{2},1,1$

3. The shortest distance from the plane $12x+4y+3z=327$ to the sphere $x^2+y^2+z^2+4x-2y-6z=155$ is **[2003]**

(a) 39 (b) 26 (c) $11\frac{4}{13}$ (d) 13.

4. The two lines $x=ay+b, z=cy+d$ and $x=a'y+b', z=c'y+d'$ will be perpendicular, if and only if **[2003]**

(a) $aa'+bb'+cc'+1=0$
(b) $aa'+bb'+cc'=0$
(c) $(a+a')(b+b')+(c+c')=0$
(d) $aa'+cc'+1=0$.

5. The lines $\frac{x-2}{1}=\frac{y-3}{1}=\frac{z-4}{-k}$ **[2003]**

and $\frac{x-1}{k}=\frac{y-4}{2}=\frac{z-5}{1}$ are coplanar if

(a) $k=3$ or -2 (b) $k=0$ or -1
(c) $k=1$ or -1 (d) $k=0$ or -3.

6. The radius of the circle in which the sphere $x^2+y^2+z^2+2x-2y-4z-19=0$ is cut by the plane $x+2y+2z+7=0$ is **[2003]**

(a) 4 (b) 1 (c) 2 (d) 3

7. Two system of rectangular axes have the same origin. If a plane cuts them at distances a,b,c and a',b',c' from the origin then **[2003]**

(a) $\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}-\frac{1}{a'^2}-\frac{1}{b'^2}-\frac{1}{c'^2}=0$

(b) $\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}+\frac{1}{a'^2}+\frac{1}{b'^2}+\frac{1}{c'^2}=0$

(c) $\frac{1}{a^2}+\frac{1}{b^2}-\frac{1}{c^2}+\frac{1}{a'^2}+\frac{1}{b'^2}-\frac{1}{c'^2}=0$

(d) $\frac{1}{a^2}-\frac{1}{b^2}-\frac{1}{c^2}+\frac{1}{a'^2}-\frac{1}{b'^2}-\frac{1}{c'^2}=0$.

8. Distance between two parallel planes $2x+y+2z=8$ and $4x+2y+4z+5=0$ is **[2004]**

(a) $\frac{9}{2}$ (b) $\frac{5}{2}$ (c) $\frac{7}{2}$ (d) $\frac{3}{2}$

9. A line with direction cosines proportional to 2, 1, 2 meets each of the lines $x=y+a=z$ and $x+a=2y=2z$. The co-ordinates of each of the points of intersection are given by **[2004]**

(a) $(2a, 3a, 3a), (2a, a, a)$
(b) $(3a, 2a, 3a), (a, a, a)$
(c) $(3a, 2a, 3a), (a, a, 2a)$
(d) $(3a, 3a, 3a), (a, a, a)$

10. If the straight lines **[2004]**
$x = 1 + s, y = -3 - \lambda s, z = 1 + \lambda s$
and $x = \frac{t}{2}, y = 1 + t, z = 2 - t,$ with parameters s and t respectively, are coplanar, then λ equals.

(a) 0 (b) –1 (c) $-\frac{1}{2}$ (d) –2

11. The intersection of the spheres
$x^2 + y^2 + z^2 + 7x - 2y - z = 13$ and
$x^2 + y^2 + z^2 - 3x + 3y + 4z = 8$ is the same as the intersection of one of the sphere and the plane **[2004]**

(a) $2x - y - z = 1$ (b) $x - 2y - z = 1$
(c) $x - y - 2z = 1$ (d) $x - y - z = 1$

12. A line makes the same angle θ, with each of the x and z axis. If the angle β, which it makes with y-axis, is such that $\sin^2\beta = 3\sin^2\theta$, then $\cos^2\theta$ equals **[2004]**

(a) $\frac{2}{5}$ (b) $\frac{1}{5}$ (c) $\frac{3}{5}$ (d) $\frac{2}{3}$

13. If the angle θ between the line $\frac{x+1}{1} = \frac{y-1}{2} = \frac{z-2}{2}$ and the plane $2x - y + \sqrt{\lambda}z + 4 = 0$ is such that $\sin\theta = \frac{1}{3}$ then the value of λ is **[2005]**

(a) $\frac{5}{3}$ (b) $\frac{-3}{5}$ (c) $\frac{3}{4}$ (d) $\frac{-4}{3}$

14. The angle between the lines $2x = 3y = -z$ and $6x = -y = -4z$ is **[2005]**

(a) 0° (b) 90° (c) 45° (d) 30°

15. If the plane $2ax - 3ay + 4az + 6 = 0$ passes through the midpoint of the line joining the centres of the spheres
$x^2 + y^2 + z^2 + 6x - 8y - 2z = 13$ and
$x^2 + y^2 + z^2 - 10x + 4y - 2z = 8$ then a equals **[2005]**

(a) – 1 (b) 1
(c) – 2 (d) 2

16. The distance between the line $\vec{r} = 2\hat{i} - 2\hat{j} + 3\hat{k} + \lambda(i - j + 4k)$ and the plane $\vec{r}.(\hat{i} + 5\hat{j} + \hat{k}) = 5$ is **[2005]**

(a) $\frac{10}{9}$ (b) $\frac{10}{3\sqrt{3}}$ (c) $\frac{3}{10}$ (d) $\frac{10}{3}$

17. Let a, b and c be distinct non- negative numbers. If the vectors $a\hat{i} + a\hat{j} + c\hat{k}$, $\hat{i} + \hat{k}$ and $c\hat{i} + c\hat{j} + b\hat{k}$ lie in a plane, then c is **[2005]**

(a) the Geometric Mean of a and b
(b) the Arithmetic Mean of a and b
(c) equal to zero
(d) the Harmonic Mean of a and b

18. The plane $x + 2y - z = 4$ cuts the sphere $x^2 + y^2 + z^2 - x + z - 2 = 0$ in a circle of radius **[2005]**

(a) 3 (b) 1 (c) 2 (d) $\sqrt{2}$

19. The two lines $x = ay + b$, $z = cy + d$; and $x = a'y + b'$, $z = c'y + d'$ are perpendicular to each other if **[2006]**

(a) $aa' + cc' = -1$ (b) $aa' + cc' = 1$
(c) $\frac{a}{a'} + \frac{c}{c'} = -1$ (d) $\frac{a}{a'} + \frac{c}{c'} = 1$

20. The image of the point (–1, 3, 4) in the plane $x - 2y = 0$ is **[2006]**

(a) $\left(-\frac{17}{3}, -\frac{19}{3}, 4\right)$ (b) $(15, 11, 4)$
(c) $\left(-\frac{17}{3}, -\frac{19}{3}, 1\right)$ (d) None of these

21. If a line makes an angle of $\pi/4$ with the positive directions of each of x- axis and y- axis, then the angle that the line makes with the positive direction of the z-axis is **[2007]**

(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{2}$
(c) $\frac{\pi}{6}$ (d) $\frac{\pi}{3}$

22. If (2, 3, 5) is one end of a diameter of the sphere $x^2 + y^2 + z^2 - 6x - 12y - 2z + 20 = 0$, then the cooordinates of the other end of the diameter are **[2007]**

(a) (4, 3, 5) (b) (4, 3, – 3)
(c) (4, 9, – 3) (d) (4, –3, 3).

23. Let L be the line of intersection of the planes $2x+3y+z=1$ and $x+3y+2z=2$. If L makes an angle α with the positive x-axis, then cos α equals **[2007]**

(a) 1 (b) $\frac{1}{\sqrt{2}}$ (c) $\frac{1}{\sqrt{3}}$ (d) $\frac{1}{2}$.

24. The vector $\vec{a} = \alpha\hat{i} + 2\hat{j} + \beta\hat{k}$ lies in the plane of the vectors $\vec{b} = \hat{i} + \hat{j}$ and $\vec{c} = \hat{j} + \hat{k}$ and bisects the angle between $\vec{b}$ and $\vec{c}$. Then which one of the following gives possible values of a and b? **[2008]**

(a) $a=2,\ b=2$ (b) $a=1,\ b=2$
(c) $a=2,\ b=1$ (d) $a=1,\ b=1$

25. The line passing through the points (5, 1, a) and (3, b, 1) crosses the yz-plane at the point $\left(0, \frac{17}{2}, \frac{-13}{2}\right)$. Then **[2008]**

(a) $a=2, b=8$ (b) $a=4, b=6$
(c) $a=6, b=4$ (d) $a=8, b=2$

26. If the straight lines $\frac{x-1}{k} = \frac{y-2}{2} = \frac{z-3}{3}$ and $\frac{x-2}{3} = \frac{y-3}{k} = \frac{z-1}{2}$ intersect at a point, then the integer k is equal to **[2008]**

(a) –5 (b) 5 (c) 2 (d) –2

27. Let the line $\frac{x-2}{3} = \frac{y-1}{-5} = \frac{z+2}{2}$ lie in the plane $x+3y-\alpha z+\beta=0$. Then (α, β) equals **[2009]**

(a) (–6, 7) (b) (5, –15)
(c) (–5, 5) (d) (6, –17)

28. **Statement -1 :** The point A(3, 1, 6) is the mirror image of the point B(1, 3, 4) in the plane $x - y + z = 5$.

Statement -2: The plane $x - y + z = 5$ bisects the line segment joining A(3, 1, 6) and B(1, 3, 4). **[2010]**

(a) Statement -1 is true, Statement -2 is true ; Statement -2 is **not** a correct explanation for Statement -1.
(b) Statement -1 is true, Statement -2 is false.
(c) Statement -1 is false, Statement -2 is true .
(d) Statement - 1 is true, Statement 2 is true ; Statement -2 is a correct explanation for Statement -1.

29. A line AB in three-dimensional space makes angles 45° and 120° with the positive x-axis and the positive y-axis respectively. If AB makes an acute angle θ with the positive z-axis, then θ equals **[2010]**

(a) 45° (b) 60° (c) 75° (d) 30°

30. If the angle between the line $x = \frac{y-1}{2} = \frac{z-3}{\lambda}$ and the plane $x + 2y + 3z = 4$ is $\cos^{-1}\left(\sqrt{\frac{5}{14}}\right)$, then λ equals **[2011]**

(a) $\frac{3}{2}$ (b) $\frac{2}{5}$

(c) $\frac{5}{3}$ (d) $\frac{2}{3}$

31. **Statement-1:** The point $A(1, 0, 7)$) is the mirror image of the point $B(1, 6, 3)$ in the line : $\frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3}$

Statement-2: The line $\frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3}$ bisects the line segment joining $A(1, 0, 7)$ and $B(1, 6, 3)$. **[2011]**

(a) Statement-1 is true, Statement-2 is true; Statement-2 is **not a** correct explanation for Statement-1.
(b) Statement-1 is true, Statement-2 is false.
(c) Statement-1 is false, Statement-2 is true.
(d) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1.

32. The distance of the point (1, – 5, 9) from the plane $x - y + z = 5$ measured along a straight $x = y = z$ is **[2011RS]**

(a) $10\sqrt{3}$ (b) $5\sqrt{3}$ (c) $3\sqrt{10}$ (d) $3\sqrt{5}$

33. The length of the perpendicular drawn from the point (3, –1, 11) to the line

$\frac{x}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ is : **[2011RS]**

(a) $\sqrt{29}$ (b) $\sqrt{33}$ (c) $\sqrt{53}$ (d) $\sqrt{66}$

34. A equation of a plane parallel to the plane $x - 2y + 2z - 5 = 0$ and at a unit distance from the origin is : **[2012]**

(a) $x - 2y + 2z - 3 = 0$ (b) $x - 2y + 2z + 1 = 0$

(c) $x - 2y + 2z - 1 = 0$ (d) $x - 2y + 2z + 5 = 0$

35. If the line $\frac{x-1}{2} = \frac{y+1}{3} = \frac{z-1}{4}$ and

$\frac{x-3}{1} = \frac{y-k}{2} = \frac{z}{1}$ intersect, then k is equal to: **[2012]**

(a) –1 (b) $\frac{2}{9}$

(c) $\frac{9}{2}$ (d) 0

36. Distance between two parallel planes $2x + y + 2z = 8$ and $4x + 2y + 4z + 5 = 0$ is **[2013]**

(a) $\frac{3}{2}$ (b) $\frac{5}{2}$

(c) $\frac{7}{2}$ (d) $\frac{9}{2}$

37. If the lines $\frac{x-2}{1} = \frac{y-3}{1} = \frac{z-4}{-k}$ and

$\frac{x-1}{k} = \frac{y-4}{2} = \frac{z-5}{1}$ are coplanar, then k can have **[2013]**

(a) any value (b) exactly one value

(c) exactly two values (d) exactly three values

38. The image of the line $\frac{x-1}{3} = \frac{y-3}{1} = \frac{z-4}{-5}$ in the plane $2x - y + z + 3 = 0$ is the line: **[2014]**

(a) $\frac{x-3}{3} = \frac{y+5}{1} = \frac{z-2}{-5}$

(b) $\frac{x-3}{-3} = \frac{y+5}{-1} = \frac{z-2}{5}$

(c) $\frac{x+3}{3} = \frac{y-5}{1} = \frac{z-2}{-5}$

(d) $\frac{x+3}{-3} = \frac{y-5}{-1} = \frac{z+2}{5}$

39. The angle between the lines whose direction cosines satisfy the equations $l + m + n = 0$ and $l^2 = m^2 + n^2$ is **[2014]**

(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{2}$

(c) $\frac{\pi}{3}$ (d) $\frac{\pi}{4}$

40. The equation of the plane containing the line $2x - 5y + z = 3$; $x + y + 4z = 5$, and parallel to the plane, $x + 3y + 6z = 1$, is : **[2015]**

(a) $x + 3y + 6z = 7$

(b) $2x + 6y + 12z = -13$

(c) $2x + 6y + 12z = 13$

(d) $x + 3y + 6z = -7$

41. The distance of the point (1, 0, 2) from the point of intersection of the line $\frac{x-2}{3} = \frac{y+1}{4} = \frac{z-2}{12}$ and the plane $x - y + z = 16$, is **[2015]**

(a) $3\sqrt{21}$ (b) 13

(c) $2\sqrt{14}$ (d) 8

42. The distance of the point (1, –5, 9) from the plane $x - y + z = 5$ measured along the line $x = y = z$ is : **[2016]**

(a) $\frac{10}{\sqrt{3}}$ (b) $\frac{20}{3}$

(c) $3\sqrt{10}$ (d) $10\sqrt{3}$

43. If the line, $\frac{x-3}{2} = \frac{y+2}{-1} = \frac{z+4}{3}$ lies in the plane, $lx + my - z = 9$, then $l^2 + m^2$ is equal to : **[2016]**

(a) 5 (b) 2

(c) 26 (d) 18

44. If the image of the point P(1, –2, 3) in the plane, 2x + 3y – 4z + 22 = 0 measured parallel to line, $\frac{x}{1}=\frac{y}{4}=\frac{z}{5}$ is Q, then PQ is equal to : **[2017]**

(a) $6\sqrt{5}$ (b) $3\sqrt{5}$

(c) $2\sqrt{42}$ (d) $\sqrt{42}$

45. The distance of the point (1, 3, –7) from the plane passing through the point (1, –1, –1), having normal perpendicular to both the lines

$\frac{x-1}{1}=\frac{y+2}{-2}=\frac{z-4}{3}$ and

$\frac{x-2}{2}=\frac{y+1}{-1}=\frac{z+7}{-1}$, is : **[2017]**

(a) $\frac{10}{\sqrt{74}}$ (b) $\frac{20}{\sqrt{74}}$ (c) $\frac{10}{\sqrt{83}}$ (d) $\frac{5}{\sqrt{83}}$

46. The length of the projection of the line segment joining the points (5, –1, 4) and (4, –1, 3) on the plane, $x+y+z=7$ is: **[2018]**

(a) $\frac{2}{3}$ (b) $\frac{1}{3}$ (c) $\sqrt{\frac{2}{3}}$ (d) $\frac{2}{\sqrt{3}}$

47. If L_1 is the line of intersection of the planes $2x-2y+3z-2=0,\ x-y+z+1=0$ and L_2 is the line of intersection of the planes $x+2y-z-3=0,\ \ 3x-y+2z-1=0$, then the distance of the origin from the plane, containing the lines L_1 and L_2, is : **[2018]**

(a) $\frac{1}{3\sqrt{2}}$ (b) $\frac{1}{2\sqrt{2}}$

(c) $\frac{1}{\sqrt{2}}$ (d) $\frac{1}{4\sqrt{2}}$

48. The equation of the line passing through (–4, 3, 1), parallel to the plane x + 2y – z – 5 = 0 and intersecting the line **[2019]**

$\frac{x+1}{-3}=\frac{y-3}{2}=\frac{z-2}{-1}$ is:

(a) $\frac{x-4}{2}=\frac{y+3}{1}=\frac{z+1}{4}$

(b) $\frac{x+4}{1}=\frac{y-3}{1}=\frac{z-1}{3}$

(c) $\frac{x+4}{3}=\frac{y-3}{-1}=\frac{z-1}{1}$

(d) $\frac{x+4}{-1}=\frac{y-3}{1}=\frac{z-1}{1}$

49. The plane through the intersection of the planes $x + y + z = 1$ and $2x + 3y - z + 4 = 0$ and parallel to y-axis also passes through the point: **[2019]**

(a) (– 3, 0, – 1) (b) (– 3, 1, 1)

(c) (3, 3, – 1) (d) (3, 2, 1)

50. If the line, $\frac{x-1}{2}=\frac{y+1}{3}=\frac{z-2}{4}$ meets the plane, x + 2y + 3z = 15 at a point P, then the distance of P from the origin is: **[2019]**

(a) $\sqrt{5}/2$ (b) $2\sqrt{5}$

(c) 9/2 (d) 7/2

51. A plane passing through the points (0, –1, 0) and (0, 0, 1) and making an angle $\frac{\pi}{4}$ with the plane y – z + 5 = 0, also passes through the point: **[2019]**

(a) $(-\sqrt{2}, 1, -4)$ (b) $(\sqrt{2}, -1, 4)$

(c) $(-\sqrt{2}, -1, -4)$ (d) $(\sqrt{2}, 1, 4)$

52. A vector $\vec{a}=\alpha\hat{i}+2\hat{j}+\beta\hat{k}$ $(\alpha, \beta \in \boldsymbol{R})$ lies in the plane of the vectors, $\vec{b}=\hat{i}+\hat{j}$ and $\vec{c}=\hat{i}-\hat{j}+4\hat{k}$. If $\vec{a}$ bisects the angle between $\vec{b}$ and $\vec{c}$, then: **[2020]**

(a) $\vec{a}\cdot\hat{i}+3=0$ (b) $\vec{a}\cdot\hat{i}+1=0$

(c) $\vec{a}\cdot\hat{k}+2=0$ (d) $\vec{a}\cdot\hat{k}+4=0$

53. Let P be a plane passing through the points (2, 1, 0), (4, 1, 1) and (5, 0, 1) and R be any point (2, 1, 6). Then the image of R in the plane P is:

(a) (6, 5, 2) (b) (6, 5, –2) **[2020]**

(c) (4, 3, 2) (d) (3, 4, –2)

Answer Key

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
(a)	(b)	(d)	(d)	(d)	(d)	(a)	(c)	(b)	(d)	(a)	(c)	(a)	(b)	(c)
16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
(b)	(a)	(b)	(a)	(d)	(b)	(c)	(c)	(d)	(c)	(a)	(a)	(a)	(b)	(d)
31	32	33	34	35	36	37	38	39	40	41	42	43	44	45
(a)	(a)	(c)	(a)	(c)	(c)	(c)	(c)	(c)	(a)	(b)	(d)	(b)	(c)	(c)
46	47	48	49	50	51	52	53							
(c)	(a)	(c)	(d)	(c)	(d)	(c)	(b)							

Solutions

1. **(a)** Since the point (3, 2, 0) lies on the given line $\dfrac{x-4}{1}=\dfrac{y-7}{5}=\dfrac{z-4}{4}$

$\therefore$ There can be infinite many planes passing through this line. We observed that only option (a) is satisfied by the coordinates of both the points (3, 2, 0) and (4, 7, 4)

$\therefore \quad x-y+z=1$ is the required plane.

2. **(b)** Equation of plane through (1, 0, 0) is

$a(x-1)+by+cz=0$ ...(i)

It is also passes through (0, 1, 0).

$\therefore \quad -a+b=0 \Rightarrow b=a;$

$$\cos 45° = \frac{a+a}{\sqrt{2(2a^2+c^2)}}$$

$\Rightarrow 2a=\sqrt{2a^2+c^2}$

$\Rightarrow 2a^2=c^2 \Rightarrow c=\sqrt{2}a$.

So d.r of normal are a, a $\sqrt{2}a$ i.e. 1, 1, $\sqrt{2}$.

3. **(d)** Centre of sphere be (–2, 1, 3) and radius 13

We know that,

Shortest distance = perpendicular distance between the plane and sphere = distance of plane from centre of sphere – radius

$$=\left|\frac{-2\times 12+4\times 1+3\times 3-327}{\sqrt{144+9+16}}\right|-13$$

$=26-13=13$

4. **(d)** $\dfrac{x-b}{a}=\dfrac{y}{1}=\dfrac{z-d}{c};\ \dfrac{x-b'}{a'}=\dfrac{y}{1}=\dfrac{z-d'}{c'}$.

For perpendicularity of lines,

$aa'+1+cc'=0$

5. **(d)** Two planes are coplanar if

$$\begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \end{vmatrix}=0$$

$$\begin{vmatrix} 1 & -1 & -1 \\ 1 & 1 & -k \\ k & 2 & 1 \end{vmatrix}=0$$

Applying $C_2 \to C_2+C_1, C_3 \to C_3+C_1$

$$\Rightarrow \begin{vmatrix} 1 & 0 & 0 \\ 1 & 2 & 1-k \\ k & k+2 & 1+k \end{vmatrix}=0$$

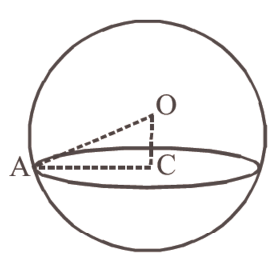
$\Rightarrow \quad 1[2+2k-(k+2)(1-k)]=0$

$\Rightarrow \quad 2+2k-(-k^2-k+2)=0$

$k^2+3k=0 \Rightarrow k(k+3)=0$

or $k=0$ or -3

6. **(d)**



Centre of sphere = (–1, 1, 2)

Radius of sphere $\sqrt{1+1+4+19}=5$

Perpendicular distance from centre to the plane

$$OC = d = \left|\frac{-1+2+4+7}{\sqrt{1+4+4}}\right| = \frac{12}{3} = 4.$$

In right, ΔAOC

$AC^2 = AO^2 - OC^2 = 5^2 - 4^2 = 9$

$\Rightarrow \quad AC = 3$

7. (a) Equation of planes in intercept form be

$\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$ & $\frac{x}{a'}+\frac{y}{b'}+\frac{z}{c'}=1$

($\perp r$ distance on plane from origin is same.)

$$\left|\frac{-1}{\sqrt{\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}}}\right| = \left|\frac{-1}{\sqrt{\frac{1}{a'^2}+\frac{1}{b'^2}+\frac{1}{c'^2}}}\right|$$

$$\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}-\frac{1}{a'^2}-\frac{1}{b'^2}-\frac{1}{c'^2}=0$$

8. (c) The planes are $2x+y+2z-8=0$. ...(1)

and $4x+2y+4z+5=0$

or $2x+y+2z+\frac{5}{2}=0$ ...(2)

Since, both planes are parallel

$\therefore$ Distance between (1) and (2)

$$= \left|\frac{\frac{5}{2}+8}{\sqrt{2^2+1^2+2^2}}\right| = \left|\frac{21}{2\sqrt{9}}\right| = \frac{7}{2}$$

9. (b) Let a point on the line $x = y + a = z = \lambda$ is $(\lambda, \lambda - a, \lambda)$ and a point on the line $x + a = 2y = 2z = \mu$ is $\left(\mu - a, \frac{\mu}{2}, \frac{\mu}{2}\right)$, then

Direction ratio of the line joining these points are $\lambda - \mu + a,\ \lambda - a - \frac{\mu}{2},\ \lambda - \frac{\mu}{2}$

If it respresents the required line whose $d \cdot r.$ be 2, 1, 2, then

$$\frac{\lambda-\mu+a}{2} = \frac{\lambda - a - \frac{\mu}{2}}{1} = \frac{\lambda - \frac{\mu}{2}}{2}$$

on solving we get $\lambda = 3a, \mu = 2a$

$\therefore$ The required points of intersection are $(3a, 3a\text{-}a, 3a)$ and $\left(2a-a, \frac{2a}{2}, \frac{2a}{2}\right)$

or $(3a, 2a, 3a)$ and (a, a, a)

10. (d) The given lines are

$x-1=\frac{y+3}{-\lambda}=\frac{z-1}{\lambda}=s$ ....(1)

and $2x = y-1 = \frac{z-2}{-1} = t$ ....(2)

The lines are coplanar, if

$$\begin{vmatrix} 0-1 & 1-(-3) & 2-1 \\ 1 & -\lambda & \lambda \\ \frac{1}{2} & 1 & -1 \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} -1 & 4 & 1 \\ 1 & -\lambda & \lambda \\ \frac{1}{2} & 1 & -1 \end{vmatrix} = 0$$

Apply $c_2 \to c_2 + c_3$; $\begin{vmatrix} -1 & 5 & 1 \\ 1 & 0 & \lambda \\ \frac{1}{2} & 0 & -1 \end{vmatrix} = 0$

$$\Rightarrow -5\left(-1-\frac{\lambda}{2}\right) = 0 \Rightarrow \lambda = -2$$

11. (a) Given that, the equations of spheres are

$S_1 : x^2 + y^2 + z^2 + 7x - 2y - z - 13 = 0$ and

$S_2 : x^2 + y^2 + z^2 - 3x + 3y + 4z - 8 = 0$

We know that eqn. of intersection plane be

$S_1 - S_2 = 0 \Rightarrow 10x - 5y - 5z - 5 = 0$

$\Rightarrow \quad 2x - y - z = 1$

12. (c) As per question the direction cosines of the line are $\cos\theta, \cos\beta, \cos\theta$

$\therefore \cos^2\theta + \cos^2\beta + \cos^2\theta = 1$

$\because 2\cos^2\theta = 1 - \cos^2\theta$

$\Rightarrow 2\cos^2\theta = \sin^2\beta = 3\sin^2\theta$ (given)

$\Rightarrow 2\cos^2\theta = 3 - 3\cos^2\theta$

$\therefore \cos^2\theta = \frac{3}{5}$

13. (a) Let θ is the angle between line and plane then

$$\sin\theta = \frac{\vec{b}\cdot\vec{n}}{|\vec{b}||\vec{n}|}$$

$$= \frac{(\hat{i}+2\hat{j}+2\hat{k}).(2\hat{i}-\hat{j}+\sqrt{\lambda}\hat{k})}{\sqrt{1+4+4}\sqrt{4+1+\lambda}}$$

$$= \frac{2-2+2\sqrt{\lambda}}{3\times\sqrt{5+\lambda}}$$

$$\Rightarrow \sin\theta = \frac{2\sqrt{\lambda}}{3\sqrt{5+\lambda}} = \frac{1}{3} \Rightarrow 4\lambda = 5+\lambda$$

$$\Rightarrow \lambda = \frac{5}{3}.$$

14. (b) The given lines are $2x = 3y = -z$

or $\frac{x}{3} = \frac{y}{2} = \frac{z}{-6}$ [Dividing by 6]

and $6x = -y = -4z$

or $\frac{x}{2} = \frac{y}{-12} = \frac{z}{-3}$ [Dividing by 12]

$\therefore$ Angle between two lines is

$$\cos\theta = \frac{a_1a_2+b_1b_2+c_1c_2}{\sqrt{a_1^2+b_1^2+c_1^2}\sqrt{a_2^2+b_2^2+c_2^2}}$$

$$\cos\theta = \frac{3.2+2.(-12)+(-6).(-3)}{\sqrt{3^2+2^2+(-6)^2}\sqrt{2^2+(-12)^2+(-3)^2}}$$

$$= \frac{6-24+18}{\sqrt{49}\sqrt{157}} = 0 \Rightarrow \theta = 90^\circ$$

15. (c) Plane $2ax-3ay+4az+6=0$ passes through the mid point of the line joining the centres of spheres

$x^2+y^2+z^2+6x-8y-2z=13$ and

$x^2+y^2+z^2-10x+4y-2z=8$

respectively centre of spheres are $c_1(-3, 4, 1)$ and $c_2(5, -2, 1)$. Mid point of c_1c_2 is $(1, 1, 1)$. Satisfying this in the equation of plane, we get

$2a-3a+4a+6=0$

$\Rightarrow a = -2.$

16. (b) The given line is

$\vec{r} = 2\vec{i}-2\vec{j}+3\vec{k}+\lambda(\vec{i}-\vec{j}+4\vec{k})$

and the plane is $\vec{r}\cdot(\vec{i}+5\vec{j}+\vec{k}) = 5$

$\Rightarrow x+5y+z=5$

Required distance

$$= \left|\frac{ax_1+by_1+cz_1+d}{\sqrt{a^2+b^2+c^2}}\right|$$

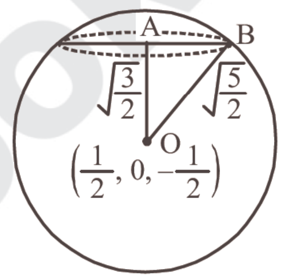
$$= \left|\frac{2-10+3-5}{\sqrt{1+25+1}}\right| = \frac{10}{3\sqrt{3}}$$

17. (a) Given that vector $a\hat{i}+a\hat{j}+c\hat{k}$, $\hat{i}+\hat{k}$ and $c\hat{i}+c\hat{j}+b\hat{k}$ are coplanar then

$$\begin{vmatrix} a & a & c \\ 1 & 0 & 1 \\ c & c & b \end{vmatrix} = 0 \Rightarrow c^2 = ab \Rightarrow c = \sqrt{ab}$$

$\therefore$ c is G.M. of a and b.

18. (b)



Centre of sphere $= \left(\frac{1}{2}, 0, -\frac{1}{2}\right)$ and radius

of sphere $= \sqrt{\frac{1}{4}+\frac{1}{4}+2} = \sqrt{\frac{5}{2}}$

Perpendicular distance OA of centre from $x+2y-z=4$ is given by

$$\frac{\left|\frac{1}{2}+\frac{1}{2}-4\right|}{\sqrt{6}} = \sqrt{\frac{3}{2}}$$

$\therefore$ radius of circle

$$AB = \sqrt{OB^2-OA^2} = \sqrt{\frac{5}{2}-\frac{3}{2}} = 1.$$

19. (a) Given equation of lines be

$$\frac{x-b}{a} = \frac{y}{1} = \frac{z-d}{c}$$

$$\frac{x-b'}{a'} = \frac{y}{1} = \frac{z-d'}{c'}$$

Given that lines are perpendicular

$\Rightarrow aa'+1+cc'=0$

20. (d) Let (α, β, γ) be the image, then mid point of (α, β, γ) and $(-1, 3, 4)$ must lie on $x-2y=0$

$\therefore \ \dfrac{\alpha-1}{2}-2\left(\dfrac{\beta+3}{2}\right)=0$

$\therefore \ \alpha-1-2\beta-6=0 \Rightarrow \alpha-2\beta=7$ ....(1)

Also line joining (α, β, γ) and $(-1, 3, 4)$ should be parallel to the normal of the plane $x-2y=0$

$\therefore \ \dfrac{\alpha+1}{1}=\dfrac{\beta-3}{-2}=\dfrac{\gamma-4}{0}=\lambda$

$\Rightarrow \ \alpha=\lambda-1, \beta=-2\lambda+3, \gamma=4$ ....(2)

From (1) and (2)

$\alpha=\dfrac{9}{5}, \ \beta=-\dfrac{13}{5}, \gamma=4$

None of the option matches.

21. (b) Let the line makes an angle θ with the positive direction of z-axis. Given that lines makes angle $\dfrac{\pi}{4}$ with x-axis and y-axis.

$\therefore \ l=\cos\dfrac{\pi}{4}, m=\cos\dfrac{\pi}{4}, n=\cos\theta$

We know that, $l^2+m^2+n^2=1$

$\therefore \ \cos^2\dfrac{\pi}{4}+\cos^2\dfrac{\pi}{4}+\cos^2\theta=1$

$\Rightarrow \ \dfrac{1}{2}+\dfrac{1}{2}+\cos^2\theta=1$

$\Rightarrow \ \cos^2\theta=0 \Rightarrow \theta=\dfrac{\pi}{2}$

Hence, angle with positive direction of the z-axis is $\dfrac{\pi}{2}$.

22. (c) We know that centre of sphere $x^2+y^2+z^2+2ux+2vy+2wz+d=0$ is $(-u, -v, -w)$

Given that, $x^2+y^2+z^2-6x-12y-2z+20=0$

$\therefore$ Centre $\equiv(3, 6, 1)$

Coordinates of one end of diameter of the sphere are (2, 3, 5). Let the coordinates of the other end of diameter are (α, β, γ)

$\therefore \ \dfrac{\alpha+2}{2}=3, \ \dfrac{\beta+3}{2}=6, \ \dfrac{\gamma+5}{2}=1$

$\Rightarrow \ \alpha=4, \beta=9$ and $\gamma=-3$

$\therefore$ Coordinate of other end of diameter are $(4, 9, -3)$

23. (c) Let the direction cosines of line L be l, m, n. Since line L lies on both planes.

$\therefore \ 2l+3m+n=0$ ....(i)

and $l+3m+2n=0$ ....(ii)

on solving equation (i) and (ii), we get

$\dfrac{l}{6-3}=\dfrac{m}{1-4}=\dfrac{n}{6-3} \Rightarrow \dfrac{l}{3}=\dfrac{m}{-3}=\dfrac{n}{3}$

Now $\dfrac{l}{3}=\dfrac{m}{-3}=\dfrac{n}{3}=\dfrac{\sqrt{l^2+m^2+n^2}}{\sqrt{3^2+(-3)^2+3^2}}$

$\because \ l^2+m^2+n^2=1$

$\therefore \ \dfrac{l}{3}=\dfrac{m}{-3}=\dfrac{n}{3}=\dfrac{1}{\sqrt{27}}$

$\Rightarrow \ l=\dfrac{3}{\sqrt{27}}=\dfrac{1}{\sqrt{3}}, m=-\dfrac{1}{\sqrt{3}}, n=\dfrac{1}{\sqrt{3}}$

Line L, makes an angle α with +ve x-axis

$\therefore \ l=\cos\alpha \Rightarrow \cos\alpha=\dfrac{1}{\sqrt{3}}$

24. (d) Given that, $\vec{a}$ lies in the plane of $\vec{b}$ and $\vec{c}$

$\therefore \ \vec{a}=\vec{b}+\lambda\vec{c}$

$\Rightarrow \ \alpha\hat{i}+2\hat{j}+\beta\hat{k}=\hat{i}+\hat{j}+\lambda(\hat{j}+\hat{k})$

Comparing both side, we get

$\Rightarrow \alpha=1, 2=1+\lambda, \beta=\lambda \Rightarrow \alpha=1, \beta=1$

25. (c) Equation of line through $(5, 1, a)$ and $(3, b, 1)$ is $\dfrac{x-5}{-2}=\dfrac{y-1}{b-1}=\dfrac{z-a}{1-a}=\lambda$

$x=-2\lambda+5$

$y=(b-1)\lambda+1$

$z=(1-a)\lambda+a$

$\therefore$ Any point on this line is a

$[-2\lambda+5, (b-1)\lambda+1, (1-a)\lambda+a]$

Given that it crosses yz plane $\therefore -2\lambda+5=0$

$\lambda=\dfrac{5}{2}$

$\therefore \left(0, (b-1)\dfrac{5}{2}+1, (1-a)\dfrac{5}{2}+a\right)$

$=\left(0, \dfrac{17}{2}, \dfrac{-13}{2}\right)$

$\Rightarrow \ (b-1)\dfrac{5}{2}+1=\dfrac{17}{2}$

and $(1-a)\frac{5}{2}+a=-\frac{13}{2}$

$\Rightarrow \quad b=4$ and $a=6$

26. (a) When the two lines intersect then shortest distance between them is zero i.e.

$$\frac{(\vec{a}_2-\vec{a}_1)\cdot\vec{b}_1\times\vec{b}_2}{\left|\vec{b}_1\times\vec{b}_2\right|}=0$$

$\Rightarrow \ (\vec{a}_2-\vec{a}_1)\cdot\vec{b}_1\times\vec{b}_2=0$

where $\vec{a}_1=\hat{i}+2\hat{j}+3\hat{k}$, $\vec{b}_1=k\hat{i}+2\hat{j}+3\hat{k}$

$\vec{a}_2=2\hat{i}+3\hat{j}+\hat{k}$, $\vec{b}_2=3\hat{i}+k\hat{j}+2\hat{k}$

$$\Rightarrow \begin{vmatrix}1 & 1 & -2\\ k & 2 & 3\\ 3 & k & 2\end{vmatrix}=0$$

$\Rightarrow \quad 1(4-3k)-1(2k-9)-2(k^2-6)=0$

$\Rightarrow \quad -2k^2-5k+25=0 \Rightarrow k=-5$ or $\frac{5}{2}$

$\because$ k is an integer, therefore $k=-5$

27. (a) Given that, the line $\frac{x-2}{3}=\frac{y-1}{-5}=\frac{z+2}{2}$

lie in the plane $x+3y-\alpha z+\beta=0$

$\therefore \quad Pt\,(2, 1, -2)$ lies on the plane

i.e. $\quad 2+3+2\alpha+\beta=0$

or $\quad 2\alpha+\beta+5=0$ ....(i)

Also normal to plane will be perpendicular to line,

$\therefore \ 3\times1-5\times3+2\times(-\alpha)=0$

$\Rightarrow \ \alpha=-6$

From equation (i) then, $\beta=7$

$\therefore (\alpha,\beta)=(-6, 7)$

28. (a) A(3, 1, 6); B = (1, 3, 4)

Putting coordinate of mid-point of AB = (2, 2, 5) in plane $x-y+z=5$ then $2-2+5=5$, satisfy

So, mid-point of AB = (2, 2, 5) lies on the plane.

d.r's of AB = (2, –2, 2)

d.r's of normal to plane = (1, –1, 1).

Direction ratio of AB and normal to the plane are proportional therefore,

AB is perpendicular to the normal of plane

$\therefore$ A is image of B

Statement-1 is correct.

Statement-2 is also correct but it is not correct explanation.

29. (b) As per question, direction cosines of the line :

$\ell=\cos45°=\frac{1}{\sqrt{2}}$, $m=\cos120°=\frac{-1}{2}$,

$n=\cos\theta$

where θ is the angle, which line makes with positive z-axis.

We know that, $\ell^2+m^2+n^2=1$

$\Rightarrow \ \frac{1}{2}+\frac{1}{4}+\cos^2\theta=1$

$\cos^2\theta=\frac{1}{4}$

$\Rightarrow \ \cos\theta=\frac{1}{2}=\cos\frac{\pi}{2}$ (θ being acute)

$\Rightarrow \ \theta=\frac{\pi}{3}$

30. (d) Let θ be the angle between the given line and plane, then

$$\sin\theta=\frac{1\times1+2\times2+\lambda\times3}{\sqrt{1^2+2^2+\lambda^2}.\sqrt{1^2+2^2+3^2}}$$

$$=\frac{5+3\lambda}{\sqrt{14}.\sqrt{5+\lambda^2}}$$

$$\Rightarrow \quad \cos\theta=\sqrt{1-\frac{(5+3\lambda)^2}{14(5+\lambda^2)}}$$

$$\Rightarrow \quad \sqrt{\frac{5}{14}}=\sqrt{1-\frac{(5+3\lambda)^2}{14(5+\lambda^2)}}$$

Squaring both sides, we get

$$\frac{5}{14}=\frac{5\lambda^2-30\lambda+45}{14(5+\lambda^2)}$$

$\Rightarrow \quad \lambda=\frac{2}{3}$

31. (a) The direction ratio of the line segment AB is 0, 6, –4 and the direction ratio of the given line is 1, 2, 3.

Clearly $1\times0+2\times6+3\times(-4)=0$

So, the given line is perpendicular to line AB.

Also , the mid point of A and B is (1, 3, 5) which satisfy the given line.

So, the image of B in the given line is A statement-1 and 2 both true but 2 is not correct explanation. of 1.

32. **(a)** Equation of line through $P(1,-5,9)$ and parallel to the line $x=y=z$ is

$$\frac{x-1}{1}=\frac{y+5}{1}=\frac{z-9}{1}=\lambda(say)$$

$Q=(x=1+\lambda, y=-5+\lambda, z=9+\lambda)$

Since Q lies on plane $x-y+z=5$

$\therefore\quad 1+\lambda+5-\lambda+9+\lambda=5$

$\Rightarrow\quad \lambda=-10$

$\therefore\quad Q=(-9,-15,-1)$

$\therefore\quad PQ=\sqrt{(1+9)^2+(15-5)^2+(9+1)^2}$

$=\sqrt{300}=10\sqrt{3}$

33. **(c)** Any point on line $\frac{x}{2}=\frac{y-2}{3}=\frac{z-3}{4}=\alpha$ is $(2\alpha, 3\alpha+2, 4\alpha+3)$

$\Rightarrow$ Direction ratio of the $\perp$ line is $2\alpha-3, 3\alpha+3, 4\alpha-8$. and

Direction ratio of the given line are 2, 3, 4

$\Rightarrow\quad 2(2\alpha-3)+3(3\alpha+3)+4(4\alpha-8)=0$

$\Rightarrow\quad 29\alpha-29=0\ \Rightarrow\ \alpha=1$

$\Rightarrow$ Foot of $\perp$ is (2, 5, 7)

$\Rightarrow$ Length $\perp$ is $\sqrt{1^2+6^2+4^2}=\sqrt{53}$

34. **(a)** Given that, equation of a plane is

$x-2y+2z-5=0$

So, Equation of parallel plane is

$x-2y+2z+d=0$

Now, it is given that distance from origin to the parallel plane is 1.

$\therefore\quad \left|\frac{d}{\sqrt{1^2+2^2+2^2}}\right|=1\ \Rightarrow\ d=\pm 3$

So equation of required plane

$x-2y+2z\pm 3=0$

35. **(c)** Given lines are $\frac{x-1}{2}=\frac{y+1}{3}=\frac{z-1}{4}$

and $\frac{x-3}{1}=\frac{y-k}{2}=\frac{z}{1}$

$\therefore\quad a_1=\hat{i}-\hat{j}+\hat{k},\ \vec{b}_1=2\hat{i}+3\hat{j}+4\hat{k}$

$a_2=3\hat{i}+k\hat{j},\ \vec{b}_2=\hat{i}+2\hat{j}+\hat{k}$

Given lies are intersect if

$$\frac{(\vec{a}_2-\vec{a}_1)\cdot(\vec{b}_1\times\vec{b}_2)}{|\vec{b}_1||\vec{b}_2|}=0$$

$\Rightarrow\quad (\vec{a}_2-\vec{a}_1)\cdot(\vec{b}_1\times\vec{b}_2)=0$

$$\Rightarrow\begin{vmatrix}2 & k+1 & -1\\ 2 & 3 & 4\\ 1 & 2 & 1\end{vmatrix}=0$$

$\Rightarrow\quad 2(3-8)-(k+1)(2-4)-1(4-3)=0$

$\Rightarrow\quad 2(-5)-(k+1)(-2)-1(1)=0$

$\Rightarrow\quad -10+2k+2-1=0\ \Rightarrow\ k=\frac{9}{2}$

36. **(c)** $2x+y+2z-8=0$ ....(Plane 1)

$2x+y+2z+\frac{5}{2}=0$ ....(Plane 2)

Distance between Plane 1 and 2

$$=\left|\frac{-8-\frac{5}{2}}{\sqrt{2^2+1^2+2^2}}\right|=\left|\frac{-21}{6}\right|=\frac{7}{2}$$

37. **(c)** Given lines will be coplanar

$$\text{If}\begin{vmatrix}-1 & 1 & 1\\ 1 & 1 & -k\\ k & 2 & 1\end{vmatrix}=0$$

$\Rightarrow\quad -1(1+2k)-(1+k^2)+1(2-k)=0$

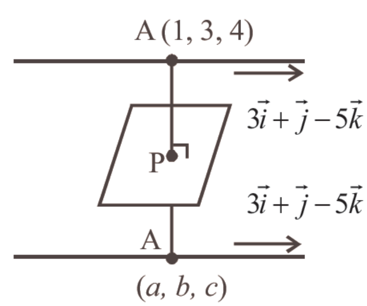
$\Rightarrow\quad k=0,-3$

38. **(c)** $\frac{a-1}{2}=\frac{b-3}{-1}=\frac{c-4}{1}=\lambda(\text{let})$

$\Rightarrow\quad a=2\lambda+1$

$b=3-\lambda$

$c=4+\lambda$



$$P=\left(\frac{a+1}{2},\frac{b+3}{2},\frac{c+4}{2}\right)$$

$$=\left(\lambda+1,\frac{6-\lambda}{2},\frac{\lambda+8}{2}\right)$$

$\therefore \quad 2(\lambda+1)-\frac{6-\lambda}{2}+\frac{\lambda+8}{2}+3=0$

$3\lambda+6=0 \Rightarrow \lambda=-2$

$a=-3, b=5, c=2$

Required line is $\frac{x+3}{3}=\frac{y-5}{1}=\frac{z-2}{-5}$

39. (c) Given, $l+m+n=0$ and $l^2=m^2+n^2$

Now, $(-m-n)^2=m^2+n^2$

$\Rightarrow \quad mn=0 \Rightarrow m=0$ or $n=0$

If $m=0$ then $l=-n$

We know $l^2+m^2+n^2=1 \Rightarrow \quad n=\pm\frac{1}{\sqrt{2}}$

i.e. $(l_1, m_1, n_1)=\left(-\frac{1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}}\right)$

If $n=0$ then $l=-m$

$l^2+m^2+n^2=1 \quad \Rightarrow \quad 2m^2=1$

$\Rightarrow \quad m=\pm\frac{1}{\sqrt{2}}$

Let $m=\frac{1}{\sqrt{2}} \quad \Rightarrow l=-\frac{1}{\sqrt{2}}$ and $n=0$

$(l_2, m_2, n_2)=\left(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0\right)$

$\therefore \quad \cos\theta=\frac{1}{2} \Rightarrow \theta=\frac{\pi}{3}$

40. (a) Equation of the plane containing the line $2x-5y+z=3$ and $x+y+4z=5$ is

$2x-5y+z-3+\lambda(x+y+4z-5)=0$

$\Rightarrow (2+\lambda)x+(-5+\lambda)y+(1+4\lambda)z+(-3-5\lambda)=0$ ...(i)

Since the plane (i) parallel to the given plane $x+3y+6z=1$

$\therefore \frac{2+\lambda}{1}=\frac{-5+\lambda}{3}=\frac{1+4\lambda}{6}$

$\Rightarrow \lambda=-\frac{11}{2}$

Hence equation of the required plane is

$$\left(2-\frac{11}{2}\right)x+\left(-5-\frac{11}{2}\right)y+\left(1-\frac{44}{2}\right)z+\left(-3+\frac{55}{2}\right)=0$$

$\Rightarrow (4-11)x+(-10-11)y+(2-44)z+(-6+55)=0$

$\Rightarrow -7x-21y-42z+49=0$

$\Rightarrow x+3y+6z-7=0$

$\Rightarrow x+3y+6z=7$

41. (b) General point on given line

$\equiv P(3r+2, 4r-1, 12r+2)$

Point P must satisfy equation of plane

$(3r+2)-(4r-1)+(12r+2)=16$

$11r+5=16$

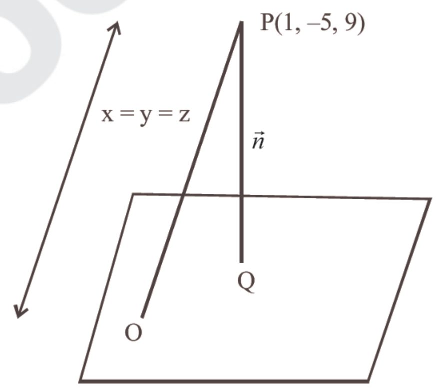
$r=1$

$P(3\times1+2, 4\times1-1, 12\times1+2)=P(5, 3, 14)$

distance between P and (1, 0, 2)

$D=\sqrt{(5-1)^2+3^2+(14-2)^2}=13$

42. (d)



Eq^n of PO : $\frac{x-1}{1}=\frac{y+5}{1}=\frac{z-9}{1}=\lambda$

$\Rightarrow x=\lambda+1;\ y=\lambda-5; z=\lambda+9.$

Putting these in eq^n of plane :

$\lambda+1-\lambda+5+\lambda+9=5$

$\Rightarrow \lambda=-10$

$\Rightarrow$ O is $(-9, -15, -1)$

$\Rightarrow$ distance OP $=10\sqrt{3}$

43. (b) Line lies in the plane $\Rightarrow (3, -2, -4)$ lie in the plane

$\Rightarrow 3\ell-2m+4=9$ or $3\ell-2m=5$ ....(1)

Also, ℓ, m, –1 are dr's of line perpendicular to plane and 2, –1, 3 are dr's of line lying in the plane

$\Rightarrow 2\ell-m-3=0$ or $2\ell-m=3$ ....(2)

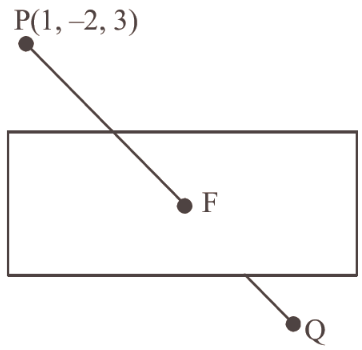
Solving (1) and (2) we get $\ell=1$ and $m=-1$

$\Rightarrow \ell^2+m^2=2.$

44. (c) Equation of line PQ is

$$\frac{x-1}{1}=\frac{y+2}{4}=\frac{z-3}{5}$$

Let F be $(\lambda+1, 4\lambda-2, 5\lambda+3)$



Since F lies on the plane

$\therefore 2(\lambda+1)+3(4\lambda-2)-4(5\lambda+3)+22=0$

$2\lambda+2+12\lambda-6-20\lambda-12+22=0$

$\Rightarrow -6\lambda+6=0 \Rightarrow \lambda=1$

$\therefore$ F is (2, 2, 8)

$PQ = 2\,PF = 2\sqrt{1^2+4^2+5^2} = 2\sqrt{42}$

45. (c) Let the plane be

$a(x-1)+b(y+1)+c(z+1)=0$

Normal vector

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -2 & 3 \\ 2 & -1 & -1 \end{vmatrix} = 5\hat{i}+7\hat{j}+3\hat{k}$$

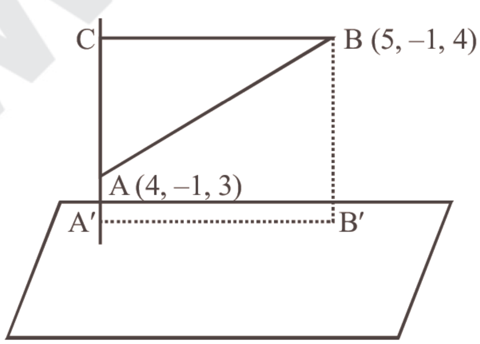
So plane is $5(x-1)+7(y+1)+3(z+1)=0$

$\Rightarrow 5x+7y+3z+5=0$

Distance of point (1, 3, –7) from the plane is

$$\frac{5+21-21+5}{\sqrt{25+49+9}}=\frac{10}{\sqrt{83}}$$

46. (c)



$$AC = \overrightarrow{AB}.\widehat{AC} = \left(\hat{i}+\hat{k}\right).\frac{\left(\hat{i}+\hat{j}+\hat{k}\right)}{\sqrt{3}} = \frac{2}{\sqrt{3}}$$

Now, $A'B' = BC = \sqrt{AB^2 - AC^2}$

$$=\sqrt{2-\frac{4}{3}}=\sqrt{\frac{2}{3}}$$

$\therefore$ Length of projection $=\sqrt{\dfrac{2}{3}}$

47. (a) Equation of plane passing through the line of intersection of first two planes is:

$(2x-2y+3z-2)+\lambda(x-y+z+1)=0$

or $x(\lambda+2)-y(2+\lambda)+z(\lambda+3)+(\lambda-2)=0$ ...(i)

is having infinite number of solution with $x+2y-z-3=0$ and $3x-y+2z-1=0$, then

$$\begin{vmatrix} (\lambda+2) & -(\lambda+2) & (\lambda+3) \\ 1 & 2 & -1 \\ 3 & -1 & 2 \end{vmatrix} = 0$$

$\Rightarrow \lambda = 5$

Now put $\lambda=5$ in (i), we get

$7x-7y+8z+3=0$

Now perpendicular distance from (0, 0, 0) to the place containing L_1 and L_2

$$=\frac{3}{\sqrt{162}}=\frac{1}{3\sqrt{2}}$$

48. (c) Let any point on the intersecting line

$$\frac{x+1}{-3}=\frac{y-3}{2}=\frac{z-2}{-1}=\lambda \text{ (say)}$$

is $(-3\lambda-1, 2\lambda+3, -\lambda+2)$

Since, the above point lies on a line which passes through the point (–4, 3, 1)

Then, direction ratio of the required line

$= [-3\lambda-1+4, 2\lambda+3-3, -\lambda+2-1]$

or $[-3\lambda+3, 2\lambda, -\lambda+1]$

Since, line is parallel to the plane

$x+2y-z-5=0$

Then, perpendicular vector to the line is

$\hat{i}+2\hat{j}-\hat{k}$

Now $(-3\lambda+3)(1)+(2\lambda)(2)+(-\lambda+1)(-1)=0$

$\Rightarrow \quad \lambda=-1$

Now direction ratio of the required line

$=[6,-2,2]$ or $[3,-1,1]$

Hence required equation of the line is

$$\frac{(x+4)}{3}=\frac{y-3}{-1}=\frac{z-1}{1}$$

49. **(d)** Since, equation of plane through intersection of planes

$x+y+z=1$ and $2x+3y-z+4=0$ is

$(2x+3y-z+4)+\lambda(x+y+z-1)=0$

$(2+\lambda)x+(3+\lambda)y+(-1+\lambda)z+(4-\lambda)=0$ ...(1)

But, the above plane is parallel to y-axis then

$(2+\lambda)\times 0+(3+\lambda)\times 1+(-1+\lambda)\times 0=0$

$\Rightarrow \lambda=-3$

Hence, the equation of required plane is

$-x-4z+7=0$

$\Rightarrow \quad x+4z-7=0$

Therefore, (3, 2, 1) the passes through the point.

50. **(c)** Let point on line be P $(2k+1, 3k-1, 4k+2)$

Since, point P lies on the plane

$x+2y+3z=15$

$\therefore \quad 2k+1+6k-2+12k+6=15$

$$\Rightarrow \quad k=\frac{1}{2} \quad \therefore \quad P\equiv\left(2,\frac{1}{2},4\right)$$

Then the distance of the point P from the origin is

$$OP=\sqrt{4+\frac{1}{4}+16}=\frac{9}{2}$$

51. **(d)** Let the required plane passing through the points (0, –1, 0) and (0, 0, 1) be

$\frac{x}{\lambda}+\frac{y}{-1}+\frac{z}{1}=1$ and the given plane is

$y-z+5=0$

$$\therefore \quad \cos\frac{\pi}{4}=\frac{|-1-1|}{\sqrt{\left(\frac{1}{\lambda^2}+1+1\right)}\sqrt{2}}$$

$$\Rightarrow \quad \lambda^2=\frac{1}{2}\Rightarrow\frac{1}{\lambda}=\pm\sqrt{2}$$

Then, the equation of plane is

$\pm\sqrt{2}x-y+z=1$

Then the point $(\sqrt{2},1,4)$ satisfies the equation of plane $-\sqrt{2}x-y+z=1$

52. **(c)** Angle bisector between $\vec{b}$ and $\vec{c}$ can be

$\vec{a}=\lambda(\hat{b}+\hat{c})$ or $\vec{a}=\mu(\hat{b}-\hat{c})$

$$\text{If } \vec{a}=\lambda\left(\frac{\hat{i}+\hat{j}}{\sqrt{2}}+\frac{\hat{i}-\hat{j}+4\hat{k}}{3\sqrt{2}}\right)$$

$$=\frac{\lambda}{3\sqrt{2}}[3\hat{i}+3\hat{j}+\hat{i}-\hat{j}+4\hat{k}]$$

$$=\frac{\lambda}{3\sqrt{2}}[4\hat{i}+2\hat{j}+4\hat{k}]$$

Compare with $\vec{a}=\alpha\hat{i}+2\hat{j}+\beta\hat{k}$

$$\frac{2\lambda}{3\sqrt{2}}=2 \Rightarrow \quad \lambda=3\sqrt{2}$$

$\vec{a}=4\hat{i}+2\hat{j}+4\hat{k}$

Not satisfy any option

$$\text{Now consider } \vec{a}=\mu\left(\frac{\hat{i}+\hat{j}}{\sqrt{2}}-\frac{\hat{i}-\hat{j}+4\hat{k}}{3\sqrt{2}}\right)$$

$$\vec{a}=\frac{\mu}{3\sqrt{2}}(3\hat{i}+3\hat{j}-\hat{i}+\hat{j}-4\hat{k})$$

$$=\frac{\mu}{3\sqrt{2}}(2\hat{i}+4\hat{j}-4\hat{k})$$

Compare with $\vec{a}=\alpha\hat{i}+2\hat{j}+\beta\hat{k}$

$$\frac{4\mu}{3\sqrt{2}}=2 \quad\Rightarrow\quad \mu=\frac{3\sqrt{2}}{2}$$

$\vec{a}=\hat{i}+2\hat{j}-2\hat{k}$

$\therefore \quad \vec{a}\cdot\vec{k}+2=0$

$-2+2=0$

53. **(b)** Equation of plane is $x+y-2z=3$

Let image of R in the plane P is (x, y, z)

$$\Rightarrow \frac{x-2}{1}=\frac{y-1}{1}=\frac{z-6}{-2}=\frac{-2(2+1-12-3)}{6}$$

$\Rightarrow \quad (x,y,z)=(6,5,-2)$

Probability

27

1. A problem in mathematics is given to three students A, B, C and their respective probability of solving the problem is $\frac{1}{2}, \frac{1}{3}$ and $\frac{1}{4}$. Probability that the problem is solved is **[2002]**

   (a) $\frac{3}{4}$ (b) $\frac{1}{2}$

   (c) $\frac{2}{3}$ (d) $\frac{1}{3}$

2. A dice is tossed 5 times. Getting an odd number is considered a success. Then the variance of distribution of success is **[2002]**

   (a) 8/3 (b) 3/8

   (c) 4/5 (d) 5/4

3. The mean and variance of a random variable X having binomial distribution are 4 and 2 respectively, then $P(X = 1)$ is **[2003]**

   (a) $\frac{1}{4}$ (b) $\frac{1}{32}$

   (c) $\frac{1}{16}$ (d) $\frac{1}{8}$

4. The probability that A speaks truth is $\frac{4}{5}$, while the probability for B is $\frac{3}{4}$. The probability that they contradict each other when asked to speak on a fact is **[2004]**

   (a) $\frac{4}{5}$ (b) $\frac{1}{5}$

   (c) $\frac{7}{20}$ (d) $\frac{3}{20}$

5. A random variable X has the probability distribution:

X:	1	2	3	4	5	6	7	8
p(X):	0.15	0.23	0.12	0.12	0.2	0.1	0.07	0.01

   For the events $E = \{X \text{ is a prime number}\}$ and $F = \{X < 4\}$, the $P(E \cup F)$ is **[2004]**

   (a) 0.50 (b) 0.77

   (c) 0.35 (d) 0.87

6. The mean and the variance of a binomial distribution are 4 and 2 respectively. Then the probability of 2 successes is **[2004]**

   (a) $\frac{28}{256}$ (b) $\frac{219}{256}$

   (c) $\frac{128}{256}$ (d) $\frac{37}{256}$

7. Three houses are available in a locality. Three persons apply for the houses. Each applies for one house without consulting others. The probability that all the three apply for the same house is **[2005]**

   (a) $\frac{2}{9}$ (b) $\frac{1}{9}$

   (c) $\frac{8}{9}$ (d) $\frac{7}{9}$

8. A random variable X has Poisson distribution with mean 2. Then $P(X > 1.5)$ equals **[2005]**

   (a) $\frac{2}{e^2}$ (b) 0

   (c) $1-\frac{3}{e^2}$ (d) $\frac{3}{e^2}$

9. At a telephone enquiry system the number of phone calls regarding relevant enquiry follow Poisson distribution with an average of 5 phone calls during 10 minute time intervals. The probability that there is at the most one phone call during a 10-minute time period is **[2006]**

(a) $\frac{6}{5^e}$ (b) $\frac{5}{6}$

(c) $\frac{6}{55}$ (d) $\frac{6}{e^5}$

10. Two aeroplanes I and II bomb a target in succession. The probabilities of I and II scoring a hit correctly are 0.3 and 0.2, respectively. The second plane will bomb only if the first misses the target. The probability that the target is hit by the second plane is **[2007]**

(a) 0.2 (b) 0.7

(c) 0.06 (d) 0.14.

11. A pair of fair dice is thrown independently three times. The probability of getting a score of exactly 9 twice is **[2007]**

(a) 8/729 (b) 8/243

(c) 1/729 (d) 8/9.

12. It is given that the events A and B are such that

$P(A)=\frac{1}{4}, P(A\,|\,B)=\frac{1}{2}$ and $P(B\,|\,A)=\frac{2}{3}$. Then $P(B)$ is **[2008]**

(a) $\frac{1}{6}$ (b) $\frac{1}{3}$

(c) $\frac{2}{3}$ (d) $\frac{1}{2}$

13. In a binomial distribution $B\left(n, p=\frac{1}{4}\right)$, if the probability of at least one success is greater than or equal to $\frac{9}{10}$, then n is greater than: **[2009]**

(a) $\frac{1}{\log_{10} 4+\log_{10} 3}$ (b) $\frac{9}{\log_{10} 4-\log_{10} 3}$

(c) $\frac{4}{\log_{10} 4-\log_{10} 3}$ (d) $\frac{1}{\log_{10} 4-\log_{10} 3}$

14. One ticket is selected at random from 50 tickets numbered 00,01,02,...,49. Then the probability that the sum of the digits on the selected ticket is 8, given that the product of these digits is zero, equals: **[2009]**

(a) $\frac{1}{7}$ (b) $\frac{5}{14}$

(c) $\frac{1}{50}$ (d) $\frac{1}{14}$

15. An urn contains nine balls of which three are red, four are blue and two are green. Three balls are drawn at random without replacement from the urn. The probability that the three balls have different colours is **[2010]**

(a) $\frac{2}{7}$ (b) $\frac{1}{21}$

(c) $\frac{2}{23}$ (d) $\frac{1}{3}$

16. Consider 5 independent Bernoulli's trials each with probability of success p. If the probability of at least one failure is greater than or equal to $\frac{31}{32}$, then p lies in the interval **[2011]**

(a) $\left(\frac{3}{4},\frac{11}{12}\right]$ (b) $\left[0,\frac{1}{2}\right]$

(c) $\left(\frac{11}{12},1\right]$ (d) $\left(\frac{1}{2},\frac{3}{4}\right]$

17. If C and D are two events such that $C \subset D$ and $P(D)\neq 0$, then the correct statement among the following is **[2011]**

(a) $P(C\,|\,D)\geq P(C)$

(b) $P(C\,|\,D)< P(C)$

(c) $P(C\,|\,D)=\frac{P(D)}{P(C)}$

(d) $P(C\,|\,D)=P(C)$

18. Let A, B, C, be pairwise independent events with P (C) > 0 and $P\left(A\cap B\cap C\right)=0$. Then $P\left(A^c \cap B^c / C\right)$. **[2011RS]**

(a) $\mathrm{P}\left(B^c\right)-\mathrm{P}\left(B\right)$ (b) $P\left(A^c\right)+P\left(B^c\right)$

(c) $P\left(A^c\right)-P\left(B^c\right)$ (d) $P\left(A^c\right)-P\left(B\right)$

19. Three numbers are chosen at random without replacement from $\{1,2,3,..8\}$. The probability that their minimum is 3, given that their maximum is 6, is : **[2012]**

(a) $\frac{3}{8}$ (b) $\frac{1}{5}$

(c) $\frac{1}{4}$ (d) $\frac{2}{5}$

20. A multiple choice examination has 5 questions. Each question has three alternative answers of which exactly one is correct. The probability that a student will get 4 or more correct answers just by guessing is: **[2013]**

(a) $\frac{17}{3^5}$ (b) $\frac{13}{3^5}$

(c) $\frac{11}{3^5}$ (d) $\frac{10}{3^5}$

21. Let A and B be two events such that $P(\overline{A \cup B}) = \frac{1}{6}$, $P(A \cap B) = \frac{1}{4}$ and $P(\overline{A}) = \frac{1}{4}$, where $\overline{A}$ stands for the complement of the event A. Then the events A and B are **[2014]**

(a) independent but not equally likely.
(b) independent and equally likely.
(c) mutually exclusive and independent.
(d) equally likely but not independent.

22. A box contains 15 green and 10 yellow balls. If 10 balls are randomly drawn, one-by-one, with replacement, then the variance of the number of green balls drawn is : **[2014]**

(a) $\frac{6}{25}$ (b) $\frac{12}{5}$

(c) 6 (d) 4

23. If two different numbers are taken from the set (0, 1, 2, 3, ..., 10), then the probability that their sum as well as absolute difference are both multiple of 4, is : **[2014]**

(a) $\frac{7}{55}$ (b) $\frac{6}{55}$

(c) $\frac{12}{55}$ (d) $\frac{14}{55}$

24. A bag contains 4 red and 6 black balls. A ball is drawn at random from the bag, its colour is observed and this ball along with two additional balls of the same colour are returned to the bag. If now a ball is drawn at random from the bag, then the probability that this drawn ball is red, is : **[2018]**

(a) $\frac{2}{5}$ (b) $\frac{1}{5}$

(c) $\frac{3}{4}$ (d) $\frac{3}{10}$

25. Two cards are drawn successively with replacement from a well-shuffled deck of 52 cards. Let X denote the random variable of number of aces obtained in the two drawn cards. Then P(X = 1) + P(X = 2) equals: **[2019]**

(a) 49/169 (b) 52/169

(c) 24/169 (d) 25/169

26. Four persons can hit a target correctly with probabilities $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}$ and $\frac{1}{8}$ respectively. If all hit at the target independently, then the probability that the target would be hit, is: **[2019]**

(a) $\frac{25}{192}$ (b) $\frac{7}{32}$

(c) $\frac{1}{192}$ (d) $\frac{25}{32}$

27. An unbiased coin is tossed 5 times. Suppose that a variable X is assigned the value k when k consecutive heads are obtained for $k = 3, 4, 5$, otherwise X takes the value -1. Then the expected value of X, is: **[2020]**

(a) $\frac{3}{16}$ (b) $\frac{1}{8}$

(c) $-\frac{3}{16}$ (d) $-\frac{1}{8}$

Answer Key

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
(a)	(d)	(b)	(c)	(b)	(a)	(b)	(c)	(d)	(d)	(b)	(b)	(d)	(d)	(a)
16	17	18	19	20	21	22	23	24	25	26	27			
(b)	(a)	(d)	(b)	(c)	(a)	(b)	(b)	(a)	(d)	(d)	(b)			

Solutions

1. **(a)** Given that $P(A)=\frac{1}{2}, P(B)=\frac{1}{3}$ and $P(C)=\frac{1}{4}$;

$P(AUBUC)=1-P(\bar{A})P(\bar{B})P(\bar{C})$

$=1-\left(1-\frac{1}{2}\right)\left(1-\frac{1}{3}\right)\left(1-\frac{1}{4}\right)$

$=1-\frac{1}{2}\times\frac{2}{3}\times\frac{3}{4}=\frac{3}{4}$

2. **(d)** The experiment follows binomial distribution with
$n=5, p=3/6=1/2$.
$q=1-p=1/2$.; $\therefore$ Variance $=npq=5/4$.

3. **(b)** Given that $np=4$ and

$npq=2 \Rightarrow q=\frac{1}{2}, p=\frac{1}{2}, n=8$

$P(X=1)={}^8C_1\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)^7$

$=8.\frac{1}{2^8}=\frac{1}{2^5}=\frac{1}{32}$

4. **(c)** A and B will contradict each other if one speaks truth and other false . So , the required probability

$P(A\cap\overline{B})+P(\overline{A}\cap B)=\frac{4}{5}\left(1-\frac{3}{4}\right)+\left(1-\frac{4}{5}\right)\frac{3}{4}$

$=\frac{4}{5}\times\frac{1}{4}+\frac{1}{5}\times\frac{3}{4}=\frac{7}{20}$

5. **(b)** $P(E)=P(2 \text{ or } 3 \text{ or } 5 \text{ or } 7)$
$=0.23+0.12+0.20+0.07=0.62$
$P(F)=P(1 \text{ or } 2 \text{ or } 3)$
$=0.15+0.23+0.12=0.50$
$P(E\cap F)=P(2 \text{ or } 3)$
$=0.23+0.12=0.35$
We know that
$P(EUF)=P(E)+P(F)-P(E\cap F)$
$=0.62+0.50-0.35=0.77$

6. **(a)** Given that mean $=np=4$ and variance $=npq=2$

$\Rightarrow p=q=\frac{1}{2}$ and $n=8$

$\therefore P(2 \text{ success})={}^8C_2\left(\frac{1}{2}\right)^6\left(\frac{1}{2}\right)^2$

$=\frac{28}{2^8}=\frac{28}{256}$

7. **(b)** Probability of particular house being selected

$=\frac{1}{3}$

P (all the persons apply for the same house)

$=\left(\frac{1}{3}\times\frac{1}{3}\times\frac{1}{3}\right)3=\frac{1}{9}$.

8. **(c)** From poission distribution, probability of getting k successes is

$P(x=k)=e^{-\lambda}\frac{\lambda^k}{k!}$

Given mean $(\lambda)=2$

$P(x\geq 2)=1-P(x=0)-P(x=1)$

$=1-e^{-\lambda}-e^{-\lambda}\left(\frac{\lambda}{1!}\right)=1-\frac{3}{e^2}$.

9. **(d)** From poission distribution

$P(X=r)=\frac{e^{-m}m^r}{r!}$

Given mean $(m)=5$
P (at most 1 phone call)
$=P(X\leq 1)=P(X=0)+P(X=1)$

$=e^{-5}+5\times e^{-5}=\frac{6}{e^5}$

10. **(d)** Given that P(I) = 0.3 and P(II) = 0.2
$\therefore P(\overline{I})=1-0.3=0.7$
$\therefore$ The required probability
$=P(\overline{I}\cap II)=P(\overline{I}).P(II)=0.7\times 0.2=0.14$

11. **(b)** The sample space of pair of fair dice is thrown, $S=(1,1),(1,2)(1,3)\ldots.=36$
Sum 9 are (5, 4), (4, 5), (6, 3), (3, 6)

P(score 9) $=\frac{4}{36}=\frac{1}{9}$
Number of trial = 3
$\therefore$ Probability of getting score 9 exactly twice

$={}^3C_2\times\left(\frac{1}{9}\right)^2.\left(1-\frac{1}{9}\right)=\frac{3!}{2!}\times\frac{1}{9}\times\frac{1}{9}\times\frac{8}{9}$

$= \frac{3.2!}{2!} \times \frac{1}{9} \times \frac{1}{9} \times \frac{8}{9} = \frac{8}{243}$

12. **(b)** Given that $P(A) = 1/4, P(A/B) = \frac{1}{2}, P(B/A) = 2/3$
By conditional probability,
$P(A \cap B) = P(A)P(B/A) = P(B)P(A/B)$

$\Rightarrow \frac{1}{4} \times \frac{2}{3} = P(B) \times \frac{1}{2} \Rightarrow P(B) = \frac{1}{3}$

13. **(d)** Given that

$P = \frac{1}{4} \Rightarrow q = 1 - \frac{1}{4} = \frac{3}{4}$

and $P(x \geq 1) \geq \frac{9}{10}$

$\Rightarrow 1 - P(x = 0) \geq \frac{9}{10}$

$\Rightarrow 1 - {}^nC_0 \left(\frac{1}{4}\right)^0 \left(\frac{3}{4}\right)^n \geq \frac{9}{10}$

$\Rightarrow 1 - \frac{9}{10} \geq \left(\frac{3}{4}\right)^n$

$\Rightarrow \left(\frac{3}{4}\right)^n \leq \left(\frac{1}{10}\right)$

Taking log at the base 3/4, on both sides, we get

$n \log_{3/4}\left(\frac{3}{4}\right) \geq \log_{3/4}\left(\frac{1}{10}\right)$

$\Rightarrow n \geq -\log_{3/4} 10 = \frac{-\log_{10} 10}{\log_{10}\left(\frac{3}{4}\right)}$

$= \frac{-1}{\log_{10} 3 - \log_{10} 4}$

$\Rightarrow n \geq \frac{1}{\log_{10} 4 - \log_{10} 3}$

14. **(d)** Let $E \equiv$ Sum of the digits is 8
$F \equiv$ Product of the digits is 0
Then E = {08, 17, 26, 35, 44}
F = {00, 01, 02, 03, 04, 05, 06, 07, 08, 09, 10, 20, 30, 40,}
$E \cap F = \{08\}$

$\therefore$ P (E/F) $= \frac{P(E \cap F)}{P(F)} = \frac{1/50}{14/50} = \frac{1}{14}$

15. **(a)** $n(S) = {}^9C_3$

$n(E) = {}^3C_1 \times {}^4C_1 \times {}^2C_1$

Probability =
$\frac{3 \times 4 \times 2}{{}^9C_3} = \frac{24 \times 3!}{9!} \times 6! = \frac{24 \times 6}{9 \times 8 \times 7} = \frac{2}{7}$

16. **(b)** Given that p (at least one failure) $\geq \frac{31}{32}$

$\Rightarrow 1 - p$ (no failure) $\geq \frac{31}{32}$

$\Rightarrow 1 - p^5 \geq \frac{31}{32}$

$\Rightarrow p^5 \leq \frac{1}{32}$

$\Rightarrow p \leq \frac{1}{2}$

But $p \geq 0$

Hence p lies in the interval $\left[0, \frac{1}{2}\right]$.

17. **(a)** We know,

$P\left(\frac{C}{D}\right) = \frac{P(C \cap D)}{P(D)} = \frac{P(C)}{P(D)}$ $[\because C \subset D]$

Where, $0 \leq P(D) \leq 1$, hence

$P\left(\frac{C}{D}\right) \geq P(C)$

18. **(d)** $P(A^c \cap B^c/C) =$

$\frac{P\left(\left(A^c \cap B^c\right) \cap C\right)}{P(C)} = \frac{P\left((A \cup B)^c \cap C\right)}{P(C)}$

$= \frac{P\left((S - A \cup B) \cap C\right)}{P(C)}$

$= \frac{P\left((S - A - B + A \cap B) \cap C\right)}{P(C)}$

$= \frac{P(C) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C)}{P(C)}$

$= \frac{P(C) - P(A).P(C) - P(B)P(C) + 0}{P(C)}$

$= 1 - P(A) - P(B)$ $[\because P(A^C) = 1 - P(A)]$

$= P\left(A^c\right) - P(B)$

19. **(b)** Given three numbers are chosen without replacement from = {1,2,3,.....,8}
Let Event
F : Maximum of three numbers is 6.
E : Minimum of three numbers is 3.
This is the case of conditional probability
We have to find P (minimum) is 3 when it is given that P (maximum) is 6.

$\therefore P\left(\frac{E}{F}\right)=\frac{P(E\cap F)}{P(F)}=\frac{{}^2C_1}{{}^5C_2}$

$=\frac{2}{10}=\frac{1}{5}$

20. **(c)** $p = p$ (correct answer), $q = p$ (wrong answer)

$\Rightarrow \; P=\frac{1}{3}, q=\frac{2}{3}, n=5$

By using Binomial distribution

Required probability

$P(x \ge 4)={}^5C_4\left(\frac{1}{3}\right)^4 \cdot \frac{2}{3}+{}^5C_5\left(\frac{1}{3}\right)^5$

$=5\cdot\frac{2}{3^5}+\frac{1}{3^5}=\frac{11}{3^5}$

21. **(a)** Given,

$P(\overline{A\cup B})=\frac{1}{6}\Rightarrow P(A\cup B)=1-\frac{1}{6}=\frac{5}{6}$

$P(\overline{A})=\frac{1}{4}\Rightarrow P(A)=1-\frac{1}{4}=\frac{3}{4}$

We know,

$P(A\cup B)=P(A)+P(B)-P(A\cap B)$

$\Rightarrow \; \frac{5}{6}=\frac{3}{4}+P(B)-\frac{1}{4}$

$\left(\because P(A\cap B)=\frac{1}{4}\right)$

$\Rightarrow \; P(B)=\frac{1}{3}$

$\because\; P(A)\ne P(B)$ so they are not equally likely.

Also $P(A)\times P(B)=\frac{3}{4}\times\frac{1}{3}=\frac{1}{4}$

$=P(A\cap B)$

So A & B are independent.

22. **(b)** We can apply binomial probability distribution

We have n = 10

p = Probability of drawing a green ball

$=\frac{15}{25}=\frac{3}{5}$

Also $q=1-\frac{3}{5}=\frac{2}{5}$

Variance = npq

$=10\times\frac{3}{5}\times\frac{2}{5}=\frac{12}{5}$

23. **(b)** Let $A \equiv \{0, 1, 2, 3, 4, ..., 10\}$

$n(S)={}^{11}C_2=55$ where 'S' denotes sample space

Let E be the given event

$\therefore E\equiv\{(0,4),(0,8),(2,6),(2,10),(4,8),(6,10)\}$

$\Rightarrow n(E)=6$

$\therefore P(E)=\frac{n(E)}{n(S)}=\frac{6}{55}$

24. **(a)** Let R_t be the even of drawing red ball in t^{th} draw and B_t be the event of drawing black ball in t^{th} draw.

Now, in the given bag there are 4 red and 6 black balls.

$\therefore \quad P(R_1)=\frac{4}{10}$ and $P(B_1)=\frac{6}{10}$

And, $P\left(\frac{R_2}{R_1}\right)=\frac{6}{12}$ and $P\left(\frac{R_2}{B_1}\right)=\frac{4}{12}$

Now, required probability

$=P(R_1)\times P\left(\frac{R_2}{R_1}\right)+P(B_1)\times P\left(\frac{R_2}{B_1}\right)$

$=\left(\frac{4}{10}\times\frac{6}{12}\right)+\left(\frac{6}{10}\times\frac{4}{12}\right)=\frac{2}{5}$

25. **(d)** X = number of aces drawn

$\therefore P(X=1)+P(X=2)$

$=\left\{\frac{4}{52}\times\frac{48}{52}+\frac{48}{52}\times\frac{4}{52}\right\}+\left\{\frac{4}{52}\times\frac{4}{52}\right\}$

$=\frac{24}{169}+\frac{1}{169}=\frac{25}{169}$

26. **(d)** P (at least one hit the target) = 1 – P (none of them hit the target)

$=1-\left(1-\frac{1}{2}\right)\left(1-\frac{1}{3}\right)\left(1-\frac{1}{4}\right)\left(1-\frac{1}{8}\right)$

$=1-\frac{1}{2}\times\frac{2}{3}\times\frac{3}{4}\times\frac{7}{8}=1-\frac{7}{32}=\frac{25}{32}$

27. **(b)**

k	0	1	2	3	4	5
$P(k)$	$\frac{1}{32}$	$\frac{12}{32}$	$\frac{11}{32}$	$\frac{5}{32}$	$\frac{2}{32}$	$\frac{1}{32}$

k = No. of times head occur consecutively

Now expectation

$=\sum xP(k)=(-1)\times\frac{1}{32}+(-1)\times\frac{12}{32}+(-1)\times\frac{11}{32}$

$+3\times\frac{5}{32}+4\times\frac{2}{32}+5\times\frac{1}{32}=\frac{1}{8}$

Properties of Triangles

28

1. The sides of a triangle are $3x+4y$, $4x+3y$ and $5x+5y$ where $x, y > 0$ then the triangle is **[2002]**
   (a) right angled (b) obtuse angled
   (c) equilateral (d) none of these

2. In a triangle with sides $a, b, c, r_1 > r_2 > r_3$ (which are the ex-radii) then **[2002]**
   (a) $a > b > c$ (b) $a < b < c$
   (c) $a > b$ and $b < c$ (d) $a < b$ and $b > c$

3. The sum of the radii of inscribed and circumscribed circles for an n sided regular polygon of side a, is **[2003]**
   (a) $\frac{a}{4}\cot\left(\frac{\pi}{2n}\right)$ (b) $a\cot\left(\frac{\pi}{n}\right)$
   (c) $\frac{a}{2}\cot\left(\frac{\pi}{2n}\right)$ (d) $a\cot\left(\frac{\pi}{2n}\right)$

4. In a triangle ABC, medians AD and BE are drawn. If AD = 4, $\angle DAB = \frac{\pi}{6}$ and $\angle ABE = \frac{\pi}{3}$, then the area of the ΔABC is **[2003]**
   (a) $\frac{64}{3}$ (b) $\frac{8}{3}$
   (c) $\frac{16}{3}$ (d) $\frac{32}{3\sqrt{3}}$

5. If in a ΔABC $a\cos^2\left(\frac{C}{2}\right)+c\cos^2\left(\frac{A}{2}\right)=\frac{3b}{2}$, then the sides a, b and c **[2003]**
   (a) satisfy $a+b=c$ (b) are in A.P
   (c) are in G.P (d) are in H.P.

6. The sides of a triangle are $\sin\alpha, \cos\alpha$ and $\sqrt{1+\sin\alpha\cos\alpha}$ for some $0 < \alpha < \frac{\pi}{2}$. Then the greatest angle of the triangle is **[2004]**
   (a) 150° (b) 90°
   (c) 120° (d) 60°

7. A person standing on the bank of a river observes that the angle of elevation of the top of a tree on the opposite bank of the river is 60° and when he retires 40 meters away from the tree the angle of elevation becomes 30°. The breadth of the river is **[2004]**
   (a) 60m (b) 30m
   (c) 40m (d) 20m

8. In a triangle ABC, let $\angle C = \frac{\pi}{2}$. If r is the inradius and R is the circumradius of the triangle ABC, then $2(r+R)$ equals **[2005]**
   (a) $b+c$ (b) $a+b$
   (c) $a+b+c$ (d) $c+a$

9. If in a ΔABC, the altitudes from the vertices A, B, C on opposite sides are in H.P, then $\sin A$, $\sin B$, $\sin C$ are in **[2005]**
   (a) G.P. (b) A. P.
   (c) A.P-G.P. (d) H. P

10. A tower stands at the centre of a circular park. A and B are two points on the boundary of the park such that $AB (= a)$ subtends an angle of 60° at the foot of the tower, and the angle of elevation of the top of the tower from A and B is 30°. The height of the tower is **[2007]**
    (a) $a/\sqrt{3}$ (b) $a\sqrt{3}$
    (c) $2a/\sqrt{3}$ (d) $2a\sqrt{3}$

11. AB is a vertical pole with B at the ground level and A at the top. A man finds that the angle of elevation of the point A from a certain point C on the ground is 60°. He moves away from the pole along the line BC to a point D such that $CD = 7$ m. From D the angle of elevation of the point A is 45°. Then the height of the pole is **[2008]**

(a) $\frac{7\sqrt{3}}{2}\frac{1}{\sqrt{3}-1}m$ (b) $\frac{7\sqrt{3}}{2}(\sqrt{3}+1)m$

(c) $\frac{7\sqrt{3}}{2}(\sqrt{3}-1)m$ (d) $\frac{7\sqrt{3}}{2}\frac{1}{\sqrt{3}+1}m$

12. For a regular polygon, let r and R be the radii of the inscribed and the circumscribed circles. A ***false*** statement among the following is **[2010]**

(a) There is a regular polygon with $\frac{r}{R}=\frac{1}{\sqrt{2}}$

(b) There is a regular polygon with $\frac{r}{R}=\frac{2}{3}$

(c) There is a regular polygon with $\frac{r}{R}=\frac{\sqrt{3}}{2}$

(d) There is a regular polygon with $\frac{r}{R}=\frac{1}{2}$

13. In a ΔPQR, If $3\sin P + 4\cos Q = 6$ and $4\sin Q + 3\cos P = 1$, then the angle R is equal to : **[2012]**

(a) $\frac{5\pi}{6}$ (b) $\frac{\pi}{6}$ (c) $\frac{\pi}{4}$ (d) $\frac{3\pi}{4}$

14. If in a triangle ABC, $\frac{b+c}{11}=\frac{c+a}{12}=\frac{a+b}{13}$, then cosA is equal to **[2012]**

(a) 5/7 (b) 1/5
(c) 35/19 (d) 19/35

15. A bird is sitting on the top of a vertical pole 20 m high and its elevation from a point O on the ground is $45°$. It flies off horizontally straight away from the point O. After one second, the elevation of the bird from O is reduced to $30°$. Then the speed (in m/s) of the bird is **[2014]**

(a) $20\sqrt{2}$ (b) $20(\sqrt{3}-1)$
(c) $40(\sqrt{2}-1)$ (d) $40(\sqrt{3}-\sqrt{2})$

16. Let a vertical tower AB have its end A on the level ground. Let C be the mid-point of AB and P be a point on the ground such that AP = 2AB. If $\angle BPC = \beta$, then $\tan\beta$ is equal to : **[2017]**

(a) $\frac{4}{9}$ (b) $\frac{6}{7}$ (c) $\frac{1}{4}$ (d) $\frac{2}{9}$

Answer Key

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
(b)	(a)	(c)	(d)	(b)	(c)	(d)	(b)	(b)	(a)	(b)	(b)	(b)	(b)	(b)
16														
(d)														

Solutions

1. **(b)** Let $a = 3x + 4y$, $b = 4x + 3y$ and $c = 5x + 5y$ as $x, y > 0$, $c = 5x + 5y$ is the largest side
$\therefore$ C is the largest angle . Now

$$\cos C = \frac{a^2+b^2-c^2}{2ab}$$

$$\cos C = \frac{(3x+4y)^2+(4x+3y)^3-(5x+5y)^2}{2(3x+4y)(4x+3y)}$$

$$= \frac{-2xy}{2(3x+4y)(4x+3y)} < 0$$

$\therefore$ C is obtuse angle $\Rightarrow \Delta ABC$ is obtuse angled

2. **(a)** We know that, $r_1 = \frac{\Delta}{s-a}$, $r_2 = \frac{\Delta}{s-b}$ and $r_3 = \frac{\Delta}{s-c}$

Given that,

$r_1 > r_2 > r_3 \Rightarrow \frac{\Delta}{s-a} > \frac{\Delta}{s-b} > \frac{\Delta}{s-c}$;

$\Rightarrow \quad s-a < s-b < s-c$

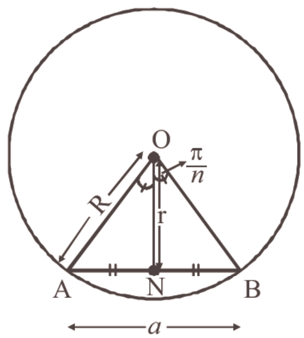
$\Rightarrow \quad -a < -b < -c \Rightarrow a > b > c$

3. **(c)** We know that,

$$\tan\left(\frac{\pi}{n}\right)=\frac{a}{2r};\ \sin\left(\frac{\pi}{n}\right)=\frac{a}{2R}$$

$$\Rightarrow \quad r=\frac{a}{2}\cot\frac{\pi}{n};\ \mathrm{R}=\frac{a}{2}\operatorname{cosec}\frac{\pi}{n}$$

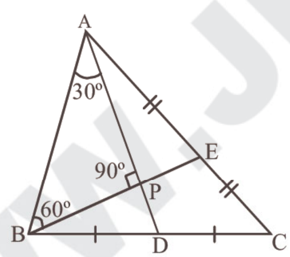
$$r+R=\frac{a}{2}\left[\cot\frac{\pi}{n}+\operatorname{cosec}\frac{\pi}{n}\right]$$



$$=\frac{a}{2}\left[\frac{\cos\frac{\pi}{n}+1}{\sin\frac{\pi}{n}}\right]=\frac{a}{2}\left[\frac{2\cos^2\frac{\pi}{2n}}{2\sin\frac{\pi}{2n}\cos\frac{\pi}{2n}}\right]$$

$$=\frac{a}{2}\cot\frac{\pi}{2\pi}$$

4. **(d)**



We know that median divides each other in ratio 2 :1

$$AP=\frac{2}{3}AD=\frac{8}{3};\quad PD=\frac{4}{3};\quad \text{Let } PB=x$$

$$\tan 60^o=\frac{8/3}{x} \text{ or } x=\frac{8}{3\sqrt{3}}$$

$$\text{Area of } \Delta ABD=\frac{1}{2}\times 4\times\frac{8}{3\sqrt{3}}=\frac{16}{3\sqrt{3}}$$

$$\therefore \text{Area of } \Delta ABC=2\times\frac{16}{3\sqrt{3}}=\frac{32}{3\sqrt{3}}$$

[$\because$ Median of a Δ divides it into two Δ's of equal area.]

5. **(b)** Given that,

$$a\cos^2\left(\frac{C}{2}\right)+c\cos^2\left(\frac{A}{2}\right)=\frac{3b}{2}$$

$a[\cos C+1]+c[\cos A+1]=3b$

$(a+c)+(a\cos C+c\cos B)=3b$

We know that, $b=a\cos C+c\cos B$

$a+c+b=3b$ or $a+c=2b$

or a, b, c are in A.P.

6. **(c)** Let $a=\sin\alpha$, $b=\cos\alpha$ and

$c=\sqrt{1+\sin\alpha\cos\alpha}$

Clearly a and $b<1$ but $c>1$ as $\sin\alpha>0$ and $\cos\alpha>0$

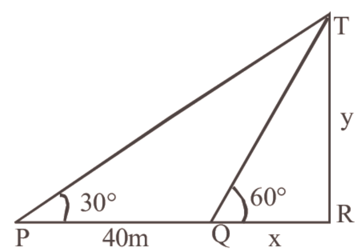
$\therefore$ c is the greatest side and greatest angle is C.

We know that, $\cos C=\dfrac{a^2+b^2-c^2}{2ab}$

$$=\frac{\sin^2\alpha+\cos^2\alpha-1-\sin\alpha\cos\alpha}{2\sin\alpha\cos\alpha}=-\frac{1}{2}$$

$\therefore C=120°$

7. **(d)**



In right ΔQTR

$$\tan 60°=\frac{y}{x}\Rightarrow y=\sqrt{3}x \qquad ....(1)$$

In right ΔPTR

$$\tan 30°=\frac{y}{x+40}\Rightarrow y=\frac{x+40}{\sqrt{3}} \qquad ....(2)$$

From (1) and (2),

$$\sqrt{3}x=\frac{x+40}{\sqrt{3}}\Rightarrow x=20m$$

8. **(b)** We know that for the circle circumscribing a right triangle, hypotenutse is the diameter

$\because \quad \angle C=90°$

$\therefore \quad 2R=c \ \Rightarrow\ R=\dfrac{c}{2}$

$$\text{also } r=\frac{\Delta}{s}=\frac{\frac{1}{2}\times a\times b}{\frac{a+b+c}{2}}$$

$\Rightarrow \quad r=\dfrac{ab}{a+b+c}$

$\therefore \quad 2r+2R=\dfrac{2ab}{a+b+c}+c$

$=\dfrac{2ab+ac+bc+c^2}{a+b+c}$

$=\dfrac{2ab+ac+bc+a^2+b^2}{a+b+c} \quad (\because c^2=a^2+b^2)$

$=\dfrac{(a+b)^2+(a+b)c}{a+b+c}=(a+b)$

9. **(b)** Let altitudes from A, B and C be p_1, p_2 and p_3 resp.

$\therefore \quad \Delta=\frac{1}{2}p_1a=\frac{1}{2}p_2b=\frac{1}{2}p_3b$

Given that, p_1, p_2, p_3, are in H.P.

$\Rightarrow \quad \dfrac{2\Delta}{a}, \dfrac{2\Delta}{b}, \dfrac{2\Delta}{c}$ are in H.P.

$\Rightarrow \quad \dfrac{1}{a}, \dfrac{1}{b}, \dfrac{1}{c}$ are in H.P.

$\Rightarrow \quad a, b, c$ are in A.P.

By sine formula

$\Rightarrow \quad K \sin A, K \sin B, K \sin C$ are in AP

$\Rightarrow \quad \sin A, \sin B, \sin C$ are in A.P.

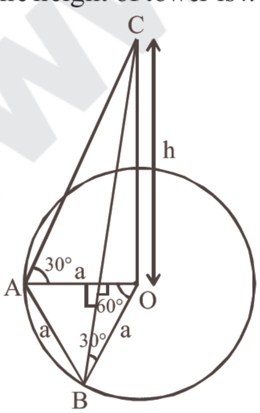
10. **(a)** In the ΔAOB given that $\angle AOB=60°$ and $OA=OB=$ radius

$\therefore \quad \angle OBA=\angle OAB=60°$

$\therefore \quad \Delta AOB$ is a equilateral triangle.

$\Rightarrow \quad OA=OB=AB=a$

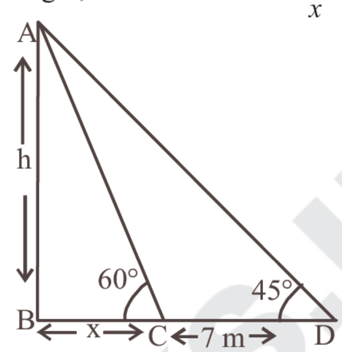
Let the height of tower is h m.



In $\Delta OAC,\ \tan 30°=\dfrac{h}{a} \Rightarrow \dfrac{1}{\sqrt{3}}=\dfrac{h}{a}$

$\Rightarrow h=\dfrac{a}{\sqrt{3}}$

11. **(b)** In right, $\Delta ABC \ \tan 60°=\dfrac{h}{x}=\sqrt{3}$



$\Rightarrow x=\dfrac{h}{\sqrt{3}}$ ....(i)

In right, ΔABD

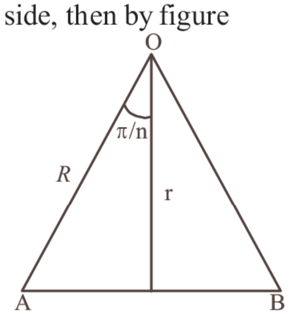
$=\tan 45°=\dfrac{h}{x+7}=1$

$\Rightarrow h=x+7 \Rightarrow h-\dfrac{h}{\sqrt{3}}=7$ [From (i)]

$\Rightarrow h=\dfrac{7\sqrt{3}}{\sqrt{3}-1}\times\dfrac{\sqrt{3}+1}{\sqrt{3}+1}$

$\Rightarrow h=\dfrac{7\sqrt{3}}{2}\left(\sqrt{3}+1\right)m$

12. **(b)** Let O is centre of polygon of n sides and AB is one of the side, then by figure



$\cos\dfrac{\pi}{n}=\dfrac{r}{R}$

$\Rightarrow \dfrac{r}{R}=\dfrac{1}{2}, \dfrac{1}{\sqrt{2}}, \dfrac{\sqrt{3}}{2}$

for $n=3, 4, 6$ respectively.

13. **(b)** Given that $3\sin P+4\cos Q=6$ ...(i)

$4\sin Q+3\cos P=1$ ...(ii)

Squaring and adding (i) & (ii) we get

$9\sin^2 P+16\cos^2 Q+24\sin P\cos Q$

$+16\sin^2 Q+9\cos^2 P+24\sin Q\cos P$

$=36+1=37$

$\Rightarrow \quad 9(\sin^2 P+\cos^2 P)+16(\sin^2 Q+\cos^2 Q)$

$+24(\sin P\cos Q+\cos P\sin Q)=37$

$\Rightarrow \quad 9+16+24\sin(P+Q)=37$

$[\because \sin^2\theta+\cos^2\theta=1$ and $\sin A\cos B+\cos A\sin B=\sin(A+B)]$

$\Rightarrow \quad \sin(P+Q)=\frac{1}{2}$

$\Rightarrow \quad P+Q=\frac{\pi}{6}$ *or* $\frac{5\pi}{6}$

$\Rightarrow \quad R=\frac{5\pi}{6}$ or $\frac{\pi}{6} \qquad (\because P+Q+R=\pi)$

If $\quad R=\frac{5\pi}{6}$ then $0<Q,\ P<\frac{\pi}{6}$

$\Rightarrow \quad \cos Q<1$ and $\sin P<\frac{1}{2}$

$\Rightarrow \quad 3\sin P+4\cos Q<\frac{11}{2}$

But given that $3\sin P+4\sin Q=6$

So, $R=\frac{\pi}{6}$

14. (b) In a triangle ABC.

Let $\frac{b+c}{11}=\frac{c+a}{12}=\frac{a+b}{13}=K$

$\Rightarrow b+c=11K,\ c+a=12K,\ a+b=13K$

On solving these equations, we get

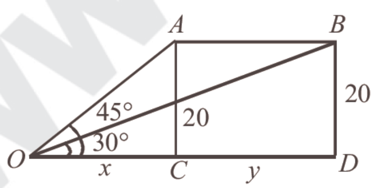
$a=7K,\ b=6K,\ c=5K$

Now we know,

$$\cos A=\frac{b^2+c^2-a^2}{2bc}$$

$$=\frac{36K^2+25K^2-49K^2}{2(6K)(5K)}=\frac{1}{5}$$

15. (b) Given that height of pole $AB=20$ m



Let O be the point on the ground such that $\angle AOC=45°$

Let $OC=x$ and $CD=y$

In right ΔAOC, $\tan 45°=\frac{20}{x}$ ...(i)

In right ΔBOD, $\tan 30°=\frac{20}{x+y}$ ...(ii)

From (i) and (ii), we have $x=20$ and $\frac{1}{\sqrt{3}}=\frac{20}{x+y}$

$\Rightarrow \quad \frac{1}{\sqrt{3}}=\frac{20}{20+y} \Rightarrow 20+y=20\sqrt{3}$

So, $y=20(\sqrt{3}-1)$ m and time = 1s (Given)

Hence, speed $=20(\sqrt{3}-1)$ m/s

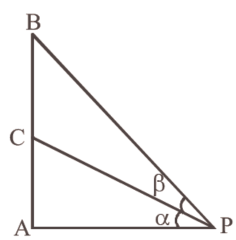
16. (d) Since AP = 2AB $\Rightarrow \frac{AB}{AP}=\frac{1}{2}$ ...(i)

Let $\angle APC=\alpha$

$\therefore \quad \tan\alpha=\frac{AC}{AP}=\frac{1}{2}\frac{AB}{AP}=\frac{1}{4}$

($\because$ C is the mid point) $(\therefore AC=\frac{1}{2}AB)$

$\Rightarrow \tan\alpha=\frac{1}{4}$



As $\tan(\alpha+\beta)=\frac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}$

$\Rightarrow \quad \frac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}=\frac{1}{2}$

$\left[\begin{array}{l}\because \tan(\alpha+\beta)=\frac{AB}{AP}\\ \tan(\alpha+\beta)=\frac{1}{2}\ [\text{From}(1)]\end{array}\right]$

$\Rightarrow \quad \frac{\frac{1}{4}+\tan\beta}{1-\frac{1}{4}\tan\beta}=\frac{1}{2} \qquad \therefore \tan\beta=\frac{2}{9}$